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MATHEMATICS 9

Update Answer Key

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Chapter 1 Number Systems

MULTIPLE CHOICE QUESTION

1. (a) $\sqrt[3]{3} = 1.44$

$\sqrt[3]{4} = 1.59$

$\sqrt[3]{2} = 1.26$

$\therefore$ 2 is the greatest

2. (c) $\sqrt{(729)^{5/3}} = \sqrt{(9^3)^{5/3}} = \sqrt{9^5} = \sqrt{(3^2)^5}$

$= \sqrt{3^{10}} = (3^{10})^{\frac{1}{2}} = 3^5 = 243$

3. (b) $\dfrac{\sqrt{5}-2}{\sqrt{5}+2} - \dfrac{\sqrt{5}+2}{\sqrt{5}-2} = \dfrac{(\sqrt{5}-2)^2-(\sqrt{5}+2)^2}{(\sqrt{5}+2)(\sqrt{5}-2)}$

$= \dfrac{5+4-4\sqrt{5}-(5+4+4\sqrt{5})}{5-4} = -8\sqrt{5}$

4. (d) $\dfrac{3+\sqrt{7}}{3-\sqrt{7}} = a+b\sqrt{7}$

$\dfrac{3+\sqrt{7}}{3-\sqrt{7}} \times \dfrac{3+\sqrt{7}}{3+\sqrt{7}} = a+b\sqrt{7}$

$\Rightarrow \quad \dfrac{(3+\sqrt{7})^2}{9-7} = a+b\sqrt{7}$

$\Rightarrow \quad \dfrac{9+7+6\sqrt{7}}{2} = a+b\sqrt{7}$

$\Rightarrow \quad 8+3\sqrt{7} = a+b\sqrt{7}$

$\Rightarrow \quad$ (a, b) = (8, 3)

5. (c) $\sqrt{\sqrt{2}\times\sqrt{2}\times\sqrt{2}} = \sqrt{2^{1/2}\times 2^{1/2}\times 2^{1/2}}$

$= \sqrt{2^{3/2}} = (2^{3/2})^{\frac{1}{2}} = 2^{3/4}$

WORKSHEET 1: SECTION-A

1. Rationalising factor of 3 + $\sqrt{2}$ is (3 − $\sqrt{2}$).

2. $(\sqrt{5} + \sqrt{3})^2 = (\sqrt{5})^2 + (\sqrt{3})^2 + 2\sqrt{5}\sqrt{3}$

$= 5 + 3 + 2\sqrt{15}$

$= 8 + 2\sqrt{15} = 2(4 + \sqrt{15})$

3. $(3 + \sqrt{5})(3 - \sqrt{5}) = (3)^2 - (\sqrt{5})^2$

$= 9 - 5 = 4$

Therefore $(3+\sqrt{5})$ $(3-\sqrt{5})$ is a rational number.

4. An irrational number between $\sqrt{2}$ and $\sqrt{3}$

$(\sqrt{2})^2 = 2$

$(\sqrt{3})^2 = 3$

An irrational number between $\sqrt{2}$ and $\sqrt{3}$ is $\sqrt{2.5}$.

5. $\left[\left\{(64)^{-\frac{1}{2}}\right\}^{-\frac{1}{3}}\right]^2 = \left[\left\{(8^2)^{-\frac{1}{2}}\right\}^{-\frac{1}{3}}\right]^2 = (8)^{2\times-\frac{1}{2}\times-\frac{1}{3}\times 2}$

$= (8)^{\frac{2}{3}} = ((2)^3)^{\frac{2}{3}} = (2)^{3\times\frac{2}{3}} = 2^2 = 4$

6. Given: $4^{2x} = \dfrac{1}{32}$

$\Rightarrow (2^2)^{2x} = \left(\dfrac{1}{2}\right)^5$

$\Rightarrow \quad 2^{4x} = 2^{-5}$

$\Rightarrow \quad 4x = -5$

$\Rightarrow \quad x = -5/4$

7. $$\frac{\sqrt{32}+\sqrt{48}}{\sqrt{8}+\sqrt{12}} = \frac{\sqrt{2\times2\times2\times2\times2}+\sqrt{2\times2\times2\times2\times3}}{2\sqrt{2}+2\sqrt{3}}$$

$$= \frac{4\sqrt{2}+4\sqrt{3}}{2\sqrt{2}+2\sqrt{3}} = \frac{4(\sqrt{2}+\sqrt{3})}{2(\sqrt{2}+\sqrt{3})} = \frac{2(\sqrt{2}+\sqrt{3})}{(\sqrt{2}+\sqrt{3})} = 2$$

8. $$\left(\frac{64}{125}\right)^{-\frac{2}{3}} = \left(\frac{4^3}{5^3}\right)^{-\frac{2}{3}} = \left\{\left(\frac{4}{5}\right)^3\right\}^{-\frac{2}{3}}$$

$$= \left(\frac{4}{5}\right)^{3\times\frac{-2}{3}} = \left(\frac{4}{5}\right)^{-2}$$

$$= \left(\frac{5}{4}\right)^{2} = \frac{25}{16}$$

9. $$\left(\frac{32}{243}\right)^{4/5} = \left(\frac{2\times2\times2\times2\times2}{3\times3\times3\times3\times3}\right)^{4/5}$$

$$= \left\{\left(\frac{2}{3}\right)^5\right\}^{4/5}$$

$$= \left(\frac{2}{3}\right)^{5\times\frac{4}{5}} = \left(\frac{2}{3}\right)^{4}$$

10. Rationalising factor of $3-\sqrt{7}$ is $3+\sqrt{7}$.

Required product = $(3-\sqrt{7})(3+\sqrt{7})$

$= (3)^2-(\sqrt{7})^2 \quad [\because a^2-b^2 = (a+b)(a-b)]$

$= 9-7 = 2$

SECTION-B

11. Let $x = 23.\overline{47}$ ...(i)

Multiplying both sides by 100, we get

$100x = 2347.\overline{47}$ ...(ii)

Subtracting (i) from (ii), we get

$$100x - x = 2347.\overline{47} - 23.\overline{47}$$

$$99x = 2324$$

$$x = \frac{2324}{99}$$

$$23.\overline{47} = \frac{2324}{99}$$

12. $\frac{5}{7}$ and $\frac{11}{13}$

Let two rational numbers are R_1 and R_2.

$$R_1 = \frac{1}{2}\left(\frac{5}{7}+\frac{11}{13}\right)$$

$$= \frac{1}{2}\left(\frac{65+77}{91}\right) = \frac{1}{2}\times\frac{142}{91}$$

$$R_1 = \frac{71}{91}$$

$$R_2 = \frac{1}{2}\left[\frac{5}{7}+\frac{71}{91}\right]$$

$$R_2 = \frac{1}{2}\left(\frac{65+77}{91}\right) = \frac{1}{2}\times\frac{142}{91}$$

$$R_2 = \frac{1}{2}\left(\frac{136}{91}\right)$$

$$R_2 = \frac{68}{91}$$

13. $64^{2x-5} = 4 \times 8^{x-5}$

$2^{6(2x-5)} = 2^2 \times 2^{3(x-5)}$

$\Rightarrow \quad 2^{12x-30} = 2^{3x-13}$

$\Rightarrow 12x-30 = 3x-13$

$9x = 17$

$x = \frac{17}{9}$

14. If $\sqrt{2} = 1.414$

$$\frac{1}{3+\sqrt{2}} = \frac{1}{3+\sqrt{2}}\times\frac{3-\sqrt{2}}{3-\sqrt{2}}$$

$$= \frac{3-\sqrt{2}}{(3)^2-(\sqrt{2})^2} = \frac{3-\sqrt{2}}{9-2} = \frac{3-\sqrt{2}}{7}$$

$$= \frac{3-1.414}{7} = \frac{1.586}{7} = 0.2266$$

15. $(343)^{2/n} = 49$

$7^{3\times\frac{2}{n}} = 7^2$

$7^{6/n} = 7^2$

$6/n = 2$

$$\frac{6}{2} = n$$

$n = 3$

16. (a) $$\frac{3\sqrt{12}}{6\sqrt{27}} = \frac{3\sqrt{2\times2\times3}}{6\sqrt{3\times3\times3}}$$

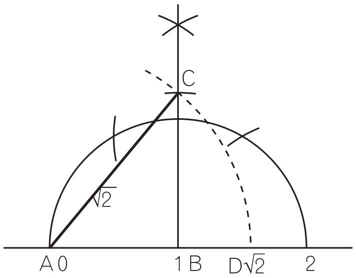
$$= \frac{3\times2\sqrt{3}}{6\times3\sqrt{3}} = \frac{6\sqrt{3}}{18\sqrt{3}} = \frac{1}{3}$$

(b) $8\sqrt{3} - 2\sqrt{3} + 4\sqrt{3}$

$= \sqrt{3}\,(8\text{-}2\text{+}4) = \sqrt{3}\,(10)$

$= 10\sqrt{3}$

17. By Pythagoras theorem:



$AC^2 = AB^2 + BC^2$

$AC^2 = 1^2 + 1^2$

$AC^2 = 2$

$AC = \sqrt{2} \Rightarrow AD = \sqrt{2}$

[Since radii of same circle are equal]

18. $(\sqrt{2} + \sqrt{3})^2 + (\sqrt{5} - \sqrt{2})^2$

$= (\sqrt{2})^2 + (\sqrt{3})^2 + 2\sqrt{2}\sqrt{3} + (\sqrt{5})^2 + (\sqrt{2})^2$

$- 2\sqrt{5}\,\sqrt{2}$

$= 2 + 3 + 2\sqrt{6} + 5 + 2 - 2\sqrt{10}$

$= 12 + 2\sqrt{6} - 2\sqrt{10}$

$= 2\,(6 + \sqrt{6} - \sqrt{10})$

19. If $x = 6 + \sqrt{2}$

$$x - \frac{1}{x} = \frac{6+\sqrt{2}}{1} - \frac{1}{6+\sqrt{2}}$$

$$= \frac{(6+\sqrt{2})^2 - 1}{6+\sqrt{2}} = \frac{36+2+2\times6\sqrt{2}-1}{6+\sqrt{2}}$$

$$= \frac{37+12\sqrt{2}}{6+\sqrt{2}}$$

SECTION-C

20. $$\frac{\sqrt{3}+\sqrt{2}}{5+\sqrt{2}} \times \frac{5-\sqrt{2}}{5-\sqrt{2}}$$

$$= \frac{\sqrt{3}(5-\sqrt{2}) + \sqrt{2}(5-\sqrt{2})}{(5)^2 - (\sqrt{2})^2}$$

$$= \frac{5\sqrt{3} - \sqrt{6} + 5\sqrt{2} - \sqrt{4}}{25-2}$$

$$= \frac{5\sqrt{3} - \sqrt{6} + 5\sqrt{2} - 2}{23}$$

21. $$(1^3 + 2^3 + 3^3)^{\frac{1}{2}} = (1+8+27)^{\frac{1}{2}}$$

$$= (36)^{\frac{1}{2}} \qquad \because\ (a^m)^n = a^{mn}$$

$$= (6^2)^{\frac{1}{2}} = (6)^{2\times\frac{1}{2}} = 6$$

22. $$\frac{4+\sqrt{5}}{4-\sqrt{5}} + \frac{4-\sqrt{5}}{4+\sqrt{5}}$$

$$= \frac{4+\sqrt{5}}{4-\sqrt{5}} \times \frac{4+\sqrt{5}}{4+\sqrt{5}} + \frac{4-\sqrt{5}}{4+\sqrt{5}} \times \frac{4-\sqrt{5}}{4+\sqrt{5}}$$

$$= \frac{(4+\sqrt{5})^2}{(4)^2 - (\sqrt{5})^2} + \frac{(4-\sqrt{5})^2}{(4)^2 - (\sqrt{5})^2}$$

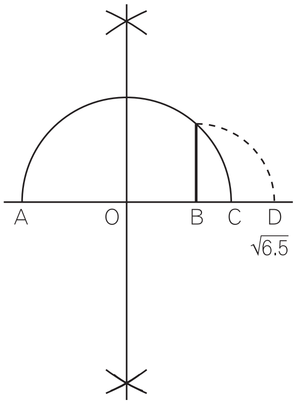
$$= \frac{16+5+2x4x\sqrt{5}}{16-5} + \frac{16+5-2x4x\sqrt{5}}{16-5}$$

$$= \frac{21+8\sqrt{5}}{11} + \frac{21-8\sqrt{5}}{11}$$

$$= \frac{21+8\sqrt{5}+21-8\sqrt{5}}{11}$$

$$= \frac{42}{11}$$

23.



24. LHS $= \left(\frac{x^{a^2}}{x^{b^2}}\right)^{\frac{1}{a+b}} \left(\frac{x^{b^2}}{x^{c^2}}\right)^{\frac{1}{b+c}} \left(\frac{x^{c^2}}{x^{a^2}}\right)^{\frac{1}{c+a}}$

$= \left((x)^{a^2-b^2}\right)^{\frac{1}{a+b}} \left((x)^{b^2-c^2}\right)^{\frac{1}{b+c}} \left((x)^{c^2-a^2}\right)^{\frac{1}{c+a}}$

$= (x)^{\frac{(a+b)(a-b)}{(a+b)}} (x)^{\frac{(b+c)(b-c)}{(b+c)}} (x)^{\frac{(c+a)(c-a)}{(c+a)}}$

$= (x)^{a-b} \times (x)^{b-c} \times (x)^{c-a}$

$= (x)^{a-b+b-c+c-a}$

$= 1$

LHS = RHS

Hence Proved.

25. $\left(\frac{2}{3}\right)^{x} \left(\frac{3}{2}\right)^{2x} = \frac{81}{16}$

$\left(\frac{2}{3}\right)^{x} \left(\frac{2}{3}\right)^{-2x} = \left(\frac{3}{2}\right)^{4}$

$\left(\frac{2}{3}\right)^{x-2x} = \left(\frac{2}{3}\right)^{-4}$

Comparing the power of (2/3), we get

$x - 2x = -4$

$-x = -4$

$x = 4$

26. $3\sqrt{45} - \sqrt{125} + \sqrt{200} - \sqrt{50}$

$= 3\sqrt{3\times3\times5} - \sqrt{5\times5\times5} + \sqrt{5\times5\times2\times2\times2}$
$-\sqrt{5\times5\times2}$

$= 3 \times 3\sqrt{5} - 5\sqrt{5} + 10\sqrt{2} - 5\sqrt{2}$

$= 9\sqrt{5} - 5\sqrt{5} + 5\sqrt{2}$

$= 4\sqrt{5} + 5\sqrt{2}$

27. $\left(\frac{81}{16}\right)^{-3/4} \times \left(\frac{25}{9}\right)^{-3/2} \times \left(\frac{2}{3}\right)^{-3}$

$= \left(\frac{3\times3\times3\times3}{2\times2\times2\times2}\right)^{\frac{-3}{4}} \times \left(\frac{5\times5}{3\times3}\right)^{\frac{-3}{2}} \times \left(\frac{2}{3}\right)^{-3}$

$= \left(\left(\frac{3}{2}\right)^{4}\right)^{\frac{-3}{4}} \times \left(\left(\frac{5}{3}\right)^{2}\right)^{\frac{-3}{2}} \times \left(\frac{2}{3}\right)^{-3}$

$= \left(\frac{3}{2}\right)^{4\times\frac{-3}{4}} \times \left(\frac{5}{3}\right)^{2\times\frac{-3}{2}} \times \left(\frac{3}{2}\right)^{3}$

$= \left(\frac{3}{2}\right)^{-3} \times \left(\frac{5}{3}\right)^{-3} \times \left(\frac{3}{2}\right)^{3}$

$= \left(\frac{3}{2}\right)^{-3} \times \left(\frac{3}{2}\right)^{3} \times \left(\frac{5}{3}\right)^{-3}$

$= \left(\frac{3}{2}\right)^{0} \times \left(\frac{3}{5}\right)^{3}$

$= 1 \times \frac{27}{125}$

$= \frac{27}{125}$

28. $x = 9 + 4\sqrt{5}$

$\Rightarrow \frac{1}{x} = \frac{1}{9+4\sqrt{5}}$

$\Rightarrow \frac{1}{x} = \frac{1}{9+4\sqrt{5}} \times \frac{9-4\sqrt{5}}{9-4\sqrt{5}}$

$\Rightarrow \frac{1}{x} = \frac{9-4\sqrt{5}}{(9)^2-\left(4\sqrt{5}\right)^2} = \frac{9-4\sqrt{5}}{81-80}$

$= 9 - 4\sqrt{5}$

Now, $\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 = (\sqrt{x})^2 - 2\sqrt{x}\frac{1}{\sqrt{x}} + \left(\frac{1}{\sqrt{x}}\right)^2$

$$\Rightarrow \left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 = x-2+\frac{1}{x}$$

$$\Rightarrow \left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 = 9+4\sqrt{5}-2+9-4\sqrt{5}$$

$$\Rightarrow \left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 = 16$$

$$\therefore \quad \sqrt{x}-\frac{1}{\sqrt{x}} = \sqrt{16} = 4$$

(By taking square root of both sides)

SECTION-D

29. $\dfrac{3}{(216)^{-2/3}}+\dfrac{2}{(256)^{3/4}}+\dfrac{2}{(243)^{1/5}}$

$$=\frac{3}{(6^3)^{-2/3}}+\frac{2}{(4^4)^{3/4}}+\frac{2}{(3^5)^{1/5}}$$

$$=\frac{3}{(6)^{3\times\frac{-2}{3}}}+\frac{2}{(4)^{4\times\frac{3}{4}}}+\frac{2}{(3)^{5\times\frac{1}{5}}}$$

$$=\frac{3}{(6)^{-2}}+\frac{2}{(4)^3}+\frac{2}{3}$$

$$=3x6^2+\frac{2}{64}+\frac{2}{3}=108+\frac{1}{32}+\frac{2}{3}$$

$$=108.70$$

30. $\dfrac{1}{1+\sqrt{2}}+\dfrac{1}{\sqrt{2}+\sqrt{3}}+\dfrac{2}{\sqrt{3}+\sqrt{5}}$

$$=\frac{1-\sqrt{2}}{(1+\sqrt{2})(1-\sqrt{2})}+\frac{\sqrt{2}-\sqrt{3}}{(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})}$$

$$+\frac{2(\sqrt{3}-\sqrt{5})}{(\sqrt{3}+\sqrt{5})(\sqrt{3}-\sqrt{5})}$$

$$=\frac{1-\sqrt{2}}{1^2-(\sqrt{2})^2}+\frac{\sqrt{2}-\sqrt{3}}{(\sqrt{2})^2-(\sqrt{3})^2}+\frac{2(\sqrt{3}-\sqrt{5})}{(\sqrt{3})^2-(\sqrt{5})^2}$$

$$=\frac{1-\sqrt{2}}{1-2}+\frac{\sqrt{2}-\sqrt{3}}{2-3}+\frac{2(\sqrt{3}-\sqrt{5})}{3-5}$$

$$=\frac{1-\sqrt{2}}{-1}+\frac{\sqrt{2}-\sqrt{3}}{-1}+\frac{2(\sqrt{3}-\sqrt{5})}{-2}$$

$$=\frac{1-\sqrt{2}+\sqrt{2}-\sqrt{3}}{-1}+\frac{2(\sqrt{3}-\sqrt{5})}{-2}$$

$$=\frac{1-\sqrt{3}}{-1}+\frac{2\sqrt{3}-2\sqrt{5}}{-2}$$

$$=\frac{2(1-\sqrt{3})}{(-1)\times 2}+\frac{2\sqrt{3}-2\sqrt{5}}{-2}$$

$$=\frac{2-2\sqrt{3}+2\sqrt{3}-2\sqrt{5}}{-2}$$

$$=\frac{2-2\sqrt{5}}{-2}$$

$$=\frac{-(-2+2\sqrt{5})}{-2}$$

$$=\frac{-2+2\sqrt{5}}{2}=\frac{-2}{2}+\frac{2\sqrt{5}}{2}=(-1+\sqrt{5})$$

31. $x = 2+\sqrt{3}$

$$\frac{1}{x}=\frac{1}{2+\sqrt{3}}$$

$$\frac{1}{x}=\frac{1}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$\Rightarrow \frac{1}{x}=\frac{2-\sqrt{3}}{(2)^2-(\sqrt{3})^2}=\frac{2-\sqrt{3}}{4-3}=(2-\sqrt{3})$$

$$x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)^3-3(x)\left(\frac{1}{x}\right)\left(x+\frac{1}{x}\right)$$

$$=(2+\sqrt{3}+2-\sqrt{3})^3-3(2+\sqrt{3}+2-\sqrt{3})$$

$$=(4)^3-3\times 4=64-12=52$$

32. $\dfrac{1}{1-x}+\dfrac{1}{1+x}=\dfrac{1+x+1-x}{1-x^2}=\dfrac{2}{1-x^2}$

$$\Rightarrow \frac{2}{1-x^2}+\frac{2}{1+x^2}=\frac{2+2x^2+2-2x^2}{1-x^4}=\frac{4}{1-x^4}$$

$$\Rightarrow \quad \frac{4}{1-x^4}+\frac{4}{1+x^4}=\frac{4+4x^4+4-4x^4}{1-x^8}=\frac{8}{1-x^8}$$

$$\Rightarrow \quad \frac{8}{1-x^8}+\frac{8}{1+x^8}=\frac{8+8x^8+8-8x^8}{1-x^{16}}=\frac{16}{1-x^{16}}$$

$$\therefore \quad \frac{1}{1-x}+\frac{1}{1+x}+\frac{2}{1+x^2}+\frac{4}{1+x^4}+\frac{8}{1+x^8}$$

$$=\frac{16}{1-x^{16}}$$

33. $x=3+\sqrt{8}$

$$\Rightarrow \quad \frac{1}{x}=\frac{1}{3+\sqrt{8}}$$

$$\Rightarrow \quad \frac{1}{x}=\frac{1}{3+\sqrt{8}}\times\frac{3-\sqrt{8}}{3-\sqrt{8}}$$

$$\Rightarrow \quad \frac{1}{x}=\frac{3-\sqrt{8}}{(3)^2-(\sqrt{8})^2}=\frac{3-\sqrt{8}}{9-8}=\frac{3-\sqrt{8}}{1}$$

$$\therefore \quad \frac{1}{x}=\frac{3-\sqrt{8}}{1}=3-\sqrt{8}$$

$$x^2+\frac{1}{x^2}=(3+\sqrt{8})^2+(3-\sqrt{8})^2$$

$$=(3)^2+2\times3\times\sqrt{8}+(\sqrt{8})^2+3^2-$$
$$2\times\sqrt{8}\times3+(\sqrt{8})^2$$

$$=9+6\sqrt{8}+8+9-6\sqrt{8}+8 \;=34$$

34. We have:

$$\frac{1}{\sqrt{2}+\sqrt{3}+\sqrt{5}}=\frac{1}{(\sqrt{2}+\sqrt{3})+\sqrt{5}}\times\frac{(\sqrt{2}+\sqrt{3})-\sqrt{5}}{(\sqrt{2}+\sqrt{3})-\sqrt{5}}$$

$$=\frac{(\sqrt{2}+\sqrt{3})-\sqrt{5}}{(\sqrt{2}+\sqrt{3})^2-(\sqrt{5})^2}=\frac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{(\sqrt{2})^2+2(\sqrt{2})(\sqrt{3})+(\sqrt{3})^2-5}$$

$$=\frac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{2+2\sqrt{6}+3-5}=\frac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{2\sqrt{6}}$$

$$=\frac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{2\sqrt{6}}\times\frac{\sqrt{6}}{\sqrt{6}}$$

$$=\frac{(\sqrt{2}+\sqrt{3}-\sqrt{5})\times\sqrt{6}}{2\times6}=\frac{\sqrt{12}+\sqrt{18}-\sqrt{30}}{12}$$

$$=\frac{2\sqrt{3}+3\sqrt{2}-\sqrt{30}}{12}$$

35. $81^{x-2} = 3 \times 27^{2x-3}$

$\Rightarrow \; 3^{4x-8} = 3 \times 3^{6x-9}$

$\Rightarrow \; 3^{4x-8} = 3^{6x-8}$

$4x-8 = 6x-8$

$2x = 0$

$x = 0$

36. $\dfrac{2\sqrt{6}}{\sqrt{2}+\sqrt{3}}+\dfrac{6\sqrt{2}}{\sqrt{6}+\sqrt{3}}-\dfrac{8\sqrt{3}}{\sqrt{6}+\sqrt{2}}$

$$\frac{2\sqrt{6}}{\sqrt{2}+\sqrt{3}}\times\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}+\frac{6\sqrt{2}}{\sqrt{6}+\sqrt{3}}\times\frac{\sqrt{6}-\sqrt{3}}{\sqrt{6}-\sqrt{3}}-\frac{8\sqrt{3}}{\sqrt{6}+\sqrt{2}}\times\frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}$$

$$=\frac{2\sqrt{6}(\sqrt{2}-\sqrt{3})}{(\sqrt{2})^2-(\sqrt{3})^2}+\frac{6\sqrt{2}(\sqrt{6}-\sqrt{3})}{(\sqrt{6})^2-(\sqrt{3})^2}-\frac{8\sqrt{3}(\sqrt{6}-\sqrt{2})}{(\sqrt{6})^2-(\sqrt{2})^2}$$

$$=\frac{2\sqrt{12}-2\sqrt{18}}{2-3}+\frac{6\sqrt{12}-6\sqrt{6}}{6-3}-\frac{8\sqrt{18}-8\sqrt{6}}{6-2}$$

$$=\frac{2\times2\sqrt{3}-2\times3\sqrt{2}}{-1}+\frac{12\sqrt{3}-6\sqrt{6}}{3}-\frac{8\times3\sqrt{2}-8\sqrt{6}}{4}$$

$$=\frac{4\sqrt{3}-6\sqrt{2}}{-1}+\frac{6(2\sqrt{3}-\sqrt{6})}{3}-\frac{8(3\sqrt{2}-\sqrt{6})}{4}$$

$$=\frac{4\sqrt{3}-6\sqrt{2}}{-1}+2(2\sqrt{3}-\sqrt{6})-(6\sqrt{2}-2\sqrt{6})$$

$$=-4\sqrt{3}+6\sqrt{2}+4\sqrt{3}-2\sqrt{6}-6\sqrt{2}+2\sqrt{6}=0$$

37. $a=\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}\times\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$$a=\frac{(\sqrt{3}-\sqrt{2})^2}{3-2}=\frac{3+2-2\sqrt{6}}{1}=5-2\sqrt{6}$$

$$b=\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}\times\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

$$=\frac{(\sqrt{3}+\sqrt{2})^2}{3-2}=\frac{3+2+2\sqrt{6}}{1}=5+2\sqrt{6}$$

$a^2 + b^2 - 5ab$

$= 49-20\sqrt{6}+49+20\sqrt{6}-5$

$=(5-2\sqrt{6})^2+(5+2\sqrt{6})^2-5(5-2\sqrt{6})(5+2\sqrt{6})$

$= 25 + 24 - 2(5)(2\sqrt{6}) + 25 + 24 + 2(5)(2\sqrt{6}) - 5\ (25 - 24)$

$= 49 - 20\ 6 + 49 + 20\ 6 - 5$

$= 98 - 5$

$= 93$

38. $0.6 + 0.\overline{7} + 0.4\overline{7}$ ...(i)

Case I: Let $x = 0.\overline{7}$

$\Rightarrow\quad x = 0.777.....$ ...(ii)

$\Rightarrow\quad 10x = 7.77.....$ ...(iii)

Subtracting (ii) from (iii), we get

$9x = 7$

$$x = \frac{7}{9}$$

Case II: Let $x = 0.4\overline{7}$

$\Rightarrow\quad x = 0.4777.....$

$\Rightarrow\quad 10x = 4.777.....$ ...(iv)

$\Rightarrow\quad 100x = 47.777.....$ ...(v)

Subtracting (iv) from (v), we get

$90x = 43$

$$x = \frac{43}{90}$$

From eqn (i), we get:

$0.6 + 0.\overline{7} + 0.4\overline{7}$

$$= \frac{6}{10} + \frac{7}{9} + \frac{43}{90}$$

$$= \frac{54 + 70 + 43}{90} = \frac{167}{90}$$

WORKSHEET 2: SECTION-A

1. Required rational number $= \frac{1}{2} \times \left(\frac{-3}{7} + \frac{4}{7}\right)$

$$= \frac{1}{2} \times \left[\frac{1}{7}\right] = \frac{1}{14}$$

2. Rationalising factor of $\frac{1}{\sqrt{7} - \sqrt{4}}$ is $\sqrt{7} + \sqrt{4}$.

3. $\frac{1}{1-\sqrt{2}} = \frac{1}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}}$

$$= \frac{1+\sqrt{2}}{(1)^2 - (\sqrt{2})^2} = \frac{1+\sqrt{2}}{1-2} = \frac{1+\sqrt{2}}{-1}$$

$$= \frac{1+1.414}{-1} = \frac{2.414}{-1} = -(2.414)$$

4. $-(81)^{3/4} = -(3^4)^{3/4} = -(3)^{4\times\frac{3}{4}}$

$= (-3)^3 = -27$

5. $(6^2 + 8^2)^{1/2}$

$$= (36 + 64)^{\frac{1}{2}}$$

$$= (100)^{\frac{1}{2}}$$

$$= (100)^{\frac{1}{2}}$$

$$= \left[(10)^2\right]^{\frac{1}{2}}$$

$$= 10^{2\times\frac{1}{2}} = 10$$

6. $(64)^{1/2}\ [64^{1/2} + 1]$

$$(8^2)^{\frac{1}{2}}\left[(8^2)^{1/2} + 1\right]$$

$$= 8^{2\times\frac{1}{2}}\left[(8)^{2\times\frac{1}{2}} + 1\right]$$

$= 8\ (8 + 1) = 8 \times 9 = 72$

7. Let R_1, R_2, R_3 and R_4 be the required four rational numbers.

$$R_1 = \frac{1}{2}\left(\frac{1}{2} + \frac{3}{4}\right) = \frac{1}{2}\left[\frac{2+3}{4}\right] = \frac{1}{2}\left[\frac{5}{4}\right] = \frac{5}{8}$$

$$R_2 = \frac{1}{2}\left(\frac{3}{4} + \frac{5}{8}\right) = \frac{1}{2}\left[\frac{6+5}{8}\right] = \frac{1}{2}\left[\frac{11}{8}\right]$$

$$R_2 = \frac{11}{16}$$

$$R_3 = \frac{1}{2}\left(\frac{11}{16}+\frac{5}{8}\right) = \frac{1}{2}\left[\frac{11+10}{16}\right] = \frac{21}{2\times16} = \frac{21}{32}$$

$$R_4 = \frac{1}{2}\left(\frac{11}{16}+\frac{21}{32}\right)$$

$$= \frac{1}{2}\times\left(\frac{22+21}{32}\right)$$

$$R_4 = \frac{1}{2}\times\frac{43}{32} = \frac{43}{64}$$

$\therefore$ Four rational numbers are $\frac{5}{8}, \frac{11}{16}, \frac{21}{32}, \frac{43}{64}$.

8. $27^{\frac{1}{3}}.4^{\frac{1}{2}}$

$$= (3^3)^{\frac{1}{3}}.(2^2)^{\frac{1}{2}}$$

$$= (3)^{3\times\frac{1}{3}}.(2)^{2\times\frac{1}{2}}$$

$$= 3 \times 2 = 6$$

9. $\left(\frac{64}{125}\right)^{-2/3} = \left(\frac{4\times4\times4}{5\times5\times5}\right)^{-2/3}$

$$= \left\{\left(\frac{4}{5}\right)^3\right\}^{-2/3} = \left(\frac{4}{5}\right)^{3\times\frac{-2}{3}} = \left(\frac{4}{5}\right)^{-2}$$

$$= \left(\frac{5}{4}\right)^2 = \left(\frac{25}{16}\right)$$

10. $\frac{(81)^{-1/4}}{(81)^{1/4}} = \frac{(3^4)^{-\frac{1}{4}}}{(3^4)^{\frac{1}{4}}} = \frac{(3)^{4\times-\frac{1}{4}}}{(3)^{4\times\frac{1}{4}}} = \frac{3^{-1}}{3}$

$$= \frac{\frac{1}{3}}{3} = \frac{1}{3}\times\frac{1}{3} = \frac{1}{9}$$

SECTION-B

11. $5\sqrt{45}\div\frac{\sqrt{75}}{\sqrt{5}}$

$$5\sqrt{3\times3\times5}\times\frac{\sqrt{5}}{\sqrt{75}}$$

$$= 5\times3\sqrt{5}\times\frac{\sqrt{5}}{\sqrt{5\times5\times3}} = 15\sqrt{5}\times\frac{\sqrt{5}}{5\sqrt{3}}$$

$$= 3\frac{\sqrt{5}.\sqrt{5}}{\sqrt{3}} = \frac{3\times5}{\sqrt{3}} = \frac{15}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{15\sqrt{3}}{3} = 5\sqrt{3}$$

12. Two rational number between 4 and 5

First rational number $= \frac{4+5}{2} = \frac{9}{2} = 4.5$

Second rational number

$$= \frac{4.5+5}{2} = \frac{9.5}{2} = 4.75$$

Two rational number between 4 and 5

= 4.5 and 4.75

13. $\frac{4}{(216)^{\frac{2}{3}}} - \frac{1}{(256)^{\frac{3}{4}}}$

$$= \frac{4}{(6^3)^{\frac{2}{3}}} - \frac{1}{(4^4)^{\frac{3}{4}}}$$

$$= \frac{4}{6^2} - \frac{1}{4^3}$$

$$= \frac{4}{36} - \frac{1}{64} = \frac{1}{9} - \frac{1}{64}$$

$$= \frac{55}{576}$$

14. $0.4\overline{7}$

0.4777..... can be expressed form p/q.

Let $x = 0.4\overline{7}$ ...(i)

$10x = 4.\overline{7}$ ...(ii)

$100x = 47.\overline{7}$ ...(iii)

Subtract eqn. (ii) from (iii), we get

90x = 43

$$X=\frac{43}{90}=p/q$$

$$\therefore \quad \begin{bmatrix} p=43 \\ q=90 \end{bmatrix}$$

15. $$\frac{36^{\frac{7}{2}}-36^{-\frac{9}{2}}}{(36)^{-5/2}}=\frac{(6^2)^{\frac{7}{2}}-(6^2)^{-\frac{9}{2}}}{(6^2)^{-5/2}}$$

$$=\frac{(6)^{2\times\frac{7}{2}}-(6)^{2\times-\frac{9}{2}}}{(6)^{2\times-5/2}}$$

$$=\frac{6^7-(6)^{-9}}{(6)^{-5}}=(6^7-(6)^{-9})6^5$$

$$=6^{12}-(6)^{-4}=6^{12}-\frac{1}{6^4}$$

$$=\frac{6^{16}-1}{6^4}$$

16. Let x = $0.\overline{456}$ ...(i)

1000x = 456.456 ...(ii)

Subtract eqn. (i) from (ii), we get

1000x – x = 456

999x = 456

$$x=\frac{456}{999}$$

17. $$\frac{25^{\frac{3}{2}}\times 343^{\frac{3}{5}}}{16^{\frac{5}{4}}\times 8^{\frac{4}{3}}\times 7^{\frac{4}{5}}}=\frac{(5^2)^{\frac{3}{2}}\times(7^3)^{\frac{3}{5}}}{(2^4)^{\frac{5}{4}}\times(2^3)^{\frac{4}{3}}\times 7^{\frac{4}{5}}}$$

$$=\frac{5^{2\times\frac{3}{2}}\times 7^{3\times\frac{3}{5}}}{2^{4\times\frac{5}{4}}\times 2^{3\times\frac{4}{3}}\times 7^{\frac{4}{5}}}$$

$$=\frac{5^3\times 7^{\frac{9}{5}}}{2^5\times 2^4\times 7^{\frac{4}{5}}}=\frac{5^3\times 7^{\frac{9}{5}-\frac{4}{5}}}{2^{5+4}}$$

$$=\frac{5^3\times 7^1}{2^9}=\frac{125\times 7}{512}=\frac{875}{512}$$

18. $[5(25^{1/2}+16^{1/2})]^3$

$$\left[5(5^2)^{\frac{1}{2}}+(4^2)^{\frac{1}{2}}\right]^3$$

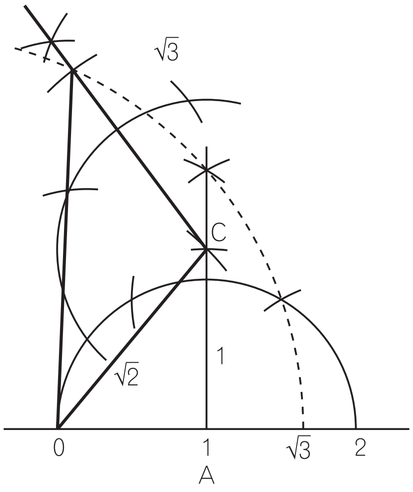
$= [5 \times (5 + 4)]^3$

$= (5 \times 9)^3$

$= (45)^3 = 91125$

SECTION-C

19.



20. $(5)^{x+1} = (125)^{3x-1}$

$(5)^{x+1} = [(5^3)]^{3x-1}$

$(5)^{x+1} = (5)^{3(3x-1)}$

$(5)^{x+1} = (5)^{9x-3}$

Comparing the power of 5, we get

x + 1 = 9x – 3

x – 9x = – 3 – 1

– 8x = – 4

$$x=\frac{4}{8}$$

$$x=\frac{1}{2}$$

21. Let x = $32.12\overline{35}$ ...(i)

Multiply both side by 100

$100\,x = 3212.\overline{35}$ ...(ii)

Again, multiply both side by 100

$10000\,x = 321235.\overline{35}$ ...(iii)

Subtract (ii) from (iii), we get

$9900\,x = 318023$

$$x=\frac{318023}{9900}$$

22. $27^{y} = \dfrac{9}{3^{y}}$

$\Rightarrow \quad (3^{3})^{y} = \dfrac{(3^{2})}{(3^{y})}$

$\Rightarrow \quad (3)^{3y} = (3)^{2-y}$

Comparing the power of 3, both sides, we get,

$$3y = 2 - y$$

$$3y + y = 2$$

$$4y = 2$$

$$y=\frac{2}{4}=\frac{1}{2}$$

23. LHS $= \sqrt{x^{-1}y^{-1}}\sqrt{xy} - \dfrac{1}{\sqrt{x^{-1}y^{-1}}}\dfrac{1}{\sqrt{xy}}$

$$= \sqrt{x^{-1}y^{-1}xy} - \frac{1}{\sqrt{x^{-1}y^{-1}xy}}$$

$$= \sqrt{x^{0}y^{0}} - \frac{1}{\sqrt{x^{0}y^{0}}} = 1-1=0$$

$\therefore$ LHS = RHS

24. $\dfrac{(\sqrt{3}-\sqrt{5})(\sqrt{5}+\sqrt{3})}{7-2\sqrt{5}}$

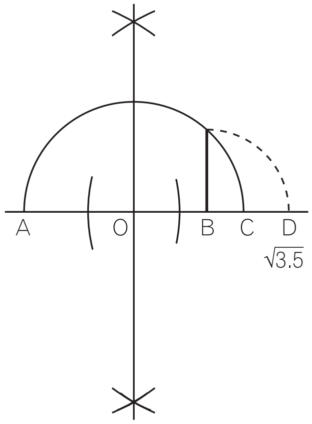
$$= \frac{\sqrt{15}+\sqrt{9}-\sqrt{25}-\sqrt{15}}{7-2\sqrt{5}} = \frac{3-5}{7-2\sqrt{5}} = \frac{-2}{7-2\sqrt{5}}$$

$$= \frac{-2}{7-2\sqrt{5}} \times \frac{7+2\sqrt{5}}{7+2\sqrt{5}}$$

$$= \frac{-2(7+2\sqrt{5})}{(7)^{2}-(2\sqrt{5})^{2}} = \frac{-14-4\sqrt{5}}{49-4\times 5}$$

$$= \frac{-14-4\sqrt{5}}{29} = \frac{-2(7+2\sqrt{5})}{29}$$

25.



26. $\dfrac{1}{1+\sqrt{2}-\sqrt{3}}$

$$= \frac{1}{1+\sqrt{2}-\sqrt{3}} \times \frac{1+\sqrt{2}+\sqrt{3}}{1+\sqrt{2}+\sqrt{3}} = \frac{1+\sqrt{2}+\sqrt{3}}{(1+\sqrt{2})^{2}-(\sqrt{3})^{2}}$$

$$= \frac{1+\sqrt{2}+\sqrt{3}}{1+2+2\sqrt{2}-3} = \frac{1+\sqrt{2}+\sqrt{3}}{2\sqrt{2}}$$

$$= \frac{1+\sqrt{2}+\sqrt{3}}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{(1+\sqrt{2}+\sqrt{3})\sqrt{2}}{4}$$

$$= \frac{\sqrt{2}+\sqrt{6}+2}{4}$$

27. $\dfrac{5\sqrt{2}-3\sqrt{5}}{\sqrt{18}+\sqrt{6}}$

$$= \frac{5\sqrt{2}-3\sqrt{5}}{\sqrt{18}+\sqrt{6}} \times \frac{\sqrt{18}-\sqrt{6}}{\sqrt{18}-\sqrt{6}}$$

$$= \frac{5\sqrt{36}-5\sqrt{12}-3\sqrt{90}+3\sqrt{30}}{(\sqrt{18})^{2}-(\sqrt{6})^{2}}$$

$$= \frac{5\times 6-5\times 2\sqrt{3}-9\sqrt{10}+3\sqrt{30}}{18-6}$$

$$= \frac{30-10\sqrt{3}-9\sqrt{10}+3\sqrt{30}}{12}$$

$$= \frac{1}{12}[30-10\sqrt{3}-9\sqrt{10}+3\sqrt{30}]$$

28. Given: $x = 1+2\sqrt{2}$

Now, $\dfrac{1}{x} = \dfrac{1}{1+2\sqrt{2}} \times \dfrac{1-2\sqrt{2}}{1-2\sqrt{2}} = \dfrac{1-2\sqrt{2}}{(1)^2-(2\sqrt{2})^2}$

$$\frac{1}{x} = \frac{1-2\sqrt{2}}{1-8} - \frac{1-2\sqrt{2}}{-7}$$

$$\frac{1}{x^2} = \left(\frac{1-2\sqrt{2}}{-7}\right)^2$$

$x^2 = (1+2\sqrt{2})^2$ $[(a+b)^2 = a^2+b^2+2ab]$

$= 1 + 4 \times 2 + 4\sqrt{2} = 1 + 8 + 4\sqrt{2}$

$= 9 + 4\sqrt{2}$

$$x^2 + \frac{1}{x^2} = 9 + 4\sqrt{2} + \left(\frac{1-2\sqrt{2}}{-7}\right)^2$$

$$= 9 + 4\sqrt{2} + \frac{1+4\times 2-4\sqrt{2}}{49}$$

$$= 9 + 4\sqrt{2} + \frac{9-4\sqrt{2}}{49}$$

SECTION-D

29. $\dfrac{3\sqrt{2}}{\sqrt{6}-\sqrt{3}} + \dfrac{2\sqrt{3}}{\sqrt{6}+2} - \dfrac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}}$

$$= \frac{3\sqrt{2}(\sqrt{6}+\sqrt{3})}{(\sqrt{6}-\sqrt{3})(\sqrt{6}+\sqrt{3})} + \frac{2\sqrt{3}(\sqrt{6}-2)}{(\sqrt{6}+2)(\sqrt{6}-2)}$$

$$-\frac{4\sqrt{3}(\sqrt{6}+\sqrt{2})}{(\sqrt{6}-\sqrt{2})(\sqrt{6}+2)}$$

$$= \frac{3\sqrt{2}(\sqrt{6}+\sqrt{3})}{3} + \frac{2\sqrt{3}(\sqrt{6}-2)}{2} - \frac{4\sqrt{3}(\sqrt{6}+\sqrt{2})}{4}$$

$$= \sqrt{2}(\sqrt{6}+\sqrt{3}) + \sqrt{3}(\sqrt{6}-2) - \sqrt{3}(\sqrt{6}+\sqrt{2})$$

$$= \sqrt{12} + \sqrt{6} + \sqrt{18} - 2\sqrt{3} - \sqrt{18} - \sqrt{6}$$

$$= \sqrt{12} - 2\sqrt{3} = 2\sqrt{3} - 2\sqrt{3} = 0$$

30. Let $x = 0.5\overline{41}$

Multiply both side by 10

$10x = 5.41$ ...(i)

Multiply both side by 100

$1000x = 541.41$ ...(ii)

subtract (i) from (ii)

$1000x - 10x = 541.41 - 5.41$

$990x = 536$

$$x = \frac{536}{990} = \frac{268}{495}$$

31. $\dfrac{\sqrt{7}-1}{\sqrt{7}+1} - \dfrac{\sqrt{7}+1}{\sqrt{7}-1} = a + b\sqrt{7}$

$$= \frac{\sqrt{7}-1}{\sqrt{7}+1} \times \frac{\sqrt{7}-1}{\sqrt{7}-1} - \frac{\sqrt{7}+1}{\sqrt{7}-1} \times \frac{\sqrt{7}+1}{\sqrt{7}+1}$$

$$= \frac{(\sqrt{7}-1)^2}{(\sqrt{7})^2-(1)^2} - \frac{(\sqrt{7}+1)^2}{(\sqrt{7})^2-(1)^2}$$

$$= \frac{7+1-2\sqrt{7}}{7-1} - \frac{7+1+2\sqrt{7}}{7-1}$$

$$= \frac{8-2\sqrt{7}}{6} - \frac{8+2\sqrt{7}}{6}$$

$$= \frac{8-2\sqrt{7}-8-2\sqrt{7}}{6}$$

$$= -\frac{4\sqrt{7}}{6} = a + b\sqrt{7}$$

$\therefore$ $a = 0$

$$b = -\frac{4}{6} = -\frac{2}{3}$$

32. Given: $\dfrac{\sqrt{5}+2}{\sqrt{5}-2} - \dfrac{\sqrt{5}-2}{\sqrt{5}+2}$

$$\left(\frac{\sqrt{5}+2}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2}\right) - \left(\frac{\sqrt{5}-2}{\sqrt{5}+2} - \frac{\sqrt{5}-2}{\sqrt{5}-2}\right)$$

$$= \frac{(\sqrt{5}+2)^2}{(\sqrt{5})^2-2^2} - \frac{(\sqrt{5}-2)^2}{(\sqrt{5})^2-(2)^2}$$

$$= \frac{(\sqrt{5}+2)^2}{1} - \frac{(\sqrt{5}-2)^2}{1}$$

$$= (\sqrt{5}+2)^2 - (\sqrt{5}-2)^2$$

$$= (5+4+4\sqrt{5}) - (5+4-4\sqrt{5})$$

$$= 9+4\sqrt{5}-9+4\sqrt{5} = 8\sqrt{5}$$

33. (a) $\dfrac{\sqrt{3}-1}{\sqrt{3}+1} = a - b\sqrt{3}$

$$\frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{(\sqrt{3}-1)^2}{(\sqrt{3})^2-(1)^2} = \frac{3+1-2\sqrt{3}}{3-1}$$

$$= \frac{4-2\sqrt{3}}{2} = 2-\sqrt{3} = a - b\sqrt{3}$$

$\therefore$ a = 2, b = 1

(b) $\dfrac{2+\sqrt{5}}{3-\sqrt{5}} = a + b\sqrt{5}$

$$\frac{2+\sqrt{5}}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}} = \frac{6+2\sqrt{5}+3\sqrt{5}+5}{(3)^2-(\sqrt{5})^2}$$

$$= \frac{11+5\sqrt{5}}{9-5} = \frac{11}{4} + \frac{5\sqrt{5}}{4} = a + b\sqrt{5}$$

$\therefore$ $a = \dfrac{11}{4}$, $b = \dfrac{5}{4}$

34. $\dfrac{5+\sqrt{11}}{3-2\sqrt{11}} = x + y\sqrt{11}$

$$\text{LHS} = \frac{5+\sqrt{11}}{3-2\sqrt{11}} \times \frac{3+2\sqrt{11}}{3+2\sqrt{11}}$$

$$= \frac{(5+\sqrt{11})(3+2\sqrt{11})}{(3)^2-(2\sqrt{11})^2}$$

$$= \frac{15+10\sqrt{11}+3\sqrt{11}+2\times 11}{9-4\times 11}$$

$$= \frac{37+13\sqrt{11}}{9-44} = \frac{37+13\sqrt{11}}{-35}$$

$$\frac{37}{-35} + \frac{13}{-35}\sqrt{11} = x + y\sqrt{11}$$

$\therefore$ $x = \dfrac{-37}{35}$, $y = \dfrac{-13}{35}$

35. (a) $\dfrac{25}{8}$

=3.125

8)25(3.125
24
10
8
20
16
40
40
0

(b) $\dfrac{29}{12}$

$= 2.41\bar{6}$

12)29(2.4166
24
50
48
20
12
80
72
8

(c) 11 ÷ 24

$= 0.458\bar{3}$

24)11(0.45833
0
110
96
140
120
200
192
80
72
80
72
8

(d) $4 \div 7$

$= 0.\overline{571428}$

$$\begin{array}{l}
7\overline{)4}(0.5714285 \\
\;\underline{0\;\;\;\;} \\
\;40 \\
\;\underline{35\;\;} \\
\;\;\;50 \\
\;\;\underline{49\;\;} \\
\;\;\;\;10 \\
\;\;\;\;\underline{7\;\;\;} \\
\;\;\;\;30 \\
\;\;\;\;\underline{28\;\;} \\
\;\;\;\;\;\;20 \\
\;\;\;\;\;\underline{14\;\;} \\
\;\;\;\;\;\;\;60 \\
\;\;\;\;\;\;\underline{56\;\;} \\
\;\;\;\;\;\;\;\;40 \\
\;\;\;\;\;\;\;\;\underline{35\;} \\
\;\;\;\;\;\;\;\;\underline{\;5\;\;}
\end{array}$$

36. (a) Let $x = 0.\overline{54}$ ...(i)

Multiply both sides by 100

$100x = 54.\overline{54}$ ...(ii)

Subtract (i) from (ii)

$99x = 54$

$$x = \frac{54}{99} = \frac{18}{33} = \frac{6}{11}$$

(b) Let $x = 0.6\overline{21}$ ...(i)

Multiply both sides by 10

$10x = 6.\overline{21}$ ...(ii)

Multiply both sides by 100

$1000x = 621.\overline{21}$ ...(iii)

Subtract (ii) from (iii)

$990x = 615$

$$x = \frac{615}{990} = \frac{205}{330} = \frac{41}{66}$$

(c) Let $x = 4.\overline{7}$ ...(i)

Multiply both sides by 10

$10x = 47.\overline{7}$ ...(ii)

Subtract (i) from (ii)

$9x = 43$

$$x = \frac{43}{9}$$

(d) Let $x = 2.66\overline{5}$ ...(i)

Multiply both sides by 100

$100x = 266.\overline{5}$ ...(ii)

Multiply both sides by 10

$1000x = 2665.\overline{5}$ ...(iii)

Subtract (ii) from (iii)

$900x = 2399$

$$x = \frac{2399}{900} = \frac{2399}{900}$$

37. Given: $x = \dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$, $y = \dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

To check : $x^2 + xy + y^2 = 99$

$$x = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

$$= \frac{3+2-2\sqrt{6}}{3-2} = \frac{5-2\sqrt{6}}{1}$$

$$y = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

$$= 3+2+2\sqrt{6} = 5+2\sqrt{6}$$

Now, $x^2 + xy + y^2$

$$= (5-2\sqrt{6})^2 + (5-2\sqrt{6})(5+2\sqrt{6}) + (5+2\sqrt{6})^2$$

$$= 25+24-20\sqrt{6} + 25-24+25+24+20\sqrt{6}$$

$= 99$

38. Given : $\sqrt[4]{16} - 6\sqrt[3]{343} + 18\sqrt[5]{243} - \sqrt{196}$

$$= \sqrt[4]{(2^4)} - 6.\sqrt[3]{7^3} + 18.\sqrt[5]{3^5} - \sqrt{14^2}$$

$$= (2^4)^{\frac{1}{4}} - 6.(7^3)^{\frac{1}{3}} + 18 \times (3^5)^{\frac{1}{5}} - (14^2)^{\frac{1}{2}}$$

$$= (2)^{4\times\frac{1}{4}} - 6.(7)^{3\times\frac{1}{3}} + 18 \times (3)^{5\times\frac{1}{5}} - (14)^{2\times\frac{1}{2}}$$

$$= 2 - 42 + 54 - 14 = 0$$

CASE STUDY-1

(i) (a) $x = 0.235, \; x = \frac{235}{1000}$

(ii) (b) $2\sqrt{3}$ is a irrational number

(iii) (b) $(\sqrt{2}+5\sqrt{3})+(\sqrt{2}-3\sqrt{3})$

$$= \sqrt{2}+\sqrt{2}+5\sqrt{3}-3\sqrt{5}$$

$$\Rightarrow \quad 2\sqrt{2}+2\sqrt{3}$$

(iv) (d) π is a non terminating constant

(v) (b) $\frac{329}{400} = 0.8225$

∴ $\frac{329}{400}$ has terminating decimal expansion.

CASE STUDY-2

(i) (d) $(3+\sqrt{3})(3-\sqrt{3}) = 9-3\sqrt{3}+3\sqrt{3}-3 = 6$

(ii) (a) Rationalising factor of $\sqrt[3]{3}+\sqrt[3]{2}$ is

$$\frac{1}{\sqrt[3]{3}+\sqrt[3]{2}}$$

$a^3 + b^3 = (a + b)\,(a^2 + b^2 - ab)$

$a = \sqrt[3]{3}, \; b = \sqrt[3]{2}$

$$(\sqrt[3]{3})^3+(\sqrt[3]{2})^3 = (\sqrt[3]{3}+\sqrt[3]{2})$$

$$\left[(\sqrt[3]{3})^2+(\sqrt[3]{2})^2-\sqrt[3]{3}\times\sqrt[3]{2}\right]$$

$$\Rightarrow \quad \sqrt[3]{3}+\sqrt[3]{2} = \frac{(\sqrt[3]{3})^3+(\sqrt[3]{2})^2}{\left[(\sqrt[3]{3})^2-(\sqrt[3]{3}\times\sqrt[3]{2})+(\sqrt[3]{2})^2\right]}$$

$$\frac{1}{\sqrt[3]{3}+\sqrt[3]{2}} = \frac{\left[(\sqrt[3]{3})^2-(\sqrt[3]{3}\times\sqrt[3]{2})+(\sqrt[3]{2})^2\right]}{3+2}$$

$$= \frac{\left[(\sqrt[3]{3})^2-(\sqrt[3]{3}\times\sqrt[3]{2})+(\sqrt[3]{2})^2\right]}{5}$$

$$= \frac{1}{5}\left[3^{2/3}-6^{1/3}+2^{2/3}\right]$$

$$= \frac{\left[\sqrt[3]{3^2}-\sqrt[3]{3}\times\sqrt[3]{2}+\sqrt[3]{2^2}\right]}{5}$$

∴ Rationalising factor is $\left[\sqrt[3]{3^2}-\sqrt[3]{3}\times\sqrt[3]{2}+\sqrt[3]{2^2}\right]$

(iii) (d) $\frac{x-y}{x-3y} = \frac{\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}-1}{\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}-3}$

$$\Rightarrow \quad \frac{\sqrt{3}+\sqrt{2}-(\sqrt{3}-\sqrt{2})}{\sqrt{3}+\sqrt{2}-3(\sqrt{3}-\sqrt{2})} \Rightarrow \frac{2\sqrt{2}}{-2\sqrt{3}+4\sqrt{2}}$$

$$\Rightarrow \quad \frac{\sqrt{2}}{2\sqrt{2}-\sqrt{3}} \Rightarrow \frac{\sqrt{2}}{2\sqrt{2}-\sqrt{3}}\times\left(\frac{2\sqrt{2}+\sqrt{3}}{2\sqrt{2}+\sqrt{3}}\right)$$

$$\Rightarrow \quad \frac{4+\sqrt{6}}{8-3} = \frac{4+\sqrt{6}}{5}$$

(iv) (c) $\frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}} = a-b\sqrt{77}$

$$\frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}}\times\frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}-\sqrt{7}} = \frac{(\sqrt{11}-\sqrt{7})^2}{11-7}$$

$$= \frac{11+7-2\sqrt{77}}{4}$$

$$\frac{18-2\sqrt{7}}{4} = \frac{9}{2}-\frac{1\sqrt{7}}{2}$$

$$\frac{9}{2}-\frac{1\sqrt{7}}{2} = a-b\sqrt{77}$$

$$a=\frac{9}{2}, \; b=\frac{1}{2}$$

$$(a, b) = \left(\frac{9}{2},\frac{1}{2}\right)$$

(v) (a) $\frac{1}{\sqrt{5}+\sqrt{2}}\times\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$

$$\Rightarrow \quad \frac{\sqrt{5}-\sqrt{2}}{5-2} = \frac{\sqrt{5}-\sqrt{2}}{3}$$

Chapter 2

Polynomials

MULTIPLE CHOICE QUESTION

1. (a) Degree of zero polynomial is 0.

2. (b) $x^3 + y^3 + z^3 = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx) + 3xyz$

$= (1)[x^2 + y^2 + z^2 - (xy + yz + zx)] + 3(-1)$

$= [x^2 + y^2 + z^2 - (-1)] -3$ ...(i)

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$

$(1)^2 = (x^2 + y^2 + z^2) + 2(-1)$

$1 = x^2 + y^2 + z^2 - 2$

$\therefore \quad x^2 + y^2 + z^2 = 3$ ...(ii)

Putting the value obtained from equation (ii) in equation (i), $x^3 + y^3 + z^3 = [3 + 1] - 3 = 1$

3. (d) $[a^3 + b^3 + 3a^2b + 3ab^2] + [a^3 - b^3 - 3a^2b + 3ab^2] + 6a^3 - 6b^3 = 8a^3$

4. (c) $150 \times 98 = (100 + 50)(100 - 2)$

$= 10000 + 5000 - 200 - 100$

$= 14700$

5. (c) $25x^2 + 16y^2 + 40xy$

at $x = 1, y = -1$

$25(1) + 16(-1)^2 + 40(1)(-1) = 1$

WORKSHEET 1: SECTION-A

1. $(x - 1)(x - 2)(x - 3)(x - 4) = 0$

So,

$(x - 1) = 0$

$x = 1$

$(x - 2) = 0$

$x = 2$

$(x - 3) = 0$

$x = 3$

$(x - 4) = 0$

$x = 4$

$\therefore$ Zeros of the polynomial are 1, 2, 3, 4

2. $p(x) = x^3 - 27$

$= x^3 - 3^3$

$= (x - 3)(x^2 + 9 + 3x)$

$= (x - 3)(x^2 + 3x + 9)$

$\left[\begin{array}{l}\text{Using} \\ (a^3 - b^3) = (a - b)(a^2 + b^2 + ab)\end{array}\right]$

Here, $a = x$

$b = 3$

To find zeroes of p(x),

Put $p(x) = 0$

$(x - 3)(x^2 + 3x + 9) = 0$

$x - 3 = 0,$

$x^2 + 3x + 9 = 0$

$x = 3$

So, we have only one real root

$x = 3.$

3. $p(x) = x^3 - x^2 + x + 1$

if $x = -1$, then

$p(-1) = (-1)^3 - (-1)^2 + (-1) + 1$

$= -1 - (1) - 1 + 1$

$= -3 + 1$

$= -2$

$p(1) = (1)^3 - (1)^2 + (1) + 1$

$= 1 - 1 + 1 + 1$

$= 2$

Putting the values of p(1), p(–1), we get:

$$\frac{p(1) + p(-1)}{2}$$

$$= \frac{2 + (-2)}{2} = \frac{2 - 2}{2} = \frac{0}{2} = 0.$$

$$\therefore \quad \frac{p(-1) + p(1)}{2} = 0$$

4. $x+\frac{1}{2} \quad x+\frac{3}{2}$

$\Rightarrow x \quad x+\frac{3}{2} \quad +\frac{1}{2} \quad x+\frac{3}{2}$

$\Rightarrow x^2+\frac{3}{2}x+\frac{1}{2}x+\frac{3}{4}$

$\Rightarrow x^2+\frac{4}{2}x+\frac{3}{4}$

$\Rightarrow x^2+2x+\frac{3}{4}$

$\Rightarrow \frac{4x^2+8x+3}{4} = \frac{1}{4}(4x^2 + 8x + 3)$

5. $(-12)^3 + (7)^3 + (5)^3$

By using the identity $x^3 + y^3 + z^3 = 3xyz$

This identity is possible if

$x + y + z = 0$, then

where, $x = -12$

$y = 7$

$z = 5$

$x + y + z = -12 + 7 + 5 = 0$

Hence verified.

So, we can use the identity

$x^3 + y^3 + z^3 = 3xyz$

$3xyz = 3(-12)(7)(5)$

$= -1260$

$\therefore (-12)^3 + (7)^3 + (5)^3 = -1260$

6. $p(x) = x^3 - a^2x + x + 2$

$x - a$, is a factor of the polynomial, then

$p(a) = (a)^3 - a^2(a) + a + 2 = 0$

$\Rightarrow a^3 - a^3 + a + 2 = 0$

$\Rightarrow a + 2 = 0$

$\Rightarrow a = -2$

$\therefore$ Value of a is –2.

7. $p(x) = x^{31} + 31$

$q(x) = x + 1$

$q(x) = 0 \quad \Rightarrow \quad x = -1$

Putting the value of $x = -1$ in $p(x)$

$p(-1) = (-1)^{31} + 31$

$= -1 + 31$

$= 30.$

Remainder will be 30.

8. $2x^2 - 7x = 0$

$\Rightarrow x(2x - 7) = 0$

Zeroes will be

0 and $\frac{7}{2}$

$$\left[\begin{array}{r}\because \; 2x - 7 = 0 \\ x = \frac{7}{2}\end{array}\right]$$

9. $2x^3 - kx^2 + 7x - 1 = p(x)$

Since when p(x) is divided by x – 1, remainder is 3, So

$p(1) = 3$

$\Rightarrow 2(1)^3 - k(1)^2 + 7(1) - 1 = 3$

$\Rightarrow 2 - k + 7 - 1 = 3$

$\Rightarrow -k + 8 = 3$

$\Rightarrow 8 - 3 = k$

$\Rightarrow 5 = k$

10. Given

$p(x) = 3x^3 - 2x^2 - x + 4$

If $x = -1$, then

$p(-1) = 3(-1)^3 - 2(-1)^2 - (-1) + 4$

$= 3(-1) - 2(1) + 1 + 4$

$= -3 - 2 + 5$

$= -5 + 5$

$= 0.$

If $x = 1$, then

$p(1) = 3(1)^3 - 2(1)^2 - (1) + 4$

$= 3 - 2 - 1 + 4$

$= 1 - 1 + 4 = 4$

Putting the value of P(1) and P(–1), then

$P(1) + P(-1)$

$= 4 + 0 = 4$

$\therefore P(1) + P(-1) = 4.$

SECTION-B

11. $185 \times 185 - 15 \times 15$

$\Rightarrow (185)^2 - (15)^2$

By using the identity

$(x^2 - y^2) = (x + y)(x - y)$, we get

$\Rightarrow (185)^2 - (15)^2 = (185 + 15)(185 - 15)$

$\Rightarrow (185)^2 - (15)^2 = 200 \times 170$

$\Rightarrow (185)^2 - (15)^2 = 34000$

$\therefore 185 \times 185 - 15 \times 15 = 34000.$

12. $p(x) = 2x^2 - 3x + 7a$

Since $x = 2$ is a root of $p(x)$,

$\Rightarrow p(2) = 0$

$\Rightarrow 2(2)^2 - 3(2) + 7a = 0$

$\Rightarrow 8 - 6 + 7a = 0$

$\Rightarrow 2 + 7a = 0$

$\Rightarrow a = \frac{-2}{7}$

13. Given

$x - 1$ is a factor of the polynomial $x^2 + x + k$.

$\therefore x - 1 = 0$

$x = 1$

Putting the value $x = 1$ in the equation.

$x^2 + x + k = 0$

So,

$(1)^2 + (1) + k = 0$

$1 + 1 + k = 0$

$2 + k = 0$

$k = -2$

$\therefore$ Value of $k = -2$

14. $p(x) = x^2 - 4x + 4$

To find

$p(2) + p(-2) + p(1)$

If $x = 2$, then

$p(2) = (2)^2 - 4(2) + 4$

$= 4 - 8 + 4 = 0$ ...(1)

If $x = -2$, then

$p(-2) = (-2)^2 - 4(-2) + 4$

$= 4 + 8 + 4$

$= 16$ ...(2)

If $x = 1$

$p(1) = (1)^2 - 4(1) + 4$

$= 1 - 4 + 4$

$= 1$ ...(3)

Putting the value of p(2), p(–2), p(1) in p(2) + p(–2) + p(1), we get

$\Rightarrow 0 + 16 + 1 = 17$

$\therefore p(2) + p(-2) + p(1) = 17$

15. $(99)^3$

99 can also be expressible in the form (100 – 1), then

$(99)^3 \Rightarrow (100 - 1)^3$

Using the identity $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$,

where $x = 100$

$y = 1$

$(100 - 1)^3 = (100)^3 - (1)^3 - 3(100)(100 - 1)$

$= 1000000 - 1 - 29700$

$= 1000000 - 29701$

$= 970299$

$\therefore (99)^3 = 970299$

16. $\left(\frac{-x}{2} + y + \frac{1}{4}\right)^2$

Using the identity $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$$= \frac{-x}{2}^2 + (y)^2 + \left(\frac{1}{4}\right)^2 + 2\left(\frac{-x}{2}\right)(y) + 2(y)\left(\frac{1}{4}\right) +$$

$$2\ \frac{1}{4}\left(\frac{-x}{2}\right)$$

$$= \frac{x^2}{4} + y^2 + \frac{1}{16} + \frac{-2xy}{2} + \frac{y}{2} + \left(\frac{-x}{4}\right)$$

$$= \frac{x^2}{4} + y^2 + \frac{1}{16} - xy + \frac{y}{2} - \frac{x}{4}$$

17. $x^4 - 125\ xy^3$

$= x(x^3 - 125y^3)$

$= x(x^3 - (5y)^3)$

$= x(x - 5y)\ (x^2 + 25y^2 + 5xy)$

SECTION-C

18. $x^2 + y^2 = 58,\ x + y = 10$

To find : $x^3 + y^3$

$x^3 + y^3 = (x + y)\ (x^2 + y^2 - xy)$

$= (10)\ (58 - xy)$ ...(i)

Consider $(x + y)^2 = x^2 + y^2 + 2xy$

$(10)^2 = 58 + 2xy$

$100 - 58 = 2xy$

$21 = xy$ ...(ii)

Put (ii) in (i)

$x^3 + y^3 = 10\ (58 - 21)$

$= 370$

19. $64x^3 - 27y^3 + z^3 + 36xyz$

$= (4x)^3 + (-3y)^3 + z^3 - 3(4x)\ (-3y)z$

Using identity, $a^3 + b^3 + c^3 - 3abc = (a + b + c)\ (a^2 + b^2 + c^2 - ab - bc - ca)$

$= [4x + (-3y) + z]\ [(4x)^2 + (-3y)^2 + (z)^2 - (4x)\ (-3y) - (-3y)z - (z)\ (4x)]$

$= (4x - 3y + z)\ (16x^2 + 9y^2 + z^2 + 12xy + 3yz - 4zx)$

20. $(x - 1)^3 = 8$

$(x - 1)^3 = 8$

$(x - 1)^3 = (2)^3$

$x - 1 = 2$

$x = 2 + 1 = 3$

$(x + 1)^2 = (3 + 1)^2 = (4)^2 = 16$

21. $x^3 - 23x^2 + 142x - 120$

Put $x = 1$

$1^3 - 23\ (1)^2 + 142\ (1) - 120 = 0$

$\therefore$ $x - 1$ is a factor.

$$\begin{array}{r|l}
 & x^2 - 22x + 120 \\ \hline
x - 1 & x^3 - 23x^2 + 142x - 120 \\
 & +x^3 - x^2 \\
 & - \quad + \\ \hline
 & \quad -22x^2 + 142x - 120 \\
 & \quad -22x^2 + 22x \\
 & \quad + \qquad - \\ \hline
 & \qquad\quad +120x - 120 \\
 & \qquad\quad +120x - 120 \\
 & \qquad\quad - \qquad + \\ \hline
 & \qquad\qquad\qquad 0 \\ \hline
\end{array}$$

Therefore, $x^2 - 22x + 120$

$= (x^2 - 22x + 120)$

$= (x^2 - 12x - 10x + 120)$

$= x(x - 12) - 10\ (x - 12)$

$= (x - 12)\ (x - 10)$ are factors.

So, $x^3 - 23x^2 + 142x - 120$

$= (x - 1)\ (x - 12)\ (x - 10)$

22. We know that:

$(a + b) = 7,\ ab = 10$

$(a+b)^2 = 7^2$

$a^2 + b^2 + 2ab = 49$

$a^2 + b^2 = 49 - 20 = 29$

23. $p(x) = kx^3 + 9x^2 + 4x - 8$ divided by $(x + 3)$ leaves remainder $10 - 10k$.

$-27k + 81 - 12 - 8 = 10 - 10k$

$-27k + 61 = 10 - 10k$

$61 - 10 = -10k + 27k$

$51 = 17k$

$\frac{51}{17} = k$

$\therefore$ Value of $k = 3$

24. $p(x) = x^4 - 2x^3 + 3x^2 - mx + 3$

$p(1) = R_1$

$1 - 2 + 3 - m + 3 = R_1$

$-1 + 6 - m = R_1$

$5 - m = R_1$

$p(-1) = R_2$

$(-1)^4 - 2(-1)^3 + 3(-1)^2 - m(-1) + 3 = R_2$

$1 + 2 + 3 + m + 3 = R_2$

$9 + m = R_2$

$3R_1 + R_2 = 35$

$\Rightarrow \quad 3(5 - m) + 9 + m = 35$

$\Rightarrow \quad 15 - 3m + 9 + m = 35$

$\Rightarrow \quad -11 = 2m$

So, $m = \frac{-2}{11}$

25. $x^3 + 15px + p^3 - 125$

$= x^3 + 15x(5 - x) + (5 - x)^3 - 125$

$= x^3 + 75x - 15x^2 + 125 - x^3 - 75x + 15x^2 - 125 = 0$

So, $x^3 + 15px + p^3 - 125 = 0$

SECTION-D

26. $2 + ax - 2x^2 - 3x^3$

Put $x = -1$

$2 + a(-1) - 2(-1)^2 - 3(-1)^3 = 0$

$2 - a - 2 + 3 = 0$

$2 - a + 3 = 2$

$2 - a = -1$

$a = 3$

$p(x) = 2 + 3x - 2x^2 - 3x^3$

$$
\begin{array}{r|l}
 & -3x^2 + x + 2 \\
\hline
x + 1 & -3x^3 - 2x^2 + 3x + 2 \\
 & -3x^3 - 3x^2 \\
 & + \quad\ + \\
\hline
 & \qquad x^2 + 3x + 2 \\
 & \qquad + x^2 + x \\
 & \qquad - \quad\ - \\
\hline
 & \qquad\qquad 2x + 2 \\
 & \qquad\qquad +2x + 2 \\
 & \qquad\qquad - \quad\ - \\
\hline
 & \qquad\qquad\qquad 0 \\
\hline
\end{array}
$$

So, $p(x) = (x + 1)(-3x^2 + x + 2)$

$= (x + 1)(-3x^2 + 3x - 2x + 2)$

$= (x + 1)[-3x(x - 1) - 2(x - 1)]$

$= (x + 1)(x - 1)(-3x - 2)$

27. Put $a = -b$ in $(a + b + c)^3 - (a^3 + b^3 + c^3)$

$[-b + b + c]^3 - [-b^3 + b^3 + c^3]$

$= c^3 - c^3 = 0$

$\Rightarrow$ $(a + b)$ is a factor of $(a + b + c)^3 - (a^3 + b^3 + c^3)$.

Put $b = -c$

$[a - c + c]^3 - [a^3 - c^3 + c^3]$

$= a^3 - a^3 = 0$

$\Rightarrow$ $(b + c)$ is a factor of $(a + b + c)^3 - (a^3 + b^3 + c^3)$.

Put $c = -a$

$[a + b - a]^3 - [a^3 + b^3 - a^3]$

$= b^3 - b^3 = 0$

$\Rightarrow$ $(c + a)$ is a factor of $(a + b + c)^3 - (a^3 + b^3 + c^3)$.

28. $2x^4 + 3x^3 - 26x^2 - 5x + 6$

Put $x = 3$

$2(3)^4 + 3(3)^3 - 26(3)^2 - 5(3) + 6$

$= 162 + 81 - 234 - 15 + 6$

$249 - 249 = 0$

29. $(a + b + c)^3 - a^3 - b^3 - c^3$

$= (a + b + c)^3 - a^3 - b^3 - c^3$

$= (a + b)^3 + c^3 + 3c(a + b)(a + b + c) - a^3 - b^3 - c^3$

$= a^3 + b^3 + 3ab(a + b) + c^3 + 3c(a + b)(a + b + c) - a^3 - b^3 - c^3$

$= 3ab(a + b) + 3c(a + b)(a + b + c)$

$= 3(a + b)(ab + c(a + b + c))$

$= 3(a + b)(a(b + c) + c(b + c))$

$= 3(a + b)(a + c)(b + c)$

30. $p(x) = x^4 - 2x^3 + 3x^2 - ax + b$

$p(1) = 5$

$1 - 2 + 3 - a + b = 5$

$2 - a + b = 5$

$-a + b = +3 \qquad$ ...(i)

$p(-1) = 19$

$(-1)^4 - 2(-1)^3 + 3(-1)^2 - a(-1) + b = 19$

$1 + 2 + 3 + a + b = 19$

$a + b = 13 \qquad$ ...(ii)

Solving (i) and (ii), we get a = 5 and b = 8

So, $p(x) = x^4 - 2x^3 + 3x^2 - 5x + 8$

Now $p(2) = 2^4 - 2(2)^3 + 3(2)^2 - 5(2) + 8$

$= 2^4 - 2^4 + 12 - 10 + 8$

$= 10$

⇒ Remainder is 10 when p(x) is divided by x –2.

31. (a) $x + \frac{1}{x} = 6$ find $x^2 + \frac{1}{x^2}$

$$\left(x + \frac{1}{x}\right)^2 = (6)^2 = x^2 + \frac{1}{x^2} + 2 = 36$$

$$x^2 + \frac{1}{x^2} = 34$$

(b) To find $x^4 + \frac{1}{x^4}$

$$\text{Now, } \left(x^2 + \frac{1}{x^2}\right)^2 = x^4 + \frac{1}{x^4} + 2$$

$$\Rightarrow \quad (34)^2 = x^2 + \frac{1}{x^4} + 2$$

$$1154 = x^4 + \frac{1}{x^4}$$

32. $x = 2y + 6$, $x^3 - 8y^3 - 36xy - 216$

$= (2y + 6)^3 - 8y^3 - 36(2y + 6)y - 216$

$= 8y^3 + 216 + 72y^2 + 216y - 8y^3 - 72y^2 - 216y - 216 = 0$

33. $p(1) = 0$

$1^3 - a(1)^2 + 13(1) + b = 0$

$1 - a + 13 + b = 0$

$a - b = 14 \quad ...(1)$

$p(-3) = 0$

$(-3)^3 - a(-3)^2 + 13(-3) + b = 0$

$-27 - 9a - 39 + b = 0$

$-9a + b = 66 \quad ...(2)$

Solve (1) and (2), $a = -10$, $b = -24$

WORKSHEET 2: SECTION-A

1. $3x^2 + x = x(3x + 1)$

2. $20x^2 - 9x + 1$

⇒ $20x^2 - (5 + 4)x + 1$

⇒ $20x^2 - 5x - 4x + 1$

⇒ $5x(4x - 1) - 1(4x - 1)$

⇒ $(4x - 1)(5x - 1)$

3. Evaluate: $(9)^3 + (-3)^3 + (-6)^3$

$a = 9, \quad b = -3, \quad c = -6$

$a^3 + b^3 + c^3 = 3abc$

if $(a + b + c) = 0$

$= 3(9) \times (-3) \times (-6)$

$= 27 \times 18$

$= 486$

4.

$$\begin{array}{r|l} & 2x + 1 \\ \hline 3x - 1 & 6x^2 + x - 1 \\ & 6x^2 - 2x \\ & - \quad\ + \\ \hline & \quad 3x - 1 \\ & \quad 3x - 1 \\ & \quad - \quad + \\ \hline & \qquad 0 \\ \hline \end{array}$$

So the other factor is 2x + 1.

5. Find the value:

$(28)^3 + (-15)^3 + (-13)^3$

$a^3 + b^3 + c^3 = 3abc$

$= 3(28) \times (-15) \times (-13)$

$= 84 \times 195 = 16380$

6. $x^2 + 8x + k$

Put $x = -1$

⇒ $(-1)^2 + 8(-1) + k = 0$

⇒ $1 - 8 + k = 0$

⇒ $-7 + k = 0$

$k = 7$

7. Constant Polynomial: A polynomial of degree zero is called a constant polynomial. The degree of constant polynomial is zero.

8. $\frac{x^2}{4} - \frac{y^2}{4}$

$= \frac{x}{2}^{2} - \frac{y}{2}^{2} = \frac{x}{2} - \frac{y}{2} \quad \frac{x}{2} + \frac{y}{2}$

9. $4x^3 - 3x^2 + 2x - 4$ is divided by $x + \frac{1}{2}$

If $x + \frac{1}{2} = 0$

Put x = $-\frac{1}{2}$ in given polynomial.

Remainder

$= 4 \ \frac{-1}{2}^{3} - 3 \ \frac{-1}{2}^{2} + 2 \ - \frac{1}{2} \ - 4$

$= 4 \ \frac{-1}{8} - 3 \ \frac{1}{4} - 1 - 4$

$R \Rightarrow -\frac{1}{2} - \frac{3}{4} - \frac{5}{1}$

$\Rightarrow \frac{-2-3-20}{4}$

$R \Rightarrow \frac{-25}{4}$

$\therefore$ Remainder is $\frac{-25}{4}$

10. $(55)^3 - (25)^3 - (30)^3$

$= (55)^3 + (-25)^3 + (-30)^3$

$= 3 \times 55 \times -25 \times -30$

$= 165 \times 750 = 123750$

SECTION-B

11. $x^4 - y^4$

$= (x^2)^2 - (y^2)^2$

$= (x^2 + y^2)(x^2 - y^2)$

$[\because x^2 - y^2 = (x + y)(x - y)]$

$= (x^2 + y^2)(x + y)(x - y)$

12. $f(x) = x^3 + 12x^2 + 3x - 7$

Put $x = -3$

$f(-3) = (-3)^3 + 12(-3)^2 + 3(-3) - 7$

$f(-3) = -27 + 12 \times 9 - 9 - 7$

$= -27 - 16 + 108$

$f(-3) = -43 + 108 = 65$

13. $x + y + z = 12$

$x^2 + y^2 + z^2 = 64$

We know that,

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

$(12)^2 = 64 + 2(xy + yz + zx)$

$144 - 64 = 2 \times (xy + yz + zx)$

$\frac{80}{2} = xy + yz + zx$

$\therefore xy + yz + zx = 40$

14. $\frac{x^2 + 7x + 12}{x^2 + 2x - 3}$

$= \frac{x^2 + 4x + 3x + 12}{x^2 + 3x - x - 3} = \frac{x(x + 4) + 3(x + 4)}{x(x + 3) - 1(x + 3)}$

$= \frac{(x + 4)(x + 3)}{(x + 3)(x - 1)} = \frac{(x + 4)}{(x - 1)}$

15. $x^3 + 3x^2 - kx - 3$ ...(i)

One factor $x + 3 = 0$

$x = -3$

Put $x = -3$ in eqn. (i), we get,

$x^3 + 3x^2 - kx - 3$

$= (-3)^3 + 3(-3)^2 - k(-3) - 3$

$\Rightarrow -27 + 27 + 3k - 3 = 0$

$3k - 3 = 0$

$3k = 3$

$k = \frac{3}{3}$

$k = 1$

16. Expand: $(-3x + 5y - 23)^2$

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

$= (-3x)^2 + (5y)^2 + (-23)^2 + 2(-3x) \times 5y + 2 \times 5y \times (-23) + 2(-23) \times (-3x)$

$= 9x^2 + 25y^2 + 529 - 30xy - 230y + 138x$

17. $2\sqrt{2}a^3 - 3\sqrt{3}b^3$

$(\sqrt{2}a)^3 - (\sqrt{3}b)^3$

$= (\sqrt{2}a - \sqrt{3}b)((\sqrt{2}a)^2 + \sqrt{2}a\sqrt{3}b + (\sqrt{3}b)^2)$

$[\because x^3 - y^3 = (x - y)(x^2 + xy + y^2)]$

$= (\sqrt{2}a \times \sqrt{3}b)(2a^2 + \sqrt{6}ab + 3b^2)$

18. $\sqrt{2}x^2 + 9x + 4\sqrt{2}$

$= \sqrt{2}x^2 + (8 + 1)x + 4\sqrt{2}$

$= \sqrt{2}x^2 + 8x + 1x + 4\sqrt{2}$

$= \sqrt{2}x(x + 4\sqrt{2}) + 1(x + 4\sqrt{2})$

$= (x + 4\sqrt{2})(\sqrt{2}x + 1)$

19. $g(x) = x - 6 = 0$

$x = 6$

$p(x) = x^3 - 4x^2 + x + 6$

$p(6) = 6^3 - 4(6)^2 + 6 + 6$

$= 216 - 144 + 12$

$= 84$

$\neq 0$

So, g(x) is not a factor of p(x).

20. $x^2 - 16$

for $x = 24$

$x^2 - 16 = x^2 - (4)^2$

$\Rightarrow (x + 4)(x - 4) \Rightarrow (24 + 4)(24 - 4)$

$\Rightarrow 28 \times 20 \Rightarrow 560$

SECTION-C

21.

$$\begin{array}{r|l} & 2x^3 - 2x^2 + x + 4 \\ \hline 2x - 1 & 4x^4 - 4x^3 + 3x^2 + 7x - 2 \\ & 4x^4 - 2x^3 \\ & - \quad + \\ \hline & -2x^3 + 3x^2 \\ & -2x^3 + x^2 \\ & + \quad - \\ \hline & 2x^2 + 7x - 2 \\ & 2x^2 - x \\ & - \quad + \\ \hline & 8x - 2 \\ & 8x - 4 \\ & - \quad + \\ \hline & 2 \\ \hline \end{array}$$

Quotient $= 2x^3 - x^2 + x + 4$

Remainder $= 2$

22. $x^3 - 3x^2 - 9x - 5$ ...(i)

$$\begin{array}{r|l} & x^2 - 4x - 5 \\ \hline x + 1 & x^3 - 3x^2 - 9x - 5 \\ & x^3 + x^2 \\ & - \quad - \\ \hline & -4x^2 - 9x \\ & -4x^2 - 4x \\ & + \quad + \\ \hline & -5x - 5 \\ & -5x - 5 \\ & + \quad + \\ \hline & 0 \\ \hline \end{array}$$

$\therefore$ $(x + 1)$ is a factor of p(x).

$p(x) = (x + 1)[x^2 - 4x - 5]$

$= (x + 1)[x^2 + x - 5x - 5]$

$= (x + 1)[x(x + 1) - 5(x + 1)]$

$= (x + 1)(x + 1)(x - 5)$

23.

$$\begin{array}{r|l} & 3x^3 + 3x^2 - x - 6 \\ \hline x - 1 & 3x^4 - 4x^2 - 5x + 3 \\ & 3x^4 - 3x^3 \\ & - \quad + \\ \hline & 3x^3 - 4x^2 - 5x + 3 \\ & 3x^3 - 3x^2 \\ & - \quad + \\ \hline & -x^2 - 5x + 3 \\ & -x^2 + x \\ & + \quad - \\ \hline & -6x + 3 \\ & -6x + 6 \\ & + \quad - \\ \hline & -3 \\ \hline \end{array}$$

Using Remainder Theorem, put $x = 1$, in the given polynomial

$3(1)^4 - 4(1)^2 - 5(1) + 3$

$= 3 - 4 - 5 + 3 = -3$

24. $4x^3 - 16x^2 - x + 4$

If $2x + 1 = 0$

$2x = -1 \Rightarrow x = -\frac{1}{2}$

$$\Rightarrow 4 \; -\frac{1}{2}^{3} - 16 \; \frac{1}{2}^{2} - \; -\frac{1}{2} + 4$$

$$\Rightarrow 4 \times -\frac{1}{8} - 16 \times \frac{1}{4} + \frac{1}{2} + 4$$

$$\Rightarrow \frac{1}{2} - 4 + \frac{1}{2} + 4 = 0$$

∴ (2x + 1) is a factor of given polynomial.

$2x - 1 = 0$

$2x = 1 \Rightarrow x = \frac{1}{2}$

$$\Rightarrow 4 \; \frac{1}{2}^{3} - 16 \; \frac{1}{2}^{2} - \; \frac{1}{2} + 4$$

$$\Rightarrow 4 \times \frac{1}{8} - 16 \times \frac{1}{4} - \frac{1}{2} + 4$$

$$\Rightarrow \frac{1}{2} - 4 - \frac{1}{2} + 4 = 0$$

∴ (2x – 1) is also a factor of given polynomial.

$x - 9 = 0 \Rightarrow x = 9$

$\Rightarrow 4(9)^3 - 16(9)^2 - 9 + 4$

$\Rightarrow 4 \times 729 - 16 \times 81 - 9 + 4$

$= 2916 - 1296 - 5 = 1615 \neq 0$

So (x – 9) is not a factor of given polynomial because remainder is not zero.

25. Polynomial: $6x^3 + 11x^2 - x - 6$ ...(1)

Put x = – 1 in eqn (1), we get

$\Rightarrow 6x^3 + 11x^2 - x - 6$

$\Rightarrow 6(-1)^3 + 11(-1)^2 - (-1) - 6$

$\Rightarrow -6 + 11 + 1 - 6$

$\Rightarrow 12 - 12 = 0$

x = – 1 is a zero of given polynomial.

$x = \frac{-3}{2}$

$$\Rightarrow 6 \; \frac{-3}{2}^{3} + 11 \; \frac{-3}{2}^{2} - \; \frac{-3}{2} - 6$$

$$\Rightarrow 6 \times \frac{-27}{8} + 11 \times \frac{9}{4} + \frac{3}{2} - 6$$

$$= -\frac{81}{4} + \frac{99}{4} + \frac{3}{2} - 6$$

$$= -\frac{81}{4} - \frac{6}{1} + \frac{99}{4} + \frac{3}{2}$$

$$= \frac{-81 - 24 + 99 + 6}{4}$$

$$= \frac{-105 + 105}{4} = 0$$

$x = \frac{-3}{2}$ is a zero of given polynomial.

$$(x + 1) \quad x + \frac{3}{2} = (x + 1)\,\frac{1}{2}\,(2x + 3)$$

$$= \frac{1}{2}(x + 1)(2x + 3)$$

$$= \frac{1}{2}(2x^2 + 3x + 2x + 3)$$

$$= \frac{1}{2}(2x^2 + 5x + 3)$$

$$\begin{array}{r|l}
 & 3x - 2 \\ \hline
2x^2 + 5x^2 + 3 & 6x^3 + 11x^2 - x - 6 \\
 & 6x^3 + 15x^2 + 9x \\
 & - \quad - \quad\quad - \\ \cline{2-2}
 & -4x^2 - 10x - 6 \\
 & -4x^3 - 10x - 6 \\
 & + \quad\quad + \quad + \\ \cline{2-2}
 & 0 \\ \cline{2-2}
\end{array}$$

Put 3x – 2 = 0

$x = \frac{2}{3}$ is a zero of polynomial.

26.

$$\begin{array}{r|l}
 & 8x^2 + 7x + 9 \\ \hline
x - 1 & 8x^3 - x^2 + 2x - 3 \\
 & 8x^3 - 8x^2 \\
 & - \quad\; + \\ \cline{2-2}
 & + 7x^2 + 2x - 3 \\
 & 7x^2 - 7x \\
 & - \quad\; + \\ \cline{2-2}
 & 9x - 3 \\
 & 9x - 9 \\
 & - \quad + \\ \cline{2-2}
 & 6 \\ \cline{2-2}
\end{array}$$

Quotient = $8x^2 + 7x + 9$

Remainder = 6

27. $V = 12k(y)^2 + 8ky - 20k$

$= 4k\ (3y^2 + 2y - 5)$

$= 4k\ (3y^2 + 5y - 3y - 5)$

$= 4k\ [y(3y + 5) - 1\ (3y + 5)]$

$= 4k\ (3y + 5)\ (y - 1)$

Volume of cuboid = length × breadth × height

⇒ Possible expressions for dimensions of cuboid are

length = 4k

breadth = 3y + 5

height = y – 1

28. $p^3 - q^3 = (p - q)\ (p^2 + q^2 + pq)$

$= (p - q)\ [(p - q)^2 + 2pq + pq]$

$= (p - q)\ ((p - q)^2 + 3pq)$

$$= \frac{10}{9}\ \ \frac{10}{9}^{2} + 3\ \frac{5}{3}$$

$$= \frac{10}{9}\left(\frac{100}{81} + 5\right) = \frac{10}{9} \times \frac{505}{81} = \frac{5050}{729}$$

29. $p(x) = x^4 - 2x^3 + 3x^2 - ax + 8$

$(2)^4 - 2(2)^3 + 3(2)^2 - 2a + 8 = 10$

$16 - 16 + 12 - 2a + 8 = 10$

$20 - 2a = 10$

$2a = 20 - 10$

$a = \frac{10}{2} = 5$

SECTION-D

30. $p(x) = x^3 + 3x^2 - 3px + q$

when $x = -1$,

$(-1)^3 + 3\ (-1)^2 - 3p\ (-1) + q$

$-1 + 3 + 3p + q = 0$

$2 + 3p + q = 0$

$3p + q = -2$ ...(1)

when $x = -2$,

$(-2)^3 + 3\ (-2)^2 - 3p\ (-2) + q$

$-8 + 12 + 6p + q = 0$

$4 + 6p + q = 0$

$6p + q = -4$ ...(2)

From (1) and (2), we get,

$p = \frac{-2}{3}$

$q = 0$

31.

$$\begin{array}{r|l}
 & x^2 - \sqrt{2}x - 12 \\
\hline
x - \sqrt{2} & x^3 - 2\sqrt{2}x^2 - 10x + 12\sqrt{2} \\
 & x^3 - \sqrt{2}x^2 \\
 & - \quad + \\
\hline
 & \quad -\sqrt{2}x^2 - 10x + 12\sqrt{2} \\
 & \quad -\sqrt{2}x^2 + 2x \\
 & \quad + \qquad - \\
\hline
 & \qquad\quad -12x + 12\sqrt{2} \\
 & \qquad\quad -12x + 12\sqrt{2} \\
 & \qquad\quad + \qquad - \\
\hline
 & \qquad\qquad\quad 0 \\
\hline
\end{array}$$

Since remainder is 0, $x - \sqrt{2}$ is a factor of the given polynomial.

Therefore, $x^3 - 2\sqrt{2}x^2 - 10x + 12\sqrt{2}$

$= (x - \sqrt{2})\ (x^2 - \sqrt{2}x - 12)$

$= (x - \sqrt{2})\ (x^2 - 3\sqrt{2}x + 2\sqrt{2}x - 12)$

$= (x - \sqrt{2})\ [\ x(x - 3\sqrt{2}) + 2\sqrt{2}\ (x - 3\sqrt{2})]$

$= (x - \sqrt{2})\ (x + 2\sqrt{2})(x - 3\sqrt{2})$

32. If $p(x) = x^3 - 4x^2 + x + 6$.

$p(3) = (3)^3 - 4(3)^2 + 3 + 6$

$= 27 - 36 + 9$

$= 36 - 36 = 0$

∴ $(x - 3)$ is a factor of $p(x)$.

$$\begin{array}{r|l}
 & x^2 - x - 2 \\
\hline
x - 3 & x^3 - 4x^2 - x + 6 \\
 & +\ x^3 - 3x^2 \\
 & - \quad + \\
\hline
 & \quad -x^2 - x \\
 & \quad -x^2 + 3x \\
 & \quad + \quad - \\
\hline
 & \qquad -2x + 6 \\
 & \qquad -2x + 6 \\
 & \qquad + \quad - \\
\hline
 & \qquad\quad 0 \\
\hline
\end{array}$$

$(x - 3)(x^2 - x - 2)$

$= (x - 3)(x^2 - 2x + x - 2)$

$= (x - 3)(x + 1)(x - 2)$

33. $x + \frac{1}{x} = 7$

$\left(x+\frac{1}{x}\right)^3 = 7^3$

$x^3 + \frac{1}{x^3} + 3x\frac{1}{x}\left(x+\frac{1}{x}\right) = 343$

$x^3 + \frac{1}{x^3} = 322$

34. $125a^3 - 27b^3 + 75a^2b - 45ab^2$

$= (5a)^3 - (3b)^3 + 15ab(5a - 3b)$

Using $a^3 - b^3 = (a-b)(a^2+b^2+ab)$

$= (5a-3b)(25a^2 + 9b^2 + 15ab) + 15ab(5a-3b)$

$= (5a - 3b)(25a^2 + 9b^2 + 15ab + 15ab)$

$= (5a-3b)(25a^2 + 9b^2 + 30ab)$

35. $\frac{(b + c)^2}{3bc} + \frac{(c + a)^2}{3ca} + \frac{(a + b)^2}{3ab}$

$= \frac{(-a)^2}{3bc} + \frac{(-b)^2}{3ca} + \frac{(-c)^2}{3ab}$

$= \frac{a^2}{3bc} + \frac{b^2}{3ca} + \frac{c^2}{3ab}$

(Since $a + b + c = 0$)

$= \frac{a^2}{3bc} + \frac{b^2}{3ca} + \frac{c^2}{3ab}$

$= \frac{a^3 + b^3 + c^3}{3abc}$

$= \frac{3abc}{3abc} = 1$

$\begin{bmatrix} \text{As } a + b + c = 0 \\ a^3 + b^3 + c^3 = 3abc \end{bmatrix}$

36. $(a + b - c)^3 + (a - b + c)^3 - 8a^3$

$= (a + b - c)^3 + (a - b + c)^3 + (-2a)^3$

Hence, $x = a + b - c$

$y = a - b + c$

$z = -2a$

$x + y + z = a + b - c + a - b + c - 2a = 0$

$\therefore$ $x^3 + y^3 + z^3 = 3xyz$

i.e. $(a + b - c)^3 + (a - b + c)^3 - 8a^3$

$= 3 \times (a + b - c) \times (a - b + c) \times (-2a)$

$= -6a(a + b - c)(a - b + c)$

37. We have:

$p(x) = 3x^4 + 5x^3 - 7x^2 + 2x + 3$

$g(x) = x^2 + 3x + 1$

By Division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$p(x) - r(x) = g(x) \times q(x)$

So, if we subtract r(x) with p(x), the resulting polynomial is divisible by g(x), on dividing p(x) by g(x), we get

$$\begin{array}{r|l}
 & 3x^2 - 4x + 2 \\ \hline
x^2 + 3x + 1 & 3x^4 + 5x^3 - 7x^2 + 2x + 3 \\
 & \underset{-}{}3x^4 \underset{-}{+} 9x^3 \underset{-}{+} 3x^2 \\ \cline{2-2}
 & \qquad -4x^3 - 10x^2 + 2x + 3 \\
 & \qquad \underset{+}{-}4x^3 \underset{+}{-} 12x^2 \underset{+}{-} 4x \\ \cline{2-2}
 & \qquad\qquad 2x^2 + 6x + 3 \\
 & \qquad\qquad \underset{-}{}2x^2 \underset{-}{+} 6x \underset{-}{+} 2 \\ \cline{2-2}
 & \qquad\qquad\qquad\qquad 1 \\ \cline{2-2}
\end{array}$$

$\therefore$ Remainder is 1

Hence, if 1 is subtracted from p(x) the resulting polynomial is exactly divisible by g(x).

38. We have: $x^4 + x^2y^2 + y^4$

$= (x^4 + 2x^2y^2 + y^4) - x^2y^2$

$= (x^2 + y^2)^2 - (xy)^2$

$= (x^2 + y^2 - xy)(x^2 + y^2 + xy)$

$\therefore$ $x^4 + x^2y^2 + y^4$

$= (x^2 + y^2 - xy)(x^2 + y^2 + xy)$

39. (a) $a + b + c = 5$, $ab + bc + ca = 10$

We know that,

$(a^3 + b^3 + c^3 - 3abc)$

$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$= (a + b + c)\,[(a + b + c)^2 - 3(ab + bc + ca)]$

$= (5) \times [(5)^2 - 3 \times 10]$

$= 5 \times (25 - 30)$

$= 5 \times (-5) = -25$

$a^3 + b^3 + c^3 - 3abc = -25$

Hence Proved.

(b) Given: $p + q + r = 16$

Taking square on both sides we get

$(p + q + r)^2 = (16)^2 = 256$

Using identity: $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$p^2 + q^2 + r^2 + 2pq + 2qr + 2rp = 256$

$\Rightarrow \quad p^2 + q^2 + r^2 + 2(pq + qr + rp) = 256$

$\Rightarrow \quad p^2 + q^2 + r^2 + 2(pq + qr + rp) = 256$

$\Rightarrow \quad p^2 + q^2 + r^2 + 2 \times 25 = 256$

$\Rightarrow \quad p^2 + q^2 + r^2 = 256 - 50$

$\Rightarrow \quad p^2 + q^2 + r^2 = 206$

CASE STUDY-1

1. (i) (d) Degree is the maximum power of variables, the degree of $t^7 - 5$ is 7.

(ii) (a) Constant polynomial has 0 degree.

(iii) (c) Coefficient of a^3 is π.

(iv) (b) $4b^4 + 5b^3 - b^2 + 2$

(v) (d) $p(x) = 0$

$ax = 0$

$x = 0$

CASE STUDY-2

2. (i) (c) $p(t) = 5 - 4t + 2t^2$

$p(-2) = 5 - 4(-2) + 2(-2)^2$

$= 5 + 8 + 8 = 21$

(ii) (b) The zero of $3x + 2$ is $\frac{-2}{3}$

$f\left(\frac{-2}{3}\right)$

$= 12\ \frac{-2}{3}^{3} - 13\ \frac{-2}{3}^{2} - 5\left(\frac{-2}{3}\right) + 7 = 1$

Therefore the remainder would be 1 when f(x) is divided by $3x + 2$.

(iii) (a) The zero of $x + 2$ is $x = -2$

$p(-2) = 2(-2)^3 - k(-2)^2 + 3(-2) + 10$

$= -16 - 4k - 6 + 10$

$-4k - 12$

As p(x) is completely divisible by $x + 2$ therefore the remainder would be 0.

$p(-2) = 0$

$= -4k - 12 = 0$

$k = -3$

(iv) (d) $p(2) = (2)^3 - (2)^2 + 2 + 1$

$= 8 - 4 + 2 + 1$

$= 7$

$p(-2) = (-2)^3 - (-2)^2 + (-2) + 1$

$= -8 - 4 - 2 + 1$

$= -13$

$= \frac{p(2) + p(-2)}{6} = \frac{7 - 13}{6} = -1$

(v) (c) Zeroes are calculated by equating the polynomial to zero,

$p(x) = 0$

$3x^2 - 1 = 0$

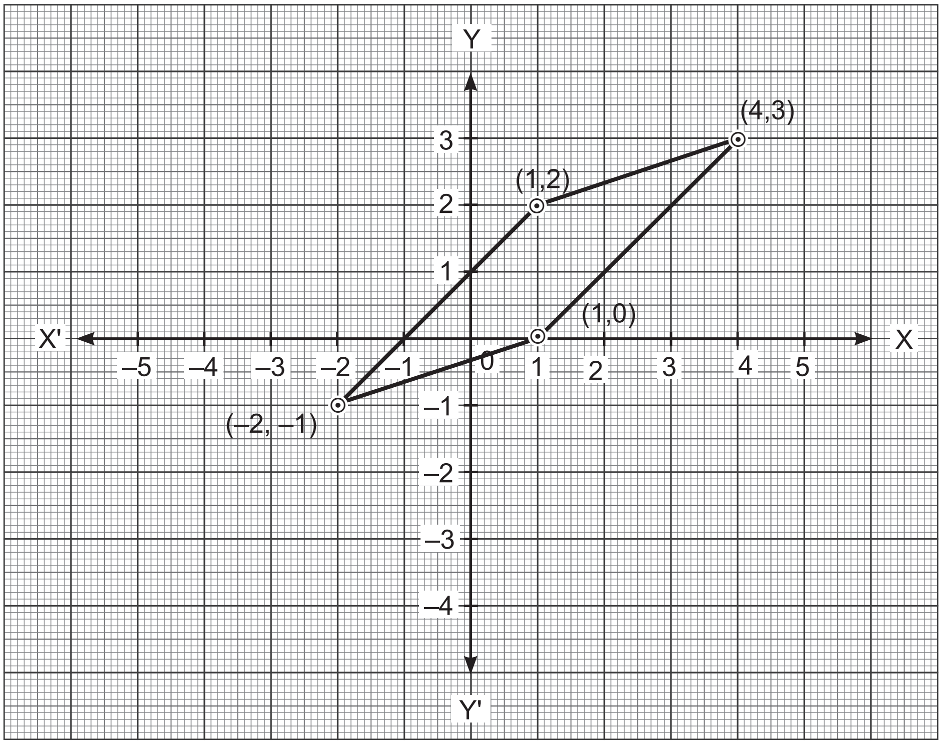
$x = \pm\frac{1}{\sqrt{3}}$

Chapter
3
Coordinate Geometry

MULTIPLE CHOICE QUESTION

1. (b) If the abscissa is zero, then the point will lie on y axis ∴ option (b) is the correct answer.

2. (b) Parallelogram



3. (b) Let the coordinate of others ands are (x, y). As center is the midpoint of Diameter.

Hence by using midpoint formula.

$$\frac{x+4}{2} = 1$$

$$x + 4 = 2$$

$$x = -2$$

$$\frac{y - 1}{2} = -3$$

$$y - 1 = -6$$

$$y = -5$$

Hence the coordinates of other end is (–2, –5)

4. (b) Absissa is the distance of a point from Y axis.

5. (c) In third quadrant, both the points are negative.

WORKSHEET 1: SECTION-A

1. The distance of the point p(2, 1) from y-axis is 2.

2. The mirror image will be coordinates (2, –5).

3. The mirror image will be (–2, 5).

4. General form of any point of y-axis is (0, y).

5. The reflection of the point (–3, –2) in y-axis will be (3, –2).

SECTION-B

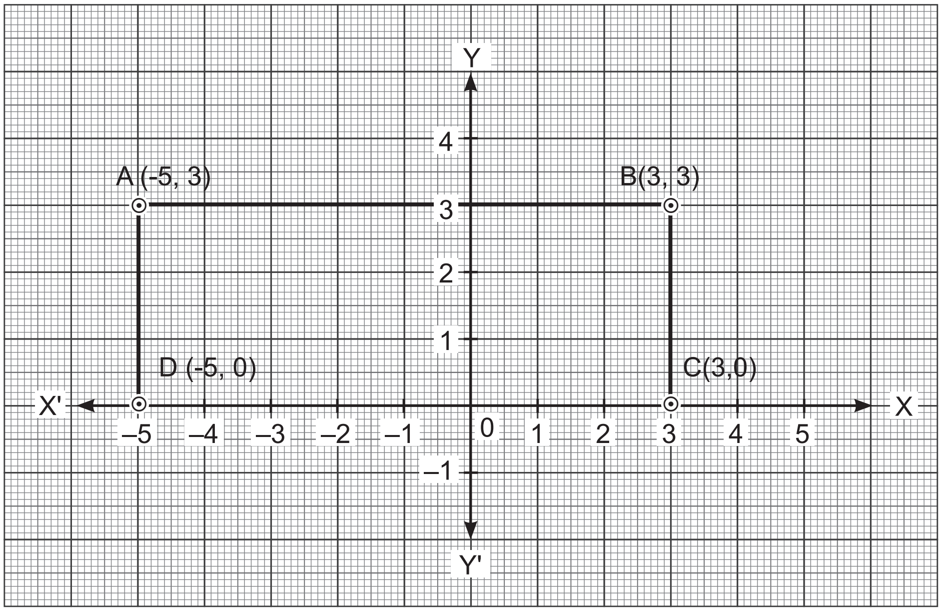
6. (a) Abscissa is – 4 and the ordinate is –2.

⇒ III Quadrant.

(b) The ordinate is 3 and abscissa is 4.

⇒ I Quadrant.

7. (a)



(b) ABCD is a rectangle.

8. Ordinates of the following are:

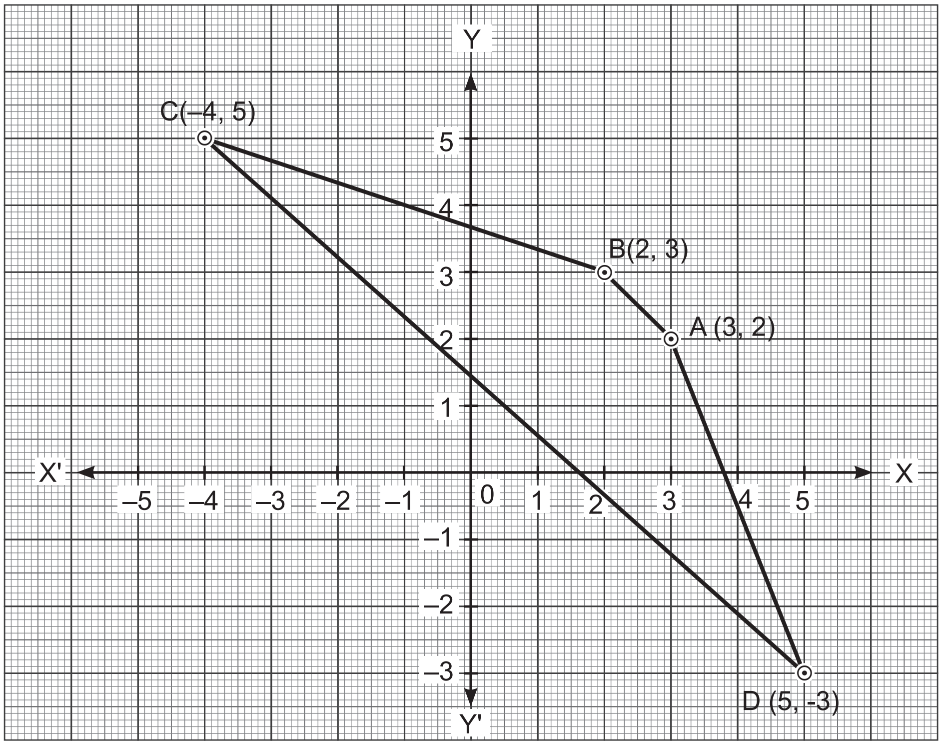
(3, 4) ordinate is 4.

(4, 0) ordinate is 0.

(0, 4) ordinate is 4.

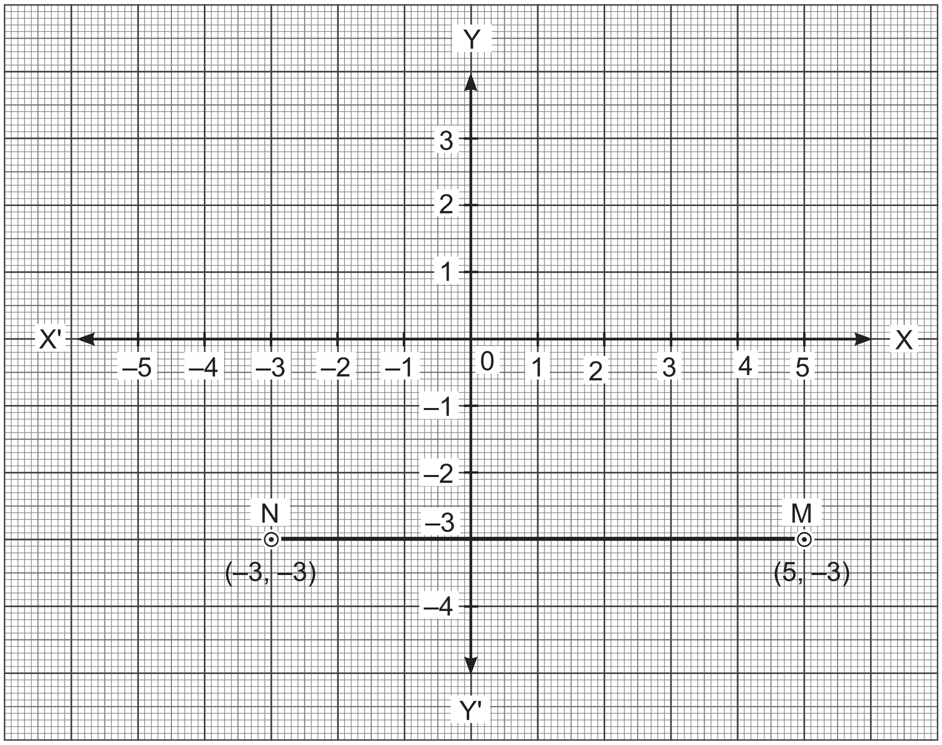
(5, –3) ordinate is –3.

9.



No, the points (3, 2) and (2, 3) are not same because they have different abscissa and ordinate lie in the same Quadrant.

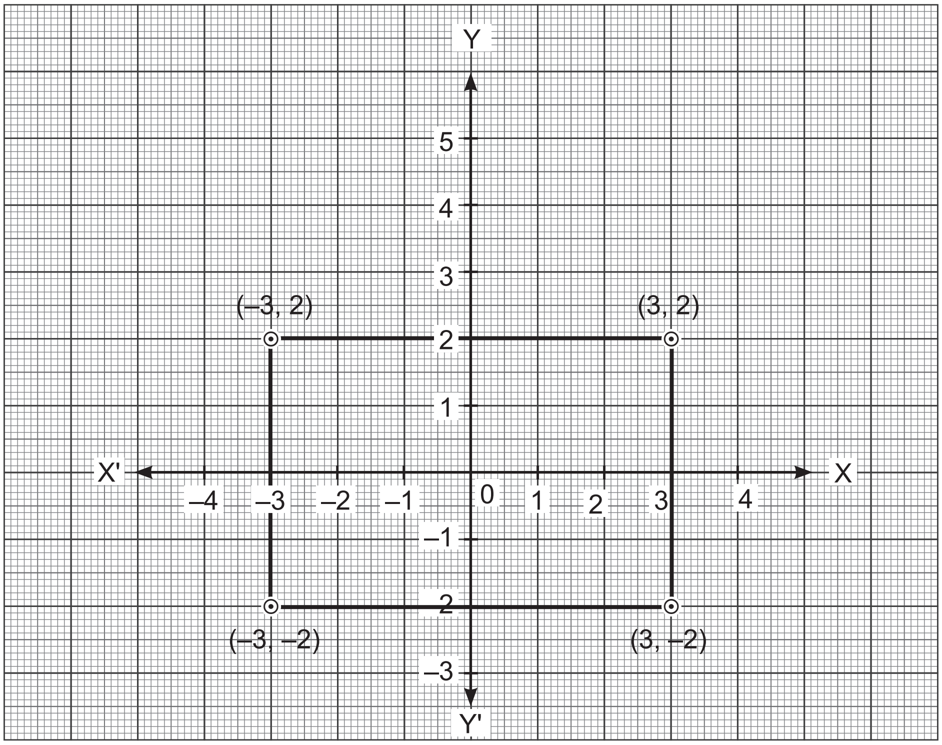
10. (a)



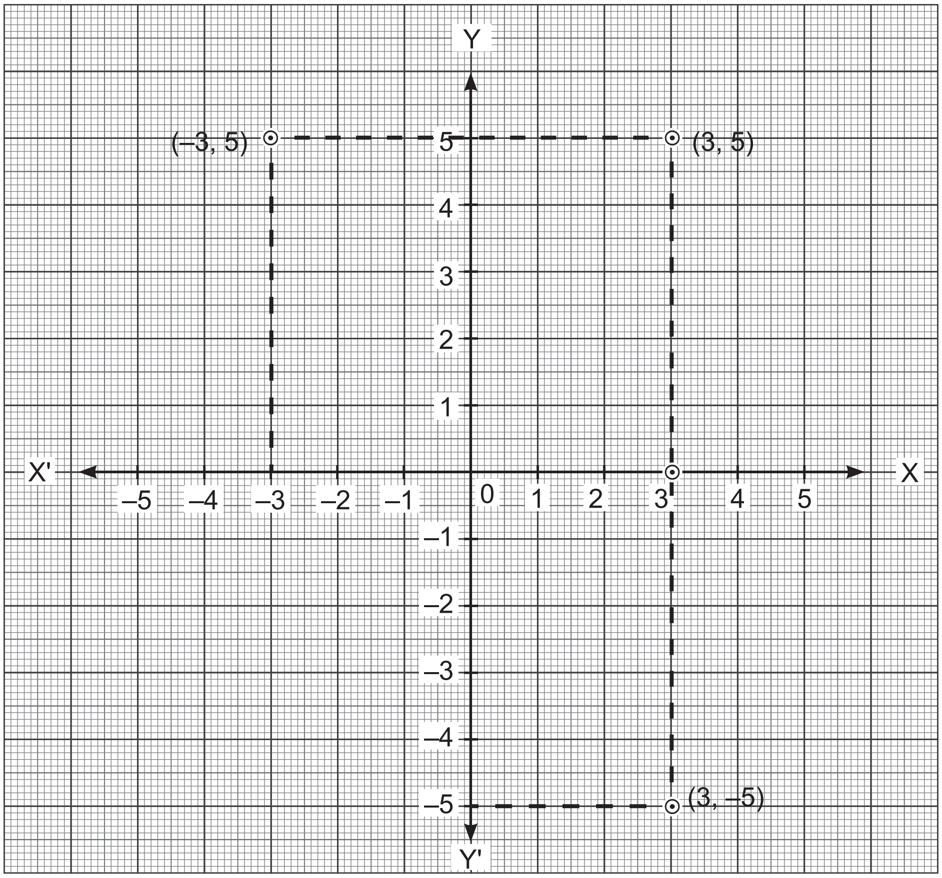
(b) MN = 8 units.

SECTION-C

11.



12. Coordinates of points which is reflection of (3, 5) in y-axis are (–3, 5).
Coordinates of points which is reflection of (3, 5) in x-axis are (3, –5).



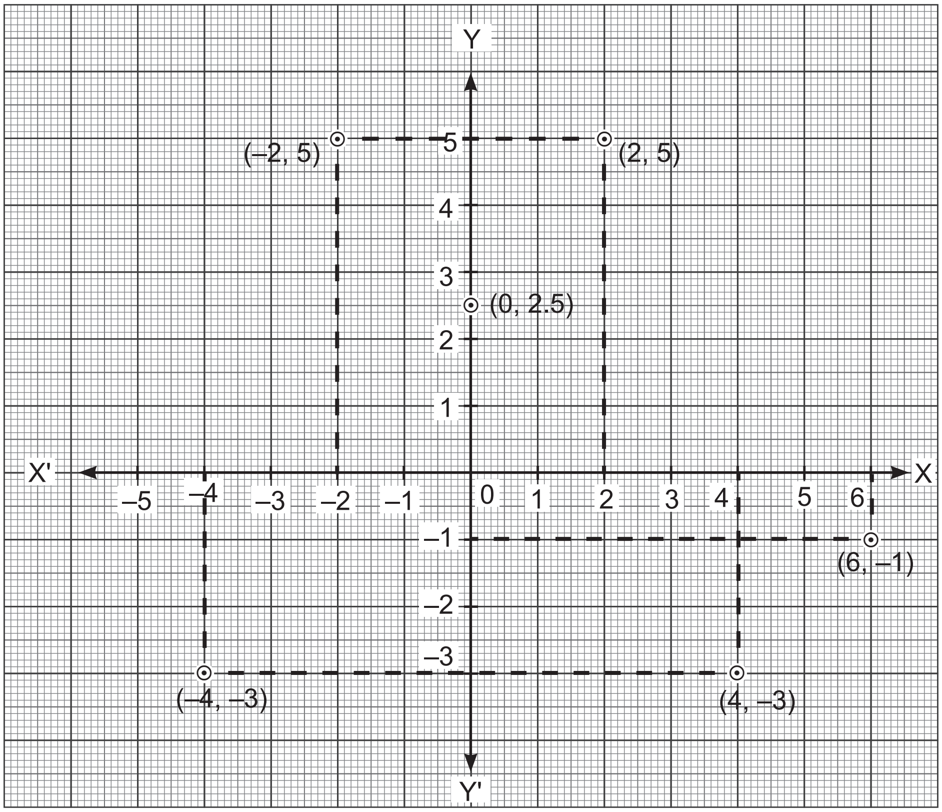
13.

Points	Quadrants
(–2, 4)	II
(3, –1)	IV
(–1, 0)	Negative x-axis
(1, 0)	Positive x-axis
(1, 2)	I
(–3, –5)	III
(0, –1)	Negative y-axis

14. If a point lie on x-axis at a distance of 9 units from y-axis then its coordinates can be (9, 0) or (–9, 0).

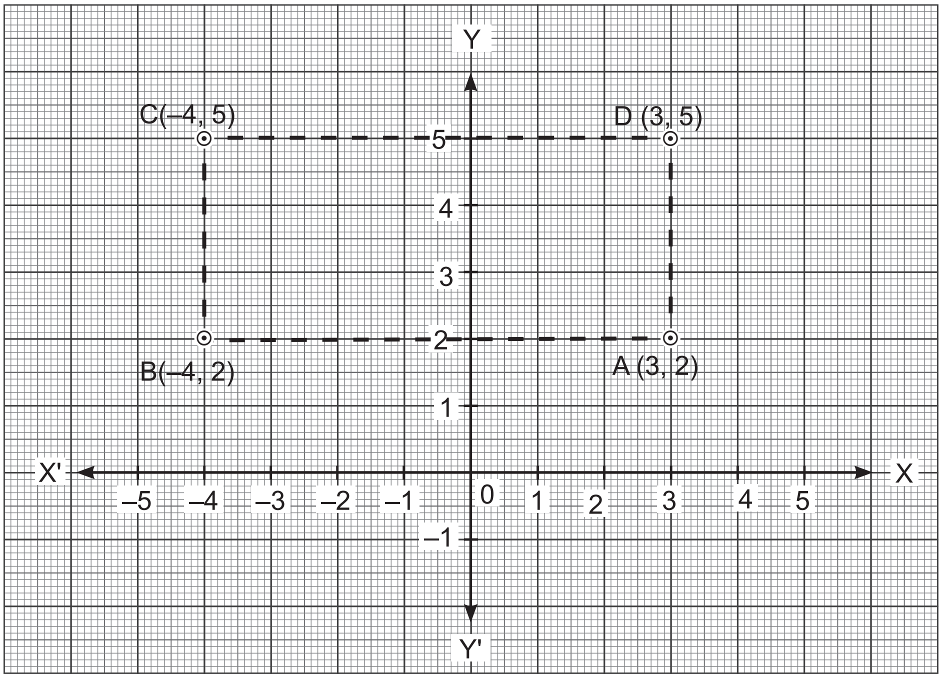
If a point lie on y-axis at a distance of 9 units from x-axis then its coordinates can be (0, 9) or (0, – 9).

15.



SECTION-D

16.

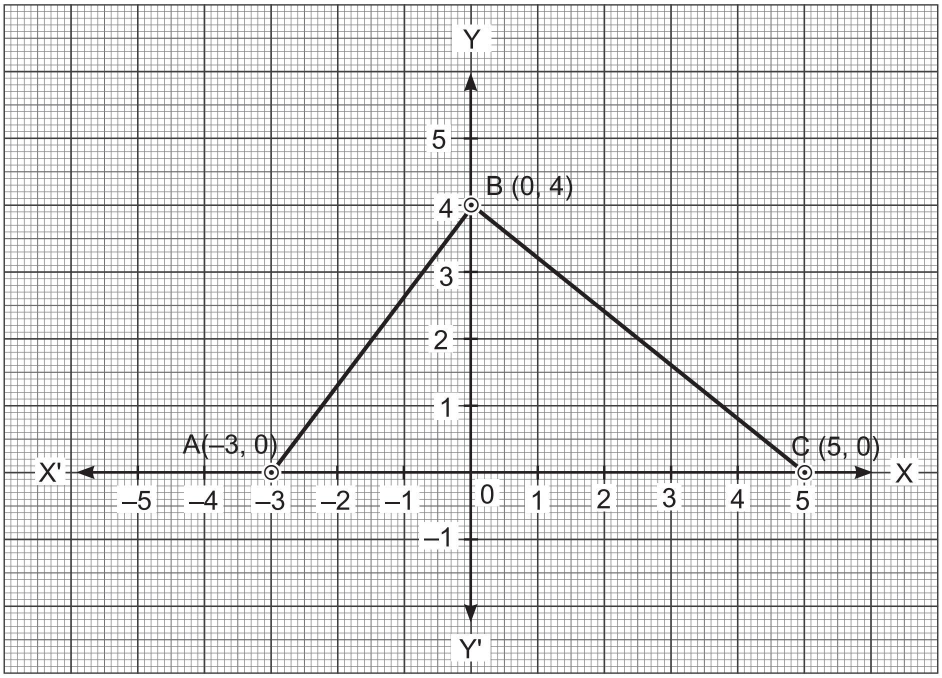


Fourth vertex i.e. D will be (3, 5) to form a rectangle.

A(3, 2) and D(3, 5) lies in I-Quadrant.

B(–4, 2) and C(–4, 5) lies in II-Quadrant.

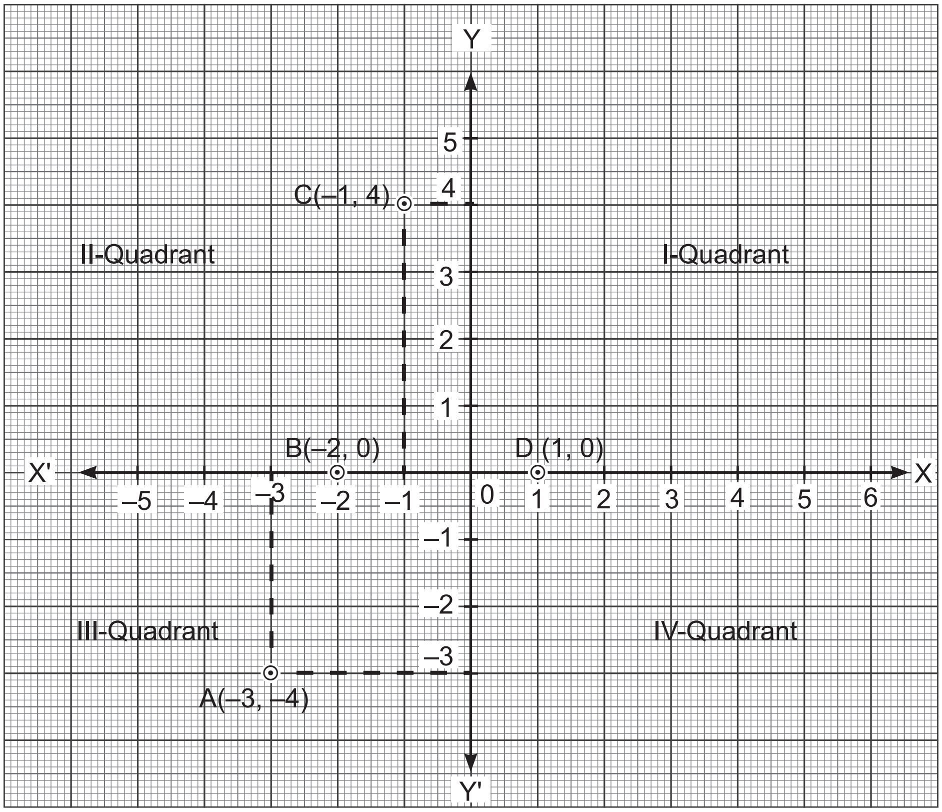
17.



Triangle is formed by joining the points A, B and C.

Area of ABC = $\frac{1}{2} \times b \times h = \frac{1}{2} \times 8 \times 4 = 16$ sq. unit

18.



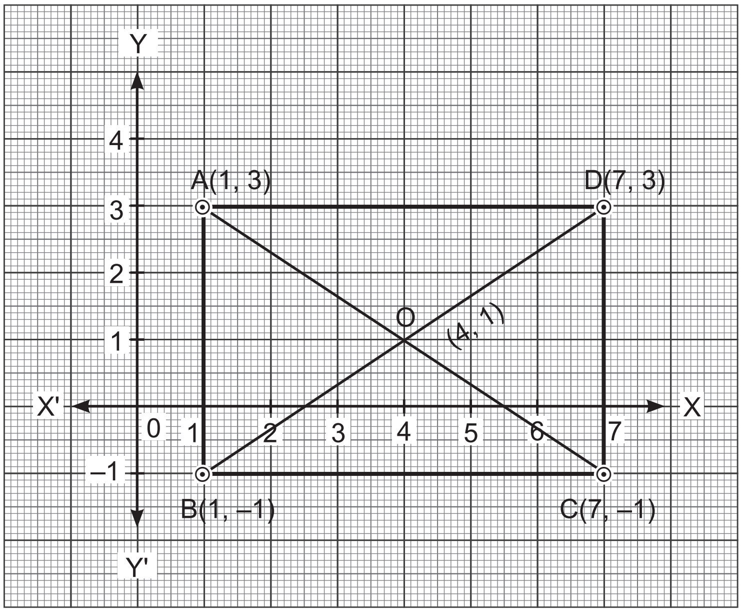
A(–3, –4) ⇒ III

C(–1, 4) ⇒ II

B(–2, 0) on negative x-axis.

D(1, 0) on positive x-axis.

19.



Diagonals intersect at point O. Coordinates of point O are (4, 1).

The obtained figure is a parallelogram.

20. Coordinates of the given points are:

P(1.5, 0.5)
Q(–3, 0.5)
R(–2, –2.5)
S(3, 0.5)
O(0, 0).
T(4, –1.5)

WORKSHEET 2: SECTION-A

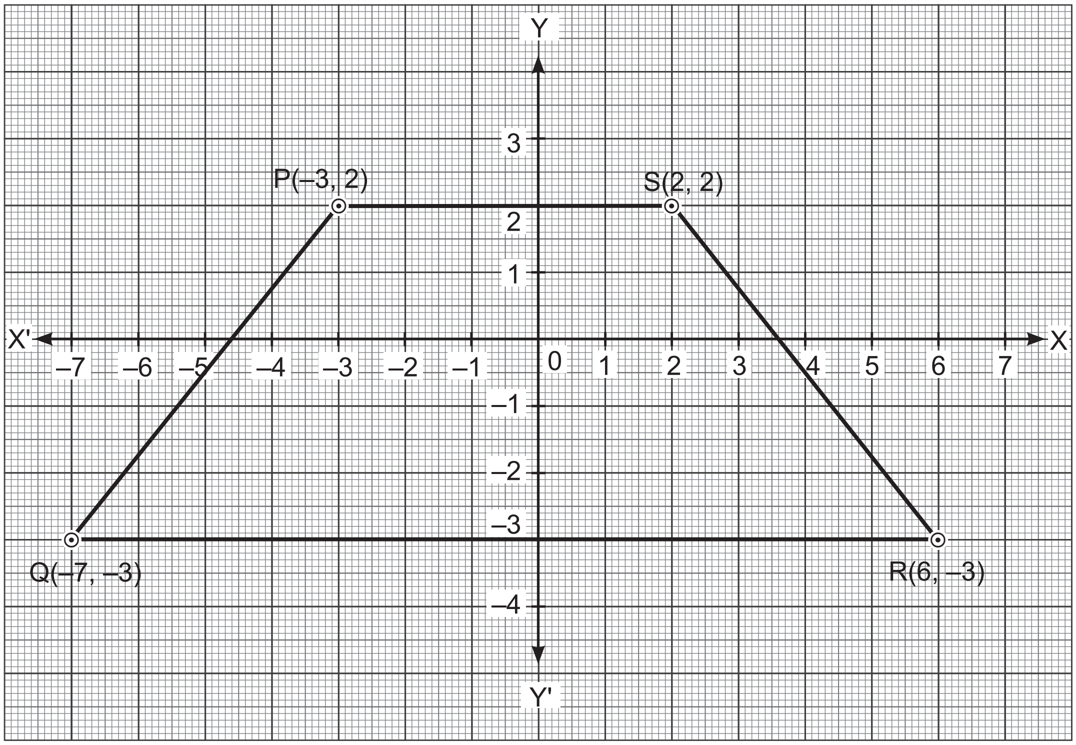
1. (–, +).
2. III Quadrant.
3. Abscissa of P (–2, 3) is –2
Abscissa of Q (–3, 5) is –3.
4. Here y is 4, distance from y-axis is 4 unit.
5. (a) y-axis
(b) x-axis.

SECTION-B

6. (a) Yes, True.

(b) No, False, it will be $1, \frac{-1}{2}$.

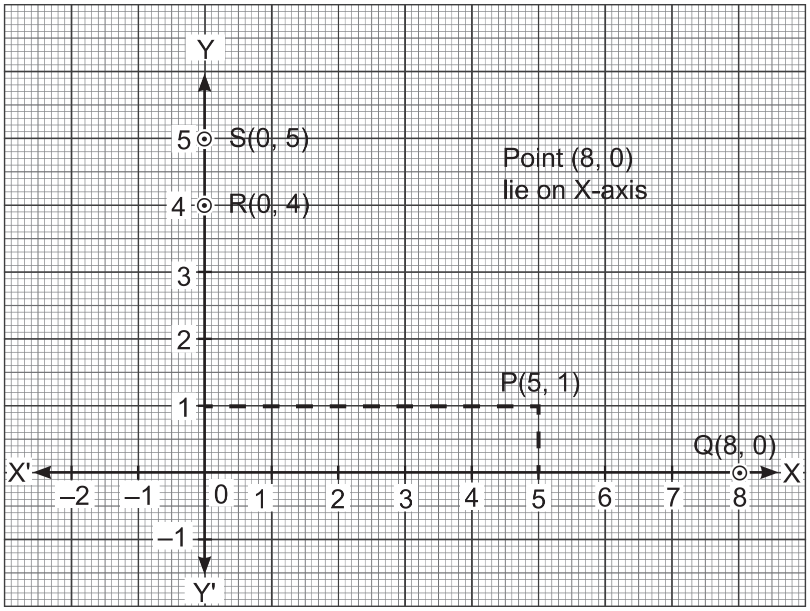
7.



Trapezium is formed by joining the points P, Q, R and S.

8. Coordinates are (7, 0) or (–7, 0). Coordinates will be (0, –7) or (0, 7).

9.



Q and O lies on x-axis.

10. Abscissa of P (–5, 3) is –5

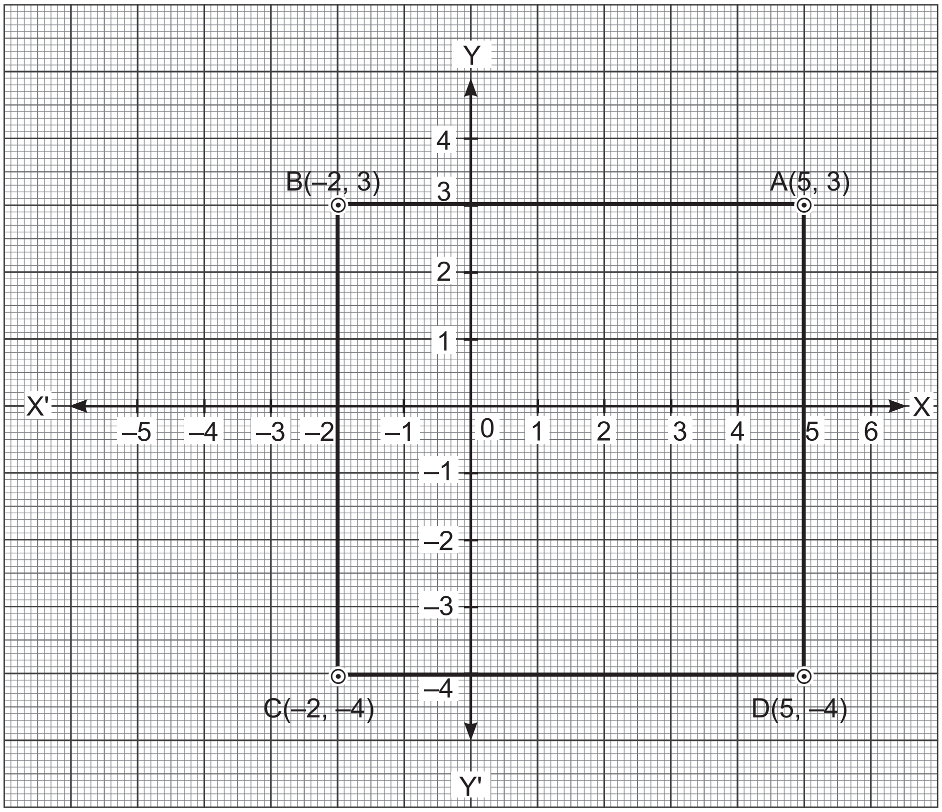
Abscissa of Q (7, 3) is 7

Ordinate of P (–5, 3) is 3

Ordinate of Q (7, 3) is 3.

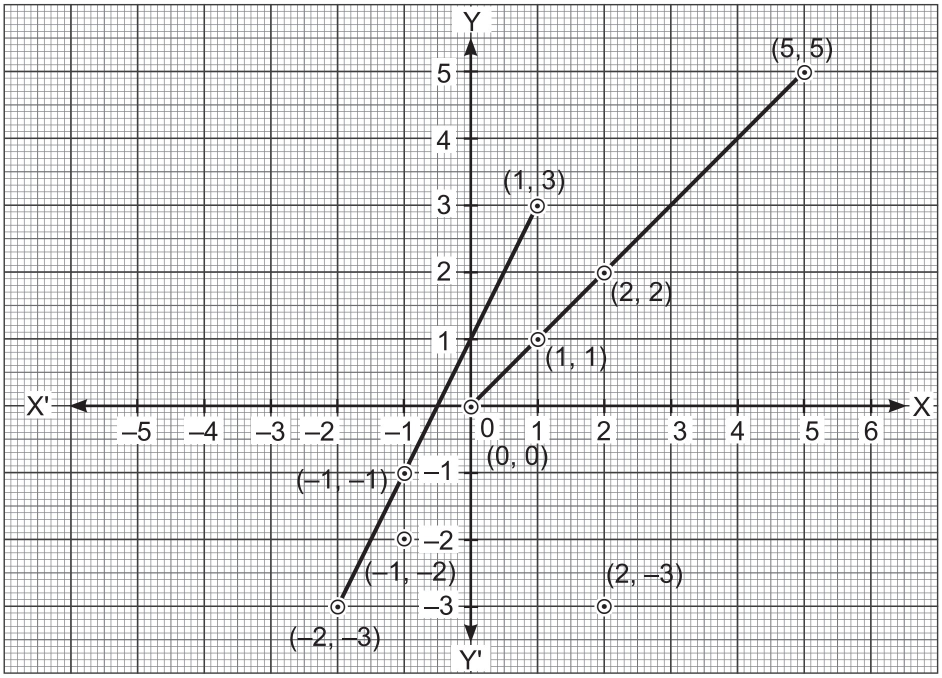
SECTION-C

11.



Coordinates of the vertex C(–2, –4).

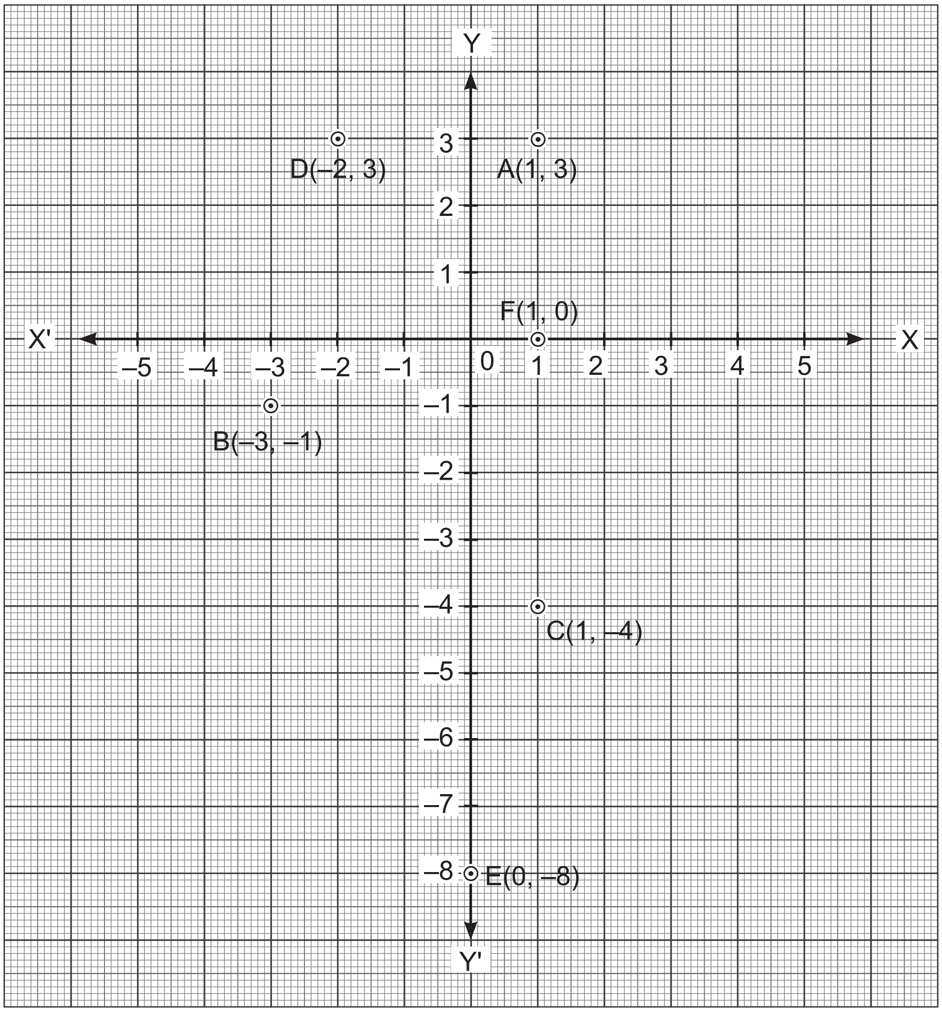
12.



(a) Yes (b) No (c) Yes.

13. (a) The origin (0, 0). (b) (0, –4). (c) (5, 0).

14.



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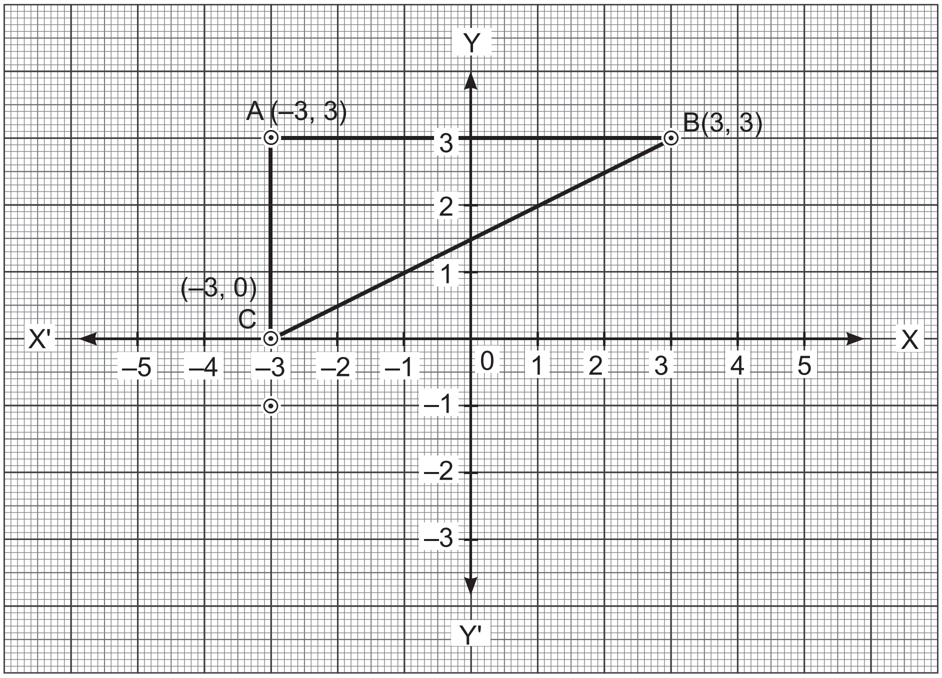
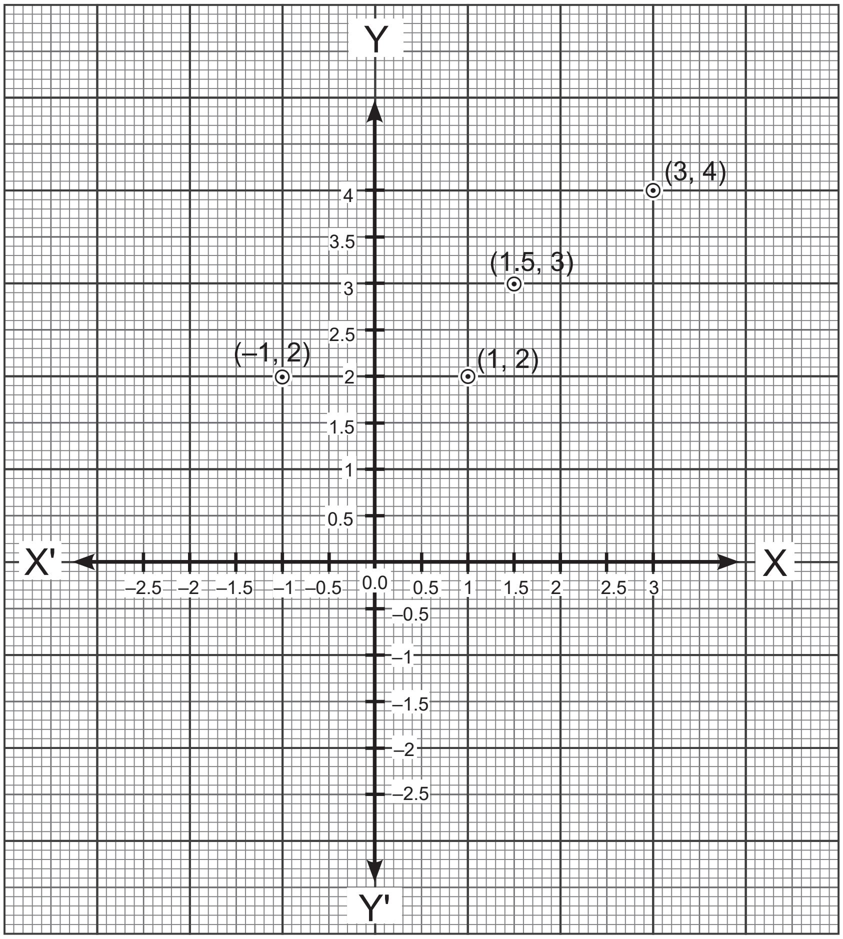


Area of ΔABC $= \frac{1}{2}$ (Base) (Height)

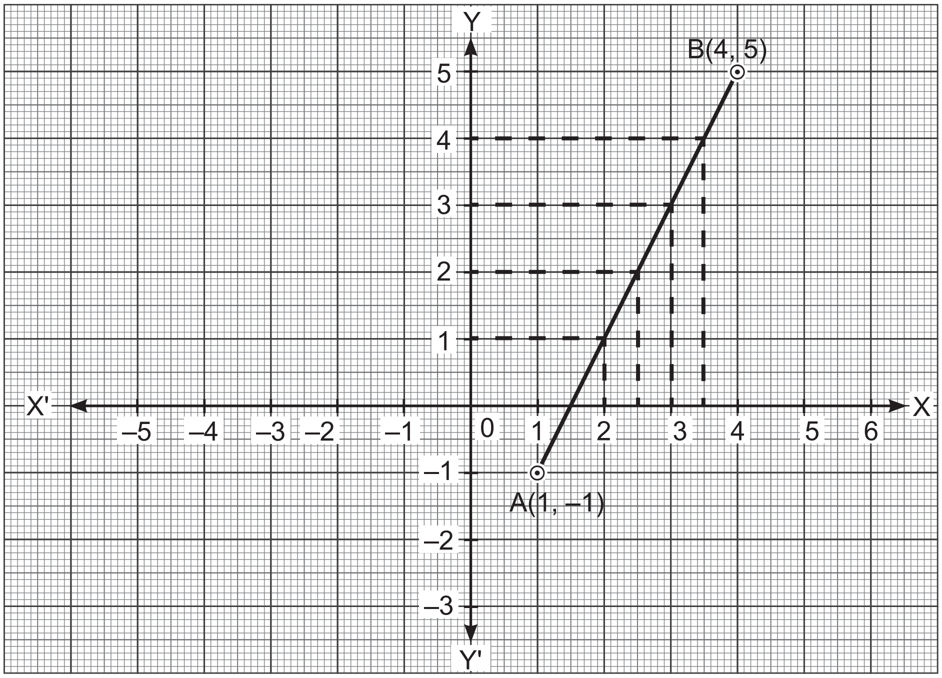
$= \frac{1}{2}$ (AC) (AB) $= \frac{1}{2}$ (3) (6) = 9 unit.

SECTION-D

16.



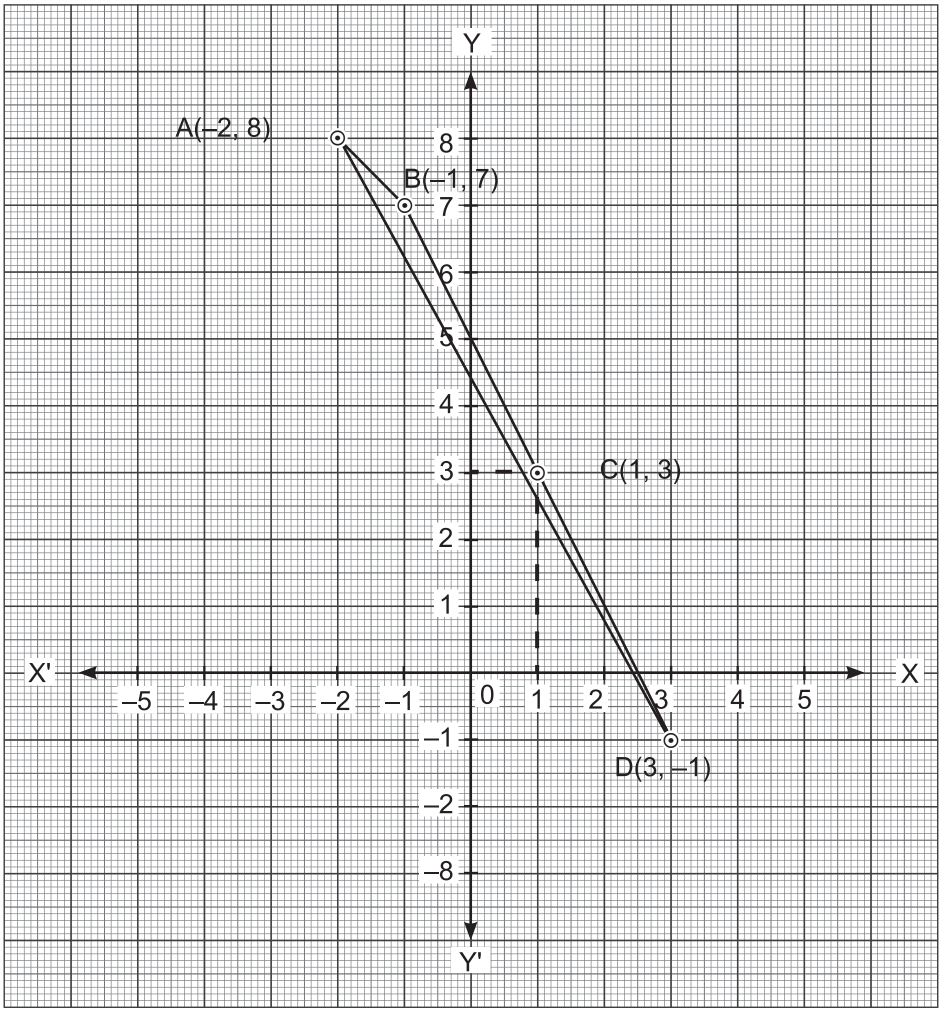
17.



(a) Coordinates of the points between the points A and B are (2, 1), (2.5, 2), (3, 3), (3.5, 4) etc. The midpoint of the segment is (2.5, 2).

(b) Ordinate of A (1, –1) is –1. Ordinate of B (4, 5) is 5.

18.



Therefore parallelogram ABCD is formed.

CASE STUDY-1

1. (i) (d) The coordinates of point A is (2, 4) and the coordinates of point D is (2, – 4).

The distance AD is calculated by using distance formula.

$$AD = \sqrt{(2-2)+\left[4-(-4)\right]^2}$$

$$= \sqrt{64} = 8 \text{ units}$$

(ii) (a) Distance AB is calculated by distance formula.

$$AB = \sqrt{\left[2-(-2)\right]^2+(4-4)^2}$$

$$= \sqrt{16}$$

AB = 4 units

(iii) (b) The figure ABCD is a rectangle.

(iv) (c) The perimeter of rectangle is 2(l + b), where "l" is the length and “b” is the breadth of rectangle.

In rectangle ABCD, length is AD and breadth is AB.

AB = 4 units

AD = 8 units

Perimeter of ABCD = 2 (4 + 8) = 24 units

(v) (c) The coordinates representing the position of Rohan lies in the third quadrant.

CASE STUDY-2

2. (i) (c) The distance of point C from X axis is 8 units from Y axis it is 4 units. Therefore the coordinates of point C are (4, 8).

(ii) (a) The ordinate of point D is 8.

(iii) (c) The abscissa of point Q is 9.

(iv) (a) The ordinates is the value of Y – coordinate, which is positive.

(v) (b) The abscissa of point B is 4.

Chapter
4
Linear Equations in Two Variables

MULTIPLE CHOICE QUESTION

1. (c) Let the number is x.

$$\frac{1}{4}\quad\frac{1}{3}\quad\frac{x}{2} = 12$$

x = 288

2. (d) x – 2y = 2

At x = 4, y = 1

4 – 2 = 2

At x = 2, y = 0

2 – 0 = 2

At x = 6, y = 2

6 – 2(2) = 2

2 = 2

3. (b) $\frac{6x + 5}{4x + 7} = \frac{3x + 5}{2x + 6}$

$(6x + 5) \quad (2x + 6) = (3x + 5)\ (4x + 7)$

$12x^2 + 36x + 10x + 30$

$= 12x^2 + 21x + 20x + 35$

$46x + 30 = 41x + 35$

$5x = 5$

$x = 1$

4. (b) As x = 1, y = 1 is a solution of 9ax + 12ay = 63

Therefore x = 1, y = 1 will satisfy the equation

9a (1) + 12a (1) = 63

21a = 63

a = 3

5. (d) $\sqrt{3x} - 2 = 2\sqrt{3} + 4$

$\sqrt{3x} = 2\sqrt{3} + 4 + 2$

$\sqrt{3x} = 2\sqrt{3} + 6$

$$x = \frac{2(3) + 6\sqrt{3}}{3}$$

$$x = \frac{2(3) + 6\sqrt{3}}{3} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{2\sqrt{3} + 6\sqrt{3}}{3}$$

$$= \frac{6 + 6\sqrt{3}}{3}$$

$$= 2 + 2\sqrt{3}$$

$$= 2(1+\sqrt{3})$$

WORKSHEET 1: SECTION-A

1. x = 2, y = –1 is a solution of:

px + 3y = 15

p × 2 + 3 × –1 = 15

2p – 3 = 15

2p = 15 + 3 = 18

$p = \frac{18}{2}$

p = 9.

2. $\sqrt{2}x + 5 = \sqrt{5}y$

$\sqrt{2}x - \sqrt{5}y + 5 = 0$ ...(i)

ax + by + c = 0 ...(ii)

Comparing eqn. (i), (ii), we get:

$a = \sqrt{2}$, $b = -\sqrt{5}$, c = 5.

3. x – 3y = 4

x = 4 + 3y

x = 4 + 3 × 0 = 4

x = 4 + 3 × 1 = 4 + 3 = 7

x	4	7
y	0	1

∴ Two solutions are (4, 0), (7, 1).

4. $x = -1, y = -1$

Linear equation: $9kx + 12ky = 63$

$$9k(-1) + 12k(-1) = 63$$
$$-9k - 12k = 63$$
$$-21k = 63$$
$$k = \frac{63}{-21}$$
$$k = -3.$$

5. Point (–1, –5) lies on the graph:

$$3x = ay + 7$$
$$3(-1) = a(-5) + 7$$
$$-3 = -5a + 7$$
$$5a = 7 + 3$$
$$5a = 10$$
$$a = 2$$

6. LHS $= x - 2y$

$$= \sqrt{2} - 2(4\sqrt{2})$$
$$= \sqrt{2} - 8\sqrt{2}$$
$$= -7\sqrt{2}$$
$$\neq 4 = \text{RHS}$$

So, $(\sqrt{2}, 4\sqrt{2})$ is not a solution of the equation.

Then, $(\sqrt{2}, 4\sqrt{2})$ is not a solution of $x - 2y = 4$.

7. A linear equation has infinitely many solution.

8. The equation representing y-axis is $x = 0$.

9. $2x + 3y = 9.\overline{35}$

$2x + 3y - 9.\overline{35} = 0$ ...(i)

$ax + by + c = 0$ ...(ii)

Comparing eqns. (i), (ii), we get:

$c = -9.\overline{35}$.

10. LHS $= x - 3y$

$$= 0 - 3(2)$$
$$= -6 \neq 4 = \text{RHS}$$

So (0, 2) is not a solution of the equation.

SECTION-B

11. (a) $4x + 3y = 12$

x	0	1
y	4	8/3

$3y = 12 - 4x$ ...(i)

$$3y = 12 - 4 \times 0 = 12$$
$$y = \frac{12}{3} = 4$$
$$3y = 12 - 4 \times 1 = 12 - 4 = 8$$
$$y = 8/3$$

$\therefore$ Two solutions are (0, 4), (1, 8/3).

(b) $2x + 5y = 0$

x	0	–5
y	0	2

$$2x = -5y$$
$$2x = -5 \times 0$$
$$x = 0$$
$$2x = -5y, \quad y = 2$$
$$2x = -5 \times 2 = -10$$
$$x = \frac{10}{2} = -5$$

Therefore two solutions are (0, 0) and (–5, 2).

12. 4 solutions of $5x = -y$,

Put $y = 0$,

$5x = -0, \quad x = 0$

x	0	1/5	–2/5	–1
y	0	–1	2	5

$y = -1, \qquad 5x = -(-1) = 1, \qquad x = 1/5$

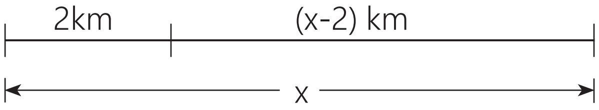
$y = 2, \qquad x = -\frac{2}{5}$

$y = 5, \qquad x = \frac{-5}{5} = -1$

Therefore 4 solutions are (0, 0), (1/5, –1) (–2/5, 2), (–1, 5).

13. For 2 kms, the fare is ₹ 19 and for subsequent distance it is ₹ 6.50 km.

Total distance covered is = x



Total fare as = ₹ y

for first 2 kms fare = ₹ 19

for next remaining (x – 2) km = 6.50

$$y = 19 + (x - 2)6.50$$
$$y = 19 + 6.50x - 13$$
$$\boxed{y = 6 + 6.50x}$$

14. x + 2y = 6 ...(i)

x = 6 – 2y

Put y = 0, x = 6 – 2 × 0 = 6

y = 1, x = 6 – 2 × 1 = 4

y = 2, x = 6 – 2 × 2 = 6 – 4 = 2

y = 6, x = 6 – 2 × 6 = 6 – 12 = – 6

x	6	4	2	–6
y	0	1	2	6

Therefore four solutions are (6, 0), (4, 1), (2, 2), (–6, 6).

15. (a) Graphical representation in one variable:

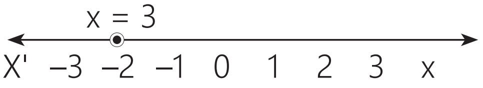
5x + 15 = 0

5x = –15

$x = \frac{-15}{5} = -3$

x = –3.

x = 3

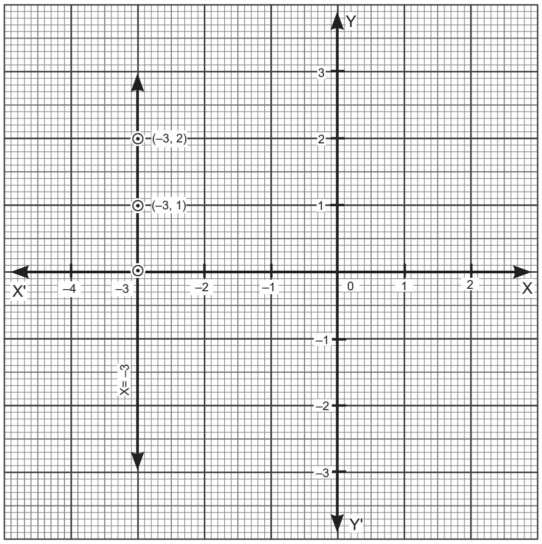


(b) In two variables: x = –3

⇒ x + 0.y + 3 = 0

Solutions are: (–3, 0), (–3, 1), (–3, 2).

x	–3	–3	–3
y	0	1	2



SECTION-C

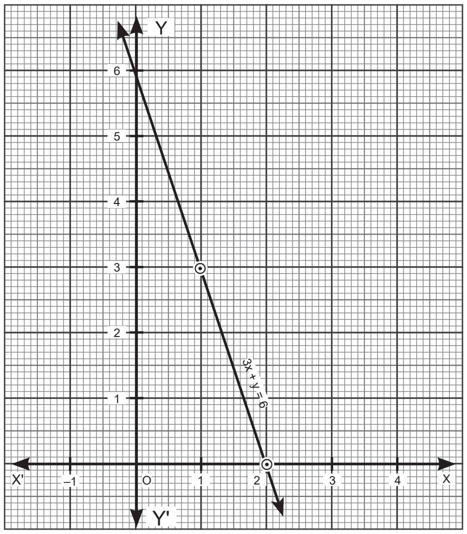
16. 3x + y = 6

y = 6 – 3x

x	1	2
y	3	0

y = 6 – 3 × 1 = 3

y = 6 – 3 × 2 = 6 – 6 = 0.



The line meet x-axis at (2, 0) and y-axis at (0, 6).

17. Let No. of hours works = ₹ x

and No. of wages = ₹ y

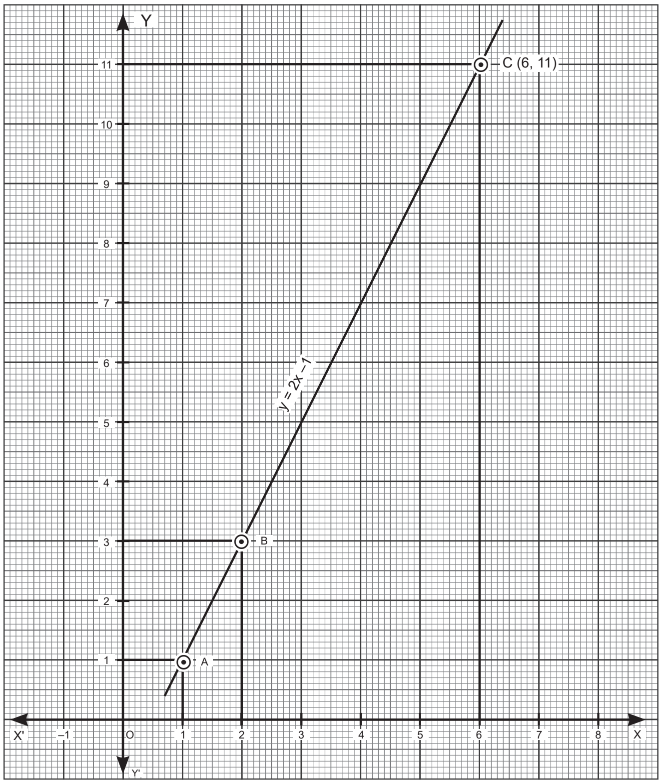
y = 2x – 1 ...(1)

x = 1, y = 2 × 1 – 1 = 1

x = 2, y = 2 × 2 – 1 = 3

x = 6, y = 2 × 6 – 1 = 12 – 1 = 11

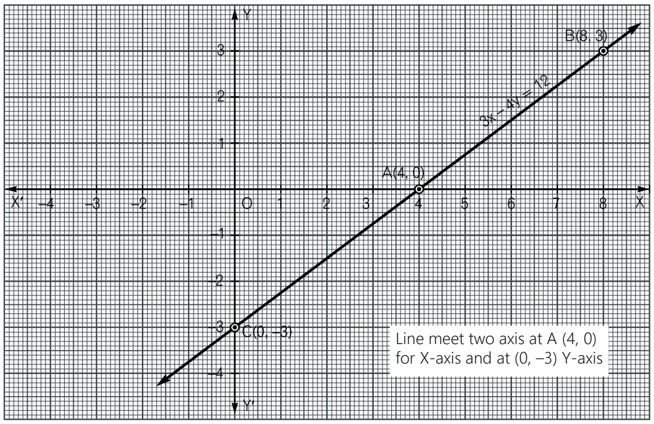
x	1	2	6
y	1	3	11



Labour works in 6 hours.

We get (6, 11), so for 6 hours he gets 11 rupees.

18.



Line meet two axis at A (4, 0) for X-axis and at (0, –3) Y-axis

Find three solutions of equation:

$3x - 4y = 12$

$3x = 12 + 4y$

Put, $y = 0$,

$3x = 12 + 4 \times 0 = 12$

$x = \frac{12}{3} = 4$

x	4	8	0
y	0	3	–3

$y = 3, \quad 3x = 12 + 4 \times 3 = 12 + 12 = 24$

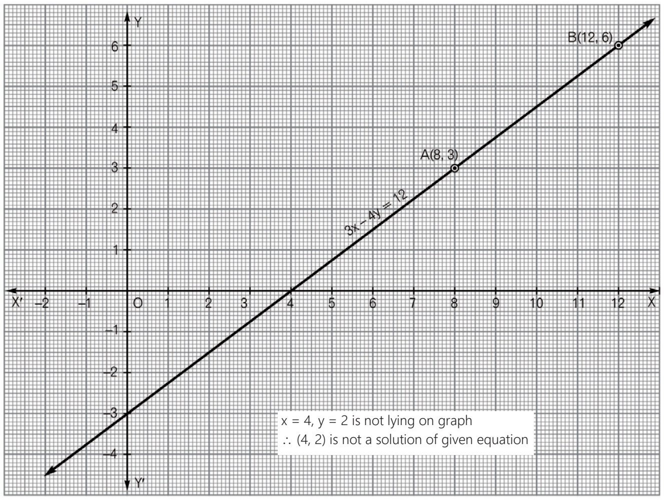
$x = \frac{24}{3} = 8$

$y = -3, \quad 3x = 12 + 4 \times -3 = 12 - 12 = 0$

$x = 0$

Line $3x - 4y = 12$ meet on x-axis at A(4, 0) and y-axis at B(0, – 3).

19. Draw the graph:



x = 4, y = 2 is not lying on graph
∴ (4, 2) is not a solution of given equation

$3x - 4y = 12$ ...(i)

$3x = 12 + 4y$

$x = \frac{12}{3} + \frac{4}{3}y$

$x = 4 + \frac{4}{3}y$

$y = 3, \quad x = 4 + \frac{4}{3} \times 3 = 8$

$y = 6, \quad x = 4 + \frac{4}{3} \times 6$

$= 4 + 8 = 12$

Since point (4, 2) does not lie on graph. So, x = 4, y = 2 is not a solution of equation $3x - 4y = 12$.

20. $3x + 7y = 20$ ...(i)

$3x = 20 - 7y$

Put $y = -1, \quad 3x = 20 - 7 \times -1$

$3x = 20 + 7$

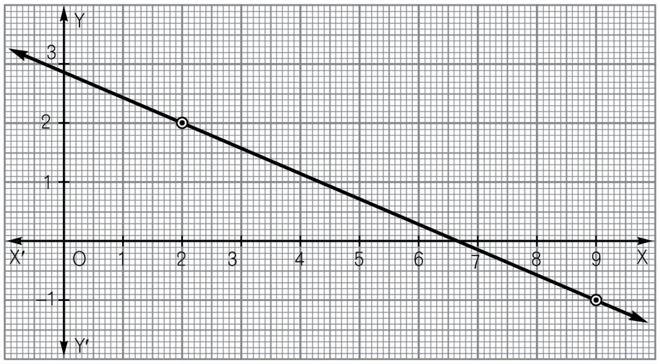
$x = \frac{27}{3} = 9$

$y = +2, \quad 3x = 20 - 7 \times 2$

$3x = 20 - 14 = 6$

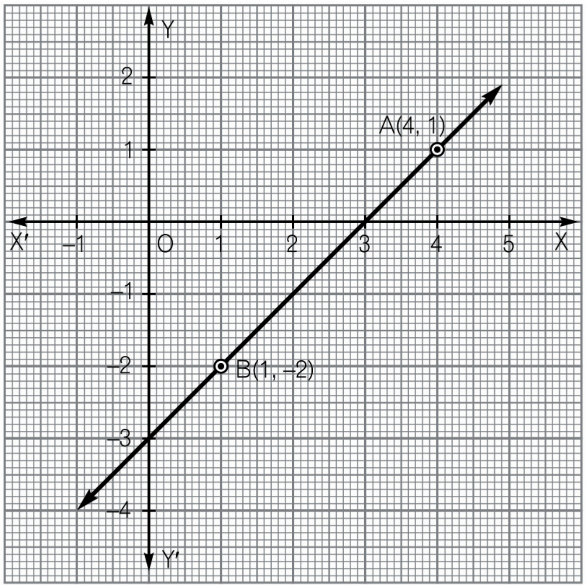
$x = \frac{6}{3} = 2$

Points are (9, –1) (2, 2) graph shown on graph paper.



The line meets x-axis at $\frac{20}{3}, 0$ and y-axis at $0, \frac{20}{7}$.

21.

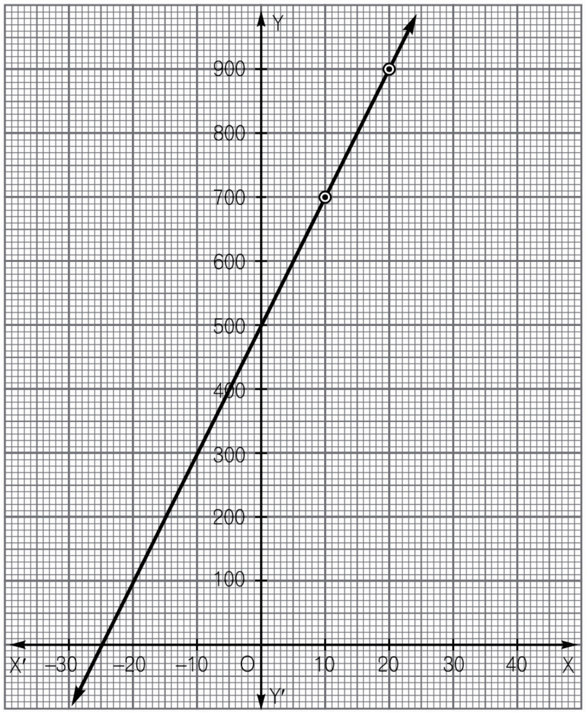


Points are plotted on above graph:
A(4, 1), B(1, –2)
Linear eqn. cut the x-axis at the point (3, 0).
Linear eqn. cut the y-axis at the point (0, – 3).

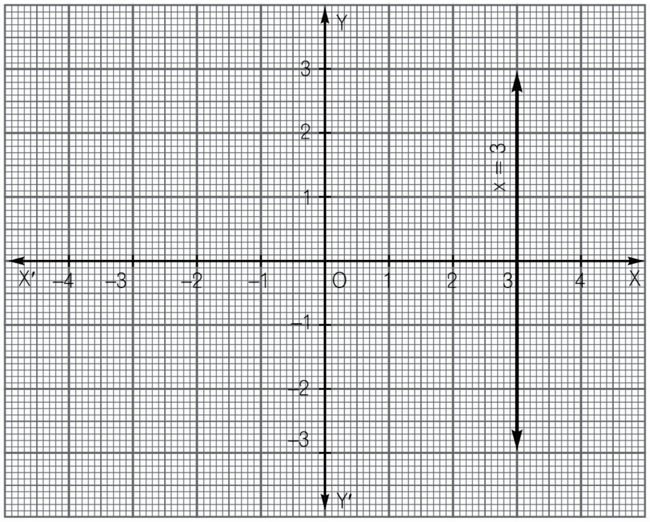
SECTION-D

22. Monthly expenditure on milk = ₹ 500
Extra quantity takes = x kg
Total expenditure = ₹ y
Linear equation: ₹ y = 500 + 20x

x	10	20
y	700	900



23. Solve for x:



$$24 - 3(x - 2) = x + 18$$

$$24 - 3x + 6 = x + 18$$

$$24 - 3x - x + 6 - 18 = 0$$

$$30 - 18 - 4x = 0$$

$$12 - 4x = 0$$

$$-4x = -12$$

$$x = \frac{12}{4}$$

$$x = 3.$$

24. $x + y = 5$ ...(i)

x	1	2
y	4	3

$y = 5 - x$

$x = 1,$ $y = 5 - 1 = 4$

$x = 2,$ $y = 5 - 2 = 3$

$2x + 2y = 12$ ...(ii)

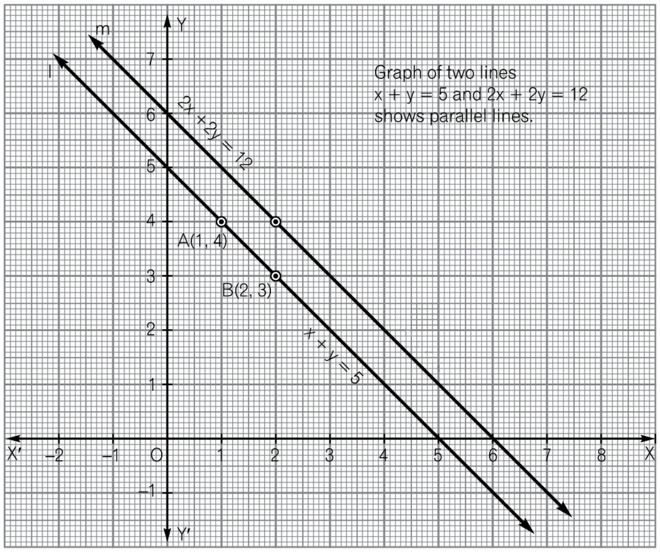
x	2	4
y	4	2

$x + y = 6$

$y = 6 - x,$

$x = 2,$ $y = 6 - 2 = 4$

$x = 4,$ $y = 6 - 4 = 2$



Graph of above two lines show that the two lines are parallel lines. i.e. l || m

25. $2x - 3y = 12$

$$-3y = 12 - 2x$$

$$3y = 2x - 12$$

$$y = \frac{2x}{3} - \frac{12}{3} = \frac{2x}{3} - 4$$

x	3	0	6
y	–2	–4	0

Let $x = 3$, $y = \frac{2}{3} \times 3 - 4 = 2 - 4 = -2$

$x = 6$, $y = \frac{2}{3} \times 6 - 4 = 4 - 4 = 0$

$x = 0$, $y = \frac{2}{3} \times 0 - 4 = -4$

The line meets x-axis at point (6, 0) and y-axis at point (0, – 4)

WORKSHEET 2: SECTION-A

1. $x = 2, y = 2$

$2x - ky + 7 = 8$

$2 \times 2 - k \times 2 + 7 = 8$

$4 + 7 - 2k = 8$

$11 - 2k = 8$

$-2k = 8 - 11 = -3$

$k = 3/2.$

2. x = 2 is a line parallel to y-axis at a distance of 2 units.

3. y = 0, representing x-axis.

4. General form of a linear equation in two variables.

$ax + by + c = 0.$

SECTION-B

5. $x + 2y = 7$

$y = 0,$ $x = 7 - 2y$

$x = 7 - 2 \times 0 = 7$

x	7	5	3	1
y	0	1	2	3

$y = 1,$ $x = 7 - 2 \times 1 = 5$

$y = 2,$ $x = 7 - 2 \times 2 = 7 - 4 = 3$

$y = 3,$ $x = 7 - 2 \times 3 = 7 - 6 = 1$

∴ 4 solutions are: (7, 0), (5, 1), (3, 2) and (1, 3).

6. P(x, y)

x = abscissa, y = ordinate

$$y = \frac{3}{2}x$$

$$3x + 4y = 18$$

$$3x + 4 \times \frac{3}{2}x = 18$$

$$3x + 6x = 18$$

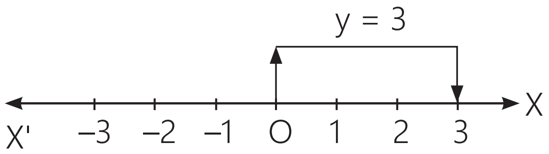
$$9x = 18$$

$$x = \frac{18}{9} = 2$$

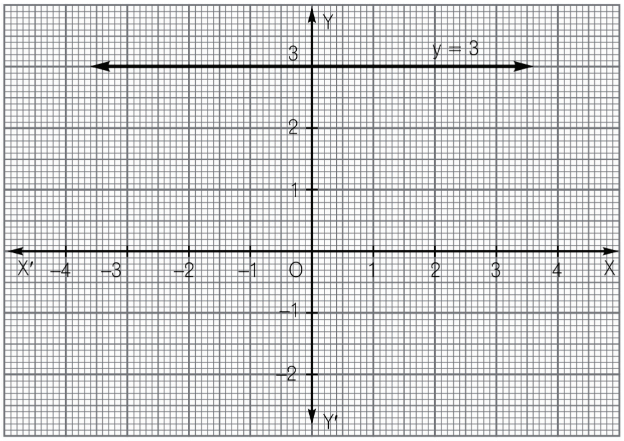
Points (2, 3).

7. y = 3

(a) In one variable:



(b) In two variables:



8. Find m: If x = –3, y = –1 is a solution of:

$$mx - 3y = 15$$

$$m(-3) - 3(-1) = 15$$

$$-3m + 3 = 15$$

$$-3m = 15 - 3 = 12$$

$$m = \frac{12}{-3}$$

$$m = -4.$$

9. Give two solutions of the equation:

$x + 3y = 8.$

$y = 1, \quad x = 8 - 3y$

$x = 8 - 3 \times 1 = 5$

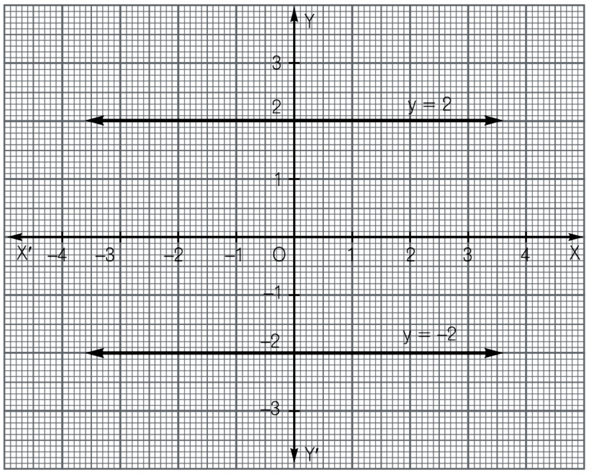
x	5	8	–1
y	1	0	3

$y = 0, \quad x = 8 - 3 \times 0 = 8$

$y = 3, \quad x = 8 - 3 \times 3 = 8 - 9 = -1$

∴ Two solutions are (5, 1), (8, 0) and (–1, 3).

10. Draw graph:



Both lines are parallel to each other.

11. $y = 9x - 7$

For x = 1, y = 2

$9x - 7 = 9 - 7 = 2 = y = \text{RHS}$

For x = 0, y = –2

$9x - 7 = 9(0) - 7 = -7 \neq -2$

So, (1, 2) is a solution of y = 9x – 7 but (0, –2) is not a solution of y = 9x – 7.

SECTION-C

12. Let larger number = x and smaller number = y

According to Q:

$5x = 2y + 9$ ...(i)

[∴ By Division Algorithm]

x	11/5	5
y	1	3

$y = 1, \quad 5x = 2 \times 1 + 9 = 11$

$x = \frac{11}{5}$

$y = 3, \quad 5x = 2 \times 3 + 9 = 15$

$x = 3$

Therefore two solutions are $\frac{11}{5}, 1$ and (3, 3).

13. $C = \frac{5F - 160}{9}$ ...(i)

(a) T = 30°C

$$30 = \frac{5F - 160}{9}$$

$$30 \times 9 = 5F - 160$$

$$270 + 160 = 5F$$

$$430 = 5F$$

$$F = \frac{430}{5} = 86$$

Temperature is 86°F.

(b) Let x be the temperature which is numerically same in both fahrenheit and celcius.

So, $x = \frac{5}{9}(x - 32)$

$9x = 5x - 32(5)$

$$9x = 5x - 160$$
$$160 = 4x$$
$$-40 = x$$

The temperature is numerically same at – 40°.

14. $4x + 3y + 6 = 0$

x	0	–3	–6
y	–2	2	6

$4x = -3y - 6$

$4x = -3(y + 2)$

$y = -2 \qquad 4x = -3(-2 + 2) = -3 \times 0 = 0$

$x = 0$

$y = 2, \qquad 4x = -3(2 + 2)$

$4x = -3 \times 4 = -12$

$x = -3$

$y = 6, \qquad 4x = -3(y + 2)$

$4x = -3(6 + 2) = -3(8)$

$4x = -24$

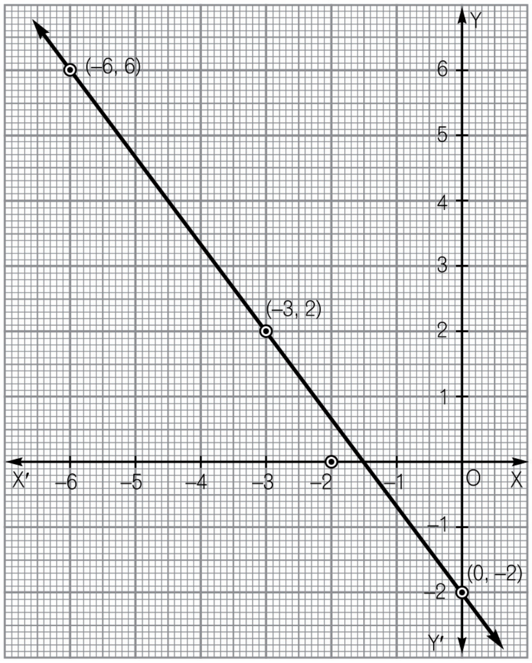
$$x = -\frac{24}{4} = -6$$

$x = -6, y = 6$

When

$y = 6, x = -6$

So, coordinates are (– 6, 6)



15. $3x - 2y = 4$ ...(i)

$$x = \frac{4 + 2y}{3}$$

x	2	4
y	1	4

$y = 1, \qquad x = \frac{4 + 2 \times 1}{3} = \frac{6}{3} = 2$

$y = 4, \qquad x = \frac{4 + 2 \times 4}{3} = \frac{12}{3} = 4$

P(2, 1), Q(4, 4)

$x + y - 3 = 0$

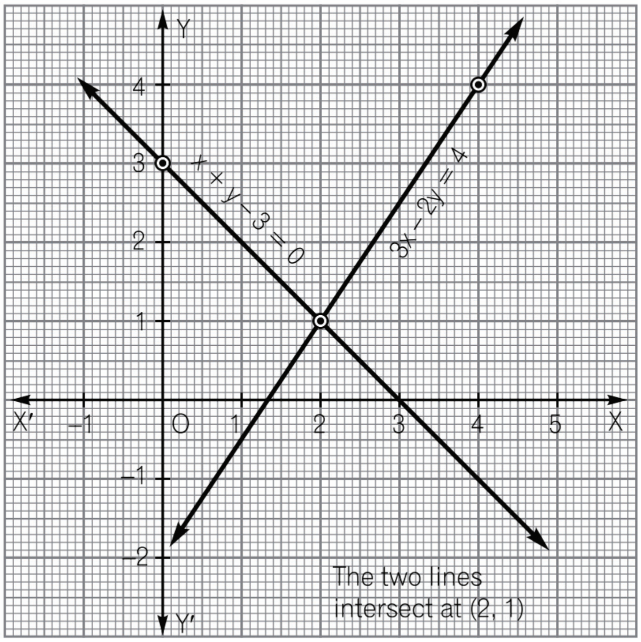
x	2	0
y	1	3

$y = 1, \qquad x = 3 - y$

$x = 3 - 1 = 2$

$y = 3, \qquad x = 3 - 3 = 0$

(2, 1), (0, 3).



The two lines intersect at (2, 1)

16. $3x - 2y = 6$ ...(i)

$$3x = 6 + 2y$$

$$x = \frac{6 + 2 \times y}{3}$$

x	4	6	–2
y	3	6	–6

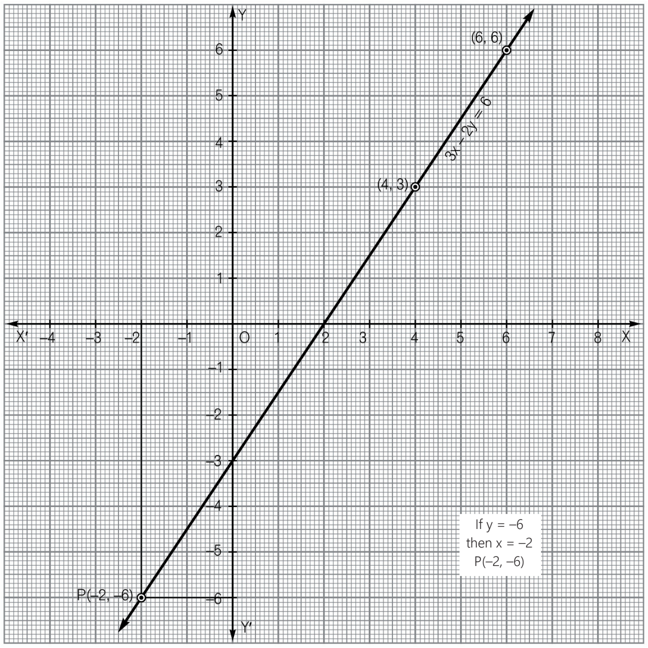
$y = 3, \qquad x = \frac{6}{3} + \frac{2}{3}y = 2 + \frac{2}{3} \times 3 = 4$

$y = 6, \qquad x = 2 + \frac{2}{3} \times 6 = 2 + 4 = 6$

$y = -6, \qquad x = 2 + \frac{2}{3} \times -6$

$x = 2 - 4 = -2$

Point (– 2, – 6).



17. P(1, –1)

$3x + 2y - 1 = 0$ ...(i)

$3x + 2y = 1$

$3x = 1 - 2y1$

$x = \frac{1 - 2y}{3}$

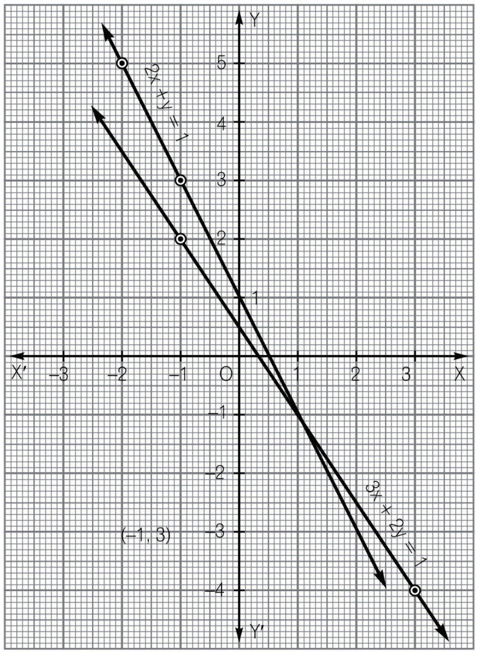
x	–1	3
y	2	–4

$2x + y = 1$ ...(ii)

$2x = 1 - y$

$x = \frac{1 - y}{2}$

x	–1	–2
y	3	5



SECTION-D

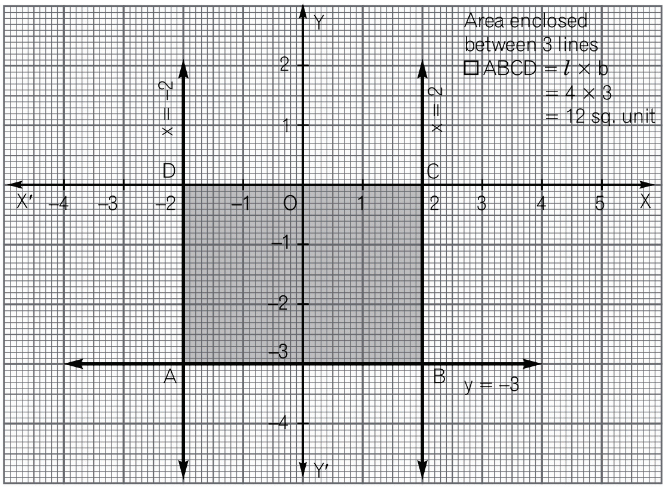
18. $x = 2,$ ...(i)

$x + 2 = 0$ ...(ii)

$\Rightarrow x = -2$

$y + 3 = 0$

$y = -3$ ...(iii)



19. Let x = distance and y = fare

₹ 50 is fixed fare.

₹ 16 per km.

$50 + 16x = y$

fare for 20 km

$= 50 + 16(20) = y$

= 50 + 320 = y

y = ₹ 370

∴ fare for 20 km = ₹ 370.

20. $3x + 5y = 15$

$x = 0, 3(0) + 5y = 15$

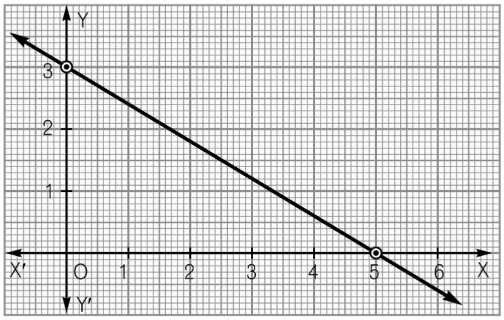
x	0	5
y	3	0

$y = 3$

$y = 0, 3x + 5(0) = 15$

$x = 5$

The figure is a triangle, ∴ the area is $\frac{1}{2}$ (5) (3) = 7.5 units



CASE STUDY-1

1. (i). (b) Subtract 9 from both side of the equation.

$3x + 2y - 9 = 9 - 9$

$3x + 2y - 9 = 0$

(ii) (c) $3y = 2x + 5$ represents a linear equa-tion with two variables and any linear equation in two variables has infinite solutions.

(iii) (a) $\pi x + y = 9$

Let $x = 1$

$\pi(1) + y = 9$

$y = 9 - \pi$

(iv) (d) $2x + 3y = k$

$x = 2$ and $y = 3$ are the solution of the given equation

Therefore

$2(2) + 3(3) = k$

$k = 13$

(v) (d) The term "b" is the coefficient of "y" which is 0 in the given equation.

CASE STUDY-2

2. (i) (b) for every value of "y" there is an equal and opposite value of x.

(ii) (a) for every value of "y" there is an equal value of x

(iii) (c) As the point (3, 4) lie on the graph, therefore the point willl satisfy the given equation

$3y = px + 7$

$3(4) = 3p + 7$

$12 = 3p + 7$

$p = \frac{5}{3}$

(iv) (c) Infinite number of lines can pass through any given set of points.

(v) (b) The line x = a is parallel to y axis.

Chapter
5

Introduction to Euclid's Geometry

MULTIPLE CHOICE QUESTION

1. (c) At least 3 lines are required to make a closed figure.

2. (b) The curves are the boundaries of a surface.

3. (b) Only one line can be drawn through two given lines.

4. (a) An angle is formed by two straight lines having a common point.

5. (d) Infinite lines can pass through one point.

WORKSHEET 1: SECTION-A

1. Things which are equal to the same things are equal to one another.

2. We can draw infinite lines passing through the point P.

3. AB = x + 3

AC = 4x – 5

BC = 2x

⇒ AC = AB + BC

4x – 5 = x + 3 + 2x

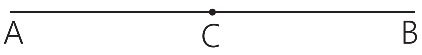
4x – 3x = 8

$\boxed{x = 8.}$

4. **Concurrent lines:** Three or more lines are said to be concurrent if there is a point which lies on all of them.

SECTION-B

5. **Given:** C is the mid-point of AB, such that AC = BC.



To prove: AC = $\frac{1}{2}$ AB

Proof: We have AC = BC ...(1)

We can write AB = AC + BC

AB = AC + AC (From Eq. 1)

AB = 2AC

⇒ $\frac{1}{2}$ AB = AC

⇒ AC = $\frac{1}{2}$ AB Hence proved.

6. Given: AB = CD

To prove: AC = BD

Proof: We have AB = CD

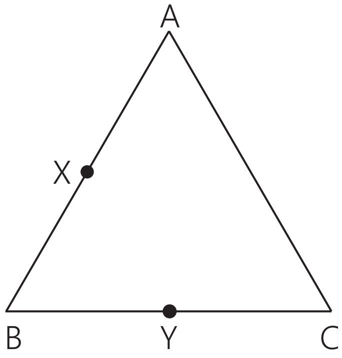
Adding BC on both sides of Eq. (1), we get

AB + BC = BC + CD

AC = BD

Axiom: If equals are added to equals, the wholes are equal.

7.



Given: BX = $\frac{1}{2}$ AB, BY= $\frac{1}{2}$ BC and

AB = BC.

To prove: BX = BY

Proof: We have

AB = BC

$BX = \frac{1}{2} AB \Rightarrow 2BX = AB$

and $BY = \frac{1}{2} BC \Rightarrow 2BY = BC$

$\Rightarrow AB = BC$

$2BX = 2BY$

$\Rightarrow BX = BY$ Hence proved.

8. Let D be another mid-point of AB.

$\therefore AD = DB$ ...(1)

Let C is the mid-point of AB.

$\therefore AC = CB$ ...(2)

Subtracting (1) from (2), we get

$AC - AD = CB - DB$

$\Rightarrow DC = -DC$

$\Rightarrow 2DC = 0$

$\Rightarrow DC = 0$

$\therefore$ C and D coincide.

Thus, every line segment has one and only one mid-point.

Hence proved.

9. If equals are subtracted from equals, the remainders are equal.

10. $AB = BC$

$AX + XB = BY + YC$

Also, $BX = BY$

$\Rightarrow (AX + XB) - BX = (BY + CY) - BY$

[If equals are subtracted from equal the remainders are equal)

$\Rightarrow AX = CY$

SECTION-C

11. The terms need to be defined are

→ Polygon is a closed figure bounded by three or more line segments.

→ Line segment is a part of line with two end points.

→ Line is an undefined term.

→ Point is an undefined term.

→ Angle in a figure is formed by two rays with one common initial point.

→ Acute angle is an angle whose measure is between 0 to 90°. Here undefined terms are line and point. All the angles of equilateral triangle are 60° each (given)

Two line segments are equal to the third one (given)

Therefore, all three sides of an equilateral triangles are equal (according to Euclid's axiom, things which are equal to the same thing are equal to one another)

12. Euclid's fifth postulate states that 'given any two lines and a transversal, if the angles on the same side of transversal are less than 180 degrees they will meet on that side of transversal'.

The statement given here is that two intersecting lines cannot be perpendicular to the same line. i.e., parallel. This statement is stating that the co-interior angles of intersecting lines cannot be 90 degrees in measure, and it is almost equivalent to Euclid postulates.

13. We know that if a transversal intersects two parallel lines, then each pair of corresponding angles are equal, which is a theorem. So, statement (i) is false and not an axiom. Also, we know that, if a transversal intersect two parallel line then each pair of alternate interior angles are equal. It is also a theorem. So, statement (ii) is true and an axiom. Thus, in given statements, (i) is false and (ii) is axiom. Hence, system of axiom is not consistent.

14. We know that if two lines intersect each other, then the vertically opposite angles are equal. It is a theorem, so given system (i) is false and not an axiom.

Also, we know that, if a ray stands on a line, then the sum of two adjacent angles so formed is equal to 180°. It is an axiom. So, given statement (ii) is true and an axiom.

Thus in given statements, (i) is false and (ii) is true.

Hence given system of axioms is not consistent.

15. The given system of axioms are consistent.

SECTION-D

16. OX = PX

⇒ 2OX = 2PX

⇒ XY = XZ

17. (i) AB = BC

$\frac{1}{2}$ AB = $\frac{1}{2}$ BC

AM = NC

$$\left[\begin{array}{l}\text{Since, M is midpoint of AB}\\ \text{AM} = \frac{1}{2}\text{AB}\\ \text{Also, N is midpoint of BC}\\ \text{CN} = \frac{1}{2}\text{BC}\end{array}\right]$$

(ii) Since, M is midpoint of AB

BM = $\frac{1}{2}$ AB

Also, N is midpoint of BC

BN = $\frac{1}{2}$ BC

As, BM = BN

⇒ $\frac{1}{2}$ AB = $\frac{1}{2}$ BC

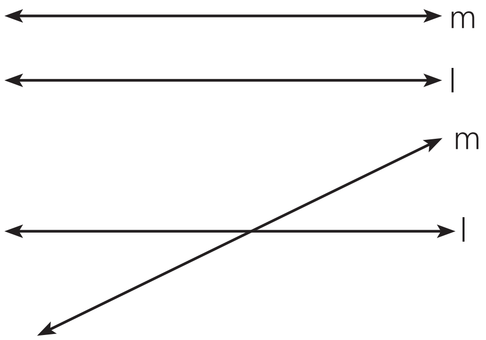
⇒ AB = BC

WORKSHEET 2: SECTION-A

1. equal
2. A straight line may be drawn from one point to another point.
3. Two lines AB and CD lying the same plane are said to be perpendicular, if they form a right angle. AB ⊥ CD.
4. For every line l and for every point p not lying on l, there exists a unique line m passing through p and parallel to l.

SECTION-B

5. Given: Two distinct lines l and m.



To prove: l ∩ m contains at the most one point.

Proof: Let l ∩ m contains two distinct points A and B.

Then l contains both points A and B.

And m also contains both points A and B.

But, there is one and only one line passing through two distinct points.

∴ l = m

This contradicts the fact.

Thus our assumption is wrong.

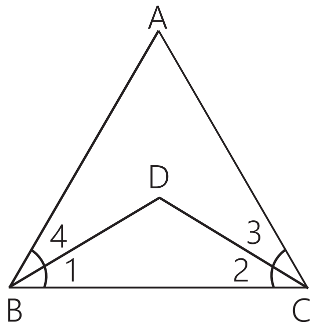
Hence, two distinct lines cannot have more than one point in common.

6. A system of axioms is called consistent, when it is impossible to deduce any statement from these axioms, which contradicts any axiom or previously proved statement.

7. We have ∠ABC = ∠ACB

and ∠3 = ∠4

To prove: DB = DC



Proof: We have

$$\angle ABC = \angle ACB \quad ...(1)$$

and $$\angle 3 = \angle 4 \quad ...(2)$$

Subtracting eq. (2) from (1), we get

$$\angle ABC - \angle 4 = \angle ACB - \angle 3$$

$$\angle 1 = \angle 2$$

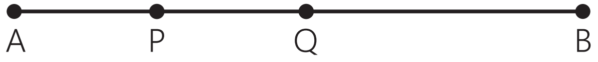
$\Rightarrow$ $DC = BD$

(sides opposite to equal angles are equals)

Hence proved.

8. PQ, PR, PS, QR, RS, QS etc.

9. Q is the mid-point of AB.



$\Rightarrow$ $AQ = QB$

$\Rightarrow$ $AB = AQ + QB$

$AB = QB + QB$

$AB = 2QB \quad ...(1)$

Also, P is the mid-point of AQ.

$\Rightarrow$ $AP = PQ$

$AQ = PQ + AP$

and $AQ = PQ + PQ$

$AQ = 2PQ \quad ...(2)$

We can write

$$AB = AQ + BQ$$

$$AB = 2PQ + \frac{1}{2} AB$$

$$AB - \frac{1}{2} AB = 2PQ$$

$$\frac{2AB - AB}{2} = 2PQ$$

$$AB = 4PQ$$

$$\boxed{\frac{1}{4} AB = PQ}$$

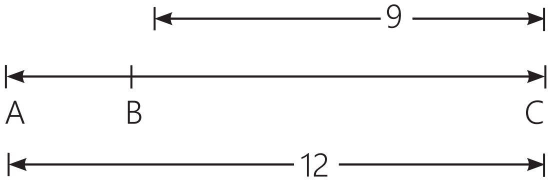
Hence proved.

10. We can say that all four lines have only are common point A. And we can not draw 4 different lines as they are parallel to each other.But if A, B, C, D and E are on same line than they can satisfied our given condition and as well as properties of parallel line.

$\Rightarrow$ A, B, C, D and E are collinear.

SECTION-C

11.



$AC = 12$ cm

$AB + BC = 12$

$AB + 9 = 12$

$AB + 9 - 9 = 12 - 9$

$AB = 3$

12. Angle is the figure formed by two rays called the sides of the angle sharing a common endpoint called the vertex of the angle. Two lines are said to be congruent if they have the same length.

13. Equilateral triangle is a triangle with all sides equal. We prove this by geometry.

→ Draw a line segment AB of any length.

→ Take compass, put the pointy end at point A and pencil at point B.

→ Draw an arc.

→ Here we draw an arc of radii AB.

→ Now put the pointy end at B and pencil at A radii AB

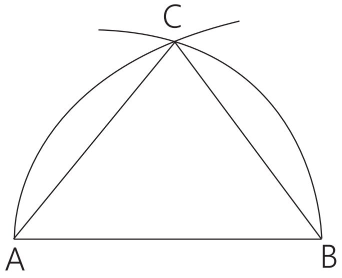
→ Draw another arc. Here are draw an arc of

→ Here we draw an arc of radii BA.

→ Mark the intersecting point as C.

→ Join point A to point C by a straight line.

→ Joint point B to point C by a straight line.



We need to prove

AB = AC = BC

Now AC = AB (radii of same circle)

and BC = AB (radii of same circle)

From Euclid's axiom, things which are equal to the same thing are equal,

So, AC = BC

So, we get AB = AC = BC,

∴ ABC is an equilateral triangle.

14. AB = PQ, PQ = XY

As, we know that things which are equal to same things are equal to one another. So, AB = XY.

15. An axiom is a rule or statement that is accepted as true without proof.

Postulate is a statement that is considered to be self-evident.

SECTION-D

16. (i) Parallel lines: Two lines in a plane that do not intersect or touch each other at any point are said to be parallel.

(ii) A line meeting another line at a right angle or 90° is said to be perpendicular to it.

17. A line extends forever in both directions.

A line segment is just part of a line. It has two endpoints.

A ray starts at one point and continues on forever in one direction.

Chapter

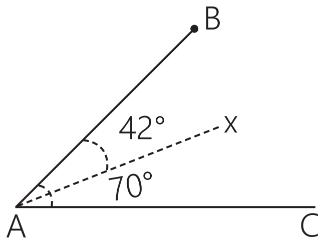
6 Lines and Angles

MULTIPLE CHOICE QUESTION

1. (c) The sum of angles in a linear pair is 180°, therefore one of the angle must be either equal to 90° or greater than 90°.

2. (d) Alternate interior angles are equal. Also the sum of interior angle on same side of transversal is 180°. Corresponding angles are equal to each other.

3. (a)



$\angle BAC = \angle XAC + \angle BAX$

$70° = \angle XAC + 42°$

$\angle XAC = 28°$

4. (d) Let the angle be x.

Complement of x is $(90 - x)°$.

Supplement of complement of x is $180 - (90 - x)°$.

<u>ATQ</u>

$180 - (90 - x) = 3\,(90 - x)$

$x = 45°$

5. (b) The angle between the bisectors of adjacent supplementry angles is 90°.

WORKSHEET 1: SECTION-A

1. Let angle be = x

So, $x = 25 + (90 - x)$

$2x = 115$

$x = 57.5$

2. Let an angle = x°

According to question

$x = 4(180° - x)$

$x = 720 - 4x$

$x + 4x = 720$

$5x = 720$

$x = \frac{720}{5}$

$x = 144°$

$180° - 144° = 36°$.

3. Let the angles be x and y.

According to question

$x - y = 40°$ ...(1)

and $x + y = 90°$ ...(2)

Adding eqns. (1) and (2), we get:

$2x = 130°$

$x = 65°$

Put in eq. (1)

$65° - y = 40°$

$65° - 40° = y$

$y = 25$

$x = 65°$ and $y = 25°$.

4. Let the angles be x and y.

We know that $x + y = 180°$ ...(1)

According to question

$x - y = 90°$ ...(2)

Adding (1) and (2)

$2x = 270°$

$x = 135°$

Put in (1),

$135° + y = 180°$

$y = 45°$

$x = 135°$ and $y = 45°$.

5. a and b form a linear pair.

$\Rightarrow$ $a + b = 180°$ ...(1)

and it is given that

$a - 2b = 30°$

$a = 30° + 2b$ ...(2)

Putting this value of a in Eq. (1), we get

$30° + 2b + b = 180°$

$3b = 150°$

$\boxed{b = 50°}$

Putting this value of b in Eq. (2)

$a = 30° + 2 \times 50°$

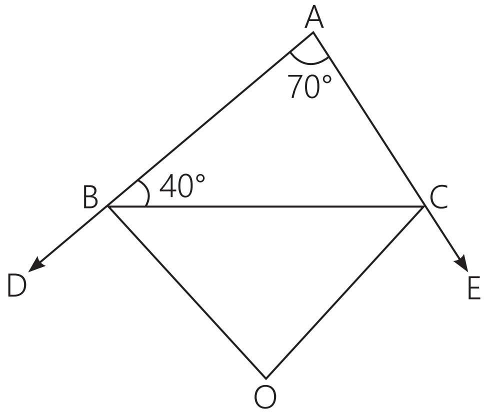
$a = 30° + 100°$

$\boxed{a = 130°}$

SECTION-B

6. Since BO and CO are bisectors of $\angle DBC$ and $\angle ECB$.

$\Rightarrow$ $\angle DBO = \angle CBO$ and $\angle ECO = \angle BCO$



In ΔABC,

$\angle ABC + \angle BAC + \angle ACB = 180°$ (A.S.P.)

$40° + 70° + \angle ACB = 180°$

$\Rightarrow$ $\angle ACB = 180° - 110°$

$\Rightarrow$ $\angle ACB = 70°$

Now

$\angle ABC + \angle DBO + \angle CBO = 180°$

$40° + 2\angle CBO = 180°$

$2\angle CBO = 140°$

$\angle CBO = 70°$

Similarly,

$\angle ACB + \angle BCO + \angle ECO = 180°$

$70° + 2\angle BCO = 180°$

$2\angle BCO = 110°$

$\angle BCO = 55°$

In ΔBOC,

$\angle BOC + \angle CBO + \angle BCO = 180°$ (A.S.P.)

$\angle BOC + 70° + 55° = 180°$

$\angle BOC = 180° - 125°$

$\boxed{\angle BOC = 55°}$

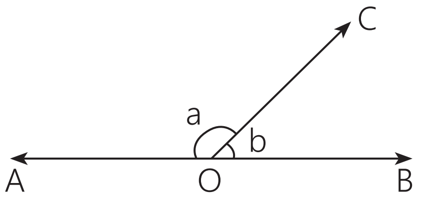
7. We have $a - b = 80°$ ...(1)

We know that $a + b = 180°$ (C.P.) ...(2)

Adding eqns. (1) and (2), we get;

$2a = 260°$

$\boxed{a = 130°}$

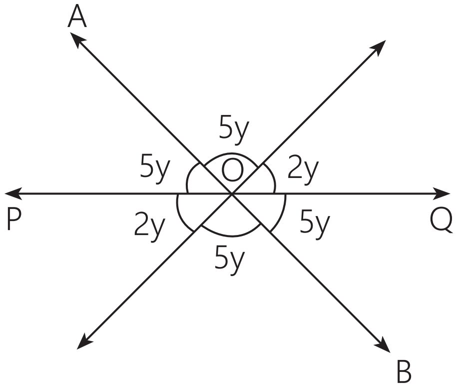


Put in (1), $130° - b = 80°$

$b = 130° - 80°$

$\boxed{b = 50°}$

8. Since POQ is a straight line.



So, $5y + 5y + 2y = 180°$ (L.P.)

$12y = 180°$

$y = \frac{180°}{12}$

$\boxed{y = 15°}$

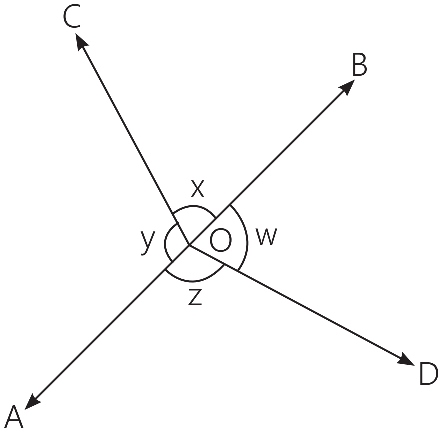
9. Let the angles be 2x and 3x. Sum of supplementary angles = 180°

A.T.Q.

$$2x + 3x = 180°$$
$$5x = 180°$$
$$x = \frac{180°}{5}$$
$$x = 36°$$
$$2x = 2 \times 36° = 72°$$
$$3x = 3 \times 36° = 108°$$

10. $x + y = w + z$

Since the sum of all the angles round a pair is equal to 360°.



$$(\angle BOC + \angle COA) + (\angle BOD + \angle AOD) = 360°$$
$$\Rightarrow (x + y) + (z + w) = 360°$$

But $x + y = w + z$

$$(x + y) + (x + y) = 360°$$
$$2(x + y) = 360°$$
$$x + y = 180°$$

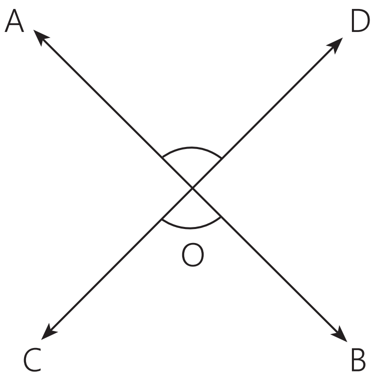
Thus, ∠BOC and ∠COA as well as ∠BOD and ∠AOD form linear pairs.

Consequently, OA and OB are two opposite rays.

Therefore, AOB is a straight line.

SECTION-C

11. Given: Two lines AB and CD intersect at O.



To prove: ∠AOC = ∠BOD

and ∠AOD = ∠BOC

Proof: Since a ray OC stands on the line AB,

$\angle AOC + \angle COB = 180°$ (L.P.) ...(1)

Since ray OA stands on the line CD,

$\angle AOC + \angle AOD = 180°$ (L.P.) ...(2)

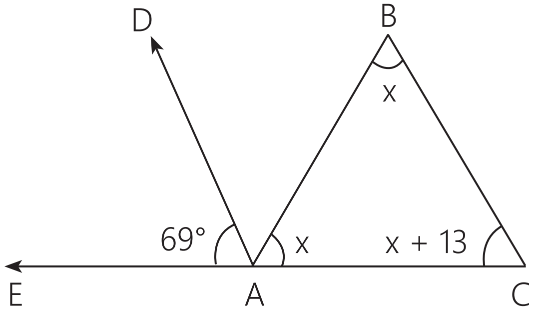
From eqns (1) and (2), we have

$$\angle AOC + \angle COB = \angle AOC + \angle AOD$$
$$\Rightarrow \angle COB = \angle AOD$$

Similarly, ∠AOC = ∠BOD.

Hence proved.

12. In ΔABC,



$\angle BAC + \angle ABC + \angle BCA = 180°$ (A.S.P.)

$$x + x + x + 13 = 180°$$
$$3x = 180° - 13°$$
$$3x = 167°$$
$$x = 55.7$$

$\angle CAB = 55.7$

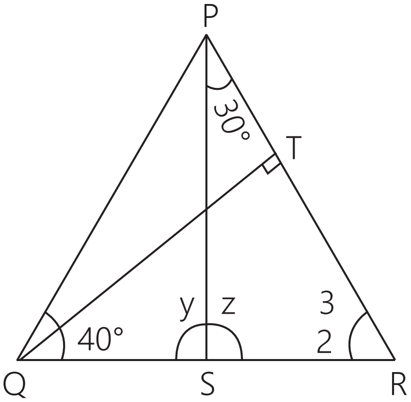
and $\angle BAD = 55.7 \times 2 = 111.3$

$\angle BAC = 55.7$

$\angle ABC = 55.7$

$\angle ACB = 55.7 + 13 = 68.7$

13. $QT \perp PR$,



$\angle TQR = 40°$ and $\angle SPR = 30°$

In ΔTQR,

$\angle TQR + \angle QTR + \angle QRT = 180°$ (A.S.P.)

$40° + 90° + \angle QRT = 180°$

$\angle QRT = 50°$

$x = 50°$

In ΔPSR,

$\angle RPS + x + z = 180°$ (A.S.P.)

$30° + 50° + z = 180°$

$z = 180° – 80°$

$\boxed{z = 100°}$

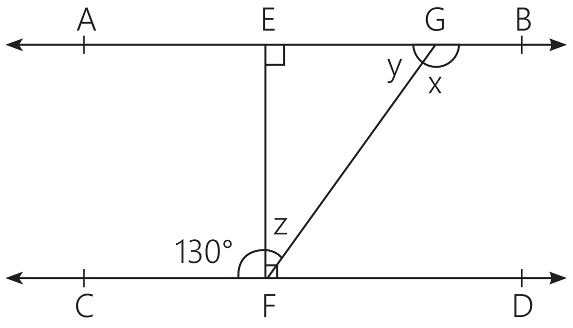
$y + z = 180°$ (L.P.)

$y + 100° = 180°$

$\boxed{y = 80°}$

$\boxed{x = 50°}$

14. AB || CD, from the given figure:



$EF \perp CD$,

$\angle GFC = 130°$

$130° + \angle GFD = 180°$ (L.P.)

$\angle GFD = 50°$

$\Rightarrow z + 50° = 90°$

$\boxed{z = 40°}$

In ΔEFG,

$90° + y + z = 180°$ (A.S.P.)

$90° + y + 40° = 180°$

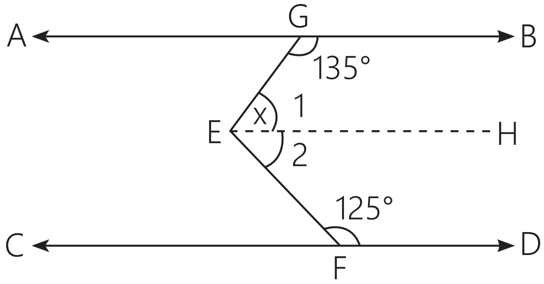
$\boxed{y = 50°}$

$x + y = 180°$ (L.P.)

$x + 50° = 180°$

$\boxed{x = 130°}$

15. AB || CD, x = ?



Draw EH || CD || AB

AB || EH and GE is a transversal.

$\angle 1 + \angle BGE = 180°$

(Consecutive interior angles)

$\angle 1 + 135° = 180°$

$\angle 1 = 45°$

Again EH || CD, EF is a transversal.

$\angle 2 + \angle DFE = 180°$

$\angle 2 + 125° = 180°$

$\angle 2 = 55°$

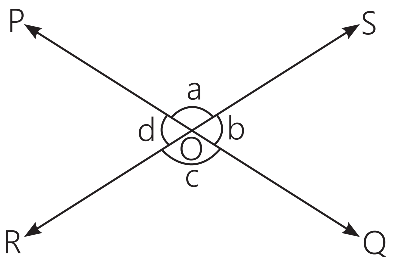
But $x = \angle 1 + \angle 2$

$= 45° + 55°$

$= 100°$.

SECTION-D

16.



$\angle POR : \angle ROQ = 5 : 7$

Let $\angle POR = 5x$ and $\angle ROQ = 7x$

and $\angle POR + \angle ROQ = 180°$ (L.P.)

$$c + d = 180°$$

$$5x + 7x = 180°$$

$$12x = 180°$$

$$x = \frac{180°}{12}\ x = 15°$$

$\Rightarrow$ $c = 7x = 7 \times 15 = 105°$

$d = 5x = 5 \times 15 = 75°$

$\Rightarrow$ a = c and b = d [Vertically opposite angles]

$\Rightarrow$ $a = c = 105°$

$d = b = 75°$

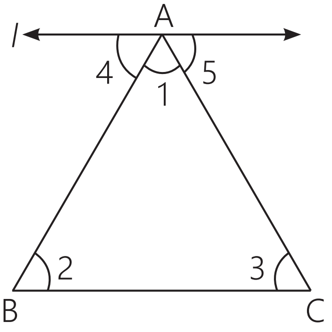
17. Given: A triangle ABC.

To prove: $\angle A + \angle B + \angle C = 180°$

i.e. $\angle 1 + \angle 2 + \angle 3 = 180°$

Construction: Through A, draw a line l parallel to BC.

Proof:



Since $l \parallel BC$

$\therefore$ $\angle 2 = \angle 4$ and $\angle 3 = \angle 5$

[Alternate interior angles]

We know that sum of angles at a point is 180°.

$\Rightarrow$ $\angle 1 + \angle 4 + \angle 5 = 180°$

$\Rightarrow$ $\angle 1 + \angle 2 + \angle 3 = 180°$

$\Rightarrow$ $\angle A + \angle B + \angle C = 180°$

Hence proved.

$$\angle A + \angle B = 120°$$

and $$\angle B + \angle C = 100°$$

$$\angle A = 120° - \angle B$$

$$\angle C = 100° - \angle B$$

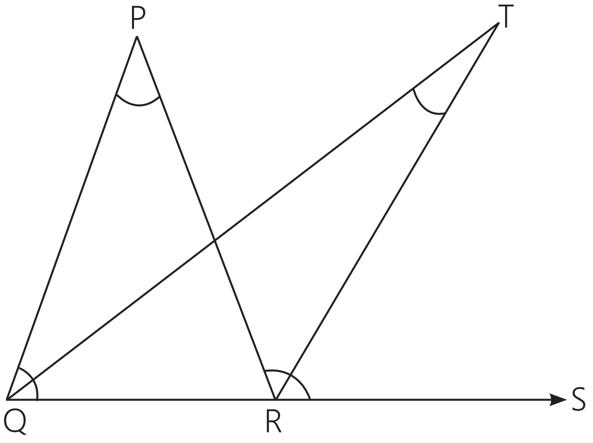
$$\angle A + \angle B + \angle C = 180°$$

$$120° - \angle B + \angle B + 100° - \angle B = 180°$$

$$220° - 180° = \angle B$$

$$\boxed{\angle B = 40°}$$

18. TQ is a bisector of $\angle PQR$.



So, $\angle PQT = \angle TQR = \frac{1}{2} \angle PQR$.

Also, TR is bisector of $\angle PRS$.

So, $\angle PRT = \angle TRS = \frac{1}{2} \angle PRS$

In ΔPQR, $\angle PRS$ is an exterior angle

$\angle PRS = \angle QPR + \angle PQR$ ...(1)

(Exterior angle property)

In ΔTQR, $\angle TRS$ is exterior angle

$\angle TRS = \angle TQR + \angle QTR$ ...(2)

(Exterior angle property)

Putting, $\angle TRS = \frac{1}{2} \angle PRS$

and $\angle TQR = \frac{1}{2} \angle PQR$

$\frac{1}{2} \angle PRS = \frac{1}{2} \angle PQR + \angle QTR$

$\frac{1}{2} \angle PRS = \frac{1}{2} \angle PQR + \angle QTR$

Putting, $\angle PRS = \angle QPR + \angle PQR$ [From (1)]

$\frac{1}{2} (\angle QPR + \angle PQR) = \frac{1}{2} \angle PQR + \angle QTR$

$\frac{1}{2} \angle QPR + \frac{1}{2} \angle PQR = \frac{1}{2} \angle PQR + \angle QTR$

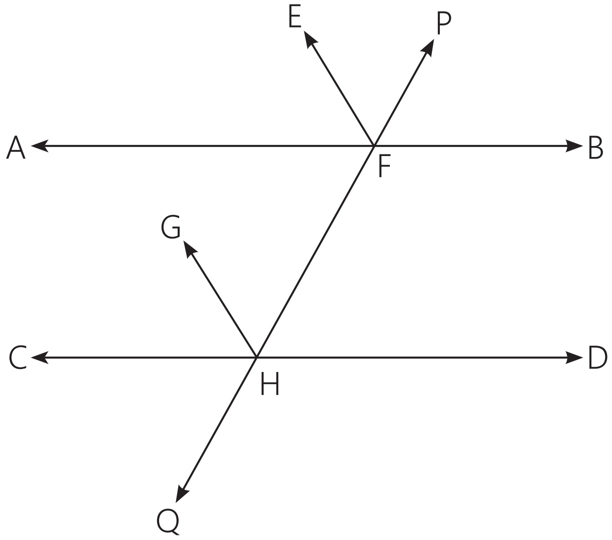
$\frac{1}{2} \angle QPR = \angle QTR$

$\Rightarrow \boxed{\angle QTR = \frac{1}{2} QPR}$

Hence proved.

19. Given: EF || GH.

To prove: AB || CD



Proof: Since, EF || GH and intersected by a transversal PQ at F and H.

$\therefore \angle EFP = \angle GHF$

$2\angle EFP = 2 \angle GHF$

$\Rightarrow \angle AFP = \angle CHF$

But these are corresponding angles formed by two lines AB and CD when transversal PQ intersects them.

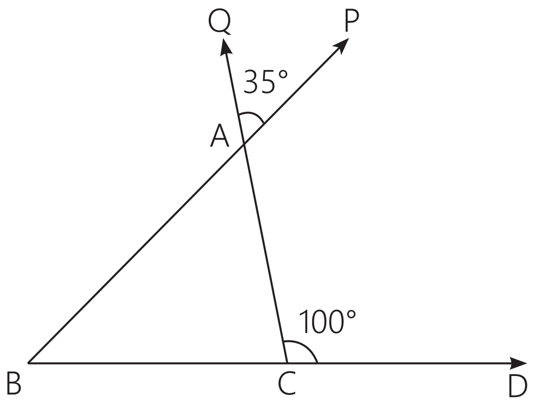
$\Rightarrow$ AB || CD

Hence proved.

20. $\angle ACD = 100°$ and $\angle QAP = 35°$

Since $\angle QAP = \angle BAC$ [Vertically opp. angles]

$\Rightarrow \boxed{\angle BAC = 35°}$



and $\angle ACB + \angle ACD = 180°$ (L.P.)

$\angle ACB + 100° = 180°$

$\angle ACB = 180° - 100°$

$\Rightarrow \boxed{\angle ACB = 80°}$

In ΔABC,

$\angle ABC + \angle BAC + \angle ACB = 180°$ (A.S.P.)

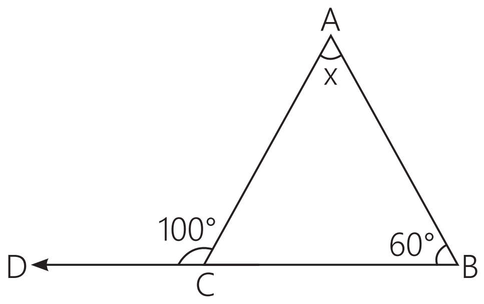
$\angle ABC + 35° + 80° = 180°$

$\angle ABC = 180° - 115°$

$\Rightarrow \boxed{\angle ABC = 65°}$

WORKSHEET 2: SECTION-A

1. By exterior angle property, exterior angle of a triangle is equal to the sum of its two opposite interior angles.



$\Rightarrow \quad \angle ACD = \angle ABC + \angle BAC$

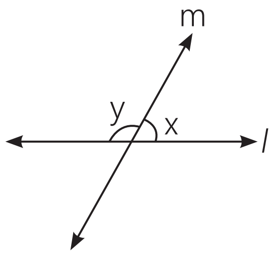
$100° = 60° + \angle BAC$

$100° - 60° = x$

$x = 40°$

2. Let $x = 1a$

and $y = 4a$



Now, $x + y = 180°$ (L.P.)

$1a + 4a = 180°$

$5a = 180°$

$a = \frac{180°}{5}$

$a = 36°$

$x = 36°$ and $y = 4 \times 36° = 144°$.

3. Sum of all angles at a point $= 180°$

$(x + 3) + (x + 20°) + (x + 7) = 180°$

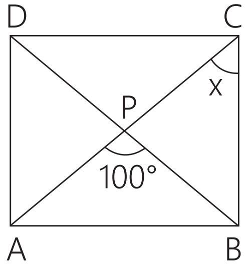
$3x + 30° = 180°$

$3x = 180° - 30°$

$= 150°$

$x = 50°$.

4. Since ABCD is a square.



$\angle B = 90°$

Since BP bisects $\angle B$

$\Rightarrow \quad \angle ABP = \angle PBC = 45°$

$\angle APB + \angle BPC = 180°$ (L.P.)

$100° + \angle BPC = 180°$

$\angle BPC = 80°$

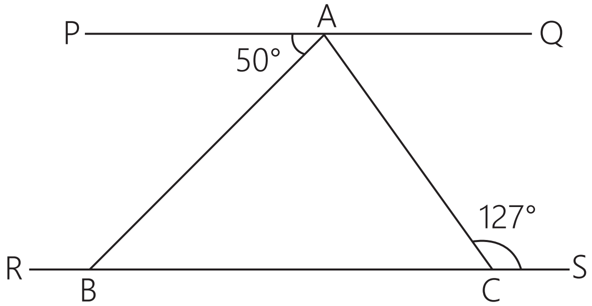
$\angle BPC + \angle PCB + \angle PBC = 180°$ (A.S.P.)

$80° + x + 45° = 180°$

$x = 180° - 125°$

$\boxed{x = 55°}$

5. PQ || RS, $\angle ACS = 127°$



and $\angle PAB = 50°$

Since, $\angle ACB + \angle ACS = 180°$ (L.P.)

$\angle ACB + 127° = 180°$

$\angle ACB = 180° - 127°$

$\angle ACB = 53°$

Since PQ || RS and AC is a transversal.

$\angle ACB = \angle QAC = 53°$

Now $\angle PAB + \angle BAC + \angle QAC = 180°$

(Sum of all angles at a point is 180°)

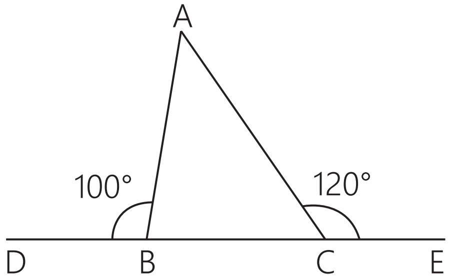
$50° + \angle BAC + 53° = 180°$

$\angle BAC = 180° - 103°$

$\boxed{\angle BAC = 77°}$

SECTION-B

6. We know that:



$\angle ABD + \angle ABC = 180^\circ$ (L.P.)

$100^\circ + \angle ABC = 180^\circ$

$\angle ABC = 80^\circ$

Similarly,

$\angle ACE + \angle ACB = 180^\circ$ (L.P.)

$120^\circ + \angle ACB = 180^\circ$

$\angle ACB = 60^\circ$

In ΔABC,

$\angle ABC + \angle BAC + \angle ACB = 180^\circ$ (A.S.P.)

$80^\circ + \angle BAC + 60^\circ = 180^\circ$

$\angle BAC = 180^\circ - 140^\circ$

$\angle BAC = 40^\circ$.

7. We know that:

$\angle AOC + \angle BOE = 70^\circ$

Also, $\angle AOC + \angle COE + \angle BOE = 180^\circ$

(Sum of all angles at a point is 180°)

$\angle AOC + \angle BOE + \angle COE = 180^\circ$

$70^\circ + \angle COE = 180^\circ$

$\boxed{\angle COE = 110^\circ}$

Now, $\angle COE + \angle BOE + \angle BOD = 180^\circ$

$110^\circ + (70^\circ - \angle AOC) + 40^\circ = 180^\circ$

$70^\circ - \angle AOE = 180^\circ - 150^\circ$

$70^\circ - \angle AOE = 30^\circ$

$70^\circ - 30^\circ = \angle AOC$

$\angle AOC = 40^\circ$

$\angle BOE = 70^\circ - \angle AOC$

$\angle BOE = 70^\circ - 40^\circ$

$\boxed{\angle BOE = 30^\circ}$

8. PQ || RS and AC is a transversal.

$\angle ACS + \angle ACB = 180^\circ$ (L.P.)

$100^\circ + \angle ACB = 180^\circ$

$\Rightarrow \angle ACB = 80^\circ$

Also, $\angle PAB = \angle ABC = 70^\circ$

(Alternate angles)

Now, in ΔABC

$\angle ABC + \angle BAC + \angle ACB = 180^\circ$ (A.S.P.)

$70^\circ + \angle BAC + 80^\circ = 180^\circ$

$\angle BAC = 180^\circ - 150^\circ$

$\boxed{\angle BAC = 30^\circ}$ and $\boxed{\angle ABC = 70^\circ}$

9. $\angle APQ = 60^\circ$ and $\angle PRD = 137^\circ$

Since AB || CD and PQ is a transversal.

$\Rightarrow \angle APQ = \angle PQR$ (alternate angles)

$\Rightarrow x = 60^\circ$

Now $\angle PRQ + \angle PRD = 180^\circ$ (L.P.)

$\angle PRQ + 137^\circ = 180^\circ$

$\angle PRQ = 43^\circ$

In ΔPQR,

$\Rightarrow \angle PQR + \angle QPR + \angle PRQ = 180^\circ$ (A.S.P.)

$x + y + 43^\circ = 180^\circ$

$60^\circ + y + 43^\circ = 180^\circ$

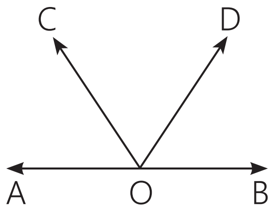
$y + 103^\circ = 180^\circ$

$y = 180^\circ - 103^\circ$

$y = 77^\circ$

$x = 60^\circ$ and $y = 77^\circ$.

10. We have, $\angle AOC + \angle BOD = 70$



Also, we know that sum of all angles at a point is 180°.

$\Rightarrow \angle AOC + \angle BOD + \angle COD = 180^\circ$

$\Rightarrow 70^\circ + \angle COD = 180^\circ$

$\Rightarrow \angle COD = 180^\circ - 70^\circ$

$\Rightarrow \boxed{\angle COD = 110^\circ}$

SECTION-C

11. $\angle PRT = 40^\circ$ and $\angle RPT = 95^\circ$

In ΔPRT,

$\angle PRT + \angle RPT + \angle PTR = 180^\circ$ (A.S.P.)

$40^\circ + 95^\circ + \angle PTR = 180^\circ$

$\angle PTR = 180^\circ - 135^\circ$

$\angle PTR = 45^\circ$

Now, $\angle PTR = \angle QTS = 45^\circ$ (V.O.A.)

In ΔTQS,

$\angle TQS + \angle TSQ + \angle QTS = 180^\circ$ (A.S.P.)

$\angle SQT + 45^\circ + 75^\circ = 180^\circ$

$\angle SQT = 180^\circ - 120^\circ$

$\boxed{\angle SQT = 60^\circ}$

12. An exterior angle of a triangle is equal to the sum of two interior opposite angles of the triangle.

Let the other interior angle = x

According to question

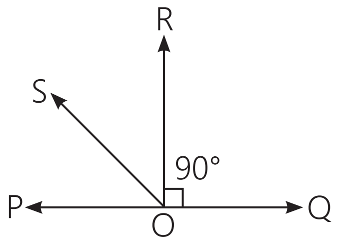
$x + 30^\circ = 110^\circ$

$x = 110^\circ - 30^\circ$

$x = 80^\circ$.

13. Since $OR \perp PQ$,

Hence $\angle ROP = 90^\circ$ and $\angle ROQ = 90^\circ$



We can say that

$ROP = \angle ROQ$

(As OR is perpendicular to line PQ)

$\angle POS + \angle ROS = \angle ROQ$

$\angle POS + \angle ROS = \angle QOS - \angle ROS$

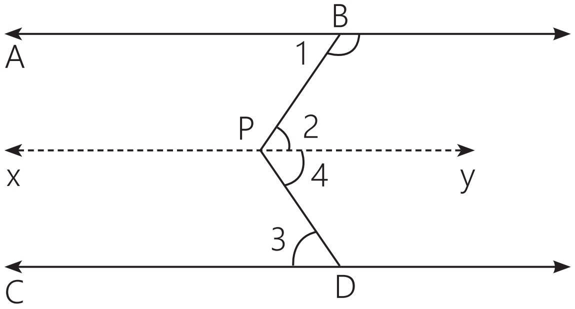
$\angle SOR + \angle ROS = \angle QOS - \angle POS$

$2\angle ROS = \angle QOS - \angle POS$

$\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$

Hence proved.

14.



Given: AB || CD and P is any point between AB and CD.

To prove: $\angle ABP + \angle CDP = \angle DPB$

Construction: Draw a line xy || AB and xy || CD.

Proof: Since AB || xy

$\angle 1 = \angle 2$ (Alternate int. angles) ...(1)

Also xy || CD

$\angle 3 = \angle 4$ (Alternate int. angles) ...(2)

Adding Eqns. (1) and (2), we get

$\angle 1 + \angle 3 = \angle 2 + \angle 4$

$\angle ABP + \angle CDP = \angle DPB$

Hence proved.

15. $AD \perp AB$ and AD || BC.

$\angle BDC = 30^\circ$ and $x : y = 11 : 19$

Let $x = 11a$ and $y = 19a$

In ΔABD,

$\angle ABD + \angle BAD + \angle ADB = 180^\circ$ (A.S.P.)

$y + 90^\circ + x = 180^\circ$

$19a + 90^\circ + 11a = 180^\circ$

$30a = 90^\circ$

$a = 3$

$\therefore$ $x = 11 \times 3 = 33^\circ$

$y = 19 \times 3 = 57^\circ$

Now, $\angle ADC = \angle DCE$

(AD || BC and DC is a transversal, so alt. angles are equal)

Now, ∠ADC = x + 30°

= 33° + 30°

= 63°

⇒ $\boxed{\angle DCE = 63°}$

SECTION-D

16. We know that sum of three angles of a triangle is 180°.

So, (2x – 7)° + (x + 25)° + (3x + 12)° = 180°

6x + 30° = 180°

6x = 150°

x = 25°

Angles are

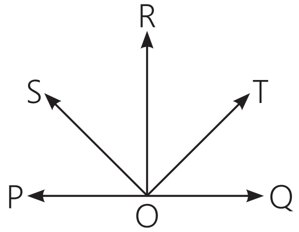
∠A = 2x – 7 = 2 × 25 – 7 = 50° – 7 = 43°

∠B = x + 25° = 25° + 25° = 50°

∠C = 3x + 12° = 3 × 25 + 12° = 75 + 12° = 87°.

17. Since POQ is a straight line.

OS bisects ∠POR and OT bisects ∠QOR.



To prove: ∠SOT is a right angle.

Proof: We know that ∠POR + ∠QOR = 180º (L.P.)

Since OS bisects ∠POR

⇒ ∠POS = ∠ROS

Also, ∠POS + ∠ROS = ∠POR

⇒ 2∠ROS = ∠POR ...(1)

Similarly OT bisects QOR

⇒ ∠QOT = ∠ROT

Also, ∠QOT + ∠ROT = ∠QOR

⇒ 2∠ROT = ∠QOR ...(2)

Now, ∠POR + ∠QOR = 180º

From eqns. (1) and (2), we get;

2∠ROS + 2 ∠ROT = 180°

2 (∠ROS + ∠ROT) = 180°

$$\angle ROS + \angle ROT = \frac{180°}{2}$$

⇒ ∠ROS + ∠ROT = 90°

and ∠SOT = ∠ROS + ∠ROT = 90°

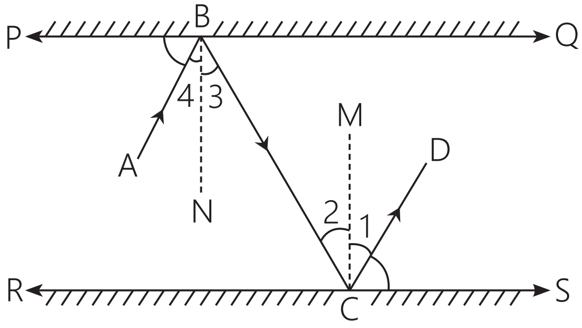
⇒ ∠SOT = 90°

⇒ ∠SOT is a right angle.

Hence proved.

18. Given: Mirror PQ || Mirror RS.

An incident ray AB strikes the mirror RQ at B, the reflected ray moves along BC and strikes the mirror RS at C and again reflects back along CD.



To prove: AB || CD

Construction: Draw BN ⊥ PQ and CM ⊥RS.

Since BN || CM

⇒ ∠3 = ∠2

⇒ 2∠3 = 2∠2

⇒ ∠3 + ∠3 = ∠2 + ∠2

(As corresponding angles are equal, lines are parallel and as angle of incidence equal to angle of reflection.)

∠1 = ∠2 and ∠3 = ∠4

⇒ ∠3 + ∠4 = ∠2 + ∠1

∠ABC = ∠BCD

AB || CD

CASE STUDY-1

(i) (d) $\angle ABL$ is the angle of incident and $\angle CBL$ is the angle of reflection.

$\therefore$ $\angle ABL = \angle CBL$

(ii) (c) $\angle BCM$ is the angle of incident and $\angle DCM$ is the angle of reflection.

$\therefore$ $\angle BCM = \angle DCM$

(iii) (b) The lines BL and CM are normal to the surfaces PQ and RS.

(iv) (b) Ray AB is parallel to Ray CD and Ray BC is transversal.

$\therefore$ $\angle ABC = \angle BCD$ (Alternate Interior angles)

(v) (a) AB is parallel to CD.

CASE STUDY-2

(i) (b) $\angle QRS + \angle QRT = 180^\circ$ (Linear Pair)

$\angle QRS = 180 - 65^\circ$

$= 115^\circ$

$\angle QRS + \angle RSQ + \angle SQR = 180^\circ$

(Angle sum property of triangle)

$115^\circ + \angle RSQ + 28^\circ = 180$

$\angle RSQ = 37^\circ$

$\angle x = \angle RSQ$

(Alternate Interior angle)

(ii) (a) $\angle y + \angle x + 90^\circ = 180^\circ$

(Angle sum property)

$\angle y + 37^\circ + 90^\circ = 180^\circ$

$\angle y = 53^\circ$

(iii) (c) $\angle QRS + \angle QRT = 180^\circ$

(Angle sum property)

$\angle QRS = 115^\circ$

(iv) (a) $\angle PSR = \angle RSQ + \angle y$

$= 37^\circ + 53 = 90^\circ$

(v) (b) $\angle RSQ = \angle x$ (Alternate Interior angle)

$= 37^\circ$

$\angle PQR = \angle x + \angle RQS$

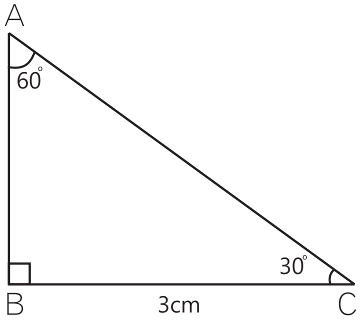
$= 37^\circ + 28^\circ = 65^\circ$

$\angle QSR = 37^\circ$

Chapter
7
Triangles

MULTIPLE CHOICE QUESTION

1. (b) Consider the triangle ABC with BC = 3cm



$\frac{AB}{BC} = \tan 30^\circ$

$\frac{AB}{3} = \frac{1}{\sqrt{3}} \Rightarrow AB = \sqrt{3}\,cm$

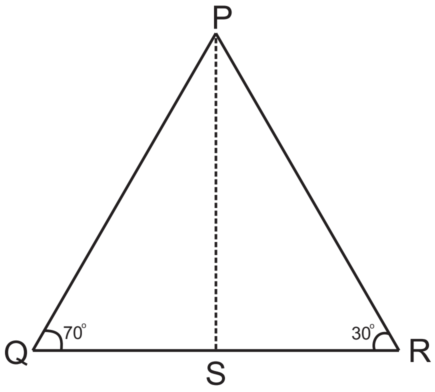
$\frac{BC}{AC} = \text{Cos } 30$

$\frac{3}{AC} = \frac{\sqrt{3}}{2}$

$AC = 2\sqrt{3}$

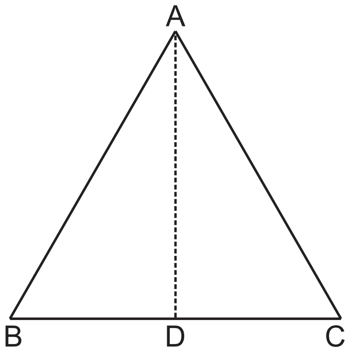
AC is the hypotenuse which is double the smallest side AB.

2. (b)



PQ < PR (Side opposite to smaller angle is smaller).

3. (a)



In ΔABD

AB + BD > AD ...(i)

(Sum of two sides of triangle is greater than the third side)

In ΔACD

AC + CD > AD ...(ii)

Adding (i) and (ii)

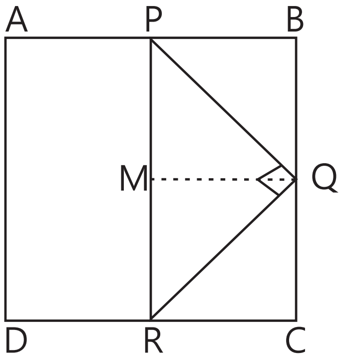
AB + BD + AC + CD > 2AD

BD + CD = BC

∴ AB + AC + BC > 2AD

4. (b) In an equilateral triangle, all the three sides are equal.

5. (a)



Construction : Join Q to M such that PM = RM and

$\angle PMQ = \angle QMR = 90^\circ$

In ΔPQM and ΔMQR

$QM = QM$ (Common)

$PM = MR$ (M is the bisector of PR)

$\angle PMQ = \angle QMR\ (90^o)$

ΔPQM and ΔMQR are congruent by SAS rule.

$\therefore$ $PQ = MR$ (CPCT)

$\angle QPR = \angle QRP$

(Angles opposite to the equal sides are equal)

In ΔPQR

$$\angle PQR + \angle QRP + \angle QPR = 180^o$$

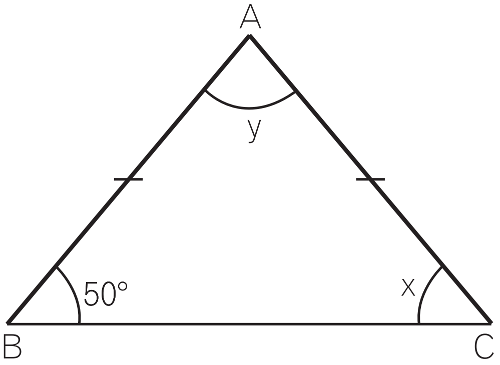
$$\angle QRP = \angle QPR$$

$$2\angle QPR = 180^o - 90^o$$

$$\angle QPR = 45^o$$

WORKSHEET 1: SECTION-A

1. $AB = AC, \angle B = \angle C, x = 50°$



We know that:

$\angle A + \angle B + \angle C = 180°$

$50° + 50° + \angle A = 180°$

Angles opposite to equal sides are equal.

$100° + \angle A = 180°$

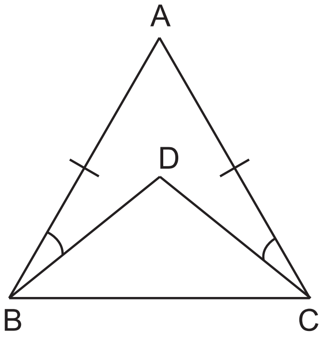
$\angle A = 180° - 100° = 80° = y°$

$\therefore$ $x = 50°, y = 80°$

2. $\angle ABD : \angle ACD$

As $AB = AC$

$\therefore$ $\angle ABC = \angle ACB$



then $\frac{1}{2}(\angle ABC) = \frac{1}{2}(\angle ACB)$

$\Rightarrow$ $\angle ABD = \angle ACD$ i.e. 1 : 1

3. In ΔABD, we get:

$$\angle BAD + \angle ABD + \angle ADB = 180°$$

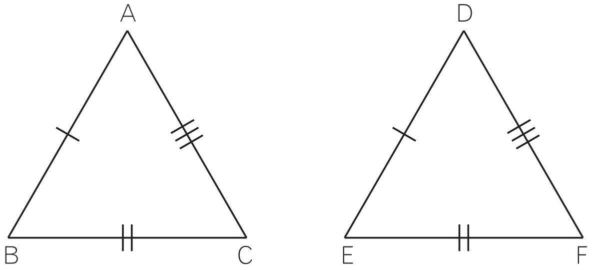
$$\angle BAD + 35° + 90° = 180°$$

$$\angle BAD + 125° = 180°$$

$$\angle BAD = 180° - 125°$$

$$\angle BAD = 55°$$

4.



From ΔABC and ΔDEF,

$\Delta ABC \cong \Delta DEF$

Because $AB = DE$

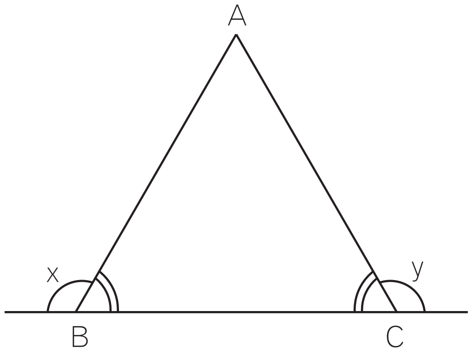
$BC = EF$

$AC = DF$

From, S.S.S. congruent rule.

$\Delta ABC \cong \Delta DEF$

5.



$\angle B = 180 - x$

$\angle C = 180 - y$

Now, $\angle B > \angle C$

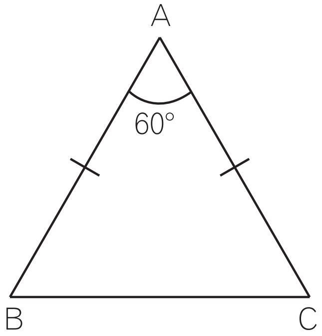
$\Rightarrow 180 - x > 180 - y$

$\Rightarrow -x > -y$

$\Rightarrow x < y$

SECTION-B

6.



Since AB = AC

$\angle B = \angle C$

Now in ΔABC

$\angle A + \angle B + \angle C = 180$

$60 + \angle B + \angle C = 180$

$\angle B + \angle C = 120$

$\angle B + \angle B = 120$

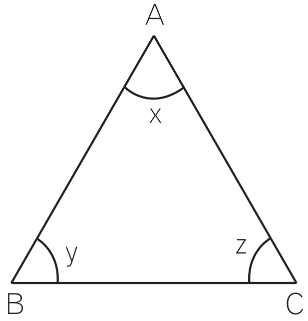
$2\angle B = 120$

$\angle B = 60$

$\angle C = \angle B = 60$

Since each of the angle of ΔABC is of 60° so it is an equilateral triangle.

7.



Given $x + y = z$

In ΔABC, $x + y + z = 180°$

$z + z = 180°$

$2z = 180°$

$\therefore$ Third angle is $z = 90°$

8. $x > y \Rightarrow -x < -y$

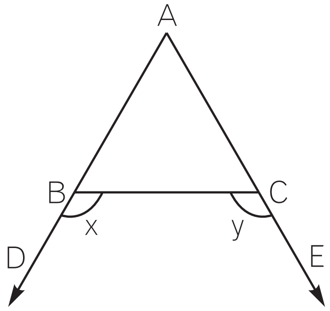
$\Rightarrow (180° - x) < (180° - y)$

$\Rightarrow \angle ABC < \angle ACB$

$\Rightarrow \angle ACB > \angle ABC$

$\Rightarrow AB > AC$

[$\because$ Side opposite to larger angle is larger.]



Hence, AB > AC

9. Consider ΔABC and ΔABD

AC = AD (Given)

$\angle CAB = \angle DAB$ (Since AB bisects $\angle A$)

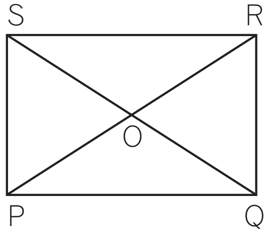
AB = AB (common)

$\therefore \Delta ABC \cong \Delta ABD$ (SAS)

$\Rightarrow$ BC = BD (CPCT)

Hence Proved.

10.



In ΔPQR

PQ + QR > PR ...(1)

In ΔRQS

SR + RQ > SQ ...(2)

In ΔPSR, SR + PS > RP ...(3)

[Since in sum of two sides is greater than the third side]

In ΔPSQ, $PS + PQ > SQ$ ...(4)

Add (1), (2), (3), (4)

$PQ + QR + SR + RQ + SR + PS + PS + PQ > PR + SQ + RP + SQ$

$\Rightarrow 2PQ + 2QR + 2SR + 2PS > 2SQ + 2PR$

$\Rightarrow 2\,(PQ + QR + SR + PS) > 2\,(PR + SQ)$

$\Rightarrow PQ + QR + SR + PS > PR + SQ$

Hence Proved.

SECTION-C

11. From the given figure:

(a) In ΔAPB and ΔCQD

$\angle CDQ = \angle ABQ$ [Alternate angles]

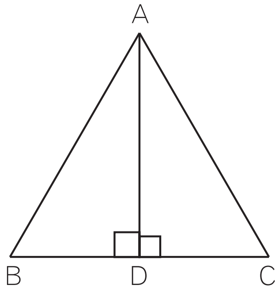
$\angle P = \angle Q$

$AB = CD$

$\therefore \Delta APB \cong \Delta CQD$ [$\because$ AAS congruent rule]

(b) $AP = CQ$ [C.P.C.T.]

12. From the given figure:



In ΔADB and ΔADC, we get:

$BD = DC$ [AD is the median of a triangle]

$AD = AD$ [Common line segment]

$\angle ADB = \angle ADC = 90°$

$\therefore \Delta ABD \cong \Delta ADC$ [By SAS congruent rule]

Hence Area (ΔABD) = Area (ΔADC)

Hence Proved.

13. $\angle DCA = \angle ECB$

$\Rightarrow \angle DCA + \angle DCE = \angle ECB + \angle DCE$

$\Rightarrow \angle ACE = \angle DCB$

Also, $\angle A = \angle B$, $AC = BC$

$\Rightarrow \Delta DBC \cong \Delta EAC$ (ASA)

$DC = EC$ (CPCT)

Hence Proved.

14. $\angle PQR = \angle S + \angle SPQ$

Since $\angle PQR = \angle R$

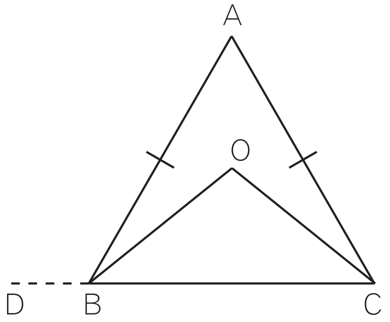
$\Rightarrow \angle R = \angle S + \angle SPQ$

$\Rightarrow \angle R > \angle S$

$PS > PR$

Hence Proved.

15. Consider the figure:



Here,

$AB = AC$

$\Rightarrow \angle ACB = \angle ABC$

$\Rightarrow \frac{1}{2}\angle ACB = \frac{1}{2}\angle ABC$

$\Rightarrow \angle OCB = \angle OBC$

In ΔBOC, we have

$\angle OBC + \angle OCB + \angle BOC = 180°$

$\Rightarrow 2\angle OBC + \angle BOC = 180°$

$\Rightarrow \angle ABC + \angle BOC = 180°$

$\Rightarrow 180° - \angle DBA + \angle BOC = 180°$

$\Rightarrow \angle DBA = \angle BOC$

SECTION-D

16. Construction: Join PR

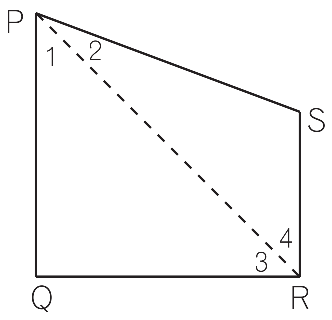
To prove: $\angle R > \angle P$

Proof: Consider the quadrilateral PQRS, drawn below.

Since PQ is the longest side

$\Rightarrow PQ > QR$

$\Rightarrow \angle 3 > \angle 1$ ...(1)



Also, Since SR is the shortest side

⇒ SR < PS

⇒ PS > SR

⇒ ∠4 > ∠2 ...(2)

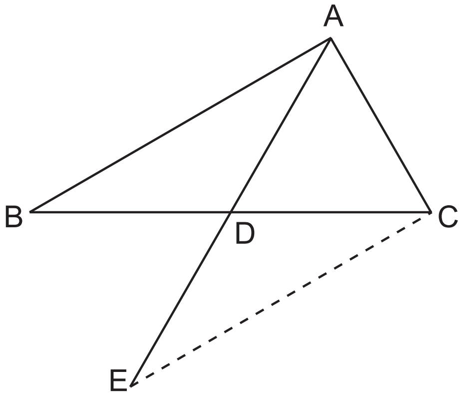
Add (1) and (2)

∠3 + ∠4 > ∠1 + ∠2

∠R > ∠P

Similarly ∠S > ∠Q

17. Given: ΔABC in which AD is median.



To prove: AB + AC > 2AD

Construction: Produce AD to E

Such that AD = DE. Join EC.

Proof: In ΔADB and ΔEDC, we get;

AD = DE [By construction]

∠ADB = ∠EDC [vert. opp. angles]

BD = CD [∵ D is the mid-point of BC]

∴ ΔADB ≅ ΔEDC [By SAS criteria]

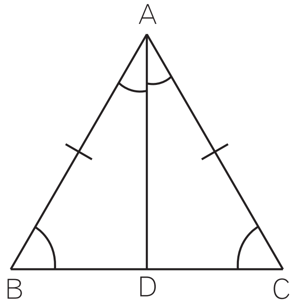
∴ AB = EC [C.P.C.T.]

EC + AC > AE [Sum of 2 sides of a Δ is greater than the III side]

AB + AC > 2AD [EC = AB, AE = 2AD]

Hence AB + AC > 2AD.

18. Given: ΔABC in which AB = AC



To Prove: ∠B = ∠C

Construction: Draw AD, the bisector of ∠A, to meet BC in D.

Proof: In ΔABD and ΔACD, we get

AB = AC (given)

AD = AD (Common line segment)

∠BAD = ∠CAD (By Construction)

∴ ΔABD ≅ ΔACD [∵ SAS criteria]

Hence, ∠B = ∠C (C.P.C.T.)

Hence Proved.

To Prove: ∠ABD = ∠ACD

In ΔABC, AB = AC ⇒ ∠ABC = ∠ACB ...(1)

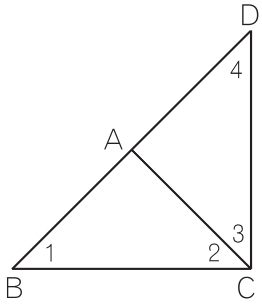
In ΔDBC, DB = DC ⇒ ∠DBC = ∠DCB ...(2)

Add (1), (2) ⇒ ∠ABC + ∠DBC

= ∠ACB + ∠DCB ⇒ ∠ABD = ∠ACD

Hence Proved.

19.



Since AB = AC ...(1)

⇒ ∠1 = ∠2

Also, AB = AD ...(2)

From (1), (2)

$\Rightarrow AC = AD$

$\Rightarrow \angle 3 = \angle 4$

Now in ΔBCD

$\angle B + \angle C + \angle D = 180^\circ$ (Angle sum property)

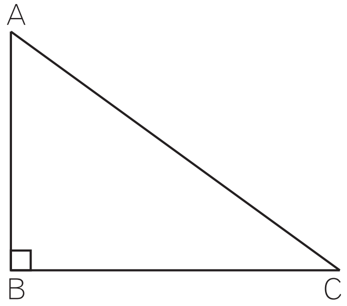
$$\begin{aligned} \angle 1 + \angle 2 + \angle 3 + \angle 4 &= 180^\circ \\ 2\angle 2 + 2\angle 3 &= 180^\circ \\ 2(\angle 2 + \angle 3) &= 180^\circ \\ \angle 2 + \angle 3 &= 90^\circ \\ \angle C &= 90^\circ \end{aligned}$$

$\Rightarrow \angle BCD$ is a right angle.

Hence Proved.

20. In right ΔABC,

$\angle B = 90^\circ$



Then $\angle A + \angle C = 90^\circ$

$\therefore \angle B > \angle A$ and $\angle B > \angle C$

$\Rightarrow AC > BC$ and $AC > AB$

[side opposite to larger angle is longer]

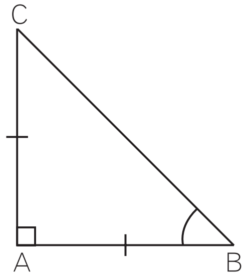
$\therefore$ AC is the longest side

Hence, in a right triangle, the hypotenuse is the longest side.

Hence Proved.

WORKSHEET 2: SECTION-A

1. ΔABC isosceles right angled triangle in which



$\angle A = 90^\circ$ and $AB = AC$

$\angle A + \angle B + \angle C = 180^\circ$

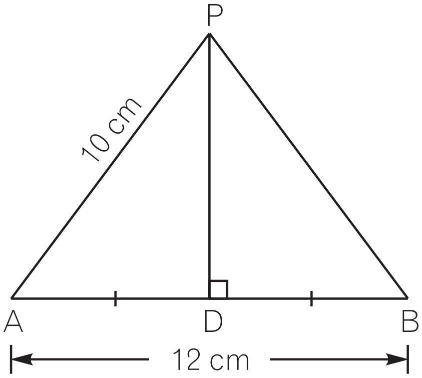
$90^\circ + \angle B + \angle B = 180^\circ$

Angles opposite to equal sides are equal.

$2\angle B = 180^\circ - 90^\circ = 90^\circ$

$\angle B = 45^\circ, \angle C = 45^\circ$

2.



To find: AD

Solution $AD = DB = \frac{1}{2}(12)$

$AD = 6$ cm

In ΔPDA

$$\begin{aligned} AP^2 &= PD^2 + AD^2 \\ (10)^2 &= PD^2 + (6)^2 \\ 100 &= PD^2 + 36 \\ PD^2 &= 100 - 36 = 64 \\ PD &= 8 \text{ cm} \end{aligned}$$

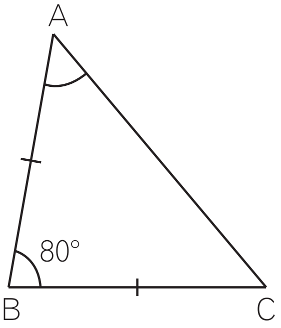
3. Yes, Sides 8 cm, 7 cm, 4 cm

ΔABC is constructed because the sum of two sides is greater than third

$7 + 4 > 8$

$11 \text{ cm} > 8 \text{ cm}$

4. In ΔABC,



$\angle A = \angle C$ [AB = BC]

In ΔABC,

$\angle A + \angle B + \angle C = 180°$

$\angle A + 80° + \angle A = 180°$

$2\angle A = 180 - 80 = 100$

$\angle A = 50°, \angle C = 50°$

SECTION-B

5. Since $AB = AD \Rightarrow \angle ABD = \angle ADB$

Also $BC = CD \Rightarrow \angle CBD = \angle CDB$

[Angle opposite to equal sides are equal]

So, $\angle ABD + \angle CBD = \angle ADB + \angle CDB$

$\Rightarrow \angle B = \angle D$

$\Rightarrow \angle ABC = \angle ADC$

6. From the given figure:

$\angle ACD = 120°$

$AB = AC$ [given]

$\angle B = \angle C$

$\angle ABC = \angle ACB = 60°$

In ΔABC,

$\angle BAC + \angle ABC + \angle ACB = 180°$

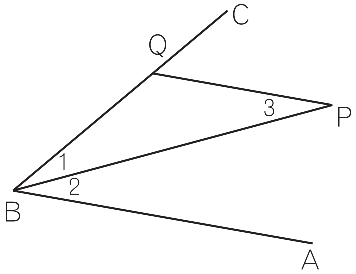
$\angle A + 60° + 60° = 180°$

$\angle A + 120° = 180°$

$\angle A = 180° - 120° = 60°$

$\angle A = 60°$

7.



Since BP bisects, $\angle B$

$\Rightarrow \angle 1 = \angle 2$

Also, $\angle 2 = \angle 3$ (Alternate interior angles)

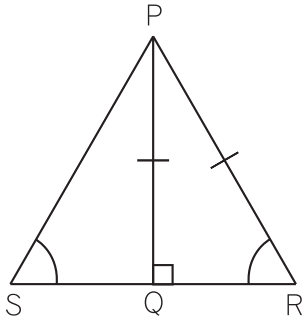
$\therefore \angle 1 = \angle 3$

$\Rightarrow BQ = PQ$

(Sides opposite to equal angles)

$\Rightarrow \Delta BQP$ is isosceles triangle.

8. Prove that: PS > PQ



When altitude is equal to one of its side then opposite side of an altitude is greater.

Given PQ = PR

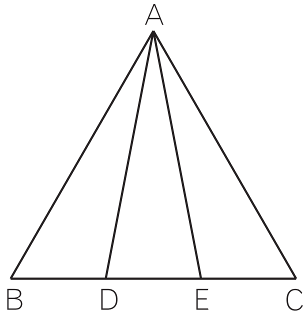
where PQ is a altitude.

$\therefore PQ < PS$

Hence Proved.

SECTION-C

9.



Since $AD = AE$

$\Rightarrow \angle ADE = \angle AED$

[Angle opposite equal sides]

$\Rightarrow 180 - \angle ADB = 180 - \angle AEC$

$\Rightarrow \angle ADB = \angle AEC$

Also, $AD = AE, BD = CE$

$\Rightarrow \Delta ABD \cong \Delta ACE$ [SAS]

10. Consider ΔDAB and ΔCBA

$AD = BC$ (Given)

$BD = AC$ (Given)

$AB = AB$ (Common)

$\Rightarrow \Delta DAB \cong \Delta CBA$ (SSS)

$\Rightarrow \angle DAB = \angle CBA$ (CPCT)

11. $AD = AC$ (Given)

$\Rightarrow \angle ACD = \angle ADC$

(Angles opposite to equal sides)

Now, $\angle ADC$ is exterior angle for ΔABD

$\angle ADC = \angle ABD + \angle BAD$

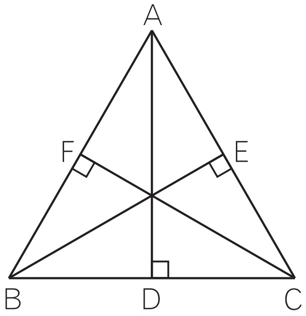
$\Rightarrow \angle ADC > \angle ABD$

$\angle ACD > \angle ABD$

$AB > AC$

$\therefore AB > AD$ (As $AC = AD$)

12. Consider ΔAEB and ΔAFC



$\angle A = \angle A$ (Common)

$\angle AEB = \angle AFC = 90°$

$BE = CF$ (Given)

$\Rightarrow \Delta AEB \cong \Delta AFC$ (AAS)

$\Rightarrow AB = AC$ ...(1) (CPCT)

Similarly $\Delta AFC \cong \Delta BEC$

$\Rightarrow AC = BC$ ...(2) (CPCT)

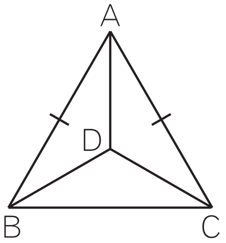
By (1), (2) $AB = BC = AC$

$\Rightarrow \Delta ABC$ is equilateral.

Hence Proved.

SECTION-D

13. Since $\angle DBC = \angle DCB$



$\Rightarrow DB = DC$ (Sides opposite to equal angles)

$AD = AD$ (Common)

$AB = AC$ (Given)

$\Rightarrow \Delta ADB \cong \Delta ADC$ (SSS)

$\Rightarrow \angle DAB = \angle DAC$ (CPCT)

$\Rightarrow AD$ bisects $\angle BAC$ of ΔABC

14. Given: $\angle C = 90°$, from given figure M is the mid-point of AB.

$PM = CM$

Prove that:

(a) $\Delta AMC \cong \Delta BMD$

(b) $\Delta DBC \cong \Delta ACB$

Proof:

(a) In ΔAMC and ΔBMD, we get:

$\angle DMB = \angle AMC$ [Vertically opp. angles]

$BM = MA$ [Given]

$MC = DM$ [Given]

$\therefore \Delta AMC \cong \Delta BMD$

[$\because$ SAS congruent criteria]

(b) Since $\Delta AMC \cong \Delta BMD$

$\Rightarrow \angle MAC = \angle MBD$ [CPCT]

$\Rightarrow DB \parallel AC$

[Since alternate angles are equal]

$\Rightarrow \angle B + \angle C = 180$

$\Rightarrow \angle B + 90 = 180$

$\Rightarrow \angle B = 90°$

Consider ΔDBC and ΔACB

$DB = AC$

[Since $\Delta AMC \cong \Delta BMD$ So, $DB = AC$ by CPCT]

$\angle B = \angle C = 90°$

$BC = BC$ [Common]

$\Rightarrow \Delta DBC \cong \Delta ACB$ [SAS]

Hence Proved.

15. (a) Consider ΔABM and ΔPQN

$AB = PQ$ [Given]

$AM = PN$ [Given]

$BC = QR \Rightarrow \frac{1}{2} BC = \frac{1}{2} QR$

[Since AM and PN are Medians]

$\Rightarrow$ $BM = QN$

$\therefore$ $\Delta ABM \cong \Delta PQN$ [SSS]

$\Rightarrow$ $\angle B = \angle Q$ [CPCT]

(b) Consider ΔABC and ΔPQR

$AB = PQ$ [Given]

$\angle B = \angle Q$ [Proved above]

$BC = QR$ [Given]

$\Rightarrow$ $\Delta ABC \cong \Delta PQR$ (SAS)

CASE STUDY-1

1. (i) (b) Consider ΔAMC and ΔDMB

$AM = BM$

[M is the midpoint of hypotenuse AB]

$MC = DM$ (Given)

$\angle DMB = \angle AMC$

[vertically opposite angle]

$\therefore$ $\Delta AMC \cong \Delta DMB$ [SAS rule]

(ii) (a) Consider ΔAMC and ΔDMB

$AM = BM$

[M is the midpoint of hypotenuse AB]

$MC = DM$ (Given)

$\angle DMB = \angle AMC$

[vertically opposite angle]

$\therefore$ $\Delta AMC \cong \Delta DMB$ [SAS rule]

Adding ΔBMC on both sides

$\Delta AMC + \Delta BMC \cong \Delta DMB + \Delta BMC$

$\Delta ABC \cong \Delta DBC$

$\therefore$ $\angle DBC = \angle ACB = 90^o$

(iii) (d) Consider ΔAMC and ΔDMB

$AM = BM$

[M is the midpoint of hypotenuse AB]

$MC = DM$ (Given)

$\angle DMB = \angle AMC$

[vertically opposite angle]

$\therefore$ $\Delta AMC \cong \Delta DMB$ [SAS rule]

Adding ΔBMC on both sides

$\Delta AMC + \Delta BMC \cong \Delta DMB + \Delta BMC$

$\Delta ABC \cong \Delta DBC$

(iv) (b) Consider ΔAMC and ΔDMB

$AM = BM$

[M is the midpoint of hypotenuse AB]

$MC = DM$ (Given)

$\angle DMB = \angle AMC$

[vertically opposite angle]

$\therefore$ $\Delta AMC \cong \Delta DMB$ [SAS rule]

As $\Delta AMC \cong \Delta BMD$

$\therefore$ $DM = AM$ [CPCT]

$DM = AM = \frac{1}{2} AB$

[M is the midpoint of AB]

$DM = MC$ [Given]

$\therefore$ $MC = \frac{1}{2} AB$

(v) (a) Consider ΔAMC and ΔDMB

$AM = BM$

[M is the midpoint of hypotenuse AB]

$MC = DM$ (Given)

$\angle DMB = \angle AMC$

[vertically opposite angle]

$\therefore$ $\Delta AMC \cong \Delta DMB$ [SAS rule]

As $\Delta AMC \cong \Delta BMD$

$\therefore$ $BM = CM$

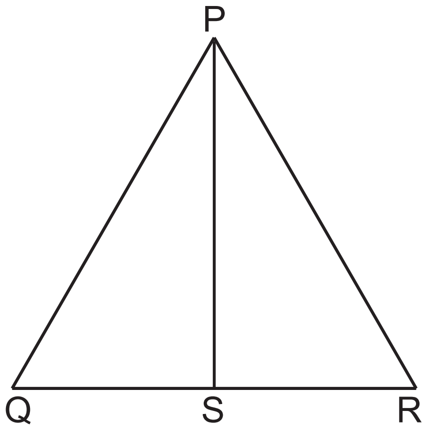
$DM = CM = \frac{1}{2} CD$

(M is the midpoint of DC)

$\therefore$ $BM = \frac{1}{2} CD$

CASE STUDY-2

2. (i) (a)



In ΔPQS

$PQ + QS > PS$ ...(i)

(Sum of two sides of triangle is greater than third side.)

In ΔSPR

$PR + SR > PS$ ...(ii)

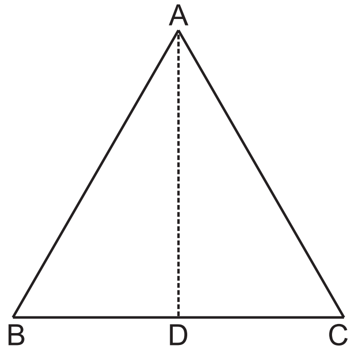
Add (i) and (ii)

$PQ + QS + PR + SR > 2PS$

$QS + SR = QR$

$\therefore$ $PQ + QR + PR > 2PS$

(ii) (b)



In ΔABD

$AB + BD > AD$ ...(i)

(Sum of two sides of triangle is greater than the third side)

In ΔACD

$AC + CD > AD$ ...(ii)

Adding (i) and (ii)

$AB + BD + AC + CD > 2AD$

$BD + CD = BC$

$\therefore$ $AB + AC + BC > 2AD$

(iii) (c) Sum of any two sides of triangle should be greater than the third side

4 cm + 3 cm > 5 cm

7 cm > 5 cm

(iv) (b) Angles opposite to smaller sides are smallest.

(v) (a) Angles opposite to equal sides are equal.

Chapter 8 Quadrilaterals

MULTIPLE CHOICE QUESTION

1. (d) $3x - 2 = 50 - x$

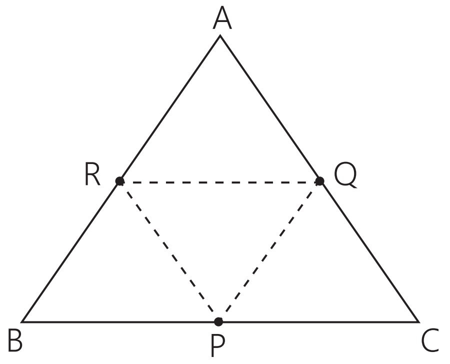
$4x = 52$

$x = 13°$

2. (d) Sum of adjacent angle in a parallelogram is 180°.

3. (b) The diagonals of rectangle are equal but not perpendicular.

4. (c)



Construction: Join RQ

$AC = 16$ cm

$AQ = QC = \frac{1}{2} AC$ [Q is midpoint of AC]

$AR = RB = \frac{1}{2} AB$ [R is midpoint of AB]

$BP = PC = \frac{BC}{2}$ [P is midpoint of BC]

In ΔARQ and ΔPRQ

$RQ = RQ$ (common)

$\angle ARQ = \angle PQR$ (Alternate Interior angles)

∴ ΔARQ and ΔPRQ are similar

$AR = PQ$ and $RP = AQ$ [CPST]

$AR = PQ = RB = \frac{1}{2} AB$

$= \frac{1}{2}(24 \text{ cm}) = 12$ cm

$RP = AQ = QC = \frac{1}{2} AC$

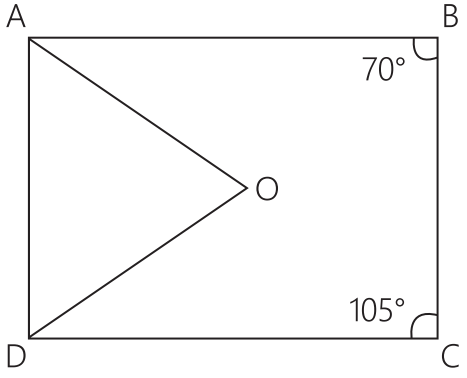
$= \frac{1}{2}(16 \text{ cm}) = 8$ cm

∴ Perimeter of quadrilateral ARPQ is given by

$AR + RP + PQ + AQ$

$= 12 + 8 + 12 + 8 = 40$ cm

5. (c)



$\angle A + \angle B = 180°$ (sum of angle on one side of straight line is 180°)

$\angle A = 180 - \angle B$

$= 180 - 70 = 110°$

$\angle BAD = \angle DAO + \angle BAO$

$[\angle DAO = \angle BAO$ as AO is bisector of $\angle A]$

$110° = 2\angle DAO$

$\angle DAO = 55°$

Similarly

$\angle ADO = 37.5°$

In ΔAOD

$\angle ADO + \angle DAO + \angle AOD = 180°$

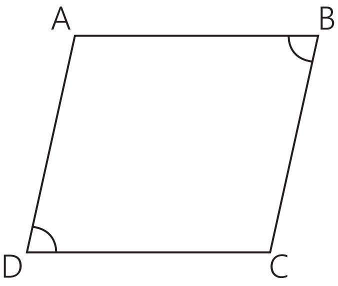
(Angle sum property)

$37.5° + 55° + \angle AOD = 180°$

$\angle AOD = 87.5°$

WORKSHEET 1: SECTION-A

1.



$\angle D = \angle B = 110°$

[Since opposite angles of parallelogram are equal].

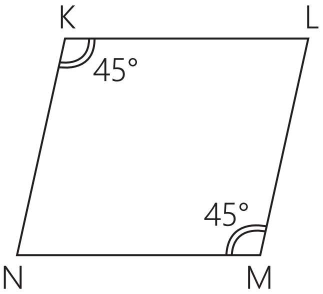
2. $110° + 85° + 45° + 65° = 305°$

But sum of angles of a quadrilateral is 360°.

So, these can not be the angles of a quadrilateral.

3. Yes, if three angles of quadrilaterals are equal then the obtained quadrilateral is parallelogram.

4.



$\angle K = \angle M$

(Opposite angles of parallelogram)

$\angle K + \angle L = 180°$ (Co-interior angles)

$45° + \angle L = 180°$

$\angle L = 180° – 45° = 135°$

$\angle N = \angle L$

(Opposite angles of parallelogram)

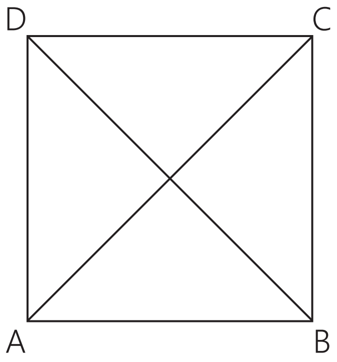
$\angle N = 135°$.

5. Perimeter = 4 + 6 + 4 + 6 = 20 cm

∴ Perimeter of a kite is 20 cm.

SECTION-B

6.



Given: Let ABCD is parallelogram where AC = BD.

To prove: ABCD is rectangle.

Proof: Rectangle is a parallelogram with one angle is 90°, we have to prove one of its interior angle is 90°.

In ΔABC and ΔDCB, we get:

AB = DC (Opp. side of parallelogram)

BC = BC (Common)

AC = DB (Given)

$\Delta ADC \cong \Delta DCB$ (SSS congruence rules)

$\angle ABC = \angle DCB$ (C.P.C.T.) ...(2)

Now AB || DC

(Opp. side of parallelogram are parallel)

and BC is transversal line

$\angle B + \angle C = 180°$

(Interior angle on same side of transversal are supplementary)

$\angle B + \angle B = 180°$ [from (1)]

$$\angle B = \frac{180}{2} = 90°$$

ABCD is parallelogram with one angle 90°.

∴ ABCD is rectangle.

7. Let a = 3x, b = 5x, c = 9x, d = 13x

Sum of angle of quadrilateral is 360°.

$a + b + c + d = 360°$

$3x + 5x + 9x + 13x = 360°$

$30x = 360°$

$x = 12°$

$a = 3 \times x = 3 \times 12 = 36°$

$b = 5 \times x = 5 \times 12 = 60°$

$c = 9 \times x = 9 \times 12 = 108°$

$d = 13 \times x = 13 \times 12 = 156°$

$\therefore$ Angles of quadrilateral are 36°, 60°, 105°, 156°.

8. $\angle C$ and $\angle D$ are adjacent angle of parallelogram. From given figure,

$\angle C + \angle D = 180°$

$x + 75° = 180°$

$x = 105°$

$y = x$ [Alt. int. angle]

$x + y = 105° + 105° = 210°$.

9. Sum $4x + 7x + 15x + 10x = 360°$

$36x = 360°$

$$x = \frac{360}{36} = 10$$

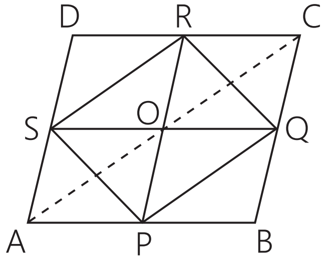
$x = 10°$

Smallest angle $= 4 \times x = 40°$

Largest angle $= 15 \times x = 150°$.

SECTION-C

10.



Given: ABCD is quadrilateral P, Q, R, S are mid-point of side AB, BC, CD, DA.

To prove: PR and SQ bisect each other

Construction: Join A and C.

Proof: In ΔADC,

S is mid-point of AD and R is mid-point of CD.

Line segment joining the mid-point of two sides AD and CD is parallel to 3rd side and half of it.

$\therefore$ SR || AC and $SR = \frac{1}{2} AC$ ...(1)

In ΔABC

P is mid-point of AB and Q is mid-point of BC.

Line segment joining the mid-point of two sides AB and BC is parallel to 3rd side and half of it.

$\therefore$ PQ || AC and $PQ = \frac{1}{2} AC$ ...(2)

From (1) and (2)

PQ = SR and PQ || SR

In PQRS

One pair of opp. side is parallel and equal.

Hence PQRS is parallelogram.

Hence proved.

11. Given: ABC is triangle D, E, F are mid-points of side AB, BC and CA.

To prove: ΔABC is divided into 4 congruent triangles.

Proof. D and F are mid-points of side. AB and AC of ΔABC.

DF || BC

(Line segment joining the mid-point of two sides of triangle is parallel to 3rd side)

Similarly we can write:

DE || AC and EF || AB

Now in DBEF

DF || BE

[Since DF || BC parts of parallel lines are parallel]

DB || EF

[Since EF || AB parts of parallel lines are parallel]

Since both pair of opp. side are parallel

DBEF is parallelogram.

DBEF is a parallelogram and DE is a diagonal.

∴ ΔDBE ≅ ΔDFE

[Diagonal of parallelogram divide to two congruent triangles] ...(1)

Similarly, DFCE is parallelogram

ΔDFE ≅ ΔCEF ...(2)

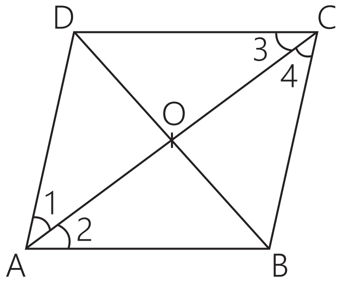
Similarly, ΔADF ≅ ΔDFE ...(3)

From (1), (2), (3), we get:

ΔDBE ≅ ΔDFE ≅ ΔCEF ≅ ΔADF

All 4 triangles are congruent.

12.



Given: ABCD is rhombus.

To prove:

i. AC bisect ∠A and ∠C.

i.e. ∠1 = ∠2

and ∠3 = ∠4

ii. BD bisects ∠B and ∠D.

Proof: In ΔADC

AB = BC (Side of rhombus are equal)

So, ∠4 = ∠2

(Angle opp. to equal sides are equal) ...(1)

Now, AD || BC

(Opp. side of rhombus are parallel and transversal AC)

∠1 = ∠4 (Alt. angle) ...(2)

Now AB || DC (Opposite sides of rhombus are parallel)

and transverse AC,

∠2 = ∠3 (alternate angle) ...(3)

From (1) and (3), we get:

∠3 = ∠4

AC bisect ∠C

From (1) and (2), we get:

∠1 = ∠2

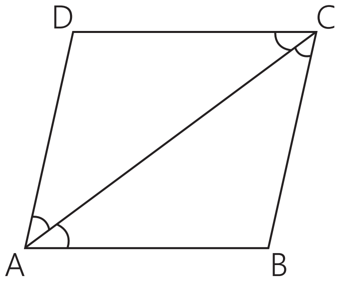
AC bisect ∠A.

Hence, AC bisect ∠C and ∠A

Similarly we can prove that BD bisect ∠B and ∠D.

Hence proved.

13.



Given: ABCD is parallelogram with AC as its diagonal.

To prove: ΔABC ≅ ΔADC.

Proof: Opp. sides of parallelogram is parallel.

So, AB || DC and AD || BC

Since AB || DC and AC is transversal.

∠BAC ≅ ∠DCA (Alt. angle) ...(1)

Since AD || BC and AC is transversal.

∠DAC ≅ ∠BCA (Alt. angle) ...(2)

In ΔADC and DABC

∠BAC = ∠DCA From (1)

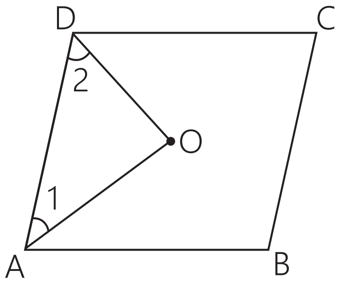
AC = AC (Common)

∠DAC = ∠BCA From (2)

∴ ΔABC ≅ ΔADC (ASA) Congruency)

Hence proved.

14.



Given: ABCD is parallelogram.

To prove: $\angle AOD = 90°$

Proof: $\angle DAB + \angle ADC = 180°$

(Angles on same side of transversal are supplementary)

$$\frac{1}{2}\angle DAB + \frac{1}{2}\angle ADC = 90°$$

$$[(\angle DAB + \angle ADC)] = 90°$$

$$\angle 1 + \angle 2 = 90°$$

$$\angle OAD + \angle ODA = 90°$$

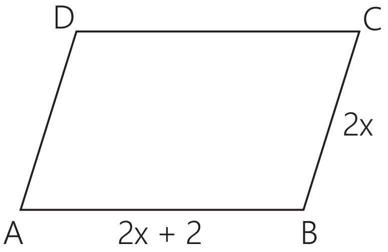
In ΔODA, we get:

$$\angle 1 + \angle 2 + \angle AOD = 180°$$

$$90° + \angle AOD = 180°$$

$$\angle AOD = 90°.$$

15.



AD = BC = 2x

CD = AB = 2x + 2

[Opposite sides of parallelogram are equal]

Now, perimeter = 40 cm

$$AB + BC + CD + AD = 40$$

$$2x + 2 + 2x + 2x + 2 + 2x = 40$$

$$8x + 4 = 40$$

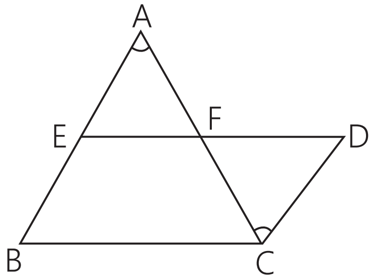
$$8x = 36$$

$$x = \frac{36}{8} = \frac{9}{2}$$

$$x = \frac{9}{2} = 4.5$$

SECTION-D

16. From mid-point theorem:



The line segment joining the mid-points of two sides of Δ is parallel to third side.

Given: ABC is Δ E and F are mid-points of AB and AC.

To prove: EF || BC.

Construction: Through C draw a line segment parallel to AB and extend EF to meet this line at D.

Proof: Since AB || CD (As AB || CD) with transversal ED.

$\angle BAC = \angle ACD$ (Alt. angle) ...(1)

In ΔAEF and ΔCDF, we get:

$\angle BAC = \angle ACD$ From (1)

AF = CF (F is mid-point AC)

$\angle AFE = \angle CFD$

(Vertically opposite angles)

∴ $\Delta AEF \cong \Delta CDF$ (AAS)

So, EA = DC (CPCT)

But EA = EB (E is mid-point of AB)

Hence, EB = DC

Now in quadrilateral EBCD

EB || CD and EB = DC

Thus, one pair of opp. sides is equal and parallel. Hence EBCD is parallelogram. Since opp. sides of parallelogram are parallel ED || BC.

∴ EF || BC

Hence proved.

17. (a) MD ⊥ AC

As MD || BC and AC is transversal

∠MDC + ∠BCD = 180°

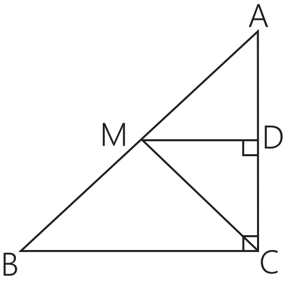
(Int. angle on same side of transversal are supplementary)

∠MDC + 90° = 180°

∠MDC = 90°

MD ⊥ AC.

(b) D is mid-point of AC.



Given: ABC is a triangle right angled at C.

∠C = 90°.

M is mid-point of AB

MD || BC

To prove: D is mid-point of AC,

Proof: In ΔABC,

M is mid-point of AB

MD || BC.

D is mid-point of AC.

(Line through mid-point of one side of triangle, parallel to another side bisect the third side)

(c) CM = MA = $\frac{1}{2}$ AB

Join MC, in ΔAMD and ΔCMD

AD = CD

(Proved in part (1) D is mid-point of AC)

∠ADM = ∠CDM (Both 90° MD ⊥ DC)

DM = DM (Common)

ΔAMD ≅ ΔCMD (SAS)

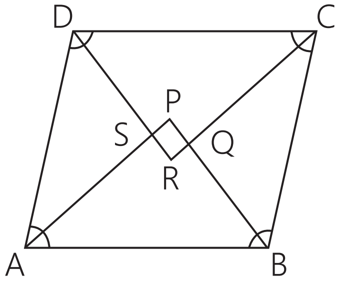
AM = CM (C.P.C.T.) ...(1)

AM = $\frac{1}{2}$ AB

[Since M is midpoint of AB] ...(2)

CM = AM = $\frac{1}{2}$ AB. [From (1) and (2)]

18. From the given figure:



Given: ABCD is parallelogram, AP, BP, DR and CR are angle bisectors of ∠A, ∠B, ∠C, ∠D.

To prove: PQRS is rectangle.

Proof: A rectangle is parallelogram with one angle 90° first we will prove PQRS is parallelogram.

Now, AB || CD

(Opp. side of parallelogram are parallel)

∠A + ∠D = 180°

(Int. angle one same side of transversal are supplementary).

Multiply by half:

$\frac{1}{2}$ ∠A + $\frac{1}{2}$ ∠D = 90°

∠DAS + ∠ADS = 90°

(DR bisect ∠D and AP bisect ∠A) ...(2)

Now, in ΔADS:

∠DAS + ∠ADS + ∠DSA = 180°

90° + ∠DSA = 180°

∠DSA = 90°

Also, line AP and DR intersect.

So, ∠PSR = ∠DSA (Vertically opp. angle)

∴ ∠PSR = 90°

Similarly, we can prove that

∠SPQ = ∠PQR = ∠SRQ = 90°

So, ∠PSR = ∠PQR and ∠SPQ = ∠SRQ

Both pair of opp. angle of PQRS are equal.

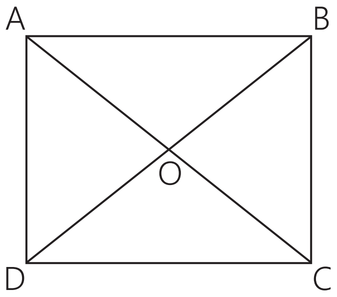
So, PQRS is parallelogram

∠PRS = ∠PQR = ∠SPQ = ∠SPQ = 90°

PQRS is parallelogram in which one angle 90°.

PQRS is rectangle.

19.



Consider ΔAOD and ΔCOB

OA = OC

OD = OB

∠AOD = ∠BOC

(vertically opposite angle)

⇒ ΔAOD ≅ ΔCOB (SAS)

⇒ AD = BC (CPCT) ...(1)

Similarly ΔAOB ≅ ΔCOD

⇒ AB = CD (CPCT) ...(2)

By (1), (2) we get ABCD is a parallelogram

[Since opposite sides are equal]

In ΔAOB and ΔCOB

AO = OC, ∠AOB = ∠COB, OB = OB

⇒ ΔAOB ≅ ΔCOB (SAS)

⇒ AB = BC ...(3)

By (1), (2), (3) AB = BC = CD = AD

In ΔDAB and ΔCBA,

AD = BC By (1)

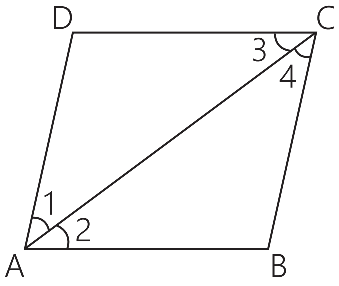
AC = BD (Given)

AB = AB (common)

ΔDAB ≅ ΔCBA

⇒ ∠A = ∠B = 90 (CPCT)

20.



Given: ABCD is rectangle.

AC bisect ∠A ∴ ∠1 = ∠2

AC bisect ∠C ∴ ∠3 = ∠4

To prove: ABCD is square.

Proof: A square is rectangle when all sides are equal.

Now, AD || BC

(Opp. side of parallelogram are parallel and AC is transversal)

∠1 = ∠4 (Alt. angle)

Also, ∠1 = ∠2 (Given)

Hence ∠4 = ∠2

In ΔABC, ∠2 = ∠4

BC = AB

(Side opp. to equal angle are equal) ...(1)

But BC = AD and AB = DC

(Opp. side of rectangle) ...(2)

From (1) and (2), we get:

AB = BC = CD = DA

So, ABCD is square.

Hence proved.

WORKSHEET 2: SECTION-A

1. ∠P – ∠R = 0 because two opp. angle of parallelogram are same.
2. No, all the angles of quadrilateral can't be obtuse. Obtuse angle is greater than 90° and sum of all angles are 360°.

3. $2x + 16 = 96 - x$

[Since opposite angles of parallelogram are equal]

$$2x + x = 96 - 16$$

$$3x = 80$$

$$x = \frac{80}{3}$$

SECTION-B

4. 2 : 4 : 5 : 7 angles of quadrilateral

$$2x + 4x + 5x + 7x = 360°$$

$$18x = 360°$$

$$\boxed{x = 20°}$$

$$2 \times 20 = 40°$$

$$7 \times 20 = 140°$$

$$5 \times 20 = 100°.$$

$$4 \times 20 = 80°$$

Greatest angle is 140° and smallest angle is 40°. ∴ The difference is 100°.

5. $4x + 5x = 180°$

$$9x = 180°$$

$$\boxed{x = 20°}$$

$$\angle A = 4 \times 20 = 80°$$

$$\angle B = 5 \times 20 = 100°.$$

6. Let length of short side = x cm

P = 38

$$2(\text{Sum of adjacent side}) = 38$$

$$2(\text{long} + \text{short}) = 38$$

$$2(11 + x) = 38$$

$$11 + x = \frac{38}{2} = 19$$

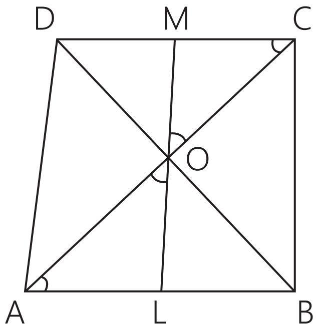
$$x = 19 - 11$$

$$x = 8$$

∴ Length of shorter side is 8 cm.

SECTION-C

7.



Given: ABCD is a parallelogram and AC and BD are diagonal intersecting at O.

Hence OA = OC and OB = OD

(Since diagonal bisect each other)

Consider ΔAOL and ΔCOM:

$\angle AOL = \angle COM$ (Vertically opp. angle)

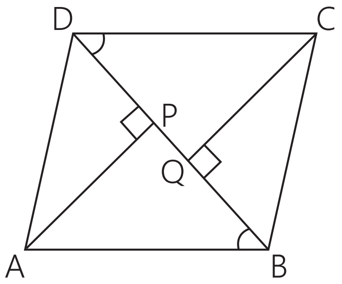
$OA = OC$ (Given)

$\angle OAL = \angle OCM$ (Alt. angle)

$\Delta AOL \cong \Delta COM$ (By ASA)

Hence $OL = OM$ (C.P.C.T.)

8.



Given: ABCD is parallelogram with AP ⊥BD and CQ ⊥BD

To Prove: AP = CQ

Proof: Now AB || DC

(Opp. side of parallelogram are parallel and transversal BD)

$\angle ABP = \angle CDQ$ (Alt angles) ...(1)

In ΔAPB and ΔCQD, we get;

$\angle APB = \angle CQD$ (Both 90°)

$\angle ABP = \angle CDQ$ (From (1))

$AB = CD$

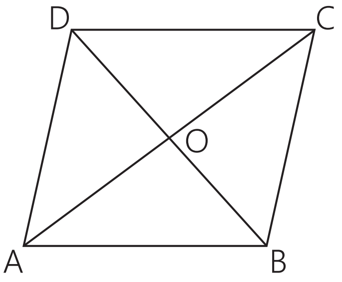
(Opp. side of parallelogram are equal)

$\therefore\ \Delta APB = \Delta CQD$ (AAS congruence rule)

$AP = CQ$ (C.P.C.T.)

Hence proved.

9.



Given: ABCD is a square. Diagonals intersect at O.

To prove: AC = BD

and AC is perpendicular to BD

Proof: ΔABC and ΔDCB

$AB = DC$ (Sides of square are equal)

$\angle ABC = \angle DCB$

(Both 90° as all angles of square are 90°)

$BC = BC$ (Common)

$\Delta ABC \cong \Delta DCB$ (SAS)

$\Rightarrow\ AC = DB$ (C.P.C.T.) ...(1)

Hence diagonal of square are equal in length.

To prove that diagonal bisect each other

AO = CO, BO = DO

In ΔAOB and ΔCOD

$\angle AOB = \angle COD$ (Vertically opp. angle)

$\angle ABO = \angle CDO$

(AB || CD and BD as transvers all alternate angles equal)

$AB = CD$

$\Delta AOB \cong \Delta COD$ (AAS) ...(2)

$AO = CO$ and $OB = OD$ (CPCT)

Hence diagonals bisect each other.

In ΔAOB and ΔCOB

$OA = OC$ (from (2))

$AB = BC$ (Side of square are equal)

$BO = BO$ (Common)

$\Delta AOB \cong \Delta COB$ (SSS)

$\therefore\ \angle AOB = \angle COB$ (C.P.C.T.) ...(3)

Now, $\angle AOB + \angle COB = 180°$ (Linear pair)

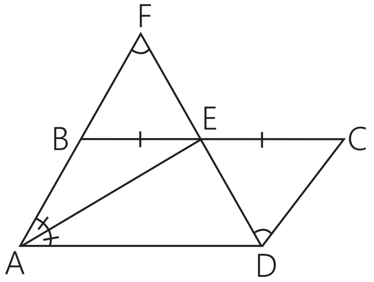
$\angle AOB + \angle AOB = 180°$ (from (3))

$2\angle AOB = 180°$

$\angle AOB = 90°$

Hence AC and BD bisect each other at right angle.

10.



Consider

ΔFEB and ΔDEC

$BE = CE$ (Given)

$\angle BFE = \angle CDE$ (Alternate angles)

$\angle BEF = \angle CED$ (Vertically opposite angles)

$\Delta FEB \cong \Delta DEC$ (AAS)

$CD = FB$ (CPCT)

But $CD = AB$ (Opposite sides of parallelogram)

$FB = AB$

$AB = \frac{1}{2}AF$ ...(1)

In ΔAFD, the angle bisector AE is also the median [FE = ED by CPCT]

$\Rightarrow\ \Delta AFD$ is isosceles

(A triangle in which angle bisector is also a median is isosceles triangle)

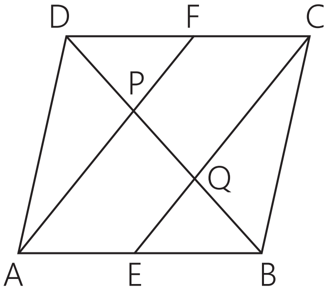
$AF = AD$ ...(2)

By (1), (2) $AB = \frac{1}{2} AF = \frac{1}{2} AD$

$AB = \frac{1}{2} AD$

SECTION-D

11.



Given: ABCD is parallelogram where E and F are the mid-points of sides AB and CD respectively.

To prove: AF and EC trisect BD

BQ = QP = DP

Proof: ABCD is a parallelogram

$AB \parallel CD$ (Opp. sides of parallelogram are parallel)

$AE \parallel CF$ (Part of parallel lines are parallel)

and AB = CD (Opp. sides of parallelogram are equal)

$\frac{1}{2} AB = \frac{1}{2} CD$ (Given F is mid-point of CD and E is mid-point of AB)

AE = CF

In AECF, $AE \parallel CF$ and AE = CF

One pair of opp. sides is equal and parallel. AECF is a parallelogram.

$\Rightarrow AF \parallel CE$

(Opp. side of parallelogram are parallel)

$\therefore PF \parallel CQ$ and $AP \parallel EQ$

(Part of parallel lines are parallel)

In ΔDQC, F is mid-point of DC and $PF \parallel CQ$

(Line drawn through mid-point of one side of triangle, parallel to another side bisect third side. P is mid-point of DQ)

PQ = DP ...(1)

In ΔABP, E is mid-point of AB and $AP \parallel EQ$

(Line drawn through mid-point of one side of triangle, parallel to another side bisect third side.)

Q is mid-point of BP

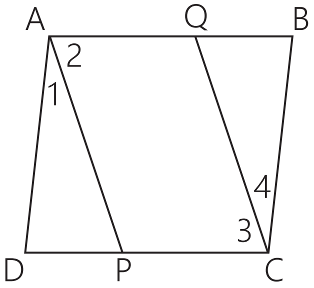
$\Rightarrow$ PQ = QB ...(2)

From (1) and (2)

DP = PQ = DQ

Hence line segment AF and EC trisect the diagonal BD.

12.



Since ABCD is a parallelogram.

$\angle A = \angle C$

(opposite angles of parallelogram)

$\frac{1}{2} \angle A = \frac{1}{2} \angle C$

$\angle 1 = \angle 2 = \angle 3 = \angle 4$ ...(1)

[Since AP is bisector of $\angle A$ and CQ is bisector of $\angle C$]

Consider ΔADP and ΔCBQ

AD = BC(opposite sides of parallelogram)

$\angle 1 = \angle 4$ (Proved in (1))

$\angle B = \angle D$

(opposite angles of parallelogram)

$\Rightarrow \Delta ADP \cong \Delta CBQ$ (ASA)

DP = BQ

P and Q are midpoints of sides CD and AB respectively.

So, we get AQ || PC

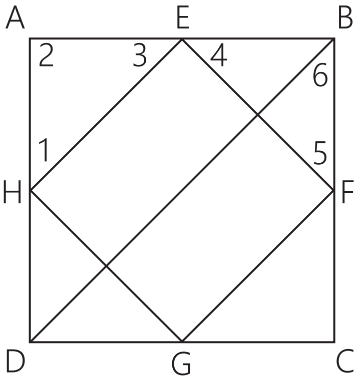
[opposite sides of parallelogram]

and AQ = PC

AQCP is a parallelogram.

$$\left(\begin{array}{l}\text{as } AB = CD \\ \Rightarrow \frac{1}{2}AB = \frac{1}{2}CD\end{array}\right)$$

13.



Construction: Join BD

Now, AB = BC = CD = AD

[As all sides of square are equal]

Also, AE = BF = CG = DH (given)

E, F, G, H are midpoints of AB, BC, CD, AD respectively.

In ΔABD, E and H are midpoints.

So, by midpoint theorem, HE || DB

and HE = $\frac{1}{2}$ DB ...(i)

Also, In ΔBCD, F and G are midpoints of sides BC and CD.

So, by midpoint theorem, FG || BD

and FG = $\frac{1}{2}$ BD ...(ii)

By (i), (ii), we get HE = GF and HE || GF

⇒ EFGH is a parallelogram

⇒ EH = FG and EF = HG ...(iii)

[since opposite sides of parallelogram are equal]

Consider ΔAEH and ΔBEF

AE = BE [since E is midpoint of AB]

∠A = ∠B = 90°

AH = BF

⇒ ΔAEH ≅ ΔBEF (SAS)

$$\left[\begin{array}{l}\text{Since } AD = BC \\ \Rightarrow \frac{1}{2}AD = \frac{1}{2}BC \\ \Rightarrow AH = BF\end{array}\right]$$

⇒ HE = EF ...(iv)

By (iii), (iv), EF = FG = GH = EH

In ΔAEH, ∠1 + ∠3 = 90°

In ΔBFE, ∠4 + ∠5 = 90

∠1 + ∠3 + ∠4 + ∠5 = 180

2∠3 + 2∠4 = 180

∠3 + ∠4 = 90

Now, ∠3 +∠HEF + ∠4 = 180

90 + ∠HEF = 180

∠HEF = 90

$$\left[\begin{array}{lr}\text{As} & AH = AE \\ \Rightarrow & \angle 1 = \angle 3 \\ \text{Also, as} & BF = BE \\ \Rightarrow & \angle 5 = \angle 4\end{array}\right]$$

So, EFGH is a parallelogram in which all sides are equal and one angle is 90°.

⇒ EFGH is a square.

CASE STUDY-1

1. (i) (a) Quadrilateral ABCD is a parallelogram as AB = DE and AB || DE.

also BC = EF and BC || EF.

(ii) (b) The Quadrilateral BCFE is a parallelogram.

(iii) (c) AD is parallel to CF.

(iv) (c) AC is parallel to DF.

(v) (d) Among all the given option only DEF is a triangle, so ΔABC is congruent to ΔDEF.

CASE STUDY-2

2. (i) (a) SR is parallel to QC.

(ii) (c) SR is half of AC as according to the midpoint theorem.

In ΔDAC, point S and R are the midpoints of DA and DC respectively.

According to midpoint theorem, the line segment in a triangle joining the midpoints of two sides of triangle is said to be parallel to its third side and also half of its length.

(iii) (d) As SR = $\frac{1}{2}$ AC [midpoint theorem]

and PQ = $\frac{1}{2}$ AC [midpoint theorem]

$\therefore$ SR = PQ

(iv) (a) In PQRS the opposite sides are equal and parallel. Therefore PQRS is a parallelogram.

(v) (b) BC = 2QB.

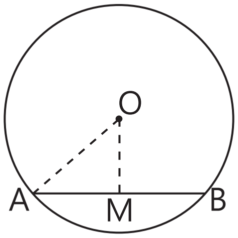
Chapter

9

Circles

MULTIPLE CHOICE QUESTION

1. (d)



Let AB is the chord and O is the center.

$$OA = \text{radius} = 10 \text{ cm}$$

$$OM = 6 \text{ cm}$$

$$AM = \sqrt{(10)^2 - (6)^2}$$

$$= 8$$

$$\therefore \quad AB = 16$$

2. (a) In a cyclic parallelogram, any two opposite angles are equal in measure and sum of any two opposite angles gives 180°.

3. (c) $\angle A + \angle C = 180°$ ...(1)

$\underline{\angle A - \angle C = 70°}$ ...(2)

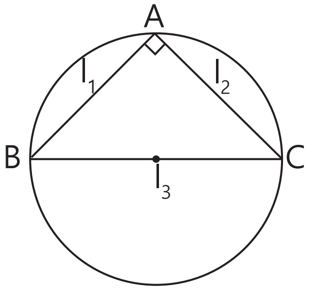
$2\angle A = 250°$

$\angle A = 125°$

$\therefore \quad \angle C = 125° - 70°$

$= \angle 55°$

4. (c)

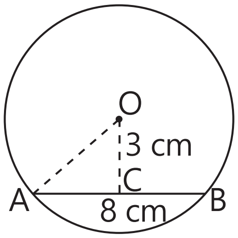


$l_3 = 2l_1$ (Given)

$$\cos B = \frac{l_1}{l_3} = \frac{1}{2}$$

$B = 60°$

5. (b)



Let AB is the chord

$$AO = \sqrt{AC^2 + OC^2}$$

$$= \sqrt{4^2 + 3^2}$$

$$AO = 5 \text{ cm}$$

The diameter of circle is 2AO or 10 cm.

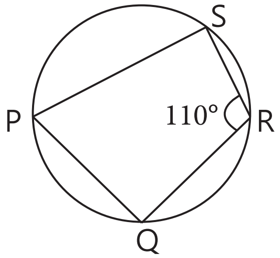
WORKSHEET 1: SECTION-A

1. PQRS is a cyclic quadrilateral.

$\angle QRS = 110°$

$\angle SPQ = ?$

We know that sum of opposite angles of a cyclic quadrilateral is 180°.



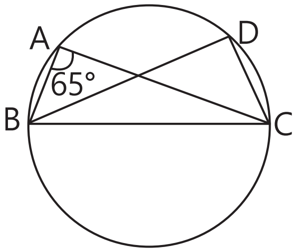
$\Rightarrow \quad \angle QRS + \angle SPQ = 180°$

$\Rightarrow \quad 110° + \angle SPQ = 180°$

$\Rightarrow \quad \angle SPQ = 180° - 110°$

$\Rightarrow \quad \angle SPQ = 70°$

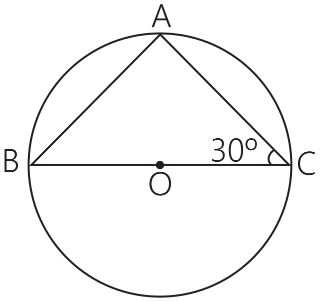
2. $\angle BAC = 65°$, $\angle BDC = ?$



$\angle BAC = 65^\circ$

$\Rightarrow$ $\angle BDC = 65^\circ$ (Angles in the same segment of a circle are equal)

3. O is the centre of the circle.



$\angle ACB = 30^\circ$, $\angle ABC = ?$

We know that angle in a semicircle is a right angle

$\Rightarrow$ $\angle BAC = 90^\circ$

Now, in ΔABC,

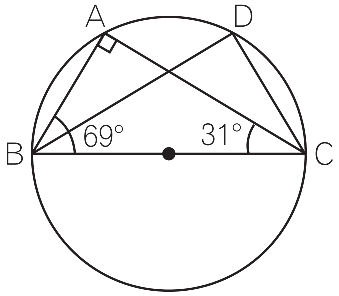
$\angle ABC + \angle BAC + \angle ACB = 180^\circ$ (A.S.P.)

$\angle ABC + 90^\circ + 30^\circ = 180^\circ$

$\angle ABC = 180^\circ - 120^\circ$

$\angle ABC = 60^\circ$

4. $\angle ABC = 69^\circ$ and $\angle ACB = 31^\circ$



$\angle BDC = ?$

In ΔABC,

$\angle ABC + \angle BAC + \angle ACB = 180^\circ$ (A.S.P)

$69^\circ + \angle BAC + 31^\circ = 180^\circ$

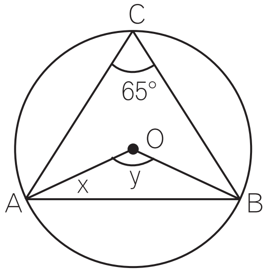
$\angle BAC = 180^\circ - 100^\circ$

$\angle BAC = 80^\circ$

Since $\angle BAC = 80^\circ \Rightarrow \angle BDC = 80^\circ$

(Angles in a same segment of a circle are equal)

5. $\angle ACB = 65^\circ$



OA = OB = Radii

$\angle OAB = \angle OBA = x$ (Angles opp. to equal sides are equal)

$\Rightarrow$ $\angle AOB = 2\angle ACB$

(Angle subtended by an arc at centre is twice the angle subtended by it on remaining part of circle)

$\Rightarrow$ $\angle AOB = 2 \times 65^\circ$

$\Rightarrow$ $\angle AOB = 130^\circ \Rightarrow y = 130^\circ$

Now in ΔOAB,

$x + y + \angle OBA = 180^\circ$ (A.S.P.)

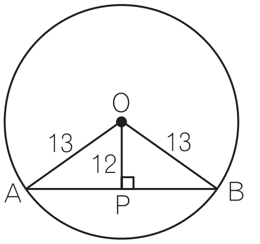
$x + 130^\circ + x = 180^\circ$

$2x = 180^\circ - 130^\circ$

$2x = 50^\circ$

$\boxed{x = 25^\circ}$ and $\boxed{y = 130^\circ}$

6. In ΔOPA and ΔOPB



OA = OB

OP = OP (common)

$\angle OPA = \angle OPB = 90^\circ$

$\Rightarrow$ $\Delta OPA \cong \Delta OPB$ (RHS)

$\Rightarrow$ AP = PB (CPCT)

In ΔOPA, $OA^2 = OP^2 + AP^2$

$13^2 = 12^2 + AP^2$

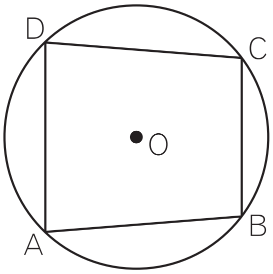
$169 - 144 = AP^2$

$25 = AP^2$

$5 = AP$

$\Rightarrow$ AB = 2AP = 10 cm

7. Let ABCD be a cyclic quadrilateral and let O be the centre of the circle. Then each of AB, BC, CD and DA being a chord of the circle, its right bisector must pass through O.

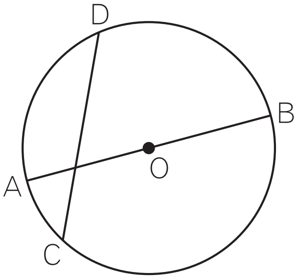


Hence, the right bisectors of AB, BC, CD and DA pass through O.

So we can say that perpendicular bisectors of the sides of a cyclic quadrilateral are concurrent.

SECTION-B

8. **Given:** AB is a diameter of circle



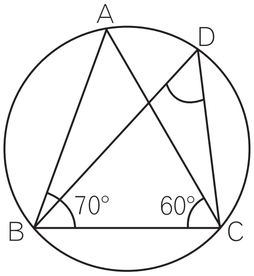
To Prove: AB is the greatest chord of a circle.

Proof: AB is nearest to centre than CD

$\therefore$ AB > CD (∵ Any two chords of a circle, the one which is nearer to centre is larger)

Hence, AB is larger than every other chord.

9. $\angle ABC = 70^\circ, \angle BCA = 60^\circ$



In ΔABC,

$\angle ABC + \angle BCA + \angle CAB = 180^\circ$ (A.S.P.)

$$70^\circ + 60^\circ + \angle CAB = 180^\circ$$

$$\angle CAB = 180^\circ - 130^\circ$$

$$\angle CAB = 50^\circ$$

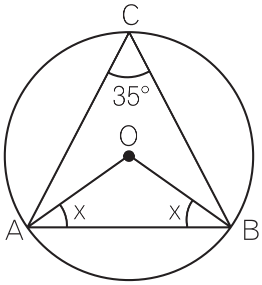
Also, $\angle BDC = 50^\circ$ (Angle in the same segment of a circle are equal)

$\therefore$ $\boxed{\angle CDB = 50^\circ}$

10. $\angle ACB = 35^\circ, \angle OAB = ?$

OA = OB = Radii

$\Rightarrow \angle OAB = \angle OBA$ (Angle opposite to equal sides of a triangle are equal)



Let $\angle OAB = \angle OBA = x$

$\angle AOB = 2\angle ACB$ (Angle subtended by an arc at the center is twice the angle subtended by the arc at any point on the circle.)

$$\angle AOB = 2 \times 35^\circ$$

$$\angle AOB = 70^\circ$$

In ΔAOB,

$x + \angle AOB + x = 180^\circ$ (A.S.P)

$$2x + 70^\circ = 180^\circ$$

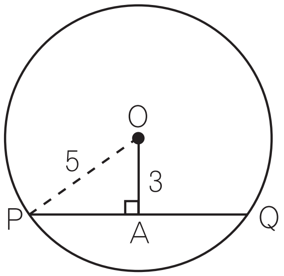
$$2x = 180^\circ - 70^\circ$$

$$= 110^\circ$$

$$x = 55^\circ$$

$\Rightarrow$ $\boxed{\angle OAB = 55^\circ}$

11. OA = 3 cm, OP = 5 cm



Find PQ.

We know that the perpendicular from centre to a chord bisect the chord.

$$\therefore \quad PA = \frac{1}{2} PQ$$

$$\Rightarrow \quad 2PA = PQ$$

$$\therefore \quad PQ = 2PA$$

In ΔOAP,

$$OP^2 = OA^2 + PA^2$$

$$(5)^2 = (3)^2 + PA^2$$

$$25 = 9 + (PA)^2$$

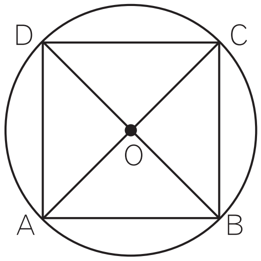
$$PA^2 = 25 - 9 = 16 = (4)^2$$

$$PA = 4 \text{ cm}$$

$$\therefore \quad PQ = 2PA = 2 \times 4 = 8 \text{ cm}$$

SECTION-C

12. Given: ABCD is a cyclic quadrilateral



To prove: $\angle A + \angle C = 180°$ and $\angle B + \angle D = 180°$

Construction: Join AC and BD.

Proof: Let AB be the chord of circle.

$\therefore \angle ACB = \angle ADB$ ...(1)

[Angle in the same segment are equal]

$\angle BAC = \angle BDC$ ...(2)

Adding eq. (1) and (2), we get;

$$\Rightarrow \quad \angle ACB + \angle BAC = \angle ADB + \angle BDC$$

$$\Rightarrow \quad \angle ACB + \angle BAC = \angle ADC$$

$$\Rightarrow \quad \angle ABC + \angle ACB + \angle BAC = \angle ABC + \angle ADC$$

(Adding $\angle ABC$ on both sides)

$$\Rightarrow \quad 180° = \angle ABC + \angle ADC$$

$$\angle B + \angle D = 180°$$

But $\angle A + \angle B + \angle C + \angle D = 360°$

$$\therefore \quad \angle A + \angle B + \angle C + \angle D = 360°$$

$$\angle A + \angle C + 180° = 360°$$

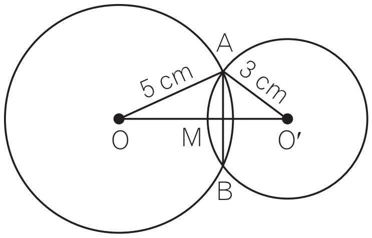
$$\angle A + \angle C = 360° - 180°$$

$$\angle A + \angle C = 180°$$

$$\text{and } \angle B + \angle D = 180°$$

Hence proved.

13. Let O and O′ be centres of circles with radii 5 cm, 3 cm, and AB be the common chord.



$\therefore$ $OO' \perp AB$ and M is the mid-point of AB.

Let $AM = y$ and $OM = x$ cm

As $OO' = 4$ cm (given)

$$\therefore \quad MO' = (4 - x) \text{ cm}$$

In ΔOMA, by Pythagoras Theorem,

$$OA^2 = OM^2 + AM^2$$

$$\Rightarrow \quad (5)^2 = x^2 + y^2$$

$$\Rightarrow \quad x^2 + y^2 = 25 \qquad ...(1)$$

In $\Delta O'MA$, by Pythagoras Theorem,

$$O'A^2 = OM'^2 + AM^2$$

$$\Rightarrow \quad (3)^2 = (4 - x)^2 + y^2 \qquad ...(2)$$

Subtracting eq. (2) from (1), we get

$$25 - 9 = x^2 - (4 - x)^2$$

$$16 = 8x - 16$$

$$32 = 8x$$

$$x = 4$$

Putting this value of x in eq. (1), we get

$$(4)^2 + y^2 = 25$$

$$y^2 = 25 - 16$$

$$y^2 = 9$$

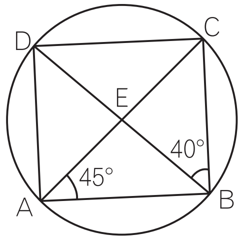
$$y = 3$$

Length of common chord,

$$AB = 2AM$$

$$= 2 \times 3 = 6 \text{ cm}$$

14. $\angle DAC = \angle DBC$



(Angle in the same segment of a circle are equal)

$\Rightarrow \quad \angle DAC = 40°$

$\therefore \quad \angle DAB = 40° + 45° = 85°$

As ABCD is a cyclic quadrilateral.

$\angle BCD + \angle DAB = 180°$

$\angle BCD + 85° = 180°$

$\angle BCD = 95°$

In ΔABC,

$AB = BC \Rightarrow \angle BCA = \angle BAC$

but $\quad \angle BAC = 45°$

$\Rightarrow \quad \angle BCA = 45°$

$\therefore \quad \angle ECD = \angle BCD - \angle BCA$

$= 95° - 45°$

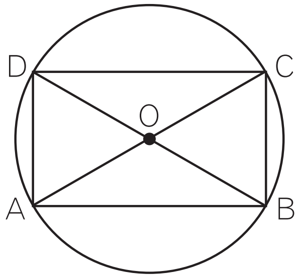
$= 50°$

$\Rightarrow \quad \boxed{\angle ECD = 50°}$

and $\quad \boxed{\angle BCD = 95°}$

SECTION-D

15. Given ABCD is a cyclic Quadrilateral and its diagonals are diameters of circle.



Now $\quad OA = OC \quad$ [Radii of same circle]

and $\quad OB = OD$

So, the diagonals AC and BD of the Quadrilateral bisect each other.

Therefore, ABCD is a Parallelogram.

Also, AC is diameter,

$\therefore \quad ABCD = 90°$

(Angle in a semicircle is a right angle)

Thus, ABCD is a Parallelogram in which one angle is 90°

$\therefore$ ABCD is Rectangle.

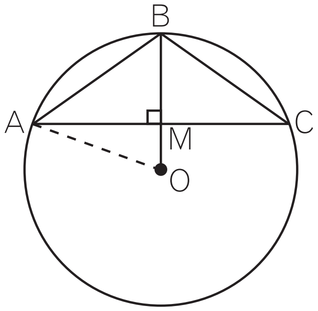
Hence proved.

16. Let A, B and C represent the three girls Reshma, Salma and Mandeep.

Then $\quad AB = 6$ m and $BC = 6$ m

If O is the centre of the circle,

then $\quad OA = \text{radius} = 5$ m



From B, draw $BM \perp AC$.

Since ABC is an isosceles triangle with AB = BC, M is mid-point of AC.

$\therefore$ BM is $\perp$lar bisector of AC, hence it passes through the centre of the circle.

Let $\quad AM = y$ and $OM = x$,

then $\quad BM = (5 - x)$

In ΔOAM, by Pythagoras Theorem,

$OA^2 = OM^2 + AM^2$

$\Rightarrow 5^2 = x^2 + y^2$

$\Rightarrow x^2 + y^2 = 25 \quad$ ...(1)

In ΔAMB, by Pythagoras Theorem,

$AB^2 = BM^2 + AM^2$

$\Rightarrow 6^2 = (5 - x)^2 + y^2$

$\Rightarrow (5 - x)^2 + y^2 = 36 \quad$ ...(2)

Subtracting eq. (2) from eq. (1), we get

$36 - 25 = (5 - x)^2 - x^2$

$\Rightarrow 11 = 25 - 10x$

$\Rightarrow 10x = 14 \Rightarrow x = \frac{7}{5}$

Substituting this value of x in eq. (1), we get

$$y^2 + \frac{49}{25} = 25 \Rightarrow y^2 = 25 - \frac{49}{25} = \frac{625 - 49}{25}$$

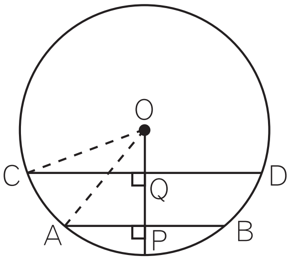
$$\Rightarrow y^2 = \frac{576}{25} \Rightarrow y = \frac{24}{5}$$

$$\therefore AC = 2AM = 2 \times y = 2 \times \frac{24}{5} = \frac{48}{5} = 9.6 \text{ m}$$

Hence, the distance between Reshma and Mandip is 9.6 m.

17. Given: Radius = 5 cm, AB = 6 cm and CD = 8 cm.

$OP \perp AB$, $OQ \perp CD$ and $AB \parallel CD$.



To find: PQ

Construction: Join OA and OC.

Sol. $AP = \frac{1}{2}AB \Rightarrow AP = \frac{1}{2}\times 6 \Rightarrow AP = 3$ cm

$CQ = \frac{1}{2}CD \Rightarrow CQ = \frac{1}{2}\times 8 \Rightarrow CQ = 4$ cm

In ΔOPA, by Pythagoras Theorem,

$$OA^2 = OP^2 + AP^2$$
$$(5)^2 = OP^2 + (3)^2$$
$$OP^2 = 25 - 9$$
$$OP^2 = 16$$
$$OP = 4$$

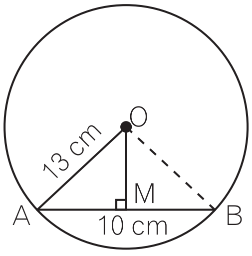
In ΔOQC, by Pythagoras Theorem,

$$OC^2 = OQ^2 + CQ^2$$
$$(5)^2 = OQ^2 + (4)^2$$
$$OQ^2 = 25 - 16$$
$$OQ^2 = 9 = (3)^2$$
$$OQ = 3$$

$$\therefore \quad PQ = OP - OQ$$
$$\Rightarrow \quad PQ = 4 - 3$$
$$\Rightarrow \quad \boxed{PQ = 1 \text{ cm}}$$

WORKSHEET 2: SECTION-A

1. AB = 10 cm



OA = OB = Radii of circle = 13 cm

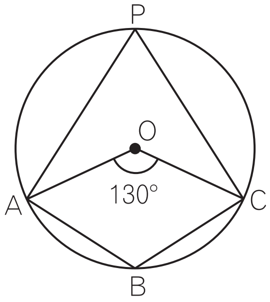
$$AM = MB = \frac{1}{2}(AB)$$
$$= \frac{1}{2}(10)$$
$$= 5 \text{ cm}$$

In ΔOAM,

$$OA^2 = AM^2 + OM^2$$
$$(13)^2 = (5)^2 + OM^2$$
$$169 - 25 = OM^2$$
$$OM^2 = 144$$
$$OM = 12 \text{ cm}$$

∴ Distance of chord from centre = 12 cm.

2. ∠AOC = 130°



Construction: Take a point P, anywhere on circle. Join AP and CP.

Sol. $\overparen{ABC}$ subtends ∠AOC at centre and ∠APC at remaining part of circle

$$\Rightarrow \angle APC = \frac{1}{2}\angle AOC = \frac{1}{2} \times 130°$$

$\angle APC = 65°$

$\angle ABC + \angle APC = 180°$

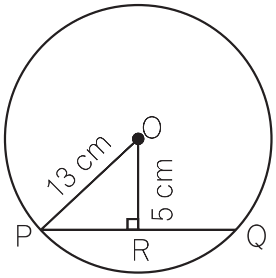
(opposite angles of cyclic Quadrilateral)

$65° + \angle ABC = 180°$

$\angle ABC = 180° - 65°$

$\angle ABC = 115°$

3. OP = radius = 13 cm



$OR = 5$ cm

In ΔOPR,

$$OP^2 = OR^2 + PR^2$$

$$(13)^2 = (5)^2 + PR^2$$

$$169 - 25 = PR^2$$

$$PR^2 = 144$$

$$PR = 12$$

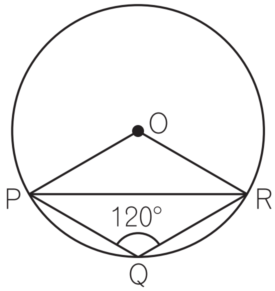
$$PQ = 2PR$$

$$PQ = 2 \times 12$$

$$PQ = 24 \text{ cm.}$$

4. $\angle PQR = 120°$

OP = OR = Radii of the circle.



Reflex $\angle POR = 2 \angle PQR$

$= 2 \times 120° = 240°$

$\therefore \quad \angle POR = 360° - 240° = 120°$

In ΔOPR, $\quad OP = OR$ (Radii of same circle)

$\therefore \quad \angle OPR = \angle ORP$

$\angle OPR + \angle ORP + \angle POR = 180°$ (A.S.P.)

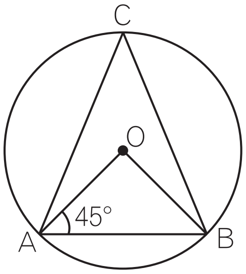
$$2\angle OPR + 120° = 180°$$

$$2\angle OPR = 60°$$

$$\angle OPR = 30°$$

SECTION-B

5. $\angle OAB = 45°$



In ΔOAB,

$OA = OB$ (Radii of circle)

$\Rightarrow \quad \angle OAB = \angle OBA$

$= 45°$ (A.S.P.)

$$\angle OAB + \angle OBA + \angle AOB = 180°$$

$$45° + 45° + \angle AOB = 180°$$

$$90° + \angle AOB = 180$$

$$\angle AOB = 90°$$

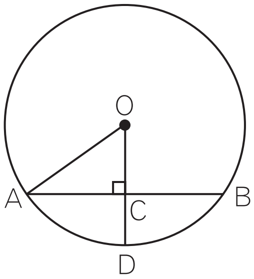
$$\angle AOB = 2 \angle ACB$$

$$90° = 2 \angle ACB$$

$$\angle ACB = \frac{90}{2}$$

$\Rightarrow \quad \angle ACB = 45°$

6. OA = 10 cm and AB = 12 cm



$OD \perp AB$

$$AB = 12 \text{ cm}$$

$$AC = BC$$

And $\quad AB = AC + BC$

$$12 = 2\,AC$$

$$AC = 6 \text{ cm}$$

In ΔAOC, by Pythagoras Theorem

$$OA^2 = OC^2 + AC^2$$

$$(10)^2 = OC^2 + (6)^2$$

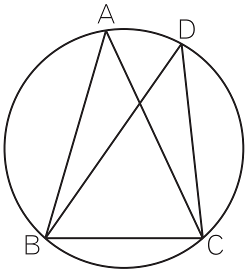
$$100 - 36 = OC^2$$
$$OC^2 = \sqrt{64}$$
$$OC = 8 \text{ cm}$$
$$\Rightarrow \quad OD = OA = 10 \text{ (radii of same circle)}$$
$$\Rightarrow \quad CD = OD - OC$$
$$= 10 - 8$$
$$= 2 \text{ cm}$$

7. We have $\angle ABC = 55°$

and $\angle BCA = 65°$

In ΔABC,

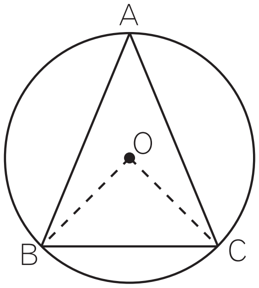
$\angle ABC + \angle BCA + \angle BAC = 180°$ (A.S.P.)



$$55° + 65° + \angle BAC = 180°$$
$$120° + \angle BAC = 180°$$
$$\angle BAC = 180° - 120°$$
$$\angle BAC = 60°$$
$$\Rightarrow \quad \angle BDC = 60°$$

(Angles in the same segment of a circle are equal)

8. The circumcentre of a triangle ABC is O. Prove that $\angle OBC + \angle BAC = 90°$



Sol. O is the circumcentre of ΔABC.

Join OB and OC.

In ΔOBC, $OB = OC$ (Radii of same circle)

$\therefore \quad \angle OBC = \angle OCB$

($\because$ angles opposite to equal sides are equal)

$$\angle BOC + \angle OBC + \angle OCB = 180°$$
$$\Rightarrow \quad \angle BOC + 2\angle OBC = 180°$$
$$\Rightarrow \quad \frac{1}{2}\angle BOC + \angle OBC = 90° \qquad \text{...(1)}$$

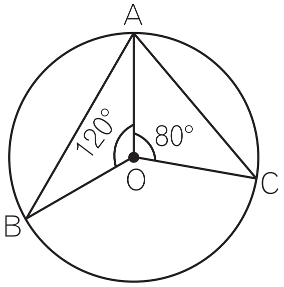
Now, $\angle BAC = \frac{1}{2}\angle BOC$

$$\Rightarrow \quad \angle BAC = 90° - \angle OBC$$
$$\Rightarrow \quad \angle OBC + \angle BAC = 90°$$

Hence proved.

SECTION-C

9. Since OA, OB and OC are radii of the circle.



In ΔOAB,

$$OA = OB$$
$$\angle OAB = \angle OBA$$
$$\angle OAB + \angle OBA + \angle AOB = 180° \qquad \text{(A.S.P.)}$$
$$2\angle OAB + 120° = 180°$$
$$2\angle OAB = 60°$$
$$\angle OAB = 30°$$

In ΔAOC, we get;

$$OA = OC$$
$$\Rightarrow \quad \angle OCA = \angle OAC$$
$$\angle OCA + \angle OAC + \angle AOC = 180° \qquad \text{(A.S.P.)}$$
$$2\angle OAC + 80° = 180°$$
$$2\angle OAC = 100°$$
$$\angle OAC = 50°$$
$$\angle BAC = \angle OAB + \angle OAC$$
$$\angle BAC = 30° + 50°$$
$$\angle BAC = 80°$$

10. In ΔAOB, $OA = OB$

$\Rightarrow \angle ABO = 30°$

$\Rightarrow$ In ΔBOC, $OB = OC$

$\Rightarrow \angle OBC = 40°$

$\therefore$ $\angle ABC = \angle ABO + \angle OBC$

$\angle ABC = 30° + 40° = 70°$

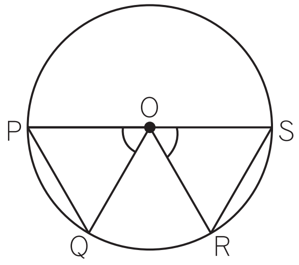
$\angle AOC = 2 \times \angle ABC$

(Angle subtended by the arc at the center is twice the angle subtended by it at any other point on remaining part of circle.)

$\angle AOC = 2 \times 70°$

$\therefore$ $\angle AOC = 140°$

11. **Given:** A circle such that Chord PQ = Chord RS



To prove: $\angle POQ = \angle ROS$

Proof: In ΔPOQ and ΔROS,

$PQ = RS$ (Given)

$OP = OR =$ radius

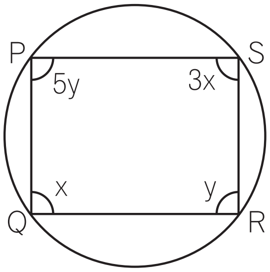
$OQ = OS =$ radius

$\therefore$ $\Delta POQ = \Delta ROS$ (by SSS)

$\therefore$ $\angle POQ = \angle ROS$ (by CPCT)

$\Rightarrow$ Angles subtended by equal Chords are equal.

12. PQRS is a cyclic quadrilateral and sum of opposite angles of cyclic quadrilateral is 180°.



$\Rightarrow$ $\angle P + \angle R = 180°$

$\Rightarrow$ $5y + y = 180°$

$\Rightarrow$ $6y = 180°$

$\Rightarrow$ $y = \dfrac{180}{6}$

$y = 30°$

$\angle P = 5y = 5 \times 30° = 150°$

$\angle R = 30°$

$\Rightarrow$ $\angle Q + \angle S = 180°$

$\Rightarrow$ $3x + x = 180°$

$4x = 180°$

$x = \dfrac{180°}{4}$

$x = 45°$

$\angle Q = 45°$

$\angle S = 3 \times 45°$

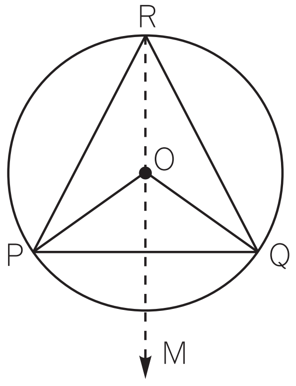
$\angle S = 135°$

SECTION-D

13. **Given:** An arc $\overset{\frown}{PQ}$ of a circle C (O, r) and a point R on the remaining part of the circle i.e., arc $\overset{\frown}{QP}$.

To prove: $\angle POQ = 2\angle PRQ$.

Construction: Join RO and produce it to a point M outside the circle.



Proof: When $\overset{\frown}{PQ}$ is a minor arc

In ΔPOR, $\angle POM$ is the exterior angle.

$\therefore$ $\angle POM = \angle OPR + \angle ORP$

[$\because$ Exterior angle property]

$\Rightarrow$ $\angle POM = \angle ORP + \angle ORP$ ...(1)

($\because$ $OP = OR = r$ $\therefore$ $\angle OPR = \angle ORP$)

$\Rightarrow$ $\angle POM = 2\angle ORP$

In ΔQOR,

$\therefore$ $\angle QOM = \angle OQR + \angle ORQ$

$\Rightarrow$ $\angle QOM = \angle ORQ + \angle ORQ$

$\Rightarrow$ $\angle QOM = 2\angle ORQ$ ...(2)

Adding (1) and (2), we get

$\angle POM + \angle QOM = 2\angle ORP + 2\angle ORQ$

$\therefore$ $\angle POM + \angle QOM = 2(\angle ORP + \angle ORQ)$

$\Rightarrow \quad \angle POQ = 2\angle PRQ$

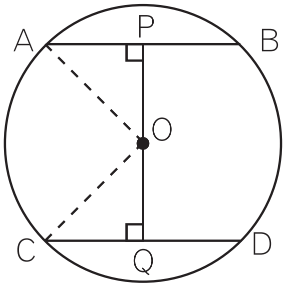
Hence proved.

14. **Given:** AB = 8 cm and CD = 6 cm

Radius = 5 cm

To find: PQ

Construction: Join OA and OC



Sol. AB = 8 cm

AP = PB = 4 cm

CD = 6 cm [∵ The perpendicular from the centre of a circle to a chord bisects the chord.]

CQ = QD = 3 cm

(The perpendicular from the centre of a circle to a chord bisects the chord.)

OA = OC = radii = 5 cm

In right ΔOAP,

By Pythagoras Theorem,

$$OP^2 = OA^2 - AP^2$$
$$= (5)^2 - (4)^2$$
$$= 25 - 16 = 9$$
$$OP = 3$$

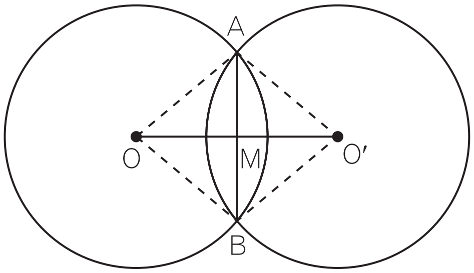
In ΔOCQ, By Pythagoras Theorem,

$$OQ^2 = OC^2 - CQ^2$$
$$= (5)^2 - (3)^2$$
$$= 25 - 9$$
$$= 16$$
$$OQ = 4$$
$$\Rightarrow \quad PQ = PO + OQ$$
$$= 3 + 4 = 7 \text{ cm}$$
$$\Rightarrow \quad PQ = 7 \text{ cm}$$

15. **Given:** Two circles C (o, r) and c(o′, r) intersect at the points A and B.

To prove: OO′ is perpendicular to AB

Construction: Join OA and OB, O′A and O′B



Proof: In ΔOAO′ and OBO′

OA = OB [Radii of circles]

O′A = O′B

OO′ = OO′ (Common)

∴ ΔOAO′ ≅ ΔOBO′ (by SSS)

⇒ ∠AOO′ = ∠BOO′ (by CPCT)

In ΔAOM and ΔBOM,

OA = OB (radii)

∠AOO′ = ∠BOO′ (Proved above)

OM = OM (Common)

∴ ΔAOM ≅ ΔBOM (by SAS)

AM = BM (by CPCT)

∠OMA = ∠OMB (by CPCT)

∴ ∠OMA + ∠OMB = 180° (L.P)

∠OMA + ∠OMA = 180°

⇒ 2∠OMA = 180°

⇒ ∠OMA = 90°

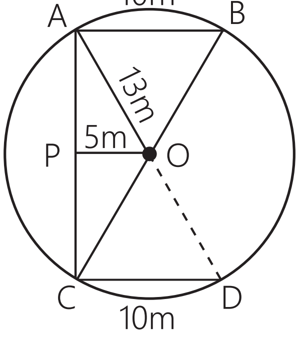
Hence, OO′ is the Perpendicular Bisector.

CASE STUDY-1

1. (i) (c) $AP^2 + OP^2 = BP^2$

$$AP^2 + 5^2 = 13^2$$
$$AP^2 = 144 \text{ cm}$$
$$AP = 12 \text{ cm}$$

(ii) (d)



Construction:- Extend line OA to meet D

$\angle AOB = \angle COD$ [vertically opposite angle]

$\angle COD = 120°$

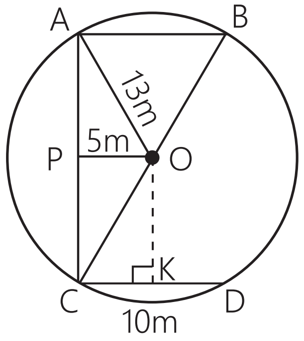
(iii) (a) $AP = PC = \frac{1}{2} AC$

$AC = 2AP$

$= 2(12m)$

$= 24\ m$

(iv) (a)



Construction:- Draw a perpendicular bisector K from O to CD such

$CK = KD = 5m$

$\angle COK = \frac{1}{2} \angle COD$

$= 60°$

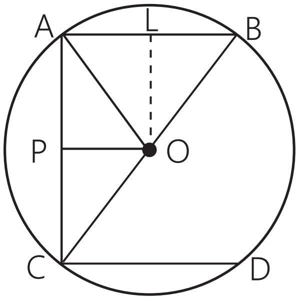
$\angle OCK + \angle COK + \angle CKO = 180°$

(Angle sum property of triangle).

$\angle OCK + 60° + 90° = 180°$

$\angle OCK = 30°$

(v) (a)



Construction:- Draw a perpendicular bisector L from O to AB such

$AL = BL = 5m$

$\angle AOL = \frac{1}{2} \angle AOB$

$= 60°$

$\angle OAL = \angle LOA + \angle ALO = 180°$

$\angle OAL + 30° + 90° = 180°$

$\angle OAL = \angle OAB$

$\angle OAL = 60°$

CASE STUDY-2

2. (i) (d) $\angle CBA + \angle CDA = 180°$

$\angle CBA = 180° - \angle CDA$

$= 180° - 80°$

$= 100°$

(ii) (b) $\angle DCB + \angle DAB = 180°$

$\angle DCB = 180° - 100°$

$= 80°$

(iii) (c) The Quadrilateral ABCD is enclosed by a circle and thus its a cyclic Quadrilateral.

(iv) (c) $\angle CBO + BCO + \angle BOC = 180°$ [Angle sum property of triangle]

$\angle CBO = \angle BCO$ (Angle in same segments of circle are equal)

$2\angle CBO = 180° - 100°$

$\angle CBO = 40°$

(v) (b) ΔOBC is an isosceles triangle as two side.

OB and OC are equal.

Chapter

10 Heron's Formula

MULTIPLE CHOICE QUESTION

1. (b) Let the sides of triangle are 3x, 4x, 5x

Perimeter = 36 cm

3x + 4x + 5x = 36

x = 3

∴ The sides are 9 cm, 12 cm, 15 cm

The semi-perimeter of the triangle is $\frac{36}{2}$ = 18 cm

Using Heron's Formula to calculate the area

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{18(18-9)(18-12)(18-15)}$$

$$= \sqrt{18 \times 9 \times 6 \times 3}$$

= 54 cm^2

2. (d) Let the sides are 3x, 4x and 5x

The semi-perimeter (S) = $\frac{3a+4x+5x}{2}$

= 6x

Area = 150 cm^2

$$150 = \sqrt{6x(6x - 3x)(6x - 4x)(6x - 5x)}$$

150 = 6x^2

x = 5

Perimeter is 2s = 2(6x)

= 2(6 × 5)

= 60 cm

3. (c) $s = \frac{15+17+8}{2} = 20$

$$\text{Area} = \sqrt{20(20-15)(20-17)(20-8)}$$

$$= \sqrt{20 \times 5 \times 3 \times 12}$$

$$= \sqrt{3600}$$

= 60 cm^2

4. (a) $S = \frac{4+4+2}{2} = 5$

$$\text{Area} = \sqrt{5(5-4)(5-4)(5-2)}$$

$$= \sqrt{5 \times 3}$$

$$= \sqrt{15} \text{ cm}^2$$

5. (a) $s = \frac{8+11+13}{2} = 16$

$$\text{Area} = \sqrt{16(16-8)(16-11)(16-13)}$$

$$= \sqrt{16 \times 8 \times 5 \times 3}$$

$$= \sqrt{64 \times 30}$$

$$= 8\sqrt{30} \text{ cm}^2$$

WORKSHEET 1: SECTION-A

1. Area of an equilateral triangle = $16\sqrt{3}$ m^2.

$$\frac{\sqrt{3}}{4}a^2 = 16\sqrt{3}$$

$$a^2 = \frac{16\sqrt{3} \times 4}{\sqrt{3}}$$

$$a^2 = 64$$

a = 8m

Perimeter = 3a

= 3 × 8= 24m

2. Side of equilateral triangle = 2a

$$\text{Area} = \frac{\sqrt{3}}{4}a^2$$

$$= \frac{\sqrt{3}}{4} \times (2a)^2$$

$$= \frac{\sqrt{3}}{4} \times 4a^2 = \sqrt{3}a^2$$

3. Perimeter of triangle = 32 cm

$\therefore$ Semi-perimeter (s) = $\frac{32}{2}$

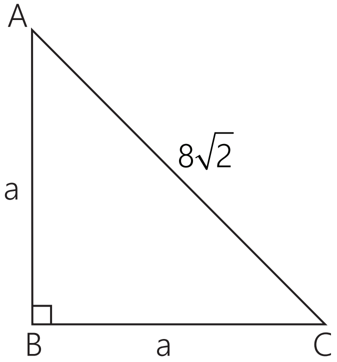
$$s = \frac{a+b+c}{2}$$

$$\frac{32}{2} = \frac{8+11+c}{2}; 32 = 19 + c$$

$$c = 32 - 19$$

$$c = 13$$

4. Area of isosceles triangle = 32 cm^2



$$\frac{1}{2}a^2 = 32$$

$a^2 = 32 \times 2$

$a^2 = 64$

$a = 8$

Hypotenuse = $8\sqrt{2}$, which is given.

So, True

5. Area of an equilateral triangle = $4\sqrt{3}$ cm^2

$$\frac{\sqrt{3}}{4}a^2 = 4\sqrt{3}$$

$$a^2 = \frac{4 \times 4\sqrt{3}}{\sqrt{3}}$$

$$a^2 = 16$$

$$a = 4 \text{ cm}$$

Perimeter = 3a

= 3 × 4 = 12 cm

Half of perimeter = $\frac{12}{2}$ = 6 cm.

SECTION-B

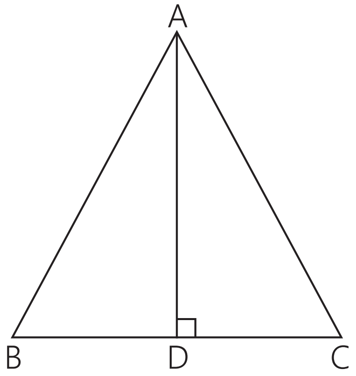
6. Base = 80 cm and area = 360 cm^2

Area of triangle = $\frac{1}{2}$ × base × height

$$360 = \frac{1}{2} \times 80 \times h$$

$$h = \frac{360}{40} \Rightarrow h = 9 \text{ cm}$$

In ΔABD, By Pythagoras theorem,



$$AB^2 = AD^2 + BD^2$$

$$AB^2 = (40)^2 + (9)^2$$

$$= 1600 + 81$$

$$= 1681$$

$$AB = 41 \text{ cm}$$

Perimeter = 2a + b

= 2 × 41 + 80

= 82 + 80 = 162 cm.

7. Height of equilateral triangle = 9 cm

Area of an equilateral triangle:

$$= \frac{h^2}{\sqrt{3}} = \frac{(9)^2}{\sqrt{3}} = \frac{81}{\sqrt{3}} = 46.767 \text{ cm}^2.$$

8. Perimeter of plot = 210 m

Let breadth of rectangular plot = x

$\therefore$ length of rectangular plot = 3x

Perimeter = 2(L + B)

$$\frac{210}{2} = x + 3x$$

$$105 = 4x$$

$$x = \frac{105}{4}$$

$$x = 26.25$$
$$\text{Length} = 3 \times 26.25 = 78.75$$
$$\text{Breadth} = 26.25$$
$$\text{Area} = L \times B$$
$$= 78.75 \times 26.25 = 2067.19\ m^2.$$

9. Area of parallelogram = 392 m^2

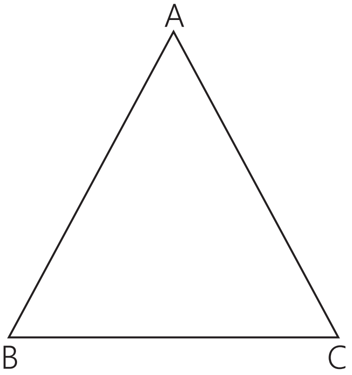
Let base = x and height = 3x

$$\text{Area} = b \times h$$
$$392 = x \times 3x$$
$$392 = 3x^2$$
$$x^2 = \frac{392}{3} = 130.67$$
$$x^2 = 131$$
$$x = \sqrt{131}$$
$$x = 11.4$$
$$\text{Base} = 11.4\ m$$
$$\therefore \quad \text{Altitude} = 3 \times 11.4 = 34.2\ m.$$

10. Let AB = 18 cm

BC = 24 cm

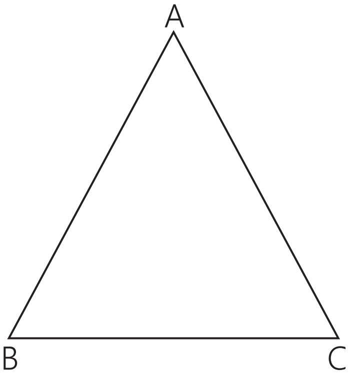
AC = 30 cm



$$s = \frac{a+b+c}{2}$$
$$s = \frac{18+24+30}{2} = \frac{72}{2} = 36\ cm$$
$$\text{Area of } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{36(36-18)(36-24)(36-30)}$$
$$= \sqrt{36 \times 18 \times 12 \times 6}$$
$$= \sqrt{9 \times 4 \times 6 \times 3 \times 6 \times 2 \times 2 \times 3}$$
$$= 3 \times 2 \times 6 \times 3 \times 2$$
$$= 9 \times 4 \times 6$$
$$= 36 \times 6 = 216\ cm^2.$$

SECTION-C

11. Perimeter = 324 m



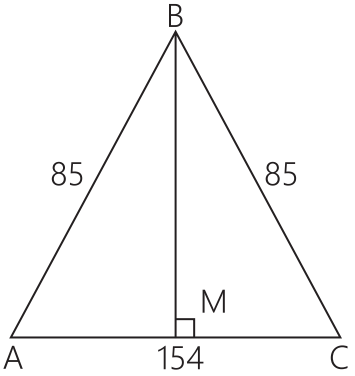
Let AB = 85 m

AC = 154 m

$$\text{Perimeter} = a + b + c$$
$$324 = 85 + 154 + c$$
$$c = 324 - 239$$
$$c = 85\ m$$
$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$S = \frac{324}{2} = 162\ m$$

(a) Area
$$= \sqrt{162(162 - 85)(162 - 154)(162 - 85)}$$
$$= \sqrt{162 \times 77 \times 8 \times 77}$$
$$= 77\sqrt{81 \times 2 \times 8}$$
$$= 77\sqrt{9 \times 9 \times 16}$$
$$= 77 \times 9 \times 4 = 2772\ m^2.$$

(b) To find AM



In ΔAMB and ΔCMB

BM = BM [Common]

AB = BC = 85 m

$\angle AMB = \angle CBM = 90°$

$\Rightarrow \quad \Delta AMB \cong \Delta CMB \qquad$ [RHS]

$\Rightarrow \quad AM = MC \qquad$ [CPCT]

$$\Rightarrow \quad AM = MC = \frac{1}{2}AC = \frac{1}{2}(154) = 77 \text{ m}$$

In ΔAMB, by pythagoras theorem

$$AB^2 = AM^2 + BM^2$$
$$(85)^2 = (77)^2 + BM^2$$
$$BM^2 = (85)^2 - (77)^2$$
$$= 7225 - 5929$$
$$= 1296$$
$$\Rightarrow \quad BM = 36 \text{ m}$$

12. Area of rhombus = 480 cm^2

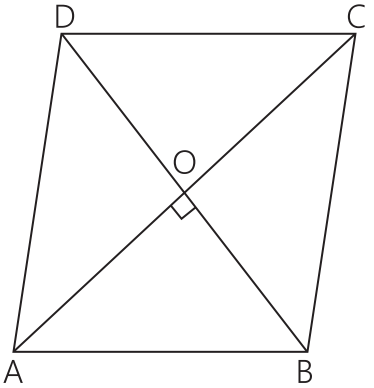
One of diagonal = 48 cm

$$\text{(a) Area of rhombus} = \frac{1}{2} \times D_1 \times D_2$$
$$480 = \frac{1}{2} \times 48 \times D_2$$
$$D_2 = \frac{480 \times 2}{48}$$
$$D_2 = 20 \text{ cm.}$$

Now, $\quad AC = 48$ cm

and $\quad BD = 20$ cm

$OA = 24$ cm, $OB = 10$ cm



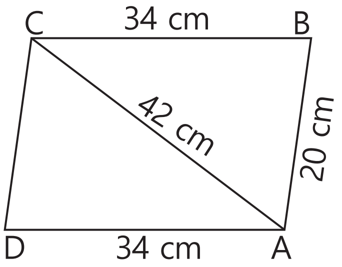
(b) In right ΔAOB,

$$AB^2 = OA^2 + OB^2$$
$$AB^2 = (24)^2 + (10)^2$$
$$AB^2 = 576 + 100$$
$$AB = \sqrt{676}$$
$$AB = 26 \text{ cm.}$$

(c) Perimeter $= 4 \times$ Side

$= 4 \times 26 = 104$ cm.

13. Diagonal AC = 42 cm



In ΔABC, we get;

$$a = 34 \text{ cm}$$
$$b = 20 \text{ cm}$$
$$c = 42 \text{ cm}$$
$$s = \frac{a+b+c}{2} = \frac{34+42+20}{2} = 48 \text{ cm}$$
$$\text{Area of } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{48(48-34)(48-42)(48-20)}$$
$$= \sqrt{48 \times 14 \times 6 \times 28} = 336 \text{ cm}^2$$

Area of parallelogram ABCD

$= 2 \times$ Area of ΔABC

$= 2 \times 336$

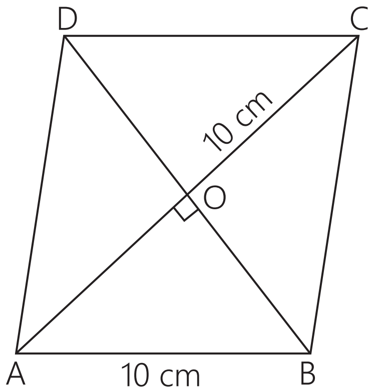
$= 672$ cm^2.

14. Area of Rhombus $= \frac{1}{2} \times D_1 \times D_2$

Length of each side = 10 cm

One of its diagonal = 10 cm

AC = 10 cm

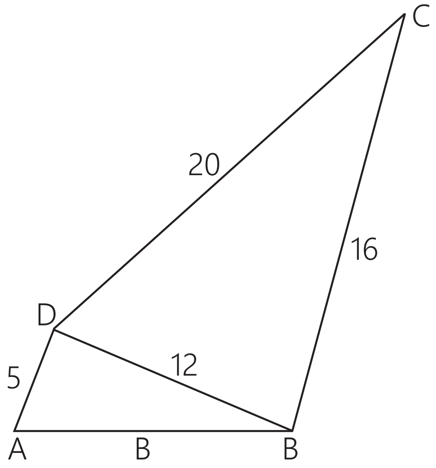


In right ΔAOB,

$$\begin{aligned} AB^2 &= OA^2 + OB^2 \\ (10)^2 &= (5)^2 + OB^2 \\ 100 &= 25 + OB^2 \\ OB^2 &= 100 - 25 \\ OB &= \sqrt{75} \\ OB &= 8.6 \text{ cm} \\ BD &= 8.6 + 8.6 \\ BD &= 17.2 \text{ cm.} \end{aligned}$$

$$\begin{aligned} \text{Area of rhombus} &= \frac{1}{2} \times AC \times BD \\ &= \frac{1}{2} \times 10 \times 17.2 \\ &= 17.2 \times 5 = 86 \text{ cm}^2. \end{aligned}$$

15. In ΔABD, AB = 13, AD = 5 and BD = 12



$$s = \frac{a+b+c}{2}$$

$$s = \frac{13+12+5}{2} = \frac{30}{2} = 15$$

$$\begin{aligned} \text{Area} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{15(15-13)\,(15-12)(15-5)} \\ &= \sqrt{15 \times 2 \times 3 \times 10} \\ &= \sqrt{3 \times 5 \times 2 \times 3 \times 5 \times 2} \\ &= 3 \times 5 \times 2 = 30 \text{ cm}^2 \end{aligned}$$

In ΔBCD,

BC = 16, CD = 20 and BD = 12

$$s = \frac{a+b+c}{2} = \frac{16+20+12}{2} = \frac{48}{2} = 24$$

$$\begin{aligned} \text{Area} &= \sqrt{24(24-16)(24-20)(24-12)} \\ &= \sqrt{24 \times 8 \times 4 \times 12} \\ &= \sqrt{12 \times 2 \times 4 \times 2 \times 4 \times 12} \\ &= 12 \times 2 \times 4 \\ &= 48 \times 2 = 96 \text{ cm}^2 \end{aligned}$$

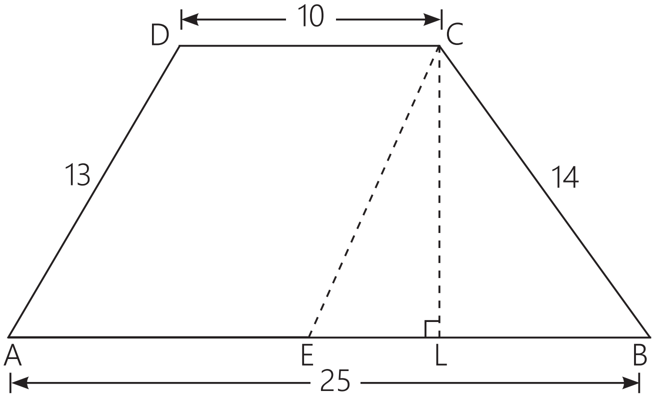
Area of Quadrilateral ABCD = Area of ΔABD + Area of ΔBCD

$$= 30 + 96 = 126 \text{ cm}^2$$

SECTION-D

16. Area of trapezium $= \frac{1}{2} \times$ (Sum of parallel sides) × Height

$$= \frac{1}{2} \times (25 + 10) \times \text{Height.}$$



CE || DA, so ADCE is a parallelogram having AD || CE and CD || AE such that

AD = 13 cm and CD = 10 cm

AE = 10 cm

CE = AD = 13 cm

BE = AB – AE = 25 – 10 = 15 cm

In ΔBCE,

$$s = \frac{a+b+c}{2} = \frac{14+13+15}{2} = \frac{42}{2} = 21 \text{m}$$

$$\begin{aligned} \text{Area of } \Delta BCE &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{21(21-14)(21-13)(21-15)} \\ &= \sqrt{21 \times 7 \times 8 \times 6} \\ &= \sqrt{7 \times 3 \times 7 \times 4 \times 2 \times 2 \times 3} \end{aligned}$$

$$= 7 \times 2 \times 3 \times 2$$

$$= 42 \times 2 = 84 \text{ cm}^2$$

$$\text{Area of } \Delta BCE = \frac{1}{2} \times BE \times CL$$

$$84 = \frac{1}{2} \times 15 \times CL$$

$$CL = \frac{168}{15} = \frac{56}{5} \text{m}$$

$$\text{Height of trapezium} = \frac{56}{6} \text{m}$$

$\therefore$ Area of trapezium

$$= \frac{1}{2} \times (25 + 10) \times \frac{56}{5}$$

$$= \frac{1}{2} \times 35 \times \frac{56}{5}$$

$$= 7 \times 28 = 196 \text{ cm}^2.$$

17. Perimeter of quadrilateral = 590 m

Let a = 5x, b = 12x, c = 17x and d = 25x

$$\text{Perimeter} = a + b + c + d$$

$$590 = 5x + 12x + 17x + 25x$$

$$590 = 59x$$

$$x = \frac{590}{2} \Rightarrow x = 10$$

$$a = 5 \times 10 = 50$$

$$b = 12 \times 10 = 120$$

$$c = 17 \times 10 = 170$$

$$d = 25 \times 10 = 250$$

$$s = \frac{a + b + c + d}{2}$$

$$= \frac{50 + 120 + 170 + 250}{2}$$

$$s = \frac{590}{2} = 295 \text{ m}$$

$$\text{Area} = \sqrt{(s - a)(s - b)(s - c)(s - d)}$$

$$= \sqrt{(295 - 50)(295 - 120)(295 - 170)(295 - 250)}$$

$$= \sqrt{245 \times 175 \times 125 \times 45}$$

$$= \sqrt{7 \times 35 \times 25 \times 7 \times 5 \times 25 \times 5 \times 9}$$

$$= 7 \times 5 \times 5 \times 5 \times 3 \sqrt{35}$$

$$= 2625\sqrt{35} \text{ m}^2.$$

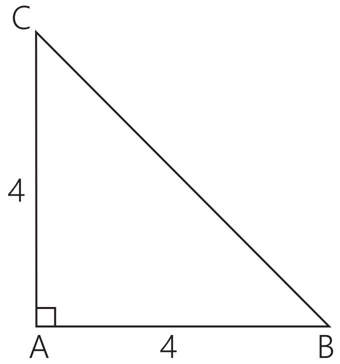
18. Base = 8 cm and height = 5.5 cm

$$\text{Area of triangle} = \frac{1}{2} \times B \times H$$

$$= \frac{1}{2} \times 8 \times 5.5 = 22 \text{ cm}^2.$$

WORKSHEET 2: SECTION-A

1. AB = 4 cm and AC = 4 cm



$$\text{Area of triangle} = \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} \times 4 \times 4 = 8\text{cm}^2.$$

2. Side of equilateral triangle = $4\sqrt{3}$ cm

$$\text{Area of equilateral triangle} = \frac{\sqrt{3}}{4}(\text{Side})^2$$

$$= \frac{\sqrt{3}}{4} \times 4\sqrt{3} \times 4\sqrt{3}$$

$$= 4 \times 3\sqrt{3}$$

$$= 12\sqrt{3} \text{ cm}^2.$$

3. Area of equilateral triangle = $20\sqrt{3}$ cm^2

$$\frac{\sqrt{3}}{4}a^2 = 20\sqrt{3}$$

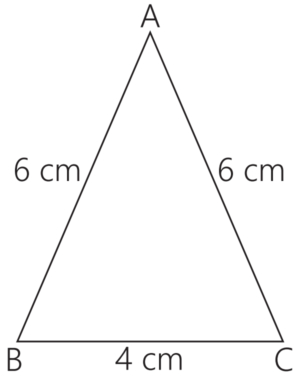
$$a^2 = 20 \times 4$$

$$a = \sqrt{80}$$

$$a = 2\sqrt{20}$$

and its given that side is 8 cm, which is false.

4. $s = \dfrac{AB + BC + AC}{2} = 8$ cm



$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{8(8-6)(8-6)(8-4)}$$
$$= \sqrt{8 \times 2 \times 2 \times 4}$$
$$= \sqrt{128}\ \text{cm}^2$$
$$= 8\sqrt{2}\ \text{cm}^2$$

5. Area of equilateral triangle = $16\sqrt{3}$ m^2

$$\frac{\sqrt{3}}{4}a^2 = 16\sqrt{3}$$
$$a^2 = 16 \times 4$$
$$a = \sqrt{64}$$
$$a = 8\text{m}$$
$$\text{Perimeter} = 3a$$
$$= 3 \times 8 = 24 \text{ m.}$$

SECTION-B

6. Area of isosceles right-angled triangle = 200 cm^2.

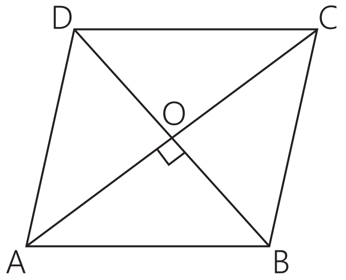
$$\frac{1}{2}a^2 = 200$$
$$a^2 = 200 \times 2$$
$$a^2 = 400$$
$$a = 20 \text{ cm}$$
$$\text{Perimeter} = 2a + \sqrt{2}\, a$$
$$= 2 \times 20 + \sqrt{2}\,(20)$$
$$= 40 + 20\sqrt{2}\ \text{cm}$$

7. a = 70 cm, b = 80 cm, c = 90 cm

$$s = \frac{a+b+c}{2} = \frac{70+80+90}{2} = \frac{240}{2} = 120$$
$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{120(120-70)(120-80)(120-90)}$$
$$= \sqrt{120 \times 50 \times 40 \times 30}$$
$$= \sqrt{12 \times 10 \times 5 \times 10 \times 4 \times 10 \times 3 \times 10}$$
$$= 12 \times 10 \times 10\sqrt{5} = 1200\sqrt{5}$$
$$= 1200 \times 2.23 = 2676 \text{ cm}^2.$$

8. Area of rhombus $= \frac{1}{2} \times D_1 \times D_2$

$$= \frac{1}{2} \times 300 \times 160 = 24{,}000 \text{ m}^2$$



Now, we have;

OB = 150 m and OA = 80 m

In ΔAOB, by Pythagoras theorem,

$$OA^2 + OB^2 = AB^2$$
$$(150)^2 + (80)^2 = AB^2$$
$$AB^2 = 6400 + 22500$$
$$AB^2 = 28900$$
$$AB = 170 \text{ m.}$$

9. By Heron's formula,

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$$s = \frac{a+b+c}{2} = \frac{41+40+9}{2}$$
$$= \frac{90}{2} = 45$$

$$\text{Area} = \sqrt{45(45-41)(45-40)(45-9)}$$

$$= \sqrt{45\times 4\times 5\times 36} = 180 \text{ m}^2$$

$$= 1800000 \text{ cm}^2$$

$$\text{Number of flower beds} = \frac{1800000}{900} \text{cm}^2$$

$$= 2000.$$

10. Perimeter of isosceles triangle = 32 cm

$$2a + h = 32$$

Let $a = 3x$ and $h = 2x$

$$3x + 3x + 2x = 32$$

$$8x = 32$$

$$x = 4$$

$$a = 12 \text{ cm}$$

$$h = 8 \text{ cm}$$

Sides of triangle are = 12 cm, 12 cm, and 8 cm

$$s = \frac{a+b+c}{2} = \frac{12+12+8}{2}$$

$$= \frac{32}{2} = 16 \text{ cm}$$

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{16(16-12)(16-12)(16-8)}$$

$$= \sqrt{16\times 4\times 4\times 8}$$

$$= 4\times 4\times 2\sqrt{2} = 32\sqrt{2} \text{ cm}^2.$$

11. Given sides are 6 cm, 8 cm and 10 cm

$$s = \frac{a+b+c}{2} = \frac{6+8+10}{2} = \frac{24}{2} = 12$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{12(12-6)(12-8)(12-10)}$$

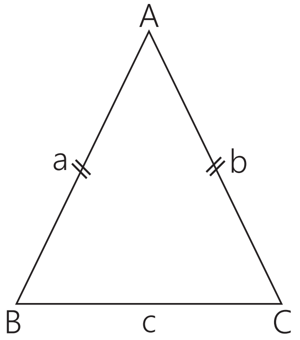
$$= \sqrt{12\times 6\times 4\times 2} = \sqrt{2\times 6\times 6\times 4\times 2}$$

$$= 2\times 6\times 2 = 24 \text{ cm}^2$$

$$\text{Cost of painting} = 0.09 \times 24$$

$$= ₹\ 2.16$$

12. Perimeter = 11 cm and Base = 5 cm



$$\text{Perimeter} = a + b + c$$

$$a = b$$

$$11 = a + a + c$$

$$11 = 2a + 5$$

$$11 - 5 = 2a$$

$$6 = 2a$$

$$a = 3$$

$$\Rightarrow \quad a = b = 3$$

$$s = \frac{a+b+c}{2} = \frac{11}{2}$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{\frac{11}{2}\left(\frac{11}{2}-3\right)\left(\frac{11}{2}-3\right)\left(\frac{11}{2}-5\right)}$$

$$= \sqrt{\frac{11}{2}\times\frac{5}{2}\times\frac{5}{2}\times\frac{1}{2}}$$

$$= \frac{5}{4}\sqrt{11} \text{ cm}^2$$

SECTION-C

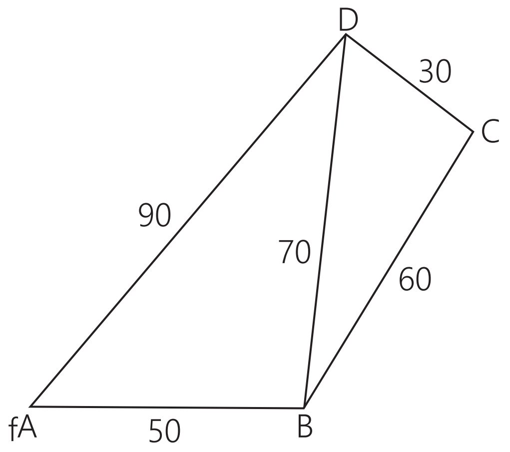
13. In ΔABD,

$$a = 90, b = 50, c = 70$$

$$s = \frac{a+b+c}{2}$$

$$= \frac{90+50+70}{2}$$

$$s = \frac{210}{2} = 105$$



Area $= \sqrt{s(s - a)(s - b)(s - c)}$

$= \sqrt{105(105 - 90)(105 - 50)(105 - 70)}$

$= \sqrt{(105 \times 15 \times 55 \times 35)}$

$= \sqrt{21 \times 5 \times 15 \times 5 + 11 \times 5 \times 7}$

$= \sqrt{3 \times 7 \times 5 \times 5 \times 3 \times 5 \times 11 \times 5 \times 7}$

$= 3 \times 5 \times 5 \times 7 \sqrt{11} = 525\sqrt{11}$ cm^2

In ΔBCD,

a = 30, B = 60, C = 70

$$s = \frac{a+b+c}{2} = \frac{30+60+70}{2} = \frac{160}{2} = 80$$

Area $= \sqrt{80(80-30)(80-60)(80-70)}$

$= \sqrt{80 \times 50 \times 20 \times 10}$

$= \sqrt{8 \times 10 \times 5 \times 10 \times 2 \times 10 \times 10}$

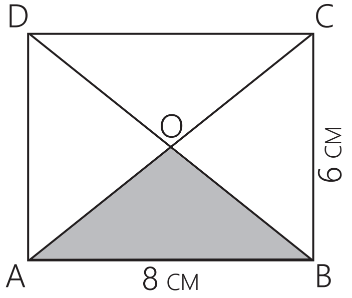
$= 10 \times 10 \times 4\sqrt{5} = 400\sqrt{5}$

Area of quadrilateral ABCD = Area of ΔABD + Area of ΔBDC

$= 525\sqrt{11} + 400\sqrt{5}$ cm^2.

14. AB = 8 cm, BC = 6 cm

In ΔABC,



By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$AC^2 = (8)^2 + (6)^2$

$AC^2 = 64 + 36$

AC = 100

AC = 10 cm

⇒ OA = 5 cm

Similarly, BD = 10 cm and OB = 5 cm

Now in ΔOAB,

OA = OB = 5 cm and AB = 8 cm

$$s = \frac{a+b+c}{2} = \frac{5+5+8}{2} = \frac{18}{2}$$

Area of triangle

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{9(9-5)(9-5)(9-8)}$

$= \sqrt{9 \times 4 \times 4 \times 1}$

= 3 × 2 × 2 = 12 cm^2.

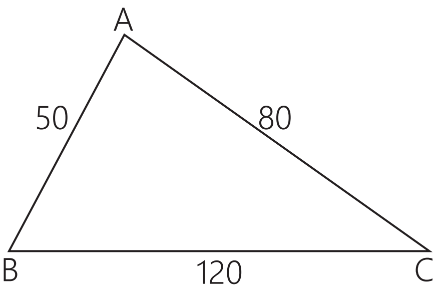
15. Area to be planted = Area of ΔABC.

Here,

a = 50 m, b = 80 m and c = 120 m

$$s = \frac{a+b+c}{2} = \frac{50+80+120}{2}$$

$$s = \frac{250}{2} = 125 \text{ m}$$



$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{125(125-80)(125-50)(125-120)}$$

$$= \sqrt{125 \times 75 \times 45 \times 5}$$

$$= \sqrt{25 \times 5 \times 25 \times 3 \times 5 \times 9 \times 5}$$

$$= 1452.4 \text{ m}^2$$

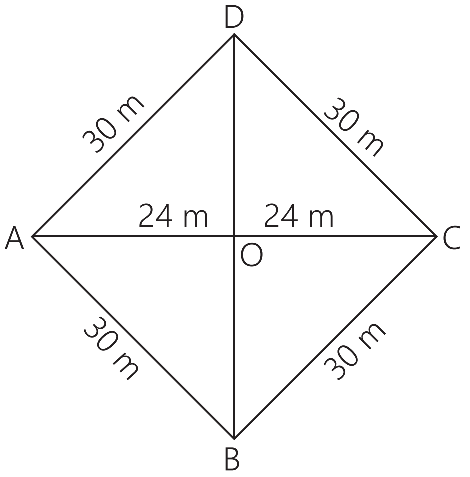
Number of meters to be fenced

= (50 + 80 + 120) – 3 = 250 – 3 = 247 m

Cost of fencing = ₹ 20 per metre

= ₹ 20 × 247 = ₹ 4940.

16. In ΔAOD,



By Pythagoras Theorem

$$AD^2 = OD^2 + OA^2$$

$$AD^2 - AO^2 = OD^2$$

$$(30)^2 - (24)^2 = OA^2$$

$$OA^2 = 900 - 576 = 324$$

$$OA = \sqrt{54 \times 6}$$

$$OA = 18 \text{ m}$$

$$\text{Area of } \Delta AOD = \frac{1}{2} \times 24 \times 18$$

$$= 12 \times 18 = 216 \text{ m}^2$$

∴ Area of rhombus = 4 × ΔAOD

= 4 × 216 = 864 m^2

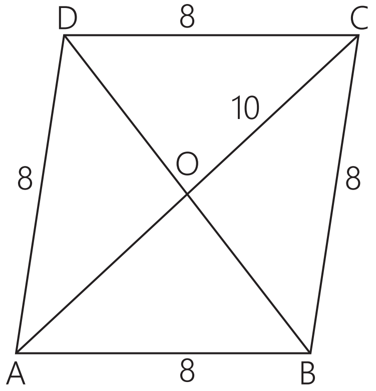
∴ Grass area for 18 cows = 864 m^2

$$\text{Grass area for 1 cow} = \frac{864}{18} \text{ m}^2 = 48\text{m}^2.$$

17. Perimeter of rhombus = 4 × side

32 = 4 × side

Side = 8 m



Diagonal of rhombus = 10 m

$$\text{Area} = \frac{1}{2} \times D_1 \times D_2;$$

$$\frac{D_1}{2}^2 + \frac{D_2}{2}^2 = (\text{Side})^2$$

Side of rhombus = Perimeter of rhombus/4

= 32 m/4

= 8m

$$\frac{10}{2}^2 + \frac{D_2}{2}^2 = (8)^2$$

$$\frac{100}{4} + \frac{D_2^2}{4} = 64$$

$$D_2^2 + 100 = 64 \times 4$$

$$D_2^2 = 256 - 100$$

$$D_2^2 = 156$$

$$D_2 = \sqrt{156} \Rightarrow D_2 = 2\sqrt{39} \text{ m}$$

$$\text{Area} = \frac{1}{2} \times 10 \times 2\sqrt{39}$$

$$= 10\sqrt{39} \text{ m}^2$$

$$= 10 \times 6.245 \text{ m}^2 = 62.45 \text{ m}^2$$

Area of both sides = 2 × 62.45 m^2 = 124.90 m^2

Cost of painting = ₹ 124.90 × 5 = ₹ 624.50.

SECTION-D

18. Let a, b, c be the sides of given triangle and s be its perimeter.

$$\therefore \quad s = \frac{a+b+c}{2} \qquad ...(1)$$

The sides of new triangle are 2a, 2b, 2c

Let s′ be its semi-perimeter

$$s' = \frac{2a+2b+2c}{2} = a + b + c$$

$$= 2s \qquad \text{[from equation (i)]}$$

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)} \qquad ...(2)$$

$$\text{Area of new } \Delta' = \sqrt{s'(s'-2a)(s'-2b)(s'-2c)}$$

$$= \sqrt{2s(2s - 2a)(2s - 2b)(2s - 2c)}$$

[using equation (1)]

$$= \sqrt{16s(s-a)(s-b)(s-c)}$$

$$= 4\Delta$$

$\therefore$ Increase in Area of $\Delta = \Delta' - \Delta = 4\,\Delta - \Delta = 3\Delta$

$$\text{Percentage increase in Area} = \left(\frac{3\Delta}{\Delta} \times 100\right)\%$$

$$= 300\%$$

19. Let sides of triangular plot = 3x, 5x and 7x

Perimeter = 300 m

3x + 5x + 7x = 300

15x = 300

x = 20 m

$\therefore$ sides are :

a = 3x = 3 × 20 = 60 m

b = 5x = 5 × 20 = 100 m

c = 7x = 7 × 20 = 140 m

$$s = \frac{a+b+c}{2} = \frac{60+100+140}{2} = \frac{300}{2}$$

= 150 m

$$\therefore \text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{150(150-60)(150-100)(150-140)}$$

$$= \sqrt{150 \times 90 \times 50 \times 10} = 1500\sqrt{3} \text{ m}^2.$$

20. Perimeter of triangle = 50 cm

Let smaller side = x

One side = x + 4

Third side = 2x – 6

Perimeter = a + b + c

50 = x + x + 4 + 2x – 6

50 + 2 = 4x

52 = 4x

x = 13

Smaller side = 13

One side = 17

$\therefore$ Third side = 20

$$s = \frac{a+b+c}{2} = \frac{13+17+20}{2}$$

$$s = \frac{50}{2} = 25$$

$$\therefore \quad \text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{25(25-13)(25-17)(25-20)}$$

$$= \sqrt{25 \times 12 \times 8 \times 5}$$

$$= \sqrt{5 \times 5 \times 4 \times 3 \times 4 \times 2 \times 5}$$

$$\text{Area} = 5 \times 4\sqrt{30} = 20\sqrt{30} \text{ cm}^2$$

CASE STUDY-1

1. (i) (b) $s = \dfrac{a+a+a}{2} = \dfrac{3a}{2}$

$$\text{Area} = \sqrt{\frac{3a}{2}\left(\frac{3a}{2} - a\right)^3}$$

$$= \sqrt{\frac{3a}{2} \times \left(\frac{a}{2}\right)^3}$$

$$= \frac{a^2}{4}\sqrt{3}$$

$$= \frac{\sqrt{3}}{4} a^2 \text{ cm}^2$$

(ii) (c) The perimeter of triangle in terms of "a" is 3a.

3a = 180

a = 60 cm

(iii) (d) Semi-perimeter is $\frac{180}{2}$ or 90 cm.

(iv) (b) Area of the triangle is $\frac{\sqrt{3}}{4}a^2$, where a = 60

$$\text{Area} = \frac{\sqrt{3}}{4}\times(60)^2 = \frac{\sqrt{3}}{4}\times 3600$$

$$= 900\sqrt{3}\ \text{cm}^2$$

(v) (a) The total area of board is $900\sqrt{3}\ \text{cm}^2$

Cost of painting 1 cm^2 board is ₹ 6

Cost of painting $900\sqrt{3}\ \text{cm}^2$ is ₹ $5400\sqrt{3}$

CASE STUDY-2

2. (i) (b) Semi-perimeter of the triangular part is $\frac{28+9+35}{2}$ = 36 cm

(ii) (a) Area of triangular part is

$$\sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{36(36-28)(36-9)(36-35)}$$

$$= \sqrt{36\times 8\times 27\times 1}$$

$$= 36\sqrt{6}\ \text{cm}^2$$

(iii) (c) Area of 1 tile is $36\sqrt{6}\ \text{cm}^2$

Area of 16 tile is $36\sqrt{6}\ \text{cm}^2 \times 16$ or $576\sqrt{6}\ \text{cm}^2$

(iv) (d) As the shaded region is half of the total region hence the area of shaded region would be half of total area.

Area of shaded region = $\frac{576\sqrt{6}}{2}$

$288\sqrt{6}\ \text{cm}^2$

(v) (d) Area of 16 part is $576\sqrt{6}\ \text{cm}^2$

cost of painting 1 cm^2 is ₹ 50

cost of painting is $576\sqrt{6}\ \text{cm}^2$

$576\sqrt{6}\times 50$ = ₹ $28800\sqrt{6}$

Chapter

11

Surface Areas and Volumes

MULTIPLE CHOICE QUESTION

1. (b) Curved surface Area of cylinder = $2\pi rh$

Let r' be the new radius

$$\text{If radius is halved } r' = \frac{r}{2}$$

Let h' be the new height $h' = 2h$

$$\text{New curved surface area} = 2\pi r' h'$$

$$= 2\pi\frac{r}{2}\times 2h$$

$$= 2\pi rh$$

Thus the curved surface area remain same.

2. (a) Volume of cone is $\frac{1}{3}\pi r^2 h$

$$\text{Volume} = \frac{1}{3}\pi\times(4)^2 h$$

$$l = \sqrt{r^2+h^2}$$

$$h = \sqrt{l^2 - r^2}$$

$$h = \sqrt{5^2 - 4^2}$$

$$h = \sqrt{9}$$

$$h = 3 \text{ cm}$$

$$\text{Volume of cone} = \frac{1}{3}\pi(4)^2(3) = 16\pi \text{ cm}^3$$

3. (c) Surface area of sphere is directly proportional to the square of radius.

Let S be the Surface Area and S′ be the New Surface Area.

Let r be the radius of sphere.

Let $r' = 2r$

$$\frac{S}{S'} = \left(\frac{r}{r'}\right) = \left(\frac{r}{2r}\right)^2$$

$$\frac{S}{S'} = \frac{1}{4}$$

4. (c) Total surface area of cube is $6a^2$, where "a" is the side of cube.

$$6a^2 = a6$$

$$a^2 = 16$$

$$a = 4$$

$$\text{Volume of cube} = a^3$$

$$a = 4$$

$$a^3 = 64$$

$$\text{Volume} = 64 \text{ cm}^3$$

5. (d) Total surface area of Hemisphere = $3\pi r^2$.

WORKSHEET 1: SECTION-A

1. Volume of cone = $\frac{1}{3}\pi r^2 h$

$r = 7$ cm and $l = 25$ cm

$$l^2 = h^2 + r^2$$

$$h^2 = l^2 - r^2$$

$$h^2 = (25)^2 - (7)^2$$

$$h^2 = 625 - 49$$

$$h = \sqrt{576}$$

$$h = 24 \text{ cm}$$

$$\text{Volume} = \frac{1}{3}\times\frac{22}{7}\times 7\times 7\times 24$$

$$V = 154 \times 8 = 1232 \text{ cm}^3.$$

2. Surface area of sphere = 154 cm^2

$$4\pi r^2 = 154$$

$$r^2 = \frac{154\times7}{22\times4}$$

$$r^2 = \frac{154\times7}{22\times4}$$

$$r^2 = \frac{49}{4}$$

$$r = \sqrt{\frac{49}{4}}$$

$$r = \frac{7}{2} \text{ cm.}$$

3. T.S.A. of cube = 216 cm^2

$$6a^2 = 216$$

$$a^2 = \frac{216}{6}$$

$$a = \sqrt{36}$$

$$a = 6 \text{ cm}$$

$$\text{Volume} = a^3$$

$$= 6 \times 6 \times 6$$

$$= 216 \text{ cm}^3.$$

4. Height of cone = ?

Diameter = 30 m

$$\text{Radius} = \frac{D}{2} = \frac{30}{2} = 15 \text{ m}$$

Slant height (l) = 25 m

$$l^2 = h^2 + r^2$$

$$h^2 = l^2 - r^2$$

$$= (25)^2 - (15)^2 = 625 - 225$$

$$= 400$$

$$h = \sqrt{400}$$

Height = 20 m.

5. Volume of cube = $27a^3$

$$(\text{side})^3 = 27a^3$$

$$\text{side} = \sqrt[3]{27a^3}$$

$$\text{side} = 3a$$

$$\text{T.S.A. of cube} = 6a^2$$

$$= 6(3a)^2$$

$$= 6 \times 9a^2 = 54a^2.$$

6. Radius of cylinder = 14 cm

Height = 14 cm

Volume of cylinder = $\pi r^2 h$

$$= \frac{22}{7}\times14\times14\times14$$

$$= 44 \times 196 = 8624 \text{ cm}^3.$$

7. Diameter of cone = 7 cm

$$\text{Radius of cone} = \frac{7}{2} \text{ cm}$$

Height of cone = 12 cm

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\times12$$

$$= 77 \times 2 = 154 \text{ cm}^3.$$

8. Radius of sphere = 5 cm

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}\times\frac{22}{7}\times5\times5\times5$$

$$= \frac{88}{21}\times125 = \frac{11000}{21}$$

$$= 262 \text{ (approx.) cm}^3.$$

9. Base area of cylinder = 154 cm^2,
Height = 5 cm

$$\pi r^2 = 154$$

$$r^2 = 154\times\frac{7}{22}$$

$$r = 7 \text{ cm}$$

Volume of cylinder= $\pi r^2 h$

$$= \frac{22}{7}\times7\times7\times5$$

$$= 154 \times 5 = 770 \text{ cm}^3$$

10. Edge of a cube = 11 cm

S.A. of a cube = $4a^2$

$= 4 \times 11 \times 11$

$= 4 \times 121 = 484 \text{ cm}^2$

SECTION-B

11. R = radius of balloon = 7 cm

r = radius of pumped balloon = 14 cm

$$\Rightarrow \frac{\text{S.A. of sphere before inflated}}{\text{S.A. of spherical balloon after inflated}}$$

$$= \frac{4\pi R^2}{4\pi r^2} = \frac{7 \times 7}{14 \times 14} = \frac{1}{4}$$

Their ratio will be 1 : 4.

12. Volume of planks = L × B × H

= 200 × 3 × 4

= 2400 cm3

Volume of wooden blocks = L × B × H

$= 600 \times 18 \times 99 = 10{,}69{,}200 \text{ cm}^3$

Number of planks

$$= \frac{\text{Volume of wooden blocks}}{\text{Volume of each planks}}$$

$$= \frac{1069200}{2400} = 445.5 = 446 \text{ (approx.)}$$

13. Circumference of cylindrical base = $2\pi r$

$$132 = 2 \times \frac{22}{7} \times r$$

$$\frac{132 \times 7}{2 \times 22} = r$$

r = 21 cm

Height = 25 cm

Volume of cylinder = $\pi r^2 h$

$$= \frac{22}{7} \times 21 \times 21 \times 25$$

$= 66 \times 21 \times 25$

$= 34{,}650 \text{ cm}^3$.

14. Height of cone = 15 cm

Volume = 1570 cm^3

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$

$$1570 = \frac{1}{3} \times 3.14 \times r^2 \times 15 \qquad (\text{Using } \pi = 3.14)$$

$$r^2 = \frac{1570 \times 3}{3.14 \times 15}$$

$$r = \sqrt{100} \Rightarrow r = 10 \text{ cm}$$

Diameter = 2r = 2 × 10 = 20 cm

15. Length = 15 cm, Breadth = 10 cm and Height = 20 cm

Surface area = 2h(l + b)

= 2 × 20(15 + 10)

$= 40 \times 25 = 1000 \text{ cm}^2$.

16. C.S.A. of cylinder = 4400 cm^2

Circumference of base = 110 cm

$$2\pi r = 110$$

$$r = \frac{110 \times 7}{22 \times 2} = \frac{35}{2} \text{ cm}$$

C.S.A. = 4400

$2\pi rh = 4400$

$$2 \times \frac{22}{7} \times \frac{35}{2} \times h = 4400$$

$$\text{Height} = \frac{4400 \times 7 \times 2}{2 \times 22 \times 35}$$

Height = 40 cm.

17. Length = 12 cm, Breadth = 12 cm,

Height = 12 cm

Volume of wooden block = L × B × H

$= 12 \times 12 \times 12 = 1728 \text{ cm}^3$

Volume of cubes of side 3 cm = $(\text{Side})^3$

$= 3 \times 3 \times 3 = 27 \text{ cm}^3$

Number of cubes formed

$$= \frac{\text{Volume of wooden cuboid}}{\text{Volume of cubes}}$$

$$= \frac{1728}{27} = 64$$

18. Radius of shot-put = 4.9 cm

$$\text{Volume of shot-put} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}\times\frac{22}{7}\times 4.9\times 4.9\times 4.9$$

$$= 493 \text{ cm}^3.$$

19. Radius of conical tin = 30 cm

$$\text{Slant height} = 50 \text{ cm}$$

$$l^2 = h^2 + r^2$$

$$(50)^2 = h^2 + (30)^2$$

$$2500 - 900 = h^2$$

$$1600 = h^2$$

$$h = 40 \text{ cm}$$

$$\text{Volume of conical tin} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\times 3.14\times 30\times 30\times 40$$

$$= 314 \times 120$$

$$= 37680 \text{ cm}^3.$$

20. Length = 2.5 m

Height = 10 m

Capacity of tank = 50,000 litre

Volume of cuboid = 50,000 litre

$$L \times B \times H = 50{,}000 \text{ litre}$$

$$2.5 \times B \times 10 = 50{,}000\times\frac{1}{1000}\text{m}^3$$

$$B = \frac{50000}{1000\times 2.5\times 10}$$

Breadth (B) = 2 m.

SECTION-C

21. Number of people = 5000

75 L water requires for 1 people.

75 × 5000 L water requires for 5000 people

375000 L water requires for 5000 people.

Volume of water in overhead tank

$$= 40 \times 25 \times 15 \text{ m}^3$$

$$= 40 \times 25 \times 15 \times 1000 \text{ L}$$

$$\text{So number of days} = \frac{40\times 25\times 15\times 1000}{375000}$$

$$= 40 \text{ days.}$$

22. Volume of cone = 100 cm^3

Height = 12 cm

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$

$$100 = \frac{1}{3}\times\frac{22}{7}\times r^2\times 12$$

$$r^2 = \frac{100\times 7}{22\times 4} = \frac{700}{88}$$

$$r = \sqrt{\frac{700}{88}} = 2.8 \text{ cm}$$

$$\text{C.S.A.} = \frac{22}{7}\times 2.8\times l = \pi r l$$

$$l = \sqrt{h^2 + r^2}$$

$$l = \sqrt{(12)^2 + (2.8)^2}$$

$$l = \sqrt{144 + 7.84}$$

$$l = \sqrt{151.84}$$

$$= 12.32 \text{ cm}$$

$$= 12 \text{ cm (approx)}$$

$$\text{C.S.A} = \pi r l$$

$$= \frac{22}{7}\times 2.8\times 12$$

$$= 105.6 \text{ cm}^2$$

23. Inner radius (r_1) of cylindrical pipe $= \frac{24}{2}$

$$= 12 \text{ cm}$$

Outer radius (r_2) of cylindrical pipe $= \frac{28}{2}$

$$= 14 \text{ cm}$$

Height of pipe = Length of pipe = 35 cm

$$\text{Volume of pipe} = \pi(r_2^2 - r_1^2)h$$

$$= \frac{22}{7}(14^2 - 12^2) \times 35$$

$$= 110 \times 52 = 5720 \text{ cm}^3$$

Mass of 1 cm^3 wood = 0.6 gm

Mass of 5720 cm^3 wood = (5720 × 0.6)g

= 3432 g

= 3.432 kg

24. Here, cube of side 12 cm is divided into 8 cubes of side a cm.

Given that their volumes are equal.

Volume of big cube side 12 cm = Volume of 8 cubes of side a cm

$$(\text{Side of big cube})^3 = 8 \times (\text{Side of small cube})^3$$

$$(12)^3 = 8 \times a^3$$

$$\frac{12 \times 12 \times 12}{8} = a^3$$

$$a^3 = \frac{12 \times 12 \times 12}{2 \times 2 \times 2}$$

$$a^3 = 6 \times 6 \times 6$$

$$a = 6 \text{ cm}$$

∴ Side of small cube = 6 cm

Ratio of their S.A.

$$= \frac{\text{S.A. of cube of side 12 cm}}{\text{S.A. of cube of side 6 cm}}$$

$$= \frac{6(\text{Side of big cube})^2}{6(\text{Side of small cube})^2}$$

$$= \frac{6 \times 12 \times 12}{6 \times 6 \times 6} = \frac{4}{1}$$

Ratio of S.A. = 4 : 1.

25. Number of tiles required

$$= \frac{\text{S.A. for tiles to cover}}{\text{Area of each tile}}$$

We put tiles on 5 faces

Area of 1 face = (Side)^2

Area of 5 faces = 5 × Area of 1 face

= 5 × (Side)^2

= 5 × (1.5)^2

= 5 × (1.5 × 100 cm)^2

= 5 × (150)^2

= 5 × 150 × 150

Now, we find area of tile

Area of 1 tile = Side × Side = 25 × 25 cm^2

$$\text{Number of tiles required} = \frac{\text{Area of 5 faces}}{\text{Area of 1 tile}}$$

$$= \frac{5 \times 150 \times 150}{25 \times 25}$$

$$= 30 \times 6 = 180$$

Given that cost of 1 dozen tiles = ₹ 360

∴ Cost of 12 tiles = ₹ 360

$$\text{Cost of 1 tile} = ₹\frac{360}{12}$$

$$\text{Cost of 180 tiles} = ₹\frac{360}{12} \times 180 = ₹\ 30 \times 180$$

= ₹ 5400.

26. Given r = 7 m

h = 24 m

$$\therefore \quad l = \sqrt{r^2 + h^2}$$

$$= \sqrt{7^2 + 24^2} = \sqrt{49 + 576}$$

$$= \sqrt{625} = 25 \text{ m}$$

$$\text{C.S.A. of cone} = \pi rl$$

$$= \frac{22}{7} \times 7 \times 25$$

$$= 22 \times 25 = 550 \text{ m}^2$$

Area of cloth used = 550 m^2

Given, width of cloth = 5 m

$$\text{Length of the cloth used} = \frac{\text{Area}}{\text{Width}}$$

$$= \frac{550}{5} = 110 \text{ m.}$$

27. Cost of painting the four walls = 15000

Cost of painting per square meter = ₹ 10

$$\text{Area of four walls painted} = \frac{15000}{10}$$

$$= 1500\ m^2$$

$$\text{Area of four walls painted} = 2h(\,l + b)$$

$$= 250\ h$$

$$1500\ m^2 = 250\ h$$

$$\Rightarrow \quad \frac{1500}{250} = h$$

$$\Rightarrow \quad h = 6\ m.$$

28. Three cubes are joined to end to end then cuboid is formed

Its length will form is 12 + 12 + 12 = 36 cm

Breadth = 12 cm, height = 12 cm

$$\text{Volume of cuboid} = l \times b \times h$$

$$= 36 \times 12 \times 12$$

$$= 5184\ cm^3.$$

29. Height of roller = 1.5 m

Diameter of roller = 84 cm

⇒ Radius of roller = 42 cm = 0.42 m

$$\text{C.S.A. of roller} = 2\pi rh$$

$$= 2 \times \frac{22}{7} \times \frac{42}{100} \times 1.5$$

$$= 3.96\ m^2$$

⇒ Area covered in one revolution = $3.96\ m^2$

⇒ Area covered in 100 revolutions

$= 100 \times 3.96 = 396\ m^2$

Cost of leveling the ground

$$= ₹ 396 \times \frac{50}{100} = ₹\ 198$$

30. Diameter of ball = 4.2 cm

⇒ Radius of ball (r) = 2.1 cm

$$\text{Volume of ball} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 2.1 \times 2.1 \times 2.1$$

$$= 38.81\ cm^3$$

SECTION-D

31. L = 20 m, B = 15 m, H = 8 m

$$\text{C.S.A. of hall} = 2H\,(L + B)$$

$$= 2 \times 8\,(15 + 20)$$

$$= 16 \times 35 = 560\ m^2$$

Cost of painting = 15 × 560 = ₹ 8400

32. Water flowing in river in 1 hour = 2 km

Water flowing in river in 1 hour = 2000 m

Water flowing in river in 60 minute = 2000 m

$$\text{Water flowing in river in 1 minute} = \frac{2000}{60} m$$

$$= \frac{100}{3}\ m$$

Now, river is in shape of cuboid

$$\text{Length} = \frac{100}{3}\ m$$

Breadth = 40 m

Height = 3 m

∴ Volume of water falling into sea = Volume of cuboid

= Length × Breadth × Height

$$= \left(\frac{100}{3} \times 40 \times 3\right) m^3 = 4000\ m^3$$

33. Inner radius = $r_1 = \frac{4}{2} = 2$ cm

Height = 77 cm

$$\text{C.S.A. of pipe} = 2\pi r_1 h$$

$$= 2 \times \frac{22}{7} \times 2 \times 77$$

$$= 968\ cm^2$$

$$\text{Outer radius} = r_2 = \frac{4.4}{2} = 2.2$$

$$\text{C.S.A. of pipe} = 2\pi r_2 h$$

$$= 2 \times \frac{22}{7} \times 2.2 \times 77$$

$$= 1064.8\ cm^2$$

T.S.A. = C.S.A. of inner cylinder + C.S.A. of outer cylinder + 2 × Area of base

Area of base $= \pi r_2^2 - \pi r_1^2$

$$= \frac{22}{7}\left[(2.2)^2 - (2)^2\right]$$

$$= \frac{22}{7}(4.84 - 4) = \frac{22}{7} \times 0.84$$

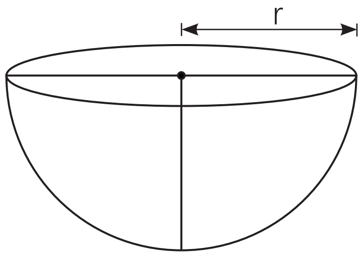
$= 2.64\ \text{cm}^2$

T.S.A. $= 968 + 1064.8 + 2 \times 2.64$

$= 2032.8 + 5.28$

$= 2038.08\ \text{cm}^2$.

34.



Circumference (c) = 17.6 cm

$2\pi r = 17.6$

$$r = \frac{17.6 \times 7}{2 \times 22} = \frac{1232}{440}$$

$$r = \frac{1232}{440} = 2.8\ \text{m}$$

$r = 280$ cm

Surface area of hemispherical dome

$= 2\pi r^2$

$$= 2 \times \frac{22}{7} \times 280 \times 280$$

$$= 44 \times \frac{12320}{7}$$

$= 1760\ \text{cm}^2$

Cost of painting $= \dfrac{5 \times 1760}{100}$ = ₹ 88

35. T.S.A. of cylinder = 6512 cm^2

Circumference of base = 88 cm

Circumference = $2\pi r$

$$88 = 2 \times \frac{22}{7} \times r$$

$$r = \frac{88 \times 7}{2 \times 22}$$

$r = 14$ cm

T.S.A. $= 2\pi r\ (h + r)$

$$6512 = 2 \times \frac{22}{7} \times 14(14 + h)$$

$6512 = 1232 + 88\ h$

$6512 - 1232 = 88\ h$

$5280 = 88\ h$

$$h = \frac{5280}{88}$$

$h = 60$ cm

Volume of cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times 14 \times 14 \times 60$$

$= 44 \times 14 \times 60$

$= 36960\ \text{cm}^2$.

36. Volume of hollow hemisphere $= \dfrac{2}{3}\pi(R^3 - r^3)$

A.T.Q.,

Volume of bowl $= \dfrac{2}{3}\pi\ [(4.5)^3 - (4)^3]$

$$= \frac{2}{3}\pi\ [(4.5 - 4)\ (4.5)^2 + (4.5 \times 4) + (4)^2]$$

$$= \frac{2}{3}\pi\ [(0.5 \times 54.25)]$$

$$= \frac{2}{3} \times 3.14 \times 27.125$$

$= 56.78\ \text{cm}^3$

37. Given height = 1 m

and capacity of cylindrical vessel = Volume of cylinder = 15.4 *l*

$$= 15.4 \times \frac{1}{1000}\text{m}^3 = 154 \times \frac{1}{10000}\text{m}^3$$

$$= \frac{154}{10000}\text{m}^3$$

Volume of cylinder = $\pi r^2 h$

$$\frac{154}{10000} = \frac{22}{7} \times r^2 \times 1$$

$$r^2 = \frac{154}{10000} \times \frac{7}{22}$$

$$r^2 = \frac{49}{10000}$$

$$r = \sqrt{\frac{49}{10000}}$$

$$r = \frac{7}{100} m$$

r = 0.07 m

T.S.A. = $2\pi r (r + h)$

$$= 2 \times \frac{22}{7} \times \frac{7}{100}\left(\frac{7}{100}+1\right)$$

$$= 2 \times \frac{22}{7} \times \frac{7}{100}\left(\frac{107}{100}\right)$$

$$= 44 \times \frac{1}{100} \times \frac{107}{100} = 0.4708 \text{ m}^2.$$

WORKSHEET 2: SECTION-A

1. Radius = $\frac{r}{2}$, Slant height = $2l$

T.S.A. of cone = $\pi r (l + r)$

$$= \frac{22}{7} \times \frac{r}{2}\left(2l + \frac{r}{2}\right) = \frac{11}{7} r\left(2l + \frac{r}{2}\right)$$

$$= \frac{11}{7} r\left(\frac{4l + r}{2}\right) = \frac{11}{14} r(4l + r)$$

2. Volume of cylinder = $\pi r^2 h$

Let original radius = r

and height = h

When radius is doubled then it will be 2r.

Volume of cylinder = $\pi(2r)^2 h$

$= \pi(4r^2)h$

$= 4\pi r^2 h$.

Volume becomes 4 times

3. Volume of cube = a^3

Volume of sphere = $\frac{4}{3}\pi r^3$

According to the question, a = 2r

Volume of sphere : Volume of cube

$\Rightarrow \frac{4}{3}\pi r^3 : a^3$

$\Rightarrow \frac{4}{3}\pi r^3 : (2r)^3$

$\Rightarrow \frac{4}{3}\pi r^3 : 8r^3$

$\Rightarrow \frac{4}{3} \times \frac{22}{7} : 8$

$\Rightarrow \frac{88}{21} : 8$

$\Rightarrow \frac{11}{21} : 1$

$\Rightarrow$ 11 : 21.

4. Radius is doubled i.e., 2r

S.A. of sphere = $4\pi r^2 = 4\pi(2r)^2$

$= 4\pi(4r^2) = 16\pi r^2$

5. Volume of sphere = $\frac{4}{3}\pi r^3$

Volume of hemisphere = $\frac{2}{3}\pi r^3$

If their radius are same.

Volume of sphere : Volume of hemisphere

$\frac{4}{3}\pi r^3 : \frac{2}{3}\pi r^3$

$\frac{4}{3} : \frac{2}{3}$

4 : 2

2 : 1

6. Diameter of cone = 30 m

Radius of cone = 15 m

Slant height = 25 m

$$l^2 = h^2 + r^2$$
$$h^2 = l^2 + r^2$$
$$h^2 = (25)^2 - (15)^2$$
$$h = \sqrt{625 - 225}$$
$$h = \sqrt{400}$$
$$h = 20 \text{ m.}$$

7. Volume of cone = $48\pi cm^3$

Height of cone = 9 cm

$$\text{Volume} = \frac{1}{3}\pi r^2 h$$
$$48\pi \times 3 = \pi r^2(9)$$
$$\frac{48 \times 3}{9} = r^2$$
$$r = \sqrt{16}$$
$$r = 4$$
$$\text{Diameter} = 2r$$
$$= 2 \times 4 = 8 \text{ cm.}$$

8. Diameter of sphere is equal to length of cube, 6 cm

Radius of sphere = 3 cm

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3} \times \frac{22}{7} \times 3 \times 3 \times 3$$
$$= \frac{36 \times 22}{7}$$
$$= \frac{792}{7} = 113.14 \text{ cm}^3$$

9. Sum of all edges of cube = 36 cm

$$\text{Length of one edge} = \frac{36}{12} = 3 \text{ cm}$$
$$\text{Volume of a cube} = a^3 = (3)^3$$
$$= 27 \text{ cm}^3.$$

10. Radius of a sphere $= \frac{1}{3}r$

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \;\frac{1}{3}r^{\;3}$$
$$= \frac{4}{3} \times \frac{22}{7} \times \frac{1}{3}r \times \frac{1}{3}r \times \frac{1}{3}r$$
$$= \frac{88}{567}r^3$$

SECTION-B

11. Side of each cube = 5 cm

Length of cuboid = 10 cm

Breadth of cuboid = 5 cm

Height of cuboid = 5 cm

$$\text{T.S.A.} = 2(lb + bh + hl)$$
$$= 2(10 \times 5 + 5 \times 5 + 5 \times 10)$$
$$= 2(50 + 25 + 50)$$
$$= 2 \times 125$$
$$= 250 \text{ cm}^2$$

12. T.S.A. of cylinder = 1628 cm^2

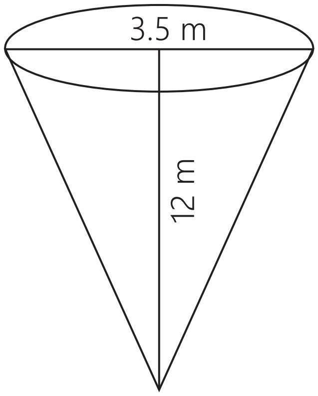
$$r + h = 37 \text{ cm}$$
$$\text{T.S.A.} = 2\pi r(r + h)$$
$$1628 = 2 \times \frac{22}{7} \times r(37)$$
$$\frac{1628 \times 7}{44 \times 37} = r \quad \Rightarrow \quad r = 7$$
$$r + h = 37$$
$$7 + h = 37$$
$$h = 30 \text{ cm}$$

13. Volume of cone = 9856 cm^2

Radius = 14 cm

$$\text{Volume} = \frac{1}{3}\pi r^2 h$$
$$9856 \times 3 = \frac{22}{7} \times 14 \times 14 \times h$$
$$\frac{9856 \times 3}{22 \times 2 \times 14} = h$$
$$h = 16 \times 3$$
$$h = 48 \text{ cm}$$

14. Height of conical pit = 12 m



Radius = $\frac{3.5}{2}$ = 1.75 m

Capacity of pit = Volume of cone

$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times (1.75)^2 \times 12$

(1 m^3 = 100 l, 1 m^3 = 1 Kl)

= 38.5 kilolitres

15. Radius of sphere = 0.63 m

Volume of sphere = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 0.63 \times 0.63 \times 0.63$

= 1.0478 m^3

SECTION-C

16. Length of a cylindrical roller = 2.5 m

Radius of a cylindrical roller = 1.75 m

Total area rolled by it = 5500 m^2

$\therefore$ Area in one revolution = C.S.A. of cylinder

$= 2\pi rh$

$= 2 \times \frac{22}{7} \times 1.75 \times 2.5$ = 27.5 m^2

No. of revolutions = $\frac{5500}{27.5}$ = 200

17. Diameter, d = 7 cm

Radius, r = $\frac{7}{2}$cm

Height, h = 12 cm

$\therefore$ V = $\pi r^2 h$

$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 12$

= 77 × 6 = 462 cm^3

Total milk for 1600 students

= 462 × 1600

= 739200 cm^2

= $\frac{739200}{1000}$ litres

= 739.2 litres

18. Diameter = 9 m, Radius = $\frac{9}{2}$ = 4.5 m, Height = 3.5 m

Volume of cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 4.5 \times 4.5 \times 3.5$

= 74.25 m^3

$l = \sqrt{h^2 + r^2}$

$= \sqrt{(4.5)^2 + (3.5)^2}$

$= \sqrt{20.25 + 12.25}$

$= \sqrt{32.5}$ = 5.7 m

C.S.A. of cone = $\pi r l$

$= \frac{22}{7} \times 4.5 \times 5.7$

= 80.614 m^2

Area of canvas required = 80.614 m^2

19. Height of solid = 8 cm

Radius of solid = 6 cm

Volume of solid = $\frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 6 \times 6 \times 8$$

$$= 301.71 \text{ cm}^3$$

20. Diameter = 8.4 cm

$$\text{Radius} = \frac{8.4}{2} = 4.2 \text{ cm}$$

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 4.2 \times 4.2 \times 4.2$$

$$= 310.464 \text{ cm}^3$$

SECTION-D

21. Let radius and height of cone be 3x and 9x.

Volume of cone = 8748π cm^3

$$\text{Volume} = \frac{1}{3}\pi r^2 h$$

$$8748 = \frac{1}{3}\pi (3x)^2 (9x)$$

$$\frac{8748 \times 3}{9 \times 9} = x^3$$

$$x^3 = 108 \times 3$$

$$x^3 = 324$$

$$x = \sqrt[3]{2 \times 2 \times 3 \times 3 \times 3 \times 3}$$

$$x = 3\sqrt[3]{12}$$

$$\text{Radius} = 3x$$

$$= 3 \times 3\sqrt[3]{12} = 9\sqrt[3]{12}$$

$$\text{Height} = 9x$$

$$= 9 \times 3\sqrt[3]{12}$$

$$= 27\sqrt[3]{12}$$

22. Thickness = 2 cm

Outer diameter = 16 cm

Outer radius (R) = 8 cm

As outer radius = Inner radius + thickness

∴ Inner radius = Outer radius – thickness

= 8 – 2 = 6 cm

∴ Volume of iron used = Volume of outer cylinder – Volume of inner cylinder

$$= \pi R^2 h - \pi r^2 h$$

$$= \frac{22}{7} \times 100 (R^2 - r^2)$$

$$= \frac{22}{7} \times 100 (64 - 36)$$

$$= \frac{22}{7} \times 100 \times 28 = 8800 \text{ cm}^3$$

23. Length = 2b, Height = 3 m

Area of four walls = 108 m^2

$\Rightarrow$ 2h(l + b) = 108

$\Rightarrow$ 2(2b + b) × 3 = 108

$\Rightarrow$ 18b = 108

$\Rightarrow$ b = 6 m

∴ *l* = 2b

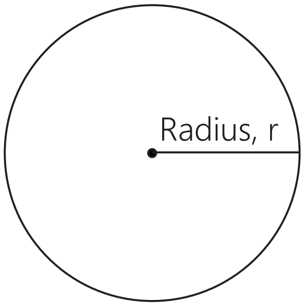
$\Rightarrow$ *l* = 2 × 6 = 12 m

∴ Volume of cold storage = l × b × h

= 12 × 6 × 3

= 216 m^3

24. Radius = r, Surface Area = S



(a) Volume of 27 sphere of radius r = Volume of big sphere of radius r^1

$$27\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi (r^1)^3$$

$$27r^3 = (r^1)^3$$

$$(r')^3 = (3r)^3$$

$$r' = 3r$$

(b) $$\frac{S}{S'} = \frac{\text{S.A. of sphere with radius r}}{\text{S.A. of sphere with radius r}'}$$

$$= \frac{4\pi r^2}{4\pi (r')^2} = \frac{r^2}{(3r)^2}$$

$$= \frac{r^2}{9r^2} = \frac{1}{9} \Rightarrow 1 : 9$$

25. Water used by total population in a day

$= 3200 \times 125\ l = 4{,}00{,}000\ l$

$$\begin{aligned}\text{Water in the tank} &= 30\text{ m} \times 10\text{ m} \times 8\text{ m}\\ &= 2400\text{ m}^3\\ &= 2400 \times 1000\ l\\ &= 24{,}00{,}000\ l\end{aligned}$$

$$\text{No. of days} = \frac{24{,}00{,}000}{4{,}00{,}000}$$

$$\text{No. of days} = \frac{24}{4} = 6.$$

CASE STUDY-1

1. (i) (b) Area of square is $(\text{side})^2$

Side = 25 cm

Area of tile = $(25)^2 = 625\text{ cm}^2$

(ii) (c) Tank is in cubical shape with length 150 cm.

The total surface area of cube is $6a^2$, where 'a' is the side of cube.

Side of tank = 150 cm

$$\begin{aligned}\text{Total surface area} &= 6a^2\\ &= 6(150)^2\\ &= 135000\text{ cm}^2\end{aligned}$$

(iii) (d) Each face of tank is a square having side = 150 cm

$$\begin{aligned}\text{Area of 1 face of tank} &= (150)^2\\ &= 22500\text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Area covered by 1 tile} &= (25)^2\\ &= 625\text{ cm}^2\end{aligned}$$

Number of tiles required for 1 face are:

$$\frac{22500}{625} = 36$$

Five faces need to be covered by tiles hence the total tiles are

$36 \times 5 = 180$

(iv) (c) Cost of 12 tiles is ₹ 360

Cost of 1 tile is $\frac{360}{12}$ = ₹ 30

(v) (b) Cost of 1 tile = ₹ 30

Cost of 180 tiles = 180×30 = ₹ 5400

CASE STUDY-2

2. (i) (a) Curved Surface Area of Cylinder is $2\pi rh$

$$\text{Area} = 2\pi\left(\frac{4}{2}\right)(77) = 967\text{ cm}^2$$

(ii) (c) Area of circular ring is $\pi(R^2 - r^2)$

$$R = \frac{4.4}{2} = 2.2\text{ cm}$$

$$r = \frac{4}{2} = 2\text{ cm}$$

Area of circular ring of 1 side

$= \pi(2.2^2 - 2^2) = 2.64\text{ cm}^2$

Area of circular rings on both sides

$2.64 \times 2 = 5.28\text{ cm}^2$

(iii) (b) Curved surface area of cylinder is $2\pi rh$ for outer cylinder, r = 2.2 cm and h = 77 cm

Area = $2\pi\ 2.2\ (77) = 1064.8\text{ cm}^2$

(iv) (a) Total surface area = Curved surface area of inner cylinder + curved surface area of outer cylinder + 2 × area of base.

Curved surface area of inner cylinder = $2\pi rh$

$= 2\pi\ (2)\ (77) = 968\text{ cm}^2$

Curved surface area of outer cylinder = 1064.8 cm^2

Area of circular rings = 5.28 cm^2

Total surface area

$= 1064.8 + 968 + 5.28$

$= 2038.08\text{ cm}^2$

(v) (d) Outer radius = $\frac{4.4}{2}$ = 2.2 cm

Inner radius = $\frac{4}{2}$ = 2 cm

Outer radius – Inner radius = 0.2 cm

Chapter

12

Statistics

MULTIPLE CHOICE QUESTION

1. (b) Let the total number of observations be "n".

Let the observation be 1, 2, 3 .... n

Let the mean is x

$$x = \frac{1+2+3+.....n}{n}$$

If each observation is increased by 5, the new mean would be

$$x' = \frac{(1+5)+(2+5)+(3+5)+.......(n+5)}{n}$$

$$= \frac{1+2+3+.....n}{n} + \frac{5n}{n}$$

$$\boxed{x' = x+5}$$

Thus new mean is increased by 5.

2. (a) Arrange the data in ascending order

22, 34, 39, 45, 54, 54, 56, 88, 78, 84

There are total 10 terms, so the median

$$\text{is } \frac{\left(\frac{n}{2}\right)^{th} + \left(\frac{n}{2}+1\right)^{th}}{2} \text{ term}$$

Where n is the number of terms

$$\text{Median } = \frac{\left(\frac{10}{2}\right)^{th} + \left(\frac{10}{2}+1\right)^{th}}{2} \text{ term}$$

$$= \frac{5^{th} + 6^{th}}{2} \text{ term}$$

The 5^{th} and 6^{th} term are 54.

$$\text{Median } = \frac{54+54}{2} = 54$$

3. (b) The total number of observations are 5.

$$\overline{x} = \frac{(x+77)+(x+7)+(x+5)+(x+3)+(x-2)}{5}$$

$$= \frac{5x+90}{5}$$

$$= x + 18$$

4. (c) 50 is the lower limit of class interval 50 – 60.

5. (a) Let the sum of five numbers is represented by $\sum_{i=1}^{5} x_i$

Let the sum of four numbers is represented by $\sum_{i=1}^{4} x_i$

Let the excluded number is y.

$$\frac{\sum_{i=1}^{5} x_i}{5} = 30$$

$$\sum_{i=1}^{5} x_i = 150$$

$$\text{Also, } \quad \frac{\sum_{i=1}^{4} x_i}{4} = 28$$

$$\sum_{i=1}^{4} x_i = 112$$

$$\sum_{i=1}^{4} x_i + y = \sum_{i=1}^{5} x_i$$

$$112 + y = 150$$

$$y = 38$$

WORKSHEET 1: SECTION-A

1. Maximum value = 70

Range = $\frac{\text{Maximum value}}{2} = \frac{70}{2} = 35$

Range = Maximum value – Minimum value

35 = 70 – Minimum value

Minimum value = 35.

2. Let upper limit = x and lower limit = y

Now, $\frac{x+y}{2} = 42$

⇒ $x + y = 84$ ...(1)

Class size = 10

⇒ $x - y = 10$ ...(2)

Adding eq. (1) and (2)

$2x = 94$

$x = 47$ and $y = 37$

Lower limit = 37 and upper limit = 47.

3. Mean = 17

According to question,

$$\frac{10+12+16+20+P+26}{6} = 17$$

$84 + P = 17 \times 6$

$P = 102 - 84$

$P = 18$

4. Class mark $= \frac{\text{Upper limit + Lower limit}}{2}$

$= \frac{100+120}{2}$

$= \frac{220}{2} = 110$

5. First five natural numbers are 1, 2, 3, 4 and 5.

Mean $= \frac{\text{Sum of all observations}}{\text{Total no. of observations}}$

$= \frac{1+2+3+4+5}{5}$

$= \frac{15}{5} = 3.$

Mean = 3.

6. Given data: 5, 8, 4, 5, 5, 8, 4, 7, 8, x

Mode of data = 5

Mode = The value in a set of data that appears most often i.e. = 5

Set of data = 5, 8, 4, 5, 5, 8, 4, 7, 8, x

5 and 8 both are repeating thrice.

But it is given that mode = 5

⇒ x = 5

SECTION-B

7. Mean of 10 numbers = 20

The sum of 10 numbers = 10 × 20 = 200

If 5 is subtracted from every number

$= 200 - (10 \times 5)$

$= 200 - 50 = 150$

So, the new mean $= \frac{150}{10} = 15.$

8. Class size = upper limit – lower limit = 10

i.e., lower class limit = 104

upper class limit = 114

9. The frequency distribution table for the data given is:

No. of Heads	No. of Frequency
0	6
1	9
2	9
3	4
Total	28

10. Given points scored by a team:

17, 2, 7, 27, 25, 5, 14, 18, 10, 24, 48, 10, 8, 7, 10, 28.

Arrange in ascending order

2, 5, 7, 7, 8, 10, 10, 10, 14, 17, 18, 24, 25, 27, 28, 48.

Value of n = even

No. of terms = 16

$$\Rightarrow \text{Value of } \frac{1}{2}\left[\left(\frac{n}{2}\right)^{th} \text{term} + \left(\frac{n}{2}+1\right)^{th} \text{term}\right]$$

will be the median.

$$= \frac{1}{2}\left[\frac{16}{2}^{th} \text{term} + \left(\frac{16}{2}+1\right)^{th} \text{term}\right]$$

$$= \frac{1}{2}\left[8^{th} \text{term} + 9^{th} \text{term}\right]$$

$$= \frac{1}{2}\left[10+14\right]$$

$$= \frac{1}{2}\times 24 = 12$$

Median = 12.

Mode is the maximum no. of repeating terms in a data, and 10 is repeating thrice.

Mode = 10.

SECTION-C

11. Given data: 8, 15, 10, 12, 20, 13, 7, 25, 15, 20, 20, 9, 20, 25, 15

$$\text{Mean} = \frac{\text{Sum of all observations}}{\text{Total no. of observations}}$$

$$= \frac{8+15+10+12+20+13+7+25+15+20+20+9+20+25+15}{15}$$

$$= \frac{234}{15} = 15.6$$

Mean = 15.6

Arrange in ascending order

7, 8, 9, 10, 12, 13, 15, 15, 15, 20, 20, 20, 20, 25, 25.

Since n = 15

$$\text{So median} = \text{Value of} \left(\frac{n+1}{2}\right)^{th} \text{term}$$

$$= \left(\frac{15+1}{2}\right)^{th} \text{term}$$

$$= \left(\frac{16}{2}\right)^{th} \text{term}$$

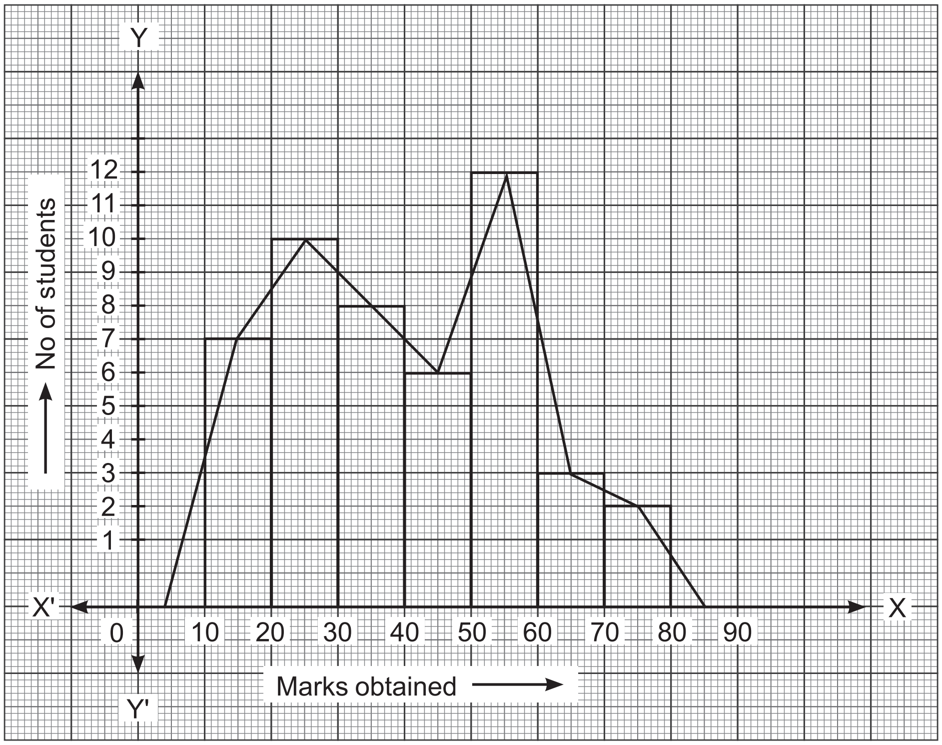
$$= 8^{th} \text{term} = 15$$

Median = 15

Mode is the maximum no. of repeating term, i.e., 20 is repeating four times.

Mode = 20

12.



First we draw a histogram from the given data and then join the mid-points of the top of the rectangles. Complete the polygon by joining the mid-points of first and last class intervals to the mid-points.

13. Mean of following distribution is 50.

x	**y**	**xy**
10	17	170
30	5a + 3	30(5a + 3)
50	32	1600
70	7a – 11	(7a – 11) 70
90	19	1710

$$\text{Mean} = \frac{\text{Sum of all observations}}{\text{No. of observations}}$$

$$\text{Mean} = \frac{\sum x_i f_i}{\sum f_i}$$

$$50 = \frac{170+1600+1710+150a+90+490a-770}{60+12a}$$

$$50\ (60 + 12a) = 640a + 2800$$

$$3000 + 600a = 640a + 2800$$

$$3000 - 2800 = 640a - 600a$$

$$200 = 40a$$

$$a = \frac{200}{40} \Rightarrow a = 5$$

Frequency of 30 = 5(5) + 3 = 25 + 3 = 28

Frequency of 70 = 7(5) – 11 = 35 – 11 = 24.

14. No. of students = 100

Initial mean = 40

$$\text{Mean} = \frac{\text{Sum of all observations}}{\text{No. of observations}}$$

Sum of observations = Mean × No. of students

= 40 × 100 = 4000

Total mistake done

= (88 + 92 + 81) – (08 + 29 + 18)

= 261 – 35 = 206

∴ Correct sum of observations

= 4000 + 206 = 4206

$$\therefore \text{Correct mean} = \frac{4206}{100} = 42.06$$

15. Median of following arranged observation = 24

1.4, 18, x + 2, x + 4, 30, 34

Since n = 6, i.e., even

$$\text{Median} = \frac{1}{2}\left[\left(\frac{n}{2}\right)^{th} \text{term} + \left(\frac{n}{2}+1\right)^{th}\right]$$

$$\text{Median} = \frac{1}{2}\left[\left(\frac{6}{2}\right)^{th} \text{term} + \left(\frac{6}{2}+1\right)^{th}\right]$$

$$24 = \frac{1}{2}\ [3^{rd} \text{ term} + 4^{th} \text{ term}]$$

$$24 \times 2 = (x + 2 + x + 4)$$

$$48 = 2x + 6$$

$$2x = 48 - 6$$

$$2x = 42$$

$$x = 21$$

$$x + 2 = 21 + 2 = 23$$

$$x + 4 = 21 + 4 = 25$$

Following data will be

14, 18, 23, 25, 30, 34

$$\text{Mean} = \frac{\text{Sum of all observations}}{\text{No. of observations}}$$

$$= \frac{14+18+23+25+30+34}{6}$$

$$= \frac{144}{6} = 24$$

SECTION-D

16. Let the total mark of boys = b

Let the total mark of girls = g

Total no. of boys = x

Total no. of girls = y

$$\text{Mean marks of boys} = \frac{b}{x} = 70$$

$$\text{Mean marks of girls} = \frac{g}{x} = 73$$

$$\text{Mean marks of total students} = \frac{(b+g)}{(x+y)} = 71$$

$b + g = 71 (x + y)$

$b + g = 71x + 71y$

Now $b = 70x$ and $g = 73y$

$\Rightarrow (70x + 73y) = 71x + 71y$

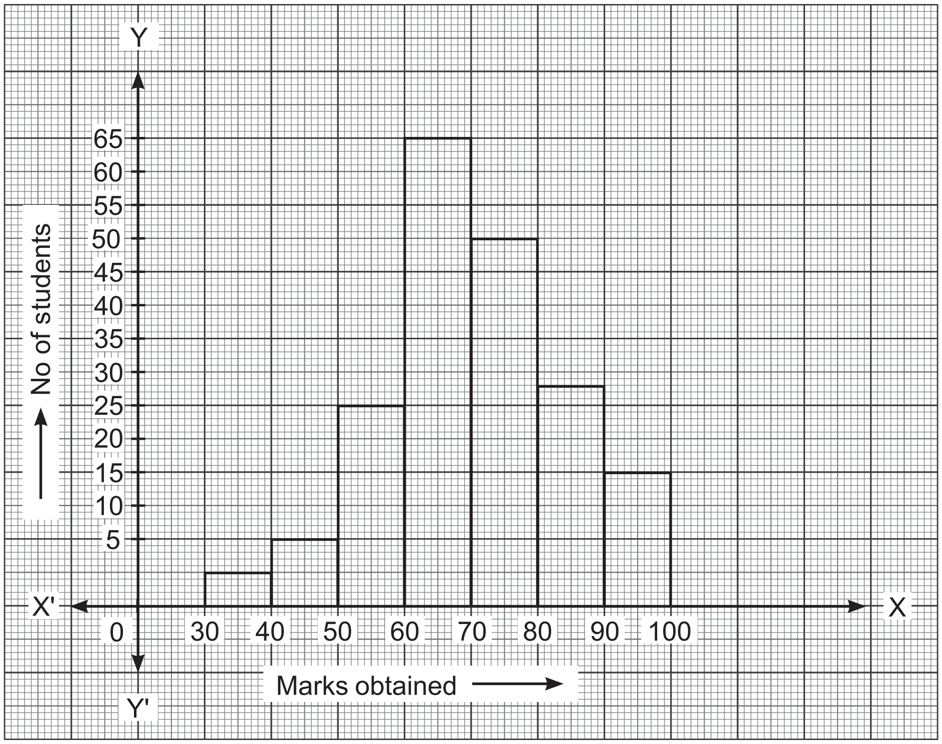
$\Rightarrow -x = -2y$

$\Rightarrow x = -2y$

$\Rightarrow \frac{x}{y} = \frac{2}{1}$

⇒ Ratio will be 2 : 1

17. Draw a histogram for the given table of marks scored by 70 students of class IX.



18. Mean of five no. = 27

So, sum of 5 numbers = 27 × 5 = 135

Let excluding number = x

sum of 4 numbers = 135 – x

$$\text{Mean} = \frac{\text{Sum of 4 numbers}}{4} = \frac{135 - x}{4}$$

$\Rightarrow 27 - 2 = \frac{135 - x}{4}$

$\Rightarrow 25 \times 4 = 135 - x$

$100 - 135 = -x$

$-x = -35$

$x = 35$

Hence excluded number = 35

WORKSHEET 2: SECTION-A

1. Mid value = 10

Width = 6

$$\text{Lower limit} = \text{mid value of class} - \frac{\text{Width}}{2}$$

$$= 10 - \frac{6}{2}$$

$$= 10 - 3 = 7$$

2. The child is incorrect, because while finding median, the data should be first arranged in ascending order. 3, 5, 14, 18, 20. There are 5 terms so median = mid-term which is 14.

3. It is not correct. Because the difference between two consecutive class marks should be equal to the class size. Here, difference between two consecutive marks is 0.1 and class size of 1.55 – 1.73 is 0.18, which are not equal.

4. It is not correct. Because in a histogram, the area of each rectangle is proportional to the corresponding frequency of its class.

SECTION-B

5. Mean of p, q, r, s and t is 28.

Mean of p, r and t = 24

$$\frac{(p+q+r+s+t)}{5} = 28$$

$$\Rightarrow \quad p + q + r + s + t = 140 \qquad ...(1)$$

$$\frac{p+r+t}{3} = 24$$

$$p + r + t = 72 \qquad ...(2)$$

Substitute eq. (2) in eq. (1)

$$p + q + r + s + t = 140$$

$$72 + q + s = 140$$

$$q + s = 140 - 72$$

$$q + s = 68$$

$$\text{Mean of q and s} = \frac{q+s}{2}$$

$$= \frac{68}{2} = 34.$$

6. Factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24.

$$\text{Mean} = \frac{\text{Sum of all observations}}{\text{Total no. of observations}}$$

$$= \frac{1+2+3+4+5+6+8+12+24}{8}$$

$$= \frac{60}{8} = 7.5$$

7. Given mean of 100 observations = 50

Sum of observations = Mean × No. of observations.

$$= 50 \times 100 = 5000$$

So we need to add this 100 to the sum of the observations to get the correct sum of observations.

Hence correct sum of observations

$$= 5000 + 100 = 5100$$

$$\text{Correct mean} = \frac{5100}{100} = 51.$$

8. Class Size = 42 – 37 = 5

Class limit are 35 – 39, 40 – 44, 45 – 49, 50 – 54, 55 – 59

SECTION-C

9. Arranged data is 6, 14, 15, 17, x + 1, 2x – 13, 30, 32, 34, 43.

Total no. of observations = 10

Here, 10 is an even number.

∴ Median will be the mean of $\left(\frac{10}{2}\right)^{th}$ and $\left(\frac{10}{2}+1\right)^{th}$ observations.

$$\text{Median} = \frac{5^{th}\text{ observation} + 6^{th}\text{ observation}}{2}$$

$$24 = \frac{x+1+2x-13}{2}$$

$$48 = 3x - 12$$

$$48 + 12 = 3x$$

$$60 = 3x$$

$$x = \frac{60}{3}$$

$$x = 20$$

10. $$\text{Mean of data} = \frac{\text{Sum of all observations}}{\text{No. of observations}}$$

$$\Rightarrow = \frac{41+39+43+52+46+62+54+40 +96+52+98+40+42+52+60}{15}$$

$$\Rightarrow = \frac{817}{15} \Rightarrow = 54.47$$

Median

Arranging in Ascending order, 39, 40, 40, 41, 42, 43, 46, 52, 52, 52, 54, 60, 62, 96, 98

No. of obs. = n = 15 (odd number)

$$\text{Median} = \left(\frac{n+1}{2}\right)^{th} \text{observation}$$

$$= \left(\frac{15+1}{2}\right)^{th} \text{observation}$$

$$= \left(\frac{16}{2}\right)^{th} \text{observation}$$

$$= 8^{th} \text{observation} = 52$$

Mode = 52

11.

Salary (x_i)	**No. of workers (f_i)**	**$f_i x_i$**
3000	16	48000
4000	12	48000
5000	10	50000
6000	8	48000
7000	6	42000
8000	4	32000
9000	3	27000
10000	1	10000
Total	60	305000

$$\text{Mean} = \frac{\sum x_i f_i}{\sum f_i}$$

$$= \frac{305000}{60}$$

$$(\bar{x}) = \frac{30500}{6}$$

$$= 5083.33$$

12.

x_i	**f_i**	**$x_i f_i$**
4	4	16
6	8	48
8	14	112
10	11	110
12	3	36
Total	40	332

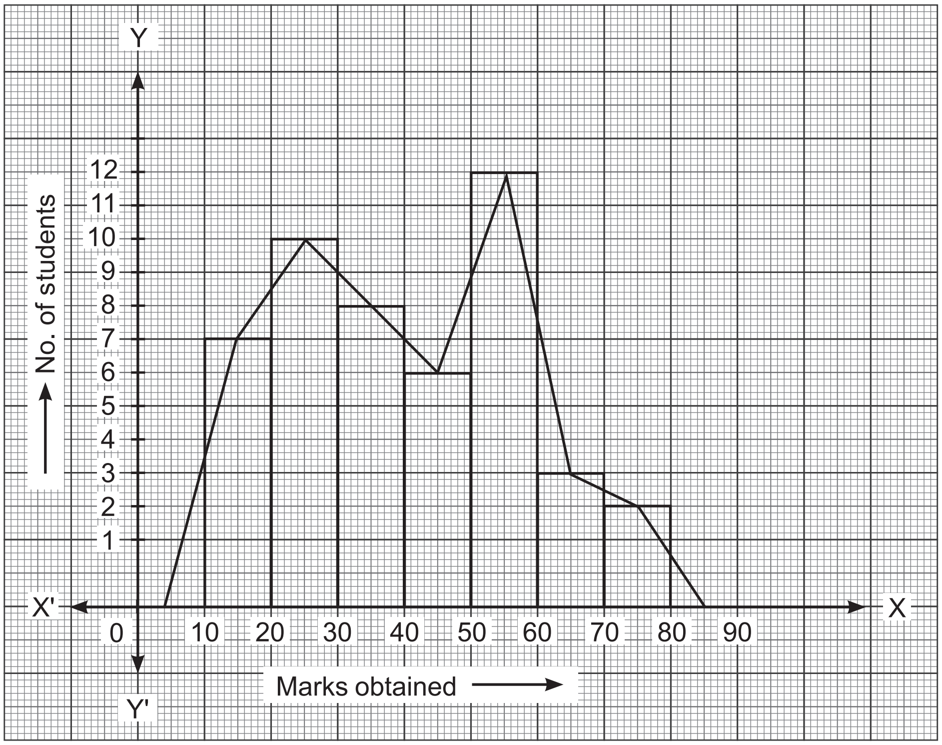
$$\text{Mean} = \frac{\sum x_i f_i}{\sum f_i}$$

$$= \frac{322}{40}$$

$$= 8.05$$

SECTION-D

13.



14. Mean of distribution = 1.46

x_i	f_i	$x_i f_i$
0	46	0
1	f_1	f_1
2	f_2	$2f_2$
3	25	75
4	10	40
5	5	25
Total	200	$140 + 2f_2 + f_1$

$$46 + f_1 + f_2 + 25 + 10 + 5 = 200$$

$$f_1 + f_2 = 114 \qquad \text{...(1)}$$

$$\text{Mean} = \frac{\sum x_i f_i}{\sum f_i}$$

$$1.46 = \frac{140 + 2f_2 + f_1}{86 + f_1 + f_2}$$

$$1.46 = \frac{140 + 2f_2 + f_1}{86 + 144}$$

$$1.46 \times 200 = 140 + f_1 + 2f_2$$

$$292 - 140 = f_1 + 2f_2$$

$$f_1 + 2f_2 = 152 \qquad \text{...(2)}$$

Subtracting eq. (1) from (2), we get

$$f_1 - f_1 + 2f_2 - f_2 = 152 - 114$$

$$f_2 = 38$$

Put in (1), we get

$$f_1 + f_2 = 114$$

$$f_1 + 38 = 114$$

$$f_1 = 114 - 38$$

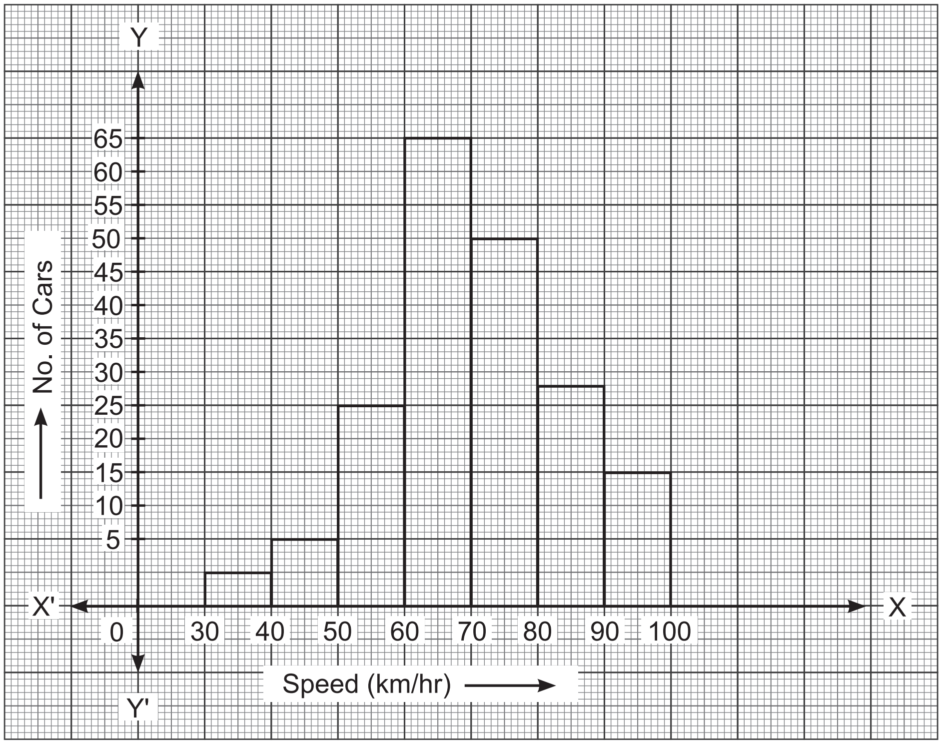
$$f_1 = 76$$

$$\Rightarrow \qquad f_1 = 76 \text{ and } f_2 = 38.$$

15.

Speed (km/h)	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No. of Cars	3	6	25	65	50	28	14

Here's a histogram of a given data:



CASE STUDY-1

1. (i) (b) 9 students have blood group A.

(ii) (a) 6 students have blood group B.

(iii) (c) 9 students have blood group A and 6 students have blood group B. Thus the total number of students having blood group AB are 15.

(iv) (c) Maximum students have blood group O, making it the most common blood group.

(v) (d) Only 3 students have blood group AB, making it the least common blood group.

CASE STUDY-2

2. (i) (a) Party A has maximum number of seats.

(ii) (b) Party D has least number of seats.

(iii) (a) Total seats are:

$70 + 60 + 40 + 20 + 40 + 50 = 280$

(iv) (c) Party C and E got equal number of seats.

(v) (d) There are minimum 20 seats.