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MATHEMATICS 10

Update Answer Key

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Chapter 1 Real Numbers

Multiple Choice Questions

1. (a) Here, a = Dividend, b = Divisor, q = Quotient and r = Remainder

Using Euclid's Division Lemma,

$$a = bq + r, \quad 0 \le r < b$$

$$a = 3q + r$$

Here $b = 3$;

So, possible values of r = 0, 1, 2.

$\therefore \quad 0 \le r < 3.$

2. (c) LCM of 23 and 33 = 23 × 33

3. (b) Given A = 2n + 13 and B = n + 7 and since n is a natural number A > B. Since A/B is not an integer we can conclude that A and B cannot have any integer in common. Therefore HCF of A and B is 1.

4. (d) Prime factorization of 102 = 2 × 3 × 17

Prime factorization of 85 = 5 × 17 = (2 + 3) × 17

Two numbers are:

(1) 3 × 17 = 51 and (2) 2 × 17 = 34.

$\therefore$ Numbers are 51 and 34.

5. (a) Since 5 and 8 are remainders, So we subtract these remainders from 70 and 125 respectively.

We get;

(i) 70 – 5 = 65 (ii) 125 – 8 = 117

Now, taking HCF of 65 and 117;

$$117 = 65 \times 1 + 52$$

$$65 = 52 \times 1 + 13$$

$$52 = 13 \times 4 + 0$$

$\therefore$ HCF = 13.

The largest number which divides 70 and 125 leaving remainders 5 and 8 respectively is 13.

WORKSHEET - 1

SECTION-A

1. Here, a is a dividend.
2. 13233343563715 is a composite number as it is also divisible by 5 besides 1 and the number itself.
3. Dividend = Divisor × Quotient + Remainder

   = 53 × 34 + 21

   = 1823
4. y = 5 × 13 = 65

   x = 3 × 195 = 585
5. HCF (k, 2k, 3k, 4k, 5k) = k
6. Smallest composite number = 4

   Smallest prime number = 2

   $\therefore$ HCF (2, 4) = 2
7. $6^n = (2 \times 3)^n$

   We know that a number ends with digit 0 only if it has both 2 and 5 as factors. As 6^n does not have 5 as a prime factor, so, 6^n does not end with digit 0.
8. LCM (p, q) = LCM $(ab, a^2b) = a^2b$
9. HCF (a, b) = HCF (x^3y^2, xy^3)

   $= xy^2$

10. LCM (a, b) = $\frac{a \times b}{HCF(a, b)}$

$= \frac{1800}{12} = 150$

SECTION-B

11. Using Euclid's Division lemma,

$a = 4q + r, \quad 0 \le r < 4$

$r = 0, \quad a = 4q$

$r = 1, \quad a = 4q + 1$

$r = 2, \quad a = 4q + 2$

$r = 3, \quad a = 4q + 3$

Hence, every positive integer can not be of form 4q + 2, it can also be of the form 4q, 4q + 1 or 4q + 3.

12. Using Euclid's Algorithm,

$240 = 228 \times 1 + 12$

$228 = 12 \times 19 + 0$

Here, remainder = 0, Divisor = 12

So, HCF (240, 228) = 12

13. LCM (253, 440) = $\frac{253 \times 440}{HCF(253, 440)}$

$\therefore$ LCM (253, 440) = $\frac{253 \times 440}{1} = 253 \times R$

So, R = 440

14. $3 \times 12 \times 101 + 4$

$= 4 \times (3 \times 3 \times 101 + 1)$

So, 4 is also a factor of $3 \times 12 \times 101 + 4$ besides 1 and the number itself.

So, $3 \times 12 \times 101 + 4$ is a composite number.

15. $\sqrt{1200} = \sqrt{2^4 \times 3 \times 5^2} = 2^2 \times 5\sqrt{3}$

So, square root of product is a rational number if we multiply 1200 by 3 such that

$\sqrt{1200 \times 3} = \sqrt{2^4 \times 3^2 \times 5^2} = 60$

16. The least number that is divisible by all the numbers from 1 to 10 is basically equal to LCM (1, 2, 3, 4, ..., 10) = 2520

17. Let x and x + 1 be two consecutive positive integers.

If x is even, x + 1 is odd, so, x (x + 1) is even

If x is odd, x + 1 is even, so, x (x + 1) is even.

Therefore, the product of two consecutive positive integers is always divisible by 2.

18. $3 \times 5 \times 13 \times 46 + 23$

$= 23 \times (3 \times 5 \times 13 \times 2 + 1)$

So, 23 is a factor of $3 \times 15 \times 13 \times 46 + 23$ besides 1 and the number itself.

Therefore, $3 \times 5 \times 13 \times 46 + 23$ is a composite number.

19. As least prime factor of a is 3, a is an odd number (because if a is even then it's least prime factor must be 2). Also, as least prime factor of b is 5, b is an odd number.

Therefore, a + b is even such that it's least prime factor is 2.

20. No, two numbers can not have 15 as their HCF and 175 as their LCM because 15 is not a factor of 175.

(HCF of two numbers is always the factor of their LCM)

SECTION-C

21. Using Euclid's Division lemma.

$a = 6q + r; \quad 0 \le r < 6$

$r = 0, \quad a = 6q = 2 (3q)$, even

$r = 1, \quad a = 6q + 1 = 2 (3q) + 1$, odd

$r = 2, \quad a = 6q + 2 = 2 (3q + 1)$, even

$r = 3, \quad a = 6q + 3 = 2 (3q + 1) + 1$, odd

$r = 4, \quad a = 6q + 4 = 2 (3q + 2)$, even

$r = 5, \quad a = 6q + 5 = 2 (3q + 2) + 1$, odd

So, any positive even integer can be written in the form of 6q, 6q + 2 or 6q + 4.

22. We know that any positive odd integer (say a) is of form 4q + 1 or 4q + 3

Case 1

a = 4q + 1

$a^2 = (4q + 1)^2 = 16q^2 + 1 + 8q = 8(2q^2 + q) + 1$

$= 8m + 1 \quad (m = 2q^2 + q)$

Case 2

a = 4q + 3

$a^2 = (4q + 3)^2 = 16q^2 + 9 + 24q$

$= 8(2q^2 + 3q + 1) + 1$

$= 8m + 1 \quad (m = 2q^2 + 3q + 1)$

so, square of an odd positive integer is of form 8m + 1.

23. First, find HCF (180, 252)

$252 = 180 \times 1 + 72$

$180 = 72 \times 2 + 36$

$72 = 36 \times 2 + 0$

Remainder = 0, Divisor = 36, So, HCF = 36

Now, find HCF (36, 324)

$324 = 36 \times 9 + 0$

So, HCF (36, 324) = 36

∴ HCF (180, 252, 324) = 36

24. Using Euclid's Division lemma,

$a = 5q + r; \quad 0 \leq r < 5$

$r = 0, \quad a = 5q, \quad a^2 = 25q^2 = 5m \quad (m = 5q^2)$

$r = 1, \quad a = 5q + 1, \quad a^2 = 25q^2 + 1 + 10q$

$= 5(5q^2 + 2q) + 1$

$= 5m + 1 \; (m = 5q^2 + 2q)$

$r = 2, \quad a = 5q + 2, \quad a^2 = 25q^2 + 4 + 20q$

$= 5(5q^2 + 4q) + 4$

$= 5m + 4 \; (m = 5q^2 + 4q)$

$r = 3, \quad a = 5q + 3, \quad a^2 = 25q^2 + 9 + 30q$

$= 5(5q^2 + 6q + 1) + 4$

$= 5m + 4 \; (m = 5q^2 + 6q + 1)$

$r = 4, \quad a = 5q + 4, \quad a^2 = 25q^2 + 16 + 40q$

$= 5(5q^2 + 8q + 3) + 1$

$= 5m + 1 \quad (m = 5q^2 + 8q + 3)$

So, square of positive integer cannot be of form 5m + 2 or 5m + 3.

25. Minimum distance each should walk so that each can cover the same distance.

= LCM (40, 42, 45)

= 2520 cm

2	40, 42, 45
2	20, 21, 45
2	10, 21, 45
3	5, 21, 45
3	5, 7, 15
5	5, 7, 5
7	1, 7, 1
	1, 1, 1

26. $7 \times 19 \times 11 + 11$

$= 11(7 \times 19 \times 1 + 1)$

So, 11 is also a factor of $7 \times 19 \times 11 + 11$ besides 1 and number itself.

So, $7 \times 19 \times 11 + 11)$ is a composite number.

$7 \times 6 \times 4 \times 3 \times 2 \times 1 + 3 = 3(7 \times 6 \times 4 \times 2 \times 1 + 1)$

So, 3 is also a factor of $7 \times 6 \times 4 \times 3 \times 2 \times 1 + 3$ besides 1 and number itself.

So, $7 \times 6 \times 4 \times 3 \times 2 \times 1 + 3$ is a composite number.

27. Here, we have to find LCM (12, 15, 18) which indicates after how long they all again toll together.

LCM (12, 15, 18) = 180

So, three bells will toll together after 180 minutes i.e. 3 hours.

2	12, 15, 18
2	6, 15, 9
3	3, 15, 9
3	1, 5, 3
5	1, 5, 1
	1, 1, 1

28. Using Euclid's division algorithm,

$$1170 = 650 \times 1 + 520$$
$$650 = 520 \times 1 + 130$$
$$520 = 130 \times 4 + 0$$

So, HCF (650, 1170) = 130

Therefore, the largest number which divides 650 and 1170 exactly is 130.

29. Consider $\frac{1}{3+2\sqrt{2}} = \frac{1}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$

$$= \frac{3-2\sqrt{2}}{3^2-\left(2\sqrt{2}\right)^2} = \frac{3-2\sqrt{2}}{1} = 3-2\sqrt{2}$$

Let if possible $3-2\sqrt{2}$ is rational

$3-2\sqrt{2} = \frac{p}{q}$, p and q are integers and $q \neq 0$

$$\frac{1}{2}\left(3-\frac{p}{q}\right) = \sqrt{2}$$

Here, $\frac{1}{2}\left(3-\frac{p}{q}\right)$ is rational but $\sqrt{2}$ is irrational which is not possible.

So, we get a contradiction.

Therefore, $3-2\sqrt{2}$ is irrational .

i.e. $\frac{1}{3+2\sqrt{2}}$ is irrational.

30. Using Euclid's division lemma,

$$117 = 65 \times 1 + 52$$
$$65 = 52 \times 1 + 13$$
$$52 = 13 \times 4 + 0$$

Here, remainder = 0, divisor = 13

So, HCF (117, 65) = 13

To find : m, n

$$13 = 65 - 52(1)$$
$$= 65 - (117 - 65(1))$$
$$= 65(2) + 117(-1)$$
$$= 65m + 117n$$

So, m = 2, n = –1

SECTION-D

31. Using Euclid's Division Algorithm, we get

$$256 = 36 \times 7 + 4$$
$$36 = 4 \times 9 + 0$$

Here, remainder = 0, divisor = 4

So, HCF (256, 36) = 4

Now,

2	256, 36
2	128, 18
2	64, 9
2	32, 9
2	16, 9
2	8, 9
2	4, 9
2	2, 9
3	1, 9
3	1, 3
	1, 1

Now, LCM (256, 36) = $2^8 \times 3^2 = 2304$

HCF × LCM = 4 × 2304
= 9216

Product of numbers = 256 × 36
= 9216

So, HCF × LCM = Product of numbers

32. We know that every positive even integer is of form 2q and every positive odd integer is of form 2q + 1.

Case 1 n = 2q

Consider $n^2 - n = 4q^2 - 2q = 2(2q^2 - q)$

$\therefore$ $n^2 - n$ is divisible by 2

Case 2 n = 2q + 1

Consider $n^2 - n = (2q+1)^2 - (2q+1)$
$= 4q^2 + 1 + 4q - 2q - 1$
$= 4q^2 + 4q - 2q$

$= 2\ (2q^2 + 2q - q)$

$= 2\ (2q^2 + q)$

$\therefore$ $n^2 - n$ is divisible by 2.

From Case 1, Case 2, we get $n^2 - n$ is divisible by 2 for every positive integer n.

33. According to Euclid's Division lemma,

$a = 3q + r, \quad 0 \leq r < 3$

For r = 0

$a = 3q \Rightarrow a^3 = 27q^3 \Rightarrow a^3 = 9\ (3q^3)$

$= 9m\ (m = 3q^3)$

For r = 1

$a = 3q + 1 \Rightarrow a^3 = 27q^3 + 1 + 27q^2 + 9q$

$= 9\ (3q^3 + 3q^2 + q) + 1$

$= 9m + 1\ (m = 3q^3 + 3q^2 + 2)$

For r = 2

$a = 3q + 2 \Rightarrow a^3 = 27q^3 + 8 + 54q^2 + 36q$

$= 9\ (3q^3 + 6q^2 + 4q) + 8$

$= 9m + 8\ (m = 3q^3 + 6q^2 + 4q)$

Therefore, cube of any positive integer is of form 9m, 9m + 1 or 9m + 8 for some integer m.

34. (a) Greatest possible length of each plank

= HCF (42, 49, 56)

$= \text{HCF}\ (2 \times 3 \times 7, 7^2, 2^3 \times 7)$

= 7

So, greatest possible length of each plank is 7m.

(b) HCF (182, 169)

$= \text{HCF}\ (2 \times 7 \times 13, 13^2)$

= 13

2	182
7	91
13	13
	1

13	169
13	13
	1

35. In order to find the number of fruits to be put in each basket in order to have minimum number of baskets, we will find HCF (990, 945)

$990 = 2 \times 3^2 \times 5 \times 11$

$945 = 3^3 \times 5 \times 7$

$\text{HCF}\ (990, 945) = 3^2 \times 5 = 45$

Therefore, 45 fruits should be put in each basket.

Number of baskets containing apples $= \frac{990}{45}$

= 22

Number of baskets containing oranges $= \frac{945}{45}$

= 21

So, total number of baskets = 22 + 21

= 43.

36. Let the three consecutive positive integers be n, n + 1 and n +2.

If number is divided by 3, remainder can be 0, 1 or 2. i.e. $n = 3q + r, 0 \leq r < 3$

If r = 0, n = 3q divisible by 3

If r = 1, n + 2 = 3q + 1 + 2

= 3q + 3

= 3 (q + 1) divisible by 3

If r = 2, n + 1 = 3q + 2 + 1 = 3 (q + 1) divisible by 3

So, one of numbers n, n + 1, and n + 2 must be divisible by 3 i.e. n (n + 1) (n + 2) is divisible by 3

Now, if a number is divided by 2, remainder is 0 or 1

i.e. $n = 2q + r\ ; \quad 0 \leq r < 2$

If r = 1 n + 1 = 2q + 1 + 1 = 2q + 2

= 2 (q + 1) divisible by 2

If r = 2 n + 2 = 2q + 2 + 2 = 2q + 4

= 2 (q + 2) divisible by 2.

So, one of n, n + 1 or n + 2 is divisible by 2 i.e. n (n + 1) (n + 2) is divisible by 2.

Since, n (n + 1) (n + 2) is divisible by 2 and 3 implies n (n + 1) (n + 2) is divisible by 6.

37. (a)

2	420
2	210
5	105
3	21
7	7
	1

2	180
2	90
3	45
3	15
5	5
	1

2	378
3	189
3	63
3	21
7	7
	1

So, HCF (420, 180, 378)

$= \text{HCF}(2^2 \times 3 \times 5 \times 7,\ 2^2 \times 3^2 \times 5,\ 2 \times 3^3 \times 7)$

$= 2 \times 3 = 6$

2	378, 180, 420
2	189, 90, 210
3	189, 45, 105
3	63, 15, 35
3	21, 5, 35
5	7, 5, 35
7	7, 1, 7
	1, 1, 1

LCM (378, 180, 420)

$= 2^2 \times 3^3 \times 5 \times 7$

= 3780

Now HCF × LCM

= 6 × 3780 = 22680

Product of numbers = 378 × 180 × 420

= 28576800

So, HCF × LCM ≠ Product of numbers.

(b) Let if possible $2\sqrt{2}$ is rational.

$$2\sqrt{2} = \frac{p}{q},$$ p and q are integers, $q \neq 0$

$$\Rightarrow \sqrt{2} = \frac{p}{2q}$$

Here, $\frac{p}{2q}$ is rational but $\sqrt{2}$ is irrational. So, we get a contradiction.

$\therefore 2\sqrt{2}$ is irrational.

38. (a) Let if possible $\frac{2\sqrt{3}}{5}$ is rational.

$$\frac{2\sqrt{3}}{5} = \frac{p}{q};$$ p and q are integers, $q \neq 0$

$$\sqrt{3} = \frac{5p}{2q}$$

Here, $\frac{5p}{2q}$ is rational but $\sqrt{3}$ is irrational which is not possible, so we get a contradiction.

$\therefore \frac{2\sqrt{3}}{5}$ is irrational.

(b) 3 rational numbers between 1.12 and 1.13 are 1.1210, 1.1211, 1.1213.

3 irrational numbers between 1.12 and 1.13 are 1.121121112111..., 1.1221222..., 1.123123312333...

WORKSHEET - 2

SECTION-A

1. Here, denominator $= 2^2 \cdot 5^7 \cdot 7^2$. As denominator is not of the form $2^m \times 5^n$, so, the given rational number has a non-terminating repeating decimal expansion.

2. $$\frac{2\sqrt{45}+2\sqrt{20}}{2\sqrt{5}} = \frac{6\sqrt{5}+4\sqrt{5}}{2\sqrt{5}} = \frac{10\sqrt{5}}{2\sqrt{5}} = 5$$

which is rational.

3. HCF (a, b) × LCM (a, b) = a × b

$$15 \times \text{LCM} = 45 \times 105$$

$$\text{LCM} = \frac{\overset{3}{\cancel{45}} \times 105}{\cancel{15}} = 315$$

4. Decimal expansion will terminate after 4 places of decimal.

5. HCF × LCM = 100 × 170 = 17000.

6. Here, denominator $= 1500 = 2^2 \times 3 \times 5^3$

As denominator is not of the form $2^m \times 5^n$, so, it has non-terminating repeating decimal expansion.

7. HCF (a, b) × LCM (a, b) = a × b

$$9 \times 360 = a \times 45$$

$$\frac{9\times 360}{45} = a$$
$$72 = a$$

8. $\frac{7}{625} = 0.0112$

9. $\frac{95}{40}+\frac{15}{4}=\frac{95+150}{40}=\frac{245}{40} = 6.125$

10. Decimal expansion will terminate after 5 places of decimal.

SECTION-B

11.
$$\begin{array}{r|l} 8 & 3 \\ \end{array}$$
$$\begin{array}{l} \quad 0.375 \\ 8\overline{)\,3} \\ -0 \\ \hline \;30 \\ -24 \\ \hline \;\;60 \\ \;-56 \\ \hline \;\;\;40 \\ \;\;-40 \\ \hline \;\;\;\;0 \end{array}$$

$\therefore \quad \frac{3}{8} = 0.375$

12. Let if possible $5\sqrt{6}$ is rational.

$$5\sqrt{6} = \frac{p}{q}; \quad p, q \text{ are integers, } q \neq 0$$

$$\sqrt{6} = \frac{p}{5q}$$

Here, $\frac{p}{5q}$ is rational but $\sqrt{6}$ is irrational which is not possible. So, we get a contradiction i.e. $5\sqrt{6}$ is irrational.

13. Let $x = 1.\overline{41}...$ .....(*i*)

$$x \times 100 = 1.\overline{41} \times 100$$
$$100x = 141.\overline{41} \quad .....(ii)$$

On subtracting (*i*) from (*ii*), we get

$$99x = 140$$
$$x = \frac{140}{99}$$

14. Maximum capacity = HCF (850, 680)

$$= \text{HCF}(2 \times 5^2 \times 17, 2^3 \times 5 \times 17)$$
$$= 2 \times 5 \times 17$$
$$= 170\ l.$$

15. (a) $(-1) + (-1)^{2n} + (-1)^{2n+1} + (-1)^{4n+1}$
$= (-1) + (1) + (-1) + (-1)$
$= -2$

(b) $\left(2^3\right)^{\frac{-5}{3}} = 2^{3\cdot\frac{-5}{3}} = 2^{-5} = \frac{1}{2^5}$
$= \frac{1}{32}$

16. $\frac{13}{64} = \frac{13}{2^6}$

Here, Denominator = $2^6 = 2^6 \times 5^0$

i.e. of form $2^m \times 5^n$, so, it has terminating decimal expansion.

Here, highest power in $2^6 \times 5^0$ is 6, so it's decimal expansion has 6 decimal places.

17. Using Euclid's Algorithm.

$$4052 = 420 \times 9 + 272$$
$$420 = 272 \times 1 + 148$$
$$272 = 148 \times 1 + 124$$
$$148 = 124 \times 1 + 24$$
$$124 = 24 \times 5 + 4$$
$$24 = 4 \times 6 + 0$$

Here remainder = 0, Divisor = 4

So, HCF (4052, 420) = 4

18. Let if possible $\frac{3}{\sqrt{5}}$ is rational

$$\frac{3}{\sqrt{5}} = \frac{p}{q}, \quad p, q \text{ are integers, } q \neq 0$$

$$\sqrt{5} = \frac{3q}{p}$$

Here, $\frac{3q}{p}$ is rational but $\sqrt{5}$ is irrational which is not possible, so we get a contradiction.

$\therefore \quad \frac{3}{\sqrt{5}}$ is irrational.

19. Using Euclid's Division Algorithm,

$$180 = 144 \times 1 + 36$$
$$144 = 36 \times 4 + 0$$

Here, remainder = 0, divisor = 36

So, HCF (144, 180) = 36

We can write

$$\begin{aligned} 36 &= 180 - 144 (1) \\ &= 36 \\ &= 39 - 3 \\ &= 13 (3) - 3 \\ &= 13\text{ m} - 3 \\ m &= 3 \end{aligned}$$

20. $9^n = (3 \times 3)^n$

Since, prime factorization does not contain 2 and 5, so, it cannot end with digit 0.

SECTION-C

21. Let if possible $\sqrt{3} + \sqrt{5}$ is rational number.

$$\sqrt{3} + \sqrt{5} = \frac{p}{q},$$ p and q are integers and $q \neq 0$

$$\sqrt{3} = \frac{p}{q} - \sqrt{5}$$

$$\Rightarrow \quad \left(\sqrt{3}\right)^2 = \left(\frac{p}{q} - \sqrt{5}\right)^2$$

$$3 = \frac{p^2}{q^2} + 5 - \frac{2p}{q}\sqrt{5}$$

$$\frac{2p}{q}\sqrt{\ } = \frac{p^2}{q^2} + 2$$

$$\sqrt{5} = \frac{q}{2p}\left(\frac{p^2}{q^2} + 2\right)$$

Here, $\frac{q}{2p} \quad \frac{p^2}{q^2} + 2$ is rational but $\sqrt{5}$ is irrational, which is not possible.

Therefore, $\sqrt{3} + \sqrt{5}$ is irrational number.

22. Let if possible $2\sqrt{3} + \sqrt{7}$ is rational number.

$$2\sqrt{3} + \sqrt{7} = \frac{p}{q},$$ p, q are integers, $q \neq 0$

$$\sqrt{7} = \frac{p}{q} - 2\sqrt{3}$$

On squaring both sides, we get

$$7 = \frac{p^2}{q^2} + 12 - \frac{4p}{q}\sqrt{3}$$

$$\frac{4p}{q}\sqrt{3} = \frac{p^2}{q^2} + 5$$

$$\sqrt{3} = \frac{q}{4p}\ \frac{p^2}{q^2} + 5$$

Here, $\frac{q}{4p}\ \frac{p^2}{q^2} + 5$ is rational but $\sqrt{3}$ is irrational which is not possible. So, we get a contradiction

$\therefore \quad 2\sqrt{3} + \sqrt{7}$ is irrational number.

$$\left(2\sqrt{3} + \sqrt{7}\right)\left(2\sqrt{3} - \sqrt{7}\right) = \left(2\sqrt{3}\right)^2 - \left(\sqrt{7}\right)^2$$
$$= 12 - 7$$
$$= 5 \text{ which is rational number.}$$

23. $5 \times 7 \times 13 \times 17 + 289 = 17 (5 \times 7 \times 13 \times 1 + 17)$

Here, 17 is also a factor of $5 \times 7 \times 13 \times 17 + 289$ besides 1 and number itself. So, it is a composite number.

Also, $7 \times 11 \times 13 \times 15 + 225 = 15 (7 \times 11 \times 13 \times 1 + 15)$

Here, 15 is also a factor of $7 \times 11 \times 13 \times 15 + 225$ besides 1 and number itself. So, it is also a composite number.

24. LCM (20, 30, 40) = 120

So, all the three bells will toll together after 120 minutes i.e. 2 hours.

2	20, 30, 40
2	10, 15, 20
2	5, 15, 10
3	5, 15, 5
5	5, 5, 5
	1, 1, 1

25. Using Euclid's Division algorithm.

$$2058 = 378 \times 5 + 168$$
$$378 = 168 \times 2 + 42$$
$$168 = 42 \times 4 + 0$$

Here, remainder = 0, divisor = 42

So, HCF (2058, 378) = 42

Therefore, the largest number which divides 2058 and 378 is their HCF (i.e. 42).

26. Let HCF = x

$\therefore$ LCM = 14x

LCM + HCF = 600

14x + x = 600

15x = 600 $\Rightarrow$ x = 40

We know that HCF (a, b) × LCM (a, b) = a × b

$$40 \times 14 \times 40 = a \times 280$$
$$a = \frac{40 \times 14 \times 40}{280}$$
$$= 80$$

27. Using Euclid's division lemma,

a = 4q + r, $0 \leq r < 4$

r = 0,	a = 4q	= 2 (2q)	even
r = 1,	a = 4q + 1	= 2 (2q) + 1	odd
r = 2,	a = 4q + 2	= 2 (2q + 1)	even
r = 3,	a = 4q + 3	= 2 (2q + 1) + 1	odd

So, any positive odd integer is of form 4q + 1 or 4q + 3.

28. Let if possible $7 - 2\sqrt{3}$ is rational number.

$7 - 2\sqrt{3} = \frac{p}{q}$, p and q are integers and $q \neq 0$

$$2\sqrt{3} = 7 - \frac{p}{q}$$
$$\sqrt{3} = \frac{1}{2}\left(7 - \frac{p}{q}\right)$$

Here, $\frac{1}{2}\left(7 - \frac{p}{q}\right)$ is rational but $\sqrt{3}$ is irrational which is not possible.

So, we get a contradiction

$\therefore$ $7 - 2\sqrt{3}$ is irrational number.

SECTION-D

29. (a) Let if possible $\frac{1}{\sqrt{2}}$ is rational number.

$\frac{1}{\sqrt{2}} = \frac{p}{q}$, p and q are integers, $q \neq 0$

$$\sqrt{2} = \frac{q}{p}$$

Here, $\frac{q}{p}$ is rational but $\sqrt{2}$ is irrational which is not possible. So, we get a contraction.

$\therefore$ $\frac{1}{\sqrt{2}}$ is irrational number.

(b) Let if possible $7\sqrt{5}$ is rational number.

$7\sqrt{5} = \frac{p}{q}$, p and q are integers, $q \neq 0$

$$\sqrt{5} = \frac{p}{7q}$$

Here, $\frac{p}{7q}$ is rational but $\sqrt{5}$ is irrational which is not possible. So, we get a contraction.

$\therefore$ $7\sqrt{5}$ is irrational number.

30. Using Euclid's division algorithm

$$237 = 81 \times 2 + 75$$
$$81 = 75 \times 1 + 6$$
$$75 = 6 \times 12 + 3$$
$$6 = 3 \times 2 + 0$$

So, HCF (237, 81) = 3

Consider $3 = 75 - 6\,(12)$

$= (81 - 6) - 6\,(12)$

$= 81 - 13\,(6)$

$= 81 - 13\,(81 - 75)$

$= 81 - 13\ (81) + 13\ (75)$

$= 81 - 13\ (81) + 13\ (237 - 162)$

$= 81 - 81(13) + 13\ (237 - 81(2))$

$= 81\ (1 - 13 - 26) + 237\ (13)$

$= 81\ (-38) + 237\ (13)$

$= 81x + 237\ y$

where $x = -38,\ y = 13$

31. HCF (96, 240, 336)

$= \text{HCF}\ (2^5 \times 3, 2^4 \times 3 \times 5, 2^4 \times 3 \times 7)$

$= 2^4 \times 3$

$= 48$

So, number of stacks of English books $= \dfrac{96}{48}$

$= 2$

Number of stacks of Hindi books $= \dfrac{240}{48} = 5$

Number of stacks of Mathematics books

$= \dfrac{336}{48} = 7$

32. (a) 5 rational numbers between 1.1 and 1.2 are 1.11, 1.12, 1.13, 1.14, 1.15

5 irrational numbers are between 1.1 and 1.2 are 1.1121121112111..., 1.1121122111222..., 1.131331333..., 1.141441444..., 1.151551555...

(b) HCF (70 – 5, 125 – 8)

$= \text{HCF}\ (65, 117)$

$= \text{HCF}\ (5 \times 13, 3^2 \times 13)$

$= 13$

33. Let if possible $\sqrt{3}$ is rational number.

$\sqrt{3} = \dfrac{p}{q}$, p and q are integers $q \neq 0$,

HCF (p, q) = 1

$q\sqrt{3} = p$

$3q^2 = p^2$

3 divides $p^2 \Rightarrow$ 3 divides p

$p = 3c$

$p^2 = 9c^2 \Rightarrow 3q^2 = 9c^2$

$q^2 = 3c^2$

$\Rightarrow$ 3 divides $q^2 \Rightarrow$ 3 divides q

So, p and q have atleast 3 in common which is a contradiction to the fact that HCF (p, q) = 1

So, our supposition was wrong,

$\sqrt{3}$ is irrational number.

34. According to Euclid's division lemma, for any positive integer n, we have

$n = bq + r, 0 < r < b$

Take b = 5

$n = 5q + r, 0 < r < 5$

For r = 0

n = 5q, divisible by 5

n + 4 = 5q + 4, not divisible by 5

n + 8 = 5q + 8, not divisible by 5

n + 12 = 5q + 12, not divisible by 5

n + 16 = 5q + 16, not divisible by 5

So, for r = 0, only n is divisible by 5

For r = 1

n = 5q + 1, not divisible by 5

n + 4 = 5q + 1 + 4

= 5q + 5

= 5 (q + 1), divisible by 5

n + 8 = 5q + 1 + 8

= 5q + 9, not divisible by 5

n + 12 = 5q + 1 + 12

= 5q + 13, not divisible by 5

$n + 16 = 5q + 1 + 16$

$= 5q + 17$, not divisible by 5.

So, for $r = 1$, only $n + 4$ is divisible by 5

For $r = 2$,

$n = 5q + 2$, not divisible by 5

$n + 4 = 5q + 6$, not divisible by 5

$n + 8 = 5q + 10$

$= 5(q + 2)$, divisible by 5

$n + 12 = 5q + 14$, not divisible by 5

$n + 16 = 5q + 18$, not divisible by 5

So, for $r = 2$, only $n + 8$ is divisible by 5

For $r = 3$

$n = 5q + 3$, not divisible by 5

$n + 4 = 5q + 7$, not divisible by 5

$n + 8 = 5q + 11$, not divisible by 5

$n + 12 = 5q + 15$

$= 5(q + 3)$, divisible by 5

$n + 16 = 5q + 19$, not divisible by 5.

So, for $r = 3$, only $n + 12$ is divisible by 5.

For $r = 4$

$n = 5q + 4$, not divisible by 5

$n + 4 = 5q + 8$, not divisible by 5

$n + 8 = 5q + 12$, not divisible by 5

$n + 12 = 5q + 16$, not divisible by 5

$n + 16 = 5q + 20$

$= 5(q + 4)$, divisible by 5

So, for $r = 4$, only $n + 16$ is divisible by 5.

From 1, 2, 3, 4 It is clear that, one and only one out of n, n + 4, n + 12, n + 6 is divisible by 5.

35. Let if possible $n + \sqrt{m}$ is rational number.

$$n + \sqrt{m} = \frac{p}{q}; \quad p, q \text{ are integers and } q \neq 0$$

$$\therefore \quad \sqrt{m} = \frac{p}{q} - n$$

Here, $\frac{p}{q} - n$ is rational (as p, q are integers and n is rational) but $\sqrt{m}$ is irrational.

So, we get a contradiction.

Therefore, $n + \sqrt{m}$ is irrational number.

36. Let if possible $\sqrt{p} + \sqrt{q}$ is rational number.

$$\sqrt{p} + \sqrt{q} = \frac{a}{b}, \quad a, b \text{ are integers and } b \neq 0$$

$$\sqrt{p} = \frac{a}{b} - \sqrt{q}$$

On squaring both sides, we get

$$p = \frac{a^2}{b^2} + q - \frac{2a}{b}\sqrt{q}$$

$$\frac{2a}{b}\sqrt{q} = \frac{a^2}{b^2} + q - p$$

$$\sqrt{q} = \frac{b}{2a}\left(\frac{a^2}{b^2} + q - p\right)$$

Here $\frac{b}{2a}\left(\frac{a^2}{b^2} + q - p\right)$ is rational but $\sqrt{q}$ is irrational (as square root of a prime number is irrational) which is not possible.

So, we get a contradiction.

Therefore, $\sqrt{p} + \sqrt{q}$ is irrational number.

37. Circumference of circular field = 360 km

Distance traveled by I[st] cyclist = 48 km

So, number of days takes by I[st] cyclist $= \frac{360}{48}$

$= 7.5$ days

Similarly,

Number of days takes by II[nd] cyclist $= \frac{360}{60}$

$= 6$ days

Number of days taken by III[rd] cyclist $= \frac{360}{72}$

$= 5$ days

Now, we need to find LCM (7.5, 6, 5)

$$7.5 = \frac{75}{10}$$

$$6 = \frac{60}{10}$$

$$5 = \frac{50}{10}$$

So, $\text{LCM}(7.5, 6, 5) = \text{LCM}\left(\frac{75}{10}, \frac{60}{10}, \frac{50}{10}\right)$

$$= \frac{\text{LCM}(75, 60, 50)}{\text{HCF}(10, 10, 10)}$$

$$= \frac{300}{10}$$

$$= 30$$

So, all the cyclists will meet at starting point after 30 days.

38. (a) In order to find the maximum number of columns in which they can march, we will find HCF (32, 616).

$$32 = 2^5$$

$$616 = 2^3 \times 7 \times 11$$

So, HCF (32, 616) = 2^3 = 8

Hence, maximum number of columns = 8

(b) We know that for any two positive integers a and b,

LCM (a, b) × HCF (a, b) = a × b

$\Rightarrow$ LCM (306, 657) × HCF (306, 657) = 306 × 657

$\Rightarrow$ LCM (306, 657) × 9 = 306 × 657

$\Rightarrow$ LCM (306, 657) = $\frac{306 \times 657}{9}$ = 22338

39. (a) According to Euclid's Division lemma,

$a = bq + r; \quad 0 \le r < b$

Take $b = 6$

$a = 6q + r; \quad 0 \le r < 6$

For $r = 0$

$a = 6q$

$= 2(3q)$ which is even

For $r = 1$

$a = 6q + 1$

$= 2(3q) + 1$ which is odd

For $r = 2$

$a = 6q + 2$

$= 2(3q + 1)$ which is even

For $r = 3$

$a = 6q + 3$

$= 6q + 2 + 1$

$= 2(3q + 1) + 1$ which is odd

For $r = 4$

$a = 6q + 4$

$= 2(3q + 2)$ which is even

For $r = 5$

$a = 6q + 5$

$= 6q + 4 + 1$

$= 2(3q + 2) + 1$ which is odd

Therefore, every positive integer is of form 6q + 1 or 6q + 3 or 6q + 5.

(b) LCM ($x^3 y^3, x^3 y^5$)

$= x^3 y^5$

40. (a) 135 and 225

$$225 = 135 \times 1 + 90$$

$$135 = 90 \times 1 + 45$$

$$90 = 45 \times 2 + 0$$

So, HCF (135, 225) = 45

(b) 196 and 38220

$$38220 = 196 \times 195 + 0$$

So, HCF (196, 38220) = 196

(c) 867 and 255

$$867 = 255 \times 3 + 102$$

$$255 = 102 \times 2 + 51$$

$$102 = 51 \times 2 + 0$$

So, HCF (867, 255) = 51

CASE STUDY-1

(i) (c) The product of non-zero rational and irrational number is always irrational.

(ii) (d) $\frac{\sqrt{7}}{\sqrt{7}} = 1$

(iii) (a) 3 = 2 × 1 + 1

5 = 2 × 2 + 1

7 = 2 × 3 + 1

Hence every odd integer is in form of (2p + 1).

(iv) (b) The number 2 and 3 are consecutive natural numbers and their product, 6 is an even number.

(v) (c) The number 60 is divisible by 1, 2, 3, 4, 5.

CASE STUDY-2

(i) (a) The numbers which have no factor other than itself and 1 are called prime numbers.

(ii) (b) The problem can be solved using Euclid's division lemma.

(iii) (a) The maximum number of books that can be equally placed in a stack are 10.As 10 is the only common factor of 980 and 130.

(iv) (b)

2	480, 130
2	240, 65
2	120, 65
2	60, 65
2	30, 65
3	15, 65
5	5, 65
13	1, 13
	11, 1

LCM of 480 and 130 = 2 × 2 × 2 × 2 × 2 × 3 × 5 × 13 = 6240.

(v) (c) The number of stacks required for english books $= \frac{\text{Total number of books}}{\text{Number of book in each stack}}$

$= \frac{480}{10} = 48$

The number of stacks required for hindi books

$= \frac{\text{Total number of books}}{\text{Number of book in each stack}}$

$= \frac{130}{10} = 13$

Chapter 2

Polynomials

Multiple Choice Questions

1. (a) Let α, β be the zeroes of f(x)

$\therefore \quad \alpha\beta = \frac{c}{a} = 3$

$\Rightarrow \quad \frac{k}{1} = 3$

$\Rightarrow \quad k = 3$

2. (c) $\alpha + \beta = \frac{-b}{a} = \frac{-3}{4}, \quad \alpha\beta = \frac{c}{a} = \frac{7}{4}$

So, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{\frac{-b}{a}}{\frac{c}{a}} = \frac{\frac{-3}{4}}{\frac{7}{4}} = \frac{-3}{7}$

3. (d)

4. (b) Let $p(x) = 2x^2 + 2ax + 5x + 10$

As (x + a) is a factor of p(x),

$\therefore \quad p(-a) = 0$

$2(-a)^2 + 2a(-a) + 5(-a) + 10 = 0$

$2a^2 - 2a^2 - 5a + 10 = 0$

$5a = 10$

$a = 2$

5. (b) Discriminant (D) = $b^2 - 4ac$

Here, a = 4, b = 10 and c = 2

$= (10)^2 - 4(4)(2) = 100 - 32 = 68$

WORKSHEET - 1

SECTION-A

1. $b^2 - 4ac = 0$

p(x) has two equal zeroes.

2. A quadratic polynomial is of form p(x) =

$\{x^2 - (\text{sum of zeroes})\, x + \text{product of zeroes}\}$

$= \{x^2 - \left(\frac{-1}{2}\right)x + (-3)\}$

$= \frac{1}{2}\{2x^2 + x - 6\}$

3. Let $p(x) = x^4 + x^3 - 2x^2 + x + 1$

$p(1) = (1)^4 + (1)^3 - 2(1)^2 + 1 + 1$

Remainder is $p(1) = 1 + 1 - 2 + 1 + 1 = 2$

4. A binomial of degree 6 is $x^6 + 4x^2$.

5. $3x^3 - x^2 - 3x + 1$

$= x^2(3x - 1) - 1(3x - 1)$

$= (x^2 - 1)(3x - 1)$

$= (x + 1)(x - 1)(3x - 1)$

6. $a + b = \frac{-B}{A} = 11, \quad ab = \frac{C}{A} = 30$

$a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$= (a + b)[(a + b)^2 - 3ab]$

$= 11(121 - 90)$

$= 11(31)$

$= 341$

7. $p(x) = 6x^2 - 3 - 7x$

$= 6x^2 - 7x - 3$

$= 6x^2 - 9x + 2x - 3$

$= 3x(2x - 3) + 1(2x - 3)$

$= (2x - 3)(3x + 1)$

Now, $p(x) = 0 \Rightarrow x = \frac{3}{2}, \frac{-1}{3}$.

So, zeroes are $x = \frac{3}{2}, \frac{-1}{3}$.

8. $p(x) = 4x^2 - 5x - 1$

$\alpha + \beta = \frac{-b}{a} = \frac{5}{4}, \quad \alpha\beta = \frac{c}{a} = \frac{-1}{4}$

$\alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha + \beta) = \frac{-1}{4}\left(\frac{5}{4}\right) = \frac{-5}{16}$

SECTION-B

9. $p(x) = 6x^3 + 3x^2 - 5x + 1$

$\alpha + \beta + \gamma = \frac{-3}{6} = \frac{-1}{2}$

$\alpha\beta\gamma = \frac{-1}{6}$

So, $\alpha^{-1}\beta^{-1}\gamma^{-1}$

$= \frac{1}{\alpha\beta\gamma} = \frac{1}{\frac{-1}{6}} = -6$

So, $\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2$

$= \alpha\beta\gamma(\alpha + \beta + \gamma)$

$= \frac{-1}{6}\left(\frac{-1}{2}\right) = \frac{1}{12}$

10. Dividend = Divisor × Quotient + Remainder

$x^3 + 2x^2 + 4x + b = (x + 1)(x^2 + ax + 3) + (2b - 3)$

$= (x^3 + ax^2 + 3x + x^2 + ax + 3 + 2b - 3)$

On comparing coefficients of x^2 and constant terms we get,

$a + 1 = 2 \Rightarrow a = 1$

$b = 3 + 2b - 3 \Rightarrow b = 0$

11. $p(x) = 3x^2 - 6x + 4$

$\alpha + \beta = \frac{-b}{a} = 2, \quad \alpha\beta = \frac{c}{a} = \frac{4}{3}$

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta$

$= \frac{\alpha^2 + \beta^2}{\alpha\beta} + 2\,\frac{\alpha + \beta}{\alpha\beta} + 3\alpha\beta$

$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} + 2\left(\frac{\alpha + \beta}{\alpha\beta}\right) + 3\alpha\beta$

$= \frac{4 - \frac{8}{3}}{\frac{4}{3}} + 2\left(\frac{2}{\frac{4}{3}}\right) + 3\left(\frac{4}{3}\right)$

$= 1 + 3 + 4$

$= 8$

12. Let the zeroes be $\alpha, \frac{1}{\alpha}$.

$\alpha + \frac{1}{\alpha} = \frac{-13}{a^2 + 9}, \quad \alpha \times \frac{1}{\alpha} = \frac{6a}{a^2 + 9}$

So, $1 = \frac{6a}{a^2 + 9} \Rightarrow a^2 - 6a + 9 = 0$

$a^2 - 3a - 3a + 9 = 0$

$a(a - 3) - 3(a - 3) = 0$

$(a - 3)(a - 3) = 0$

$a = 3$

13. $p(t) = kt^2 + 2t + 3k$

Sum of zeroes = Product of zeroes

$\frac{-2}{k} = \frac{3k}{k}$

$k = \frac{-2}{3}$

14. Long division of $8x^4 + 14x^3 - 2x^2 + 8x - 12$ by $4x^2 + 3x - 2$:

$$\begin{array}{r|l} & 2x^2 + 2x - 1 \\ \hline 4x^2 + 3x - 2 & 8x^4 + 14x^3 - 2x^2 + 8x - 12 \\ & 8x^4 + 6x^3 - 4x^2 \\ & - \quad - \quad + \\ \hline & 8x^3 + 2x^2 + 8x - 12 \\ & 8x^3 + 6x^2 - 4x \\ & - \quad - \quad + \\ \hline & -4x^2 + 12x - 12 \\ & -4x^2 - 3x + 2 \\ & + \quad + \quad - \\ \hline & 15x - 14 \end{array}$$

15. Cubic polynomial is of form

$\{x^3 - (\alpha + \beta + \gamma) x^2 + (\alpha\beta + \beta\gamma + \alpha\gamma) x - \alpha\beta\gamma\}$

$= k \{x^3 - (5 + 6 - 1) x^2 + (30 - 6 - 5) x - (-30)\}$

$= k \{x^3 - 10x^2 + 19x + 30\}$

16. Let a – d, a, a + d be the zeroes of p(x).

$$a - d + a + a + d = -3p$$

$$3a = -3p$$

$$\boxed{a = -p}$$

$$a(a - d) + a(a + d) + (a - d)(a + d) = 3q$$

$$-p(-p - d) - p(-p + d) + (p^2 - d^2) = 3q$$

$$p^2 + pd + p^2 - pd + p^2 - d^2 = 3q$$

$$\boxed{3p^2 - d^2 = 3q} \quad \text{...(i)}$$

$$(a - d)\, a\, (a + d) = -r$$

$$a(a^2 - d^2) = -r$$

$$-p(p^2 - d^2) = -r$$

$$p^2 - d^2 = \frac{r}{p}$$

$$\boxed{d^2 = p^2 - \frac{r}{p}} \quad \text{...(ii)}$$

On putting (ii) in (i), we get

$$3p^2 - \quad p^2 - \frac{r}{p} \quad = 3q$$

$$2p^2 + \frac{r}{p} = 3q$$

SECTION-C

17. $p(x) = 2x^2 - 5x + 7$

$\alpha + \beta = \frac{5}{2}, \quad \alpha\beta = \frac{7}{2}$

Polynomial whose zeroes are $2\alpha + 3\beta$ and $3\alpha + 2\beta$ is,

$k \{x^2 - (2\alpha + 3\beta + 3\alpha + 2\beta) x + (2\alpha + 3\beta)(3\alpha + 2\beta)\}$

$= k \{x^2 - 5(\alpha + \beta) x + 6\alpha^2 + 4\alpha\beta + 9\alpha\beta + 6\beta^2\}$

$= k \{x^2 - 5(\alpha + \beta) x + [6 \{(\alpha + \beta)^2 - 2\alpha\beta\} + 13\alpha\beta]\}$

$$= k \left\{x^2 - \frac{25}{2}x + \left[6\left(\frac{25}{4} - 7\right) + \frac{91}{2}\right]\right\}$$

$$= k \quad x^2 - \frac{25}{2}x + 41$$

$$= \frac{k}{2}\{2x^2 - 25x + 82\}$$

18. Dividend = Divisor × Quotient + Remainder
$x^4 + 2x^3 - 2x^2 + x - 1 = (x^2 + 2x - 3)$ Quotient + Remainder

$$\begin{array}{r|l} & x^2 + 1 \\ \hline x^2 + 2x - 3 & x^4 + 2x^3 - 2x^2 + x - 1 \\ & x^4 + 2x^3 - 3x^2 \\ & - \quad - \quad + \\ \hline & x^2 + x - 1 \\ & x^2 + 2x - 3 \\ & - \quad - \quad + \\ \hline & -x + 2 \end{array}$$

So, $x^4 + 2x^3 - 2x^2 + x - 1 = (x^2 + 2x - 3)(x^2 + 1) + (-x + 2)$

So, $-(-x + 2) = x - 2$ must be added to the polynomial p(x).

19.

$$\begin{array}{r|l} & 2x^2 + 5 \\ \hline 3x^2 + 4x + 1 & 6x^4 + 8x^3 + 17x^2 + 21x + 7 \\ & 6x^4 + 8x^3 + 2x^2 \\ & - \quad - \quad - \\ \hline & 15x^2 + 21x + 7 \\ & 15x^2 + 20x + 5 \\ & - \quad - \quad - \\ \hline & x + 2 \end{array}$$

On comparing $x + 2$ with $ax + b$, we get
$a = 1, \quad b = 2$

20. Let the quotient be $q(x) = ax^2 + bx + c$ and remainder $r(x) = px + q$

Using division algorithm,

$p(x) = g(x)\, q(x) + r(x)$

$$3x^4 + 5x^3 - 7x^2 + 2x + 2$$
$$= (x^2 + 3x + 1)(ax^2 + bx + c) + px + q$$
$$= ax^4 + bx^3 + cx^2 + 3ax^3 + 3bx^2 + 3cx + ax^2 + bx + c + px + q$$

$a = 3$

$5 = b + 3a \Rightarrow b = 5 - 3a \Rightarrow b = -4$

$-7 = c + 3b + a$

$-7 = c - 12 + 3 \Rightarrow c = 2$

$2 = 3c + b + p$

$2 = 6 - 4 + p \Rightarrow p = 0$

$2 = q + c \Rightarrow q = 2 - 2 = 0$

So, Remainder $= px + q = 0$

As remainder is zero, $g(x)$ is a factor of $p(x)$.

21. Let $p(x) = x^3 + 2x^2 + kx + 3$

Remainder $= f(3) = 21$

$3^3 + 2(3)^2 + 3k + 3 = 21$

$27 + 18 + 3k + 3 = 21$

$3k = 21 - 48 = -27$

$k = -9$

Now, we will find the quotient.

Dividend $= x^3 + 2x^2 + kx + 3$
$= x^3 + 2x^2 - 9x + 3$

Divisor $= x - 3$

$$\begin{array}{r|l} & x^2 + 5x + 6 \\ \hline x - 3 & x^3 + 2x^2 - 9x + 3 \\ & x^3 - 3x^2 \\ & - \quad + \\ \hline & 5x^2 - 9x + 3 \\ & 5x^2 - 15x \\ & - \quad + \\ \hline & 6x + 3 \\ & 6x - 18 \\ & - \quad + \\ \hline & 21 \end{array}$$

So, quotient $= x^2 + 5x + 6$

22. Zeroes are $-\sqrt{3}$ and $\sqrt{3}$

So, factors are $(x + \sqrt{3})$, $(x - \sqrt{3})$

i.e. $(x + \sqrt{3})(x - \sqrt{3})$ is also a factor

i.e. $x^2 - 3$ is a factor of given polynomial.

$$\begin{array}{r|l} & 2x + 1 \\ \hline x^2 - 3 & 2x^3 + x^2 - 6x - 3 \\ & 2x^3 \quad - 6x \\ & - \quad + \\ \hline & x^2 - 3 \\ & x^2 - 3 \\ & - \quad + \\ \hline & 0 \end{array}$$

For the remaining zero,

put $2x + 1 = 0$

$$x = \frac{-1}{2}$$

23. As $\sqrt{2}$ is a zero of given polynomial, $x - \sqrt{2}$ is a factor of the polynomial.

$$\begin{array}{r|l} & 6x^2 + 7\sqrt{2}x + 4 \\ \hline x - \sqrt{2} & 6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2} \\ & 6x^3 - 6\sqrt{2}x^2 \\ & - \quad + \\ \hline & 7\sqrt{2}x^2 - 10x \\ & 7\sqrt{2}x^2 - 14x \\ & - \quad + \\ \hline & 4x - 4\sqrt{2} \\ & 4x - 4\sqrt{2} \\ & - \quad + \\ \hline & 0 \end{array}$$

For other zeroes,

$6x^2 + 7\sqrt{2}x + 4 = 0$

$6x^2 + 3\sqrt{2}x + 4\sqrt{2}x + 4 = 0$

$3x(2x + \sqrt{2}) + 4(\sqrt{2}x + 4) = 0$

$3\sqrt{2}x(\sqrt{2}x + 1) + 4(\sqrt{2}x + 1) = 0$

$(3\sqrt{2}x + 4)(\sqrt{2}x + 1) = 0$

$x = \frac{-4}{3\sqrt{2}} = \frac{-4\sqrt{2}}{6} = \frac{-2\sqrt{2}}{3}$

and $x = \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$

24. According to division algorithm,

Dividend = Divisor × Quotient + Remainder

$x^3 - 3x^2 + x + 2 = g(x)(x - 2) + (-2x + 4)$

$$g(x) = \frac{x^3 - 3x^2 + x + 2 + 2x - 4}{x - 2}$$

$$= \frac{x^3 - 3x^2 + 3x - 2}{x - 2}$$

$$\begin{array}{r|l} & x^2 - x + 1 \\ \hline x - 2 & x^3 - 3x^2 + 3x - 2 \\ & x^3 - 2x^2 \\ & - \quad + \\ \hline & -x^2 + 3x - 2 \\ & -x^2 + 2x \\ & + \quad - \\ \hline & x - 2 \\ & x - 2 \\ & - \quad + \\ \hline & 0 \end{array}$$

So, $g(x) = x^2 - x + 1$

SECTION-D

25. $p(x) = x^2 - px + q$

$\alpha + \beta = p, \quad \alpha\beta = q$

Consider

LHS

$$\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2} = \frac{\alpha^4 + \beta^4}{\alpha^2\beta^2}$$

$$= \frac{(\alpha^2)^2 + (\beta^2)^2}{\alpha^2\beta^2}$$

$$= \frac{[\alpha^2 + \beta^2] - \alpha^2\beta^2}{(\alpha\beta)}$$

$$= \frac{[(\alpha + \beta)^2 - 2\alpha\beta]^2 - 2(\alpha\beta)^2}{(\alpha\beta)^2}$$

$$= \frac{[p^2 - 2q]^2 - 2(q)^2}{(q)^2}$$

$$= \frac{p^4 + 4q^2 - 4p^2q - 2q^2}{q^2}$$

$$= \frac{p^4}{q^2} - \frac{4p^2}{q} + 2 \quad = \text{RHS}$$

26. Let $p(x) = x^3 - 2x^2 + qx - r$

$\alpha + \beta + \gamma = 2$

For $\alpha + \beta = 0 \Rightarrow 0 + \gamma = 2$

$\Rightarrow \gamma = 2$

Also, $\alpha\beta\gamma = r$

$2\alpha\beta = r$

$\alpha\beta + \beta\gamma + \alpha\gamma = q$

$\alpha\beta + \gamma(\alpha + \beta) = q$

$\alpha\beta + \gamma(0) = q \qquad [\text{As } \alpha + \beta = 0]$

$\alpha\beta = q$

$\frac{r}{2} = q$

$2q = r$

27.

$$
\begin{array}{r|l}
 & 2x^2 - 3x + (-8 - 2k) \\
\hline
x^2 + 2x + k & 2x^4 + x^3 - 14x^2 + 5x + 6 \\
 & 2x^4 + 4x^3 + 2kx^2 \\
 & - \quad - \quad - \\
\hline
 & -3x^3 + x^2(-14 - 2k) + 5x + 6 \\
 & -3x^3 - 6x^2 \qquad - 3kx \\
 & + \quad + \qquad + \\
\hline
 & x^2(-8 - 2k) + x(5 + 3k) + 6 \\
 & x^2(-8 - 2k) + x(-16 - 4k) + k(-8 - 2k) \\
 & - \quad - \quad - \\
\hline
 & x(5 + 3k + 16 + 4k) + 6 + 8k + 2k^2 \\
\hline
\end{array}
$$

Remainder = $(21 + 7k)x + 6 + 8k + 2k^2$

As $x^2 + 2x + k$ is a factor of

$2x^4 + x^3 - 14x^2 + 5x + 6$,

So, Remainder should be zero

$(21 + 7k)x + 6 + 8k + 2k^2 = 0$

$= 0x + 0$

On comparing coefficient of x, we get
$21 + 7k = 0$

$k = -3$

Now, we will find zeroes of the two polynomials.

$2x^4 + x^3 - 14x^2 + 5x + 6$

$= (x^2 + 2x + k)\,[2x^2 - 3x + (-8 - 2k)]$

$= (x^2 + 2x - 3)\,(2x^2 - 3x - 2)$

$= (x^2 + 3x - x - 3)\,(2x^2 - 4x + x - 2)$

$= [x(x + 3) - 1(x + 3)]\,[2x(x - 2) + 1(x - 2)]$

$= (x + 3)(x - 1)(2x + 1)(x - 2)$

So, zeroes are $-3, 1, \frac{-1}{2}, 2$.

28. $p(x) = x^2 - 2x + 3$

$\alpha + \beta = 2$

$\alpha\beta = 3$

(a) Roots are $(\alpha + 2, \beta + 2)$

Polynomial is

$k\{x^2 - (\text{sum of zeroes})\,x + \text{product of zeroes}\}$

$= k\{x^2 - (\alpha + 2 + \beta + 2)x + (\alpha + 2)(\beta + 2)\}$

$= k\{x^2 - (\alpha + \beta + 4)x + \alpha\beta + 2(\alpha + \beta) + 4\}$

$= k\{x^2 - (2 + 4)x + 3 + 2(2) + 4\}$

$= k\{x^2 - 6x + 11\}$

(b) Sum of zeroes

$$= \frac{\alpha - 1}{\alpha + 1} + \frac{\beta - 1}{\beta + 1}$$

$$= \frac{(\alpha - 1)(\beta + 1) + (\alpha + 1)(\beta - 1)}{(\alpha + 1)(\beta + 1)}$$

$$= \frac{\alpha\beta + \alpha - \beta - 1 + \alpha\beta - \alpha + \beta - 1}{\alpha\beta + \alpha + \beta + 1}$$

$$= \frac{2\alpha\beta - 2}{\alpha\beta + \alpha + \beta + 1}$$

$$= \frac{6 - 2}{3 + 2 + 1}$$

$$= \frac{4}{6} = \frac{2}{3}$$

As
$\alpha + \beta = 2$
$\alpha\beta = 3$

Product of zeroes $= \left(\frac{\alpha - 1}{\alpha + 1}\right)\left(\frac{\beta - 1}{\beta + 1}\right)$

$$= \frac{(\alpha - 1)(\beta - 1)}{(\alpha + 1)(\beta + 1)}$$

$$= \frac{\alpha\beta - (\alpha + \beta) + 1}{\alpha\beta + (\alpha + \beta) + 1}$$

$$= \frac{3 - 2 + 1}{3 + 2 + 1}$$

$$= \frac{2}{6}$$

$$= \frac{1}{3}$$

A quadratic polynomial is of form

$k\{x^2 - (\text{sum of zeroes})\,x + \text{product of zeroes}\}$

$$= k\left\{x^2 - \frac{2}{3}x + \frac{1}{3}\right\}$$

$$= \frac{k}{3}\left\{3x^2 - 2x + 1\right\}$$

29.

$$\begin{array}{r|l} & x^2 - 2\sqrt{5}x + 3 \\ \hline x-\sqrt{5} & x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5} \\ & x^3 - \sqrt{5}x^2 \\ & - \quad + \\ \hline & -2\sqrt{5}x^2 + 13x - 3\sqrt{5} \\ & -2\sqrt{5}x^2 + 10x \\ & + \quad\quad - \\ \hline & 3x - 3\sqrt{5} \\ & 3x - 3\sqrt{5} \\ & - \quad + \\ \hline & 0 \\ \hline \end{array}$$

For other zeroes,

Consider $x^2 - 2\sqrt{5}x + 3 = 0$

$$x = \frac{2\sqrt{5} \pm \sqrt{20-12}}{2}$$

$$= \frac{2\sqrt{5} \pm \sqrt{8}}{2}$$

$$= \frac{2\sqrt{5} \pm 2\sqrt{2}}{2}$$

$$= \sqrt{5} \pm \sqrt{2}$$

30. Let $p(x) = ax^3 + 3x^2 - bx - 6$

Let α, β, γ be the zeroes such that

$$\alpha = -1 \quad \text{and} \quad \beta = -2$$

$$\alpha + \beta + \gamma = \frac{-3}{a}$$

$$-1 -2 + \gamma = \frac{-3}{a}$$

$$\gamma = \frac{-3}{a} + 3$$

Also, $\alpha\beta\gamma = \frac{6}{a}$

$$2 \; \frac{-3}{a} + 3 = \frac{6}{a}$$

$$\frac{-3}{a} + 3 = \frac{3}{a}$$

$$3 = \frac{6}{a} \Rightarrow a = 2$$

So, $\gamma = \frac{-3}{a} + 3 = \frac{-3}{2} + 3 = \frac{3}{2}$

Also, $\alpha\beta + \beta\gamma + \alpha\gamma = \frac{-b}{a} = \frac{-b}{2}$

$$2 + (-2)\left(\frac{3}{2}\right) + (-1)\left(\frac{3}{2}\right) = \frac{-b}{2}$$

$$2 - 3 - \frac{3}{2} = \frac{-b}{2}$$

$$-1 - \frac{3}{2} = \frac{-b}{2}$$

$$\frac{-5}{2} = \frac{-b}{2}$$

$$b = 5$$

31. As zeroes of q(x) are also the zeroes of p(x), so, remainder should be zero. [As q(x) is a factor of p(x)].

$$\begin{array}{r|l} & x^2 - 3x + 2 \\ \hline x^3+2x^2+a & x^5 - x^4 - 4x^3 + 3x^2 + 3x + b \\ & x^5 + 2x^4 + ax^2 \\ & - \quad - \quad - \\ \hline & -3x^4 - 4x^3 + (3-a)x^2 + 3x + b \\ & -3x^4 - 6x^3 \quad\quad -3ax \\ & + \quad + \quad\quad + \\ \hline & 2x^3 + (3-a)x^2 + (3+3a)x + b \\ & 2x^3 + 4x^2 \quad\quad + 2a \\ & - \quad - \quad\quad - \\ \hline & (-a-1)x^2 + (3+3a)x + (b-2a) \\ \hline \end{array}$$

Remainder = 0

$(-a-1)x^2 + (3+3a)x + (b-2a) = 0$

$\Rightarrow \; -a-1 = 0, \quad b - 2a = 0$

$\Rightarrow \; a = -1, \quad b + 2 = 0$

$\Rightarrow \; a = -1, \quad b = -2$

Now,

$$p(x) = (x^3 + 2x^2 + a)(x^2 - 3x + 2) + 0$$

$$= (x^3 + 2x^2 - 1)(x^2 - 3x + 2)$$

For other zeroes of p(x),

Put $x^2 - 3x + 2 = 0$

$$x^2 - 2x - x + 2 = 0$$

$$x(x-2) - 1(x-2) = 0$$

$$(x-1)(x-2) = 0$$

$$x = 1, 2$$

So, x = 1, 2 are zeroes of p(x) but not of q(x).

32. (a) $f(x) = x^3 - 5x^2 - 16x + 80$

Let the two zeroes be $\alpha, -\alpha$ and the third zero be γ.

$$\alpha + (-\alpha) + \gamma = 5$$
$$\gamma = 5$$

Also $\alpha(-\alpha)\gamma = -80$

$$-\alpha^2(5) = -80$$
$$\alpha^2 = \frac{80}{5} = 16$$
$$\alpha = \pm 4$$

For $\alpha = -4$, $-\alpha = -(-4) = 4$

For $\alpha = 4$, $-\alpha = -4$.

So, zeroes are $-4, 4, 5$

(b) $p(x) = x^2 - p(x + 1) - c$

$$= x^2 - px - (p + c)$$

$$\alpha + \beta = p, \quad \alpha\beta = -(p + c)$$

Consider

$$(\alpha + 1)(\beta + 1) = \alpha\beta + (\alpha + \beta) + 1$$
$$= -(p + c) + p + 1$$
$$= 1 - c$$

WORKSHEET - 2

SECTION-A

1. p(x) has 2 real zeroes.
2. $x^2 + 7x + 12$

$= x^2 + 3x + 4x + 12$

$= x^2 + (x + 3) + 4(x + 3)$

$= (x + 3)(x + 4)$

For zeroes of polynomial,

$x + 3 = 0, \quad x + 4 = 0$

$x = -3, \quad x = -4$

3. Let $\alpha, \frac{1}{\alpha}$ be the zeroes of p(x).

$$\alpha\frac{1}{\alpha} = \frac{-a}{5}$$
$$1 = \frac{-a}{5}$$
$$a = -5$$

4. p(x) has 2 distinct real zeroes.
5. Let $p(x) = x^3 + ax^2 + bx + c$

Let α, β, γ be zeroes of p(x)

Such that $\alpha = -1$

$$\alpha\beta\gamma = -c$$
$$(-1)\beta\gamma = -c$$
$$\beta\gamma = c$$

So, product of other two zeroes = c

6. Quadratic polynomial is of form p(x) =

{x^2 – (Sum of zeroes) x + product of zeroes}

$$= \left\{x^2 - \left(\frac{2}{3} - \frac{1}{4}\right)x + \frac{2}{3}\left(\frac{-1}{4}\right)\right\}$$
$$= \left\{x^2 - \left(\frac{5}{12}\right)x - \frac{1}{6}\right\}$$
$$= \left\{\frac{12x^2 - 5x - 2}{12}\right\}$$
$$= (12x^2 - 5x - 2)$$

7. $2y^2 + 7y + 5$

$$\alpha + \beta = \frac{-7}{2}$$
$$\alpha\beta = \frac{5}{2}$$

8. The sign of c is negative.
9. $p(x) = (k^2 + 4)x^2 + 13x + 4k$

Let the two zeroes be $\alpha, \frac{1}{\alpha}$

$$1 = \alpha\left(\frac{1}{\alpha}\right) = \frac{4k}{k^2 + 4}$$
$$k^2 - 4k + 4 = 0$$
$$(k - 2)^2 = 0$$
$$k = 2$$

10. $x^2 + 99x + 127$

$\alpha + \beta = -99, \quad \alpha\beta = 127$

α, β are either both positive or both negative

If α, β are both positive then $\alpha + \beta = -99$ is not possible

So, α and β must be negative.

SECTION-B

11. $p(x) = x^2 - px + q$

$\alpha + \beta = p, \quad \alpha\beta = q$

(a) Consider $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= p^2 - 2q$

(b) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{p}{q}$

12. $p(x) = x^2 - 5x + k,$

$$\begin{aligned} \alpha + \beta &= 5 \\ \alpha - \beta &= 1 \\ \hline 2\alpha &= 6 \\ \hline \alpha &= 3 \end{aligned}$$

So, $\beta = 5 - \alpha = 5 - 3 = 2$

As, 2 is a zero of p(x)

$p(2) = 0$

$4 - 10 + k = 0$

$k = 6$

13. Quadratic polynomial is of form p(x) =

{x^2 – (sum of zeroes)x + Product of zeroes}

Sum of zeroes $= \frac{4+\sqrt{2}}{2} + \frac{4-\sqrt{2}}{2}$

$= 4$

Product of zeroes $= \left(\frac{4+\sqrt{2}}{2}\right)\left(\frac{4-\sqrt{2}}{2}\right)$

$= \frac{16-2}{4} = \frac{14}{4} = \frac{7}{2}$

So, quadratic polynomial is p(x) =

$\left\{x^2 - 4x + \frac{7}{2}\right\}$

$= \{2x^2 - 8x + 7\}$

14.

$$\begin{array}{r|l} & 3x^2 - x \\ \hline 3x^2 + x - 1 & 9x^4 - 4x^2 + 4 \\ & 9x^4 + 3x^3 - 3x^2 \\ & - \quad - \quad + \\ \hline & -3x^3 - x^2 + 4 \\ & -3x^3 - x^2 + x \\ & + \quad + \quad - \\ \hline & -x + 4 \\ \hline \end{array}$$

Quotient $= 3x^2 - x$

Remainder $= -x + 4$

15. $p(x) = x^2 - 1 = x^2 + 0x - 1$

$\alpha + \beta = 0, \quad \alpha\beta = -1$

Sum of zeroes $= \frac{2\alpha}{\beta} + \frac{2\beta}{\alpha}$

$= 2\ \frac{\alpha^2 + \beta^2}{\alpha\beta}$

$= 2\left[\frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}\right]$

$= 2\left[\frac{(0)^2 - 2x(-1)}{(-1)}\right]$

$= 2(-2) = -4$

Product of zeroes $= \frac{2\alpha}{\beta} \times \frac{2\beta}{\alpha} = 4$

A quadratic polynomial is of form p(x) = {x^2 – (sum of zeroes) x + product of zeroes}

$= \{x^2 + 4x + 4\}$

16. $p(x) = ax^2 + bx + c$

$\alpha + \beta = \frac{-b}{a}, \quad \alpha\beta = \frac{c}{a}$

Consider $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta}$

$= \frac{(\alpha+\beta)(\alpha^2 + \beta^2 - \alpha\beta)}{\alpha\beta}$

$= \frac{(\alpha+\beta)\left[(\alpha+\beta)^2 - 3\alpha\beta\right]}{\alpha\beta}$

$$= \frac{\left(\frac{-b}{a}\right)\left[\left(\frac{-b}{a}\right)^2 - 3\times\frac{c}{a}\right]}{\frac{c}{a}}$$

$$= \frac{-b}{c}\left(\frac{b^2-3ac}{a^2}\right)$$

17. As 1 is a zero of p(x),

so, (x – 1) is a factor of p(x).

$$\begin{array}{r|l} & -x^2 - x + 6 \\ \hline x - 1 & -x^3 + 7x - 6 \\ & -x^3 + x^2 \\ & \underline{+ \quad -} \\ & -x^2 + 7x - 6 \\ & -x^2 + x \\ & \underline{+ \quad -} \\ & 6x - 6 \\ & 6x - 6 \\ & \underline{- \quad +} \\ & 0 \end{array}$$

For other zeroes of p(x),

put

$$\begin{aligned} -x^2 - x + 6 &= 0 \\ x^2 + x - 6 &= 0 \\ x^2 + 3x - 2x - 6 &= 0 \\ x(x+3) - 2(x+3) &= 0 \\ (x-2)(x+3) &= 0 \\ x &= 2, \quad -3 \end{aligned}$$

So, other zeroes are x = 2, –3

18. $p(x) = x^2 - 13x + k$

Let α, β be two zeroes of p(x),

$$\alpha\beta = k = 40$$

So,

$$\begin{aligned} p(x) &= x^2 - 13x + 40 \\ &= x^2 - 5x - 8x + 40 \\ &= x(x-5) - 8(x-5) \\ &= (x-5)(x-8) \end{aligned}$$

For zeroes of p(x), put p(x) = 0

i.e. $(x-5)\quad(x-8) = 0$

$$x = 5, 8$$

19.

$$\begin{array}{r|l} & 2x^2 + 2x - 1 \\ \hline 4x^2 + 3x - 2 & 8x^4 + 14x^3 - 2x^2 + 7x - 8 \\ & 8x^4 + 6x^3 - 4x^2 \\ & \underline{- \quad - \quad +} \\ & 8x^3 + 2x^2 + 7x - 8 \\ & 8x^3 + 6x^2 - 4x \\ & \underline{- \quad - \quad +} \\ & -4x^2 + 11x - 8 \\ & -4x^2 - 3x + 2 \\ & \underline{+ \quad + \quad -} \\ & 14x - 10 \end{array}$$

So, 14x – 10 must be subtracted from $8x^4 + 14x^3 - 2x^2 + 7x - 8$. So, that the resultant polynomial is exactly divisible by $4x^2 + 3x - 2$.

20. $p(t) = t^2 - 4t + 3$

$\alpha + \beta = 4, \quad \alpha\beta = 3$

Consider

$$\begin{aligned} \alpha^4\beta^3 + \alpha^3\beta^4 &= \alpha^3\beta^3(\alpha+\beta) \\ &= (\alpha\beta)^3(\alpha+\beta) \\ &= 27\,(4) = 108 \end{aligned}$$

And

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{4}{3}$$

SECTION-C

21. Let a – d, a and a + d be the zeroes of f(x).

$$\begin{aligned} a - d + a + a + d &= 12 \\ 3a &= 12 \\ a &= 4 \end{aligned}$$

Also,

$$\begin{aligned} (a-d)\,a\,(a+d) &= 28 \\ (4-d)\,4\,(4+d) &= 28 \\ 16 - d^2 &= 7 \\ d^2 &= 9 \\ d &= \pm 3 \end{aligned}$$

Case 1

$a = 4, \quad d = 3$

So, zeroes are

$a - d, a, a + d = 1, 4, 7$

Therefore, zeroes of polynomial are 1, 4 and 7.

Case 2

$a = 4, d = -3$

So, zeroes are 7, 4, 1

22.

$$\begin{array}{r|l} & 10x^2 + \frac{19}{3}x - \frac{8}{9} \\ \hline 3x^2 - x + 1 & 30x^4 + 9x^3 + x^2 + 2 \\ & 30x^4 - 10x^3 + 10x^2 \\ & - \quad + \quad - \\ \hline & 19x^3 - 9x^2 + 2 \\ & 19x^3 - \frac{19}{3}x^2 + \frac{19}{3}x \\ & - \quad + \quad - \\ \hline & -\frac{8}{3}x^2 - \frac{19}{3}x + 2 \\ & -\frac{8}{3}x^2 + \frac{8}{9}x - \frac{8}{9} \\ & + \quad - \quad + \\ \hline & -\frac{65}{9}x + \frac{26}{9} \\ \hline \end{array}$$

$$\text{Dividend} = 30x^4 + 9x^3 + x^2 + 2$$

$$\text{Divisor} = 3x^2 - x + 1$$

$$\text{Quotient} = 10x^2 + \frac{19}{3}x - \frac{8}{9}$$

$$\text{Remainder} = -\frac{65}{9}x + \frac{26}{9}$$

According to divisor algorithm,

Dividend = Divisor × Quotient + Remainder

Consider,

Divisor × Quotient + Remainder

$$= (3x^2 - x + 1)\left(10x^2 + \frac{19}{3}x - \frac{65}{9}\right) - \frac{65}{9} + \frac{26}{9}$$

$$= 30x^4 + 19x^3 - \frac{8}{3}x^2 - 10x^3 - \frac{19}{3}x^2 + \frac{8}{9}x + 10x^2 + \frac{19}{3}x - \frac{8}{9} - \frac{65}{9}x + \frac{26}{9}$$

$$= 30x^4 + x^3(19 - 10) + x^2\left(-\frac{8}{3} - \frac{19}{3} + 10\right) + x\left(\frac{8}{9} + \frac{19}{3} - \frac{65}{9}\right) + \left(-\frac{8}{9} + \frac{26}{9}\right)$$

$$= 30x^4 + 9x^3 + x^2 + 0x + 2$$

$$= 30x^4 + 9x^3 + x^2 + 2$$

= Dividend Hence verified.

23. $p(x) = 4\sqrt{3}x^2 + 5x - 2\sqrt{3}$

$$p(x) = 4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3}$$

$$= 4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2)$$

$$= (4x - \sqrt{3})(\sqrt{3}x + 2)$$

For zeroes of p(x), put p(x) = 0

$$(4x - \sqrt{3})(\sqrt{3}x + 2) = 0$$

$$x = \frac{\sqrt{3}}{4}, \frac{-2}{\sqrt{3}}$$

$$\text{Sum of zeroes} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$= \frac{\sqrt{3}}{4} - \frac{2}{\sqrt{3}} \qquad = -\frac{5}{4\sqrt{3}}$$

$$= \frac{3-8}{4\sqrt{3}} \qquad = -\frac{5}{4\sqrt{3}}$$

$$\therefore \quad \text{Sum of zeroes} = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$\text{Product of zeroes} = \left(\frac{\sqrt{3}}{4}\right)\left(\frac{-2}{\sqrt{3}}\right)$$

$$= -\frac{1}{2}$$

$$\frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$= -\frac{2\sqrt{3}}{4\sqrt{3}}$$

$$= -\frac{1}{2}$$

$$\therefore \quad \text{Product of zeroes} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

Hence, the relationship between zeroes and its coefficient is verified.

24. $p(x) = x^2 - x - 2$

$\alpha + \beta = \frac{-b}{a} = 1, \quad \alpha\beta = \frac{c}{a} = -2$

$$\text{Sum of zeroes} = 2\alpha + 1 + 2\beta + 1 = 2(\alpha + \beta) + 2 = 2(1) + 2 = 4$$

$$\text{Product of zeroes} = (2\alpha + 1)(2\beta + 1) = 4\alpha\beta + 2(\alpha + \beta) + 1 = -8 + 2 + 1 = -5$$

Quadratic polynomial is of form p(x) =

{x^2 – (sum of zeroes) x + Product of zeroes}

$= \{x^2 - 4x - 5\}$

Now, we need to find $\alpha^3 + \beta^3$

$$= (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta)$$
$$= (\alpha + \beta)[(\alpha + \beta)^2 - 3\alpha\beta]$$
$$= 4(16 + 15)$$
$$= 4(31)$$
$$= 124$$

25. $p(x) = 3x^2 - 4x + 1$

$\alpha + \beta = \frac{-b}{a} = \frac{4}{3}, \quad \alpha\beta = \frac{c}{a} = \frac{1}{3}$

$$\text{Sum of zeroes} = \frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta} = \frac{(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)}{\alpha\beta} = \frac{(\alpha+\beta)\left[(\alpha+\beta)^2 - 3\alpha\beta\right]}{\alpha\beta} = \frac{\frac{4}{3}\left(\frac{16}{9}-1\right)}{\frac{1}{3}}$$

$$= 4\ \frac{16 - 9}{9} = \frac{28}{9}$$

$$\text{Product of zeroes} = \frac{\alpha^2\beta^2}{\alpha\beta} = \alpha\beta = \frac{1}{3}$$

Quadratic polynomial is of the form p(x) = {x^2 – (sum of zeroes) x + product of zeroes}

$$= k\ \ x^2 - \frac{28}{9}x + \frac{1}{3}$$

$$= \frac{k}{9}\{9x^2 - 28x + 3\}$$

26.

$$\begin{array}{r|l} & x^2 + x + 7 \\ x^2 + 1 & x^4 + x^3 + 8x^2 + ax + b \\ & \underline{\underset{-}{x^4} + \underset{-}{x^2}} \\ & x^3 + 7x^2 + ax + b \\ & \underline{\underset{-}{x^3} \qquad + \underset{-}{x}} \\ & 7x^2 + (a-1)x + b \\ & \underline{\underset{-}{7x^2} \qquad\qquad \underset{-}{+7}} \\ & (a-1)x + (b-7) \end{array}$$

As $x^4 + x^3 + 8x^2 + ax + b$ is exactly divisible by $x^2 + 1$

$\therefore$ Remainder = 0

$(a - 1)x + (b - 7) = 0$

$a = 1, \quad b = 7$

27. Let α, β be the zeroes of p(x)

$$\alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a}$$

Zeroes of the required polynomial are $\frac{1}{\alpha}, \frac{1}{\beta}$

Quadratic polynomial is of form p(x) =

$$\left\{x^2 - \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)x + \frac{1}{\alpha\beta}\right\}$$

$$= \quad x^2 - \frac{\alpha + \beta}{\alpha\beta}\ x + \frac{1}{\alpha\beta}$$

$$= \left\{x^2 - \left(\frac{-\frac{b}{a}}{\frac{c}{a}}\right)x + \frac{a}{c}\right\}$$

$$= \left\{x^2 + \frac{b}{c}x + \frac{a}{c}\right\}$$

$$= \{cx^2 + bx + a\}$$

28. $p(x) = x^3 - 4x^2 - 3x + 12$

As $\sqrt{3}$, $-\sqrt{3}$ are zeroes of p(x), so $(x - \sqrt{3})$ and $(x + \sqrt{3})$ are factors of p(x).

i.e. $(x - \sqrt{3})(x + \sqrt{3}) = x^2 - 3$ is a factor of p(x)

$$\begin{array}{r|l} & x - 4 \\ x^2 - 3 & x^3 - 4x^2 - 3x + 12 \\ & \underline{\overset{-}{x^3} \quad \overset{+}{-3x}} \\ & -4x^2 + 12 \\ & \underline{\overset{+}{-4x^2} \quad \overset{-}{+12}} \\ & 0 \end{array}$$

For third zero, $x - 4 = 0$

$x = 4$

29. $p(x) = 2x^2 + 5x + k$

$$\alpha + \beta = \frac{-b}{a} = -\frac{5}{2}, \quad \alpha\beta = \frac{c}{a} = \frac{k}{2}$$

Given: $\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$

$$\Rightarrow (\alpha + \beta)^2 - \alpha\beta = \frac{21}{4}$$

$$\Rightarrow \frac{25}{4} - \frac{k}{2} = \frac{21}{4}$$

$$\Rightarrow \frac{k}{2} = \frac{25}{4} - \frac{21}{4} = 1$$

$$\Rightarrow k = 2$$

30.

$$\begin{array}{r|l} & x^2 + 2x + 3 \\ x^2 + 5 & x^4 + 2x^3 + 8x^2 + 12x + 18 \\ & \underline{\overset{-}{x^4} + \overset{-}{5x^2}} \\ & 2x^3 + 3x^2 + 12x + 18 \\ & \underline{\overset{-}{2x^3} \quad \overset{-}{+10x}} \\ & 3x^2 + 2x + 18 \\ & \underline{\overset{-}{3x^2} \quad \overset{-}{+15}} \\ & 2x + 3 \end{array}$$

On comparing 2x + 3 with px + q,
we get $p = 2, \quad q = 3$

SECTION-D

31.

$$\begin{array}{r|l} & x^2 - 4x + (8 - k) \\ x^2 - 2x + k & x^4 - 6x^3 + 16x^2 - 25x + 10 \\ & \underline{\overset{-}{x^4} \overset{+}{-2x^3} \overset{-}{+kx^2}} \\ & -4x^3 + (16-k)x^2 - 25x + 10 \\ & \underline{\overset{+}{-4x^3} \overset{-}{+8x^2} \quad \overset{+}{-4kx}} \\ & (8 - k)x^2 + (-25 + 4k)x + 10 \\ & \underline{-(8 - k)x^2 - 2(8 - k)x + k(8 - k)} \\ & (-9 + 2k)x + (10 - 8k + k^2) \end{array}$$

Remainder $= (-9 + 2k)x + (10 - 8k + k^2)$

$= x + a$

$-9 + 2k = 1$ $\qquad 10 - 8k + k^2 = a$

$2k = 10$ $\qquad 10 - 40 + 25 = a$

$k = 5$ $\qquad -5 = a$

32. $p(x) = x^4 - 6x^3 - 26x^2 + 138x - 35$

Zeroes of p(x) are $2 \pm \sqrt{3}$.

So, $[x - (2 + \sqrt{3})]$, $[x - (2 - \sqrt{3})]$

are factors of p(x)

i.e. $[(x - 2) - \sqrt{3}][(x - 2) + \sqrt{3}]$ is a factor of p(x).

i.e. $(x - 2)^2 - (\sqrt{3})^2$ is a factor of p(x)

i.e. $x^2 + 4 - 4x - 3$ is a factor of p(x) i.e. $x^2 - 4x + 1$ is a factor of p(x)

$$\begin{array}{r|l} & x^2 - 2x - 35 \\ x^2 - 4x + 1 & x^4 - 6x^3 - 26x^2 + 138x - 35 \\ & x^4 - 4x^3 + x^2 \\ & - \quad + \quad - \\ \hline & -2x^3 - 27x^2 + 138x \\ & -2x^3 + 8x^2 - 2x \\ & + \quad - \quad + \\ \hline & -35x^2 + 140x - 35 \\ & -35x^2 + 140x - 35 \\ & + \quad - \quad + \\ \hline & 0 \end{array}$$

For other zeroes,

Put $x^2 - 2x - 35 = 0$

$x^2 - 7x + 5x - 35 = 0$

$x(x - 7) + 5(x - 7) = 0$

$(x + 5)(x - 7) = 0$

$x = -5, 7$

So, other zeroes are –5 and 7.

33. $p(x) = x^3 - 5x^2 - 2x + 24$

Let α, β, γ be the zeroes of p(x).

$\alpha\beta = 12$ ...(i)

$\alpha + \beta + \gamma = \frac{-b}{a} = 5$

$\alpha\beta\gamma = \frac{-d}{a} = -24 \Rightarrow 12\gamma = -24$

$\Rightarrow \gamma = -2$

Also, $\alpha + \beta + \gamma = 5 \Rightarrow \alpha + \beta - 2 = 5$

$\alpha + \beta = 7$ ...(iii)

On solving (i) and (ii), we get

$\alpha(7 - \alpha) = 12$

$7\alpha - \alpha^2 = 12$

$\alpha^2 - 7\alpha + 12 = 0$

$\alpha^2 - 3\alpha - 4\alpha + 12 = 0$

$\alpha(\alpha - 3) - 4(\alpha - 3) = 0$

$(\alpha - 3)(\alpha - 4) = 0$

$\alpha = 3, 4$

If $\alpha = 3, \beta = 7 - \alpha = 7 - 3 = 4$

If $\alpha = 4, \beta = 7 - \alpha = 7 - 4 = 3$

So, zeroes of the polynomial are 3, 4 and –2.

34. $p(x) = 2x^4 - 2x^3 - 7x^2 + 3x + 6$

$-\sqrt{\frac{3}{2}}$ and $\sqrt{\frac{3}{2}}$ are zeroes of p(x).

$\left(x + \sqrt{\frac{3}{2}}\right), \left(x - \sqrt{\frac{3}{2}}\right)$ are factors of p(x).

$\left(x + \sqrt{\frac{3}{2}}\right)\left(x - \sqrt{\frac{3}{2}}\right)$ is a factor of p(x).

$\left(x^2 - \frac{3}{2}\right)$ is a factor of p(x).

$(2x^2 - 3)$ is a factor of p(x).

$$\begin{array}{r|l} & x^2 - x - 2 \\ 2x^2 - 3 & 2x^4 - 2x^3 - 7x^2 + 3x + 6 \\ & 2x^4 - 3x^2 \\ & - \quad + \\ \hline & -2x^3 - 4x^2 + 3x + 6 \\ & -2x^3 + 3x \\ & + \quad - \\ \hline & -4x^2 + 6 \\ & -4x^2 + 6 \\ & + \quad - \\ \hline & 0 \end{array}$$

For other zeroes of p(x),

Put $x^2 - x - 2 = 0$

$x^2 - 2x + x - 2 = 0$

$x(x - 2) + 1(x - 2) = 0$

$(x + 1)(x - 2) = 0$

$x = -1, 2$

35. $p(x) = 6x^2 + x - 2$

$\alpha + \beta = \frac{-b}{a} = -\frac{1}{6}, \quad \alpha\beta = \frac{c}{a} = -\frac{1}{3}$

(a) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$

$$= \frac{\frac{1}{36}+\frac{2}{3}}{-\frac{1}{3}} = \frac{\frac{1+24}{36}}{-\frac{1}{3}}$$

$$= \frac{25}{36} \times -\frac{3}{1} = -\frac{25}{12}$$

(b) $\alpha^3 + \beta^3$

$= (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta)$

$= (\alpha + \beta)[(\alpha + \beta)^2 - 3\alpha\beta)$

$$\left(\frac{-1}{6}\right)\left[\left(\frac{-1}{6}\right)^2 - 3\times\left(\frac{-1}{3}\right)\right]$$

$$= \left(\frac{-1}{6}\right)\left(\frac{1}{36}+1\right)$$

$$= \frac{-1}{6} \times \frac{37}{36} = \frac{-37}{216}$$

(c) $$2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right) = 2\left(\frac{\alpha+\beta}{\alpha\beta}\right)$$

$$= 2\left(\frac{-\frac{1}{6}}{-\frac{1}{3}}\right)$$

$$= 2\left(\frac{1}{2}\right)$$

$= 1$

(d) $\alpha^3 \beta^3 - \alpha^5 \beta^5$

$= \alpha^3 \beta^3 (1 - \alpha^2 \beta^2)$

$$= \left(-\frac{1}{3}\right)^3\left(1-\frac{1}{9}\right)$$

$$= -\frac{1}{27}\left(\frac{9-1}{9}\right)$$

$$= -\frac{1}{27}\left(\frac{8}{9}\right) = -\frac{8}{243}$$

36. (a) Let $p(x) = 8$

$g(x) = 3$

$q(x) = 2$

$r(x) = 2$

deg $p(x)$ = deg $q(x) = 0$

(b) Let $p(x) = 15$

$g(x) = 4$

$q(x) = 2$

$r(x) = 7$

deg $q(x)$ = deg $r(x) = 0$

(c) Let $p(x) = 20$

$g(x) = 3$

$r(x) = 2$

$q(x) = 6$

Here, deg $r(x) = 0$

37. Let $p(x) = 2x^3 + x^2 - 5x + 2$

$$p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) + 2$$

$$= 2\left(\frac{1}{8}\right) + \frac{1}{4} - \frac{5}{2} + 2$$

$$= \frac{1}{4} + \frac{1}{4} - \frac{5}{2} + 2$$

$$= \frac{1}{2} + 2 - \frac{5}{2}$$

$$= \frac{5}{2} - \frac{5}{2} = 0$$

So, $\frac{1}{2}$ is a zero of $p(x)$.

$p(1) = 2 + 1 - 5 + 2 = 0$

So, 1 is a zero of $p(x)$.

$p(-2) = 2(-8) + 4 + 10 + 2$

$= -16 + 16$

$= 0$

So, -2 is a zero of $p(x)$.

$$\text{Sum of zeroes} = \frac{-\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$$

$$= \frac{1}{2} + 1 - 2 = -\frac{1}{2}$$

$$= -\frac{1}{2}$$

$$\text{So, Sum of zeroes} = \frac{-\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$$

Sum of product of zeroes taken two at a time = $\dfrac{\text{Coefficient of x}}{\text{Coefficient of } x^3}$

$= \frac{1}{2}(1) + 1(-2) + \frac{1}{2}(-2) \quad = -\frac{5}{2}$

$= \frac{1}{2} - 2 - 1$

$= \frac{1}{2} - 3$

$= -\frac{5}{2}$

So, sum of product of zeroes taken two at a time = $\dfrac{\text{Coefficient of x}}{\text{Coefficient of } x^3}$

Product of zeroes $= \dfrac{-\text{Constant term}}{\text{Coefficient of } x^3}$

$= \frac{1}{2}(1)(-2) \qquad = -\frac{2}{2}$

$= -1 \qquad = -1$

So, product of zeroes = $\dfrac{-\text{Constant term}}{\text{Coefficient of } x^3}$

Hence, relationship between zeroes and the coefficients is verified.

38. $p(x) = x^3 + 13x^2 + 32x + 20$

$p(-2) = (-2)^3 + 13(-2)^2 + 32(-2) + 20$

$= -8 + 52 - 64 + 20$

$= 12 - 12$

$= 0$

$\Rightarrow$ x + 2 is a factor of p(x).

$$
\begin{array}{r|l}
 & x^2 + 11x + 10 \\
\hline
x + 2 & x^3 + 13x^2 + 32x + 20 \\
 & x^3 + 2x^2 \\
 & - \quad - \\
\hline
 & 11x^2 + 32x + 20 \\
 & 11x^2 + 22x \\
 & - \quad - \\
\hline
 & 10x + 20 \\
 & 10x + 20 \\
 & - \quad - \\
\hline
 & 0
\end{array}
$$

For other zeroes of p(x),

put $x^2 + 11x + 10 = 0$

$x^2 + 10x + x + 10 = 0$

$x(x + 10) + 1(x + 10) = 0$

$(x + 1)(x + 10) = 0$

$x = -1, -10$

So, zeroes of p(x) are –2, –1, –10.

39. $p(x) = ax^2 + bx + c$

$\alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a}$

(a) $\alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha + \beta)$

$= \frac{c}{a}\left(-\frac{b}{a}\right) = -\frac{bc}{a^2}$

(b) $\alpha^4 + \beta^4 = (\alpha^2)^2 + (\beta^2)^2$

$= [\alpha^2 + \beta^2]^2 - 2\alpha^2\beta^2$

$= [(\alpha + \beta)^2 - 2\alpha\beta]^2 - 2(\alpha\beta)^2$

$= \frac{b^2}{a^2} - \frac{2c}{a}^{2} - \frac{2c^2}{a^2}$

$= \frac{1}{a^4}\left(b^2 - 2ac\right)^2 - \frac{2c^2}{a^2}$

$= \frac{1}{a^4}\left(b^4 + 4a^2c^2 - 4ab^2c\right) - \frac{2c^2}{a^2}$

$= \frac{b^4}{a^4} + \frac{4c^2}{a^2} - \frac{4b^2c}{a^3} - \frac{2c^2}{a^2}$

$= \frac{b^4}{a^4} - \frac{4b^2c}{a^3} + \frac{2c^2}{a^2}$

40. $p(x) = x^3 - 6x^2 + 3x + 10$

$a + (a + b) + (a + 2b) = 6$

$3a + 3b = 6$

$a + b = 2$
$b = 2 - a$

$a(a + b)(a + 2b) = -10$

$a(2)(4 - a) = -10$

$2a(4 - a) = -10$

$8a - 2a^2 = -10$

$$2a^2 - 8a - 10 = 0$$
$$a^2 - 4a - 5 = 0$$
$$a^2 - 5a + a - 5 = 0$$
$$a(a - 5) + 1(a - 5) = 0$$
$$(a + 1)(a - 5) = 0$$
$$a = -1, 5$$

For $a = -1$, $b = 2 - a = 2 - (-1) = 3$

For $a = 5$, $b = 2 - a = 2 - 5 = -3$

$a = -1$, $b = 3$

zeroes are $a, a + b, a + 2b$

$= -1, -1 + 3, -1 + 6$

$= -1, 2, 5$

for $a = 5, b = -3$

zeroes are $a, a + b, a + 2b$

$= 5, 5 - 3, 5 - 6$

$= 5, 2, -1$

So, zeroes of the given polynomial are -1, 2 and 5.

CASE STUDY-1

(i) (b) As the graph of $y = p(x)$ does not touch x axis hence there are no zeroes available for the graph.

(ii) (a) The graph which does not intersect with x axis has no zeroes.

(iii) (c) The graph I cut x axis at 3 points hence it has maximum of 3 zeroes.

(iv) (a) Those points where the graph of $y = p(x)$ cuts the x axis are called zeroes of $y = p(x)$.

(v) (b) $p(x) = 2x^2 + 5x + 3$

To find zero, equate p(x) to zero

$p(x) = 0$

$2x^2 + 5x + 3 = 0$

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(2)(3)}}{2(2)}$$

$$= \frac{-5 \pm \sqrt{25 - 24}}{4}$$

$$= \frac{-5 \pm 1}{4}$$

$$x = \frac{-5 + 1}{4} = -1 \text{ and}$$

$$x = \frac{-5 - 1}{4} = -1.5$$

CASE STUDY-2

(i) (b) The curve BCD represents a parabola.

(ii) (c)
$$x^2 - 7x + 12 = 0$$
$$x^2 - 4x - 3x + 12 = 0$$
$$x(x - 4) - 3(x - 4) = 0$$
$$(x - 3)(x - 4) = 0$$

$x - 3 = 0$	$x - 4 = 0$
$x = 3$	$x = 4$

(iii) (b) For the Quadratic equation $ax^2 + bx + c = 0$ the products of roots α, β is

$$\alpha - \beta = \frac{c}{a}$$

For the polynormial $2x^2 - 7x + k$, the root are reciprocal therefore the product of roots will be 1 and is equal to $\frac{k}{2}$

$$\therefore \frac{k}{2} = 1$$

$\Rightarrow k = 2$

(iv) (b) The zeroes of the given curve are $-4, 0, 2$ and 4.

The sum of $-4, 0, 2$ and 4 is 2.

(v) (a) For cubic equation $ax^3 + bx^2 + cx + d = 0$.

The sum of roots α, β, γ is $\frac{-b}{a}$

The product of roots $\alpha\beta\gamma$ is $\frac{-d}{a}$

The sum product of roots taken 2 at a time is $\frac{c}{a}$.

ATQ:

Comparing the given polynomial with standard form of quadratic polynomial.

$a = 1, b = -2, c = q, d = -r$

$\therefore \quad \alpha + \beta + \gamma = -2$

As $\quad \alpha + \beta = 0$

$\therefore \quad \gamma = 2 \quad \text{...(i)}$

As $\alpha + \beta = 0 \therefore \alpha = -\beta$

$$\alpha\beta\gamma = -(-r)$$

$$\alpha\beta\,(2) = r$$

$$\alpha\beta = \frac{r}{2} \quad \text{... (ii)}$$

$$\alpha\beta + \beta\gamma + \alpha\gamma = q$$

$$\frac{r}{2} + \beta 2 + 2\alpha = q \quad \text{[From (ii)]}$$

$$\frac{r}{2} + 2\beta - 2\beta = q$$

$$r = 2q$$

Chapter 3

Pair of Linear Equations in Two Variables

Multiple Choice Questions

1. (c) The system of equations has no solution if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

i.e. $\frac{1}{2} = \frac{1}{k} \neq \frac{4}{3}$

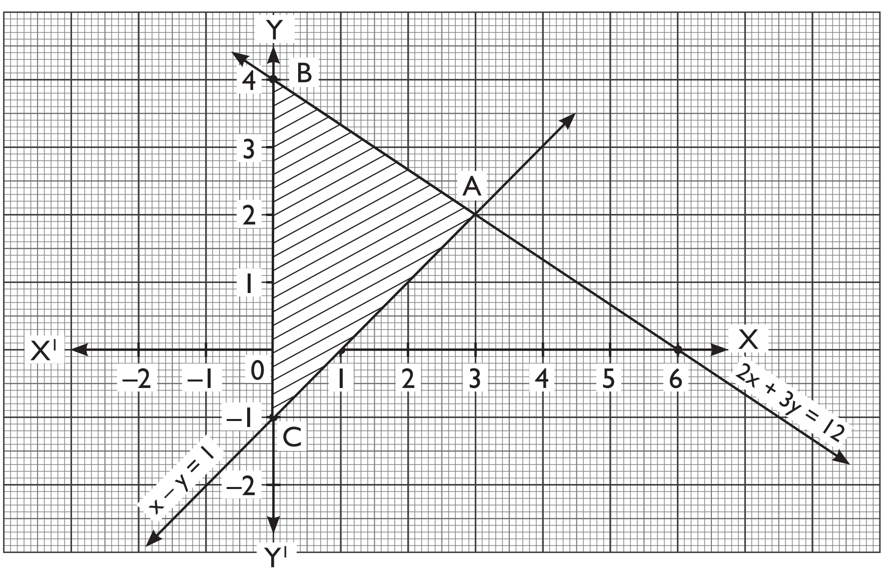
i.e. $k = 2$

2. (d) For no solution, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

i.e. $\frac{k}{6} = -\frac{5}{2} \neq \frac{2}{7}$

i.e. $k = -15$

3. (a)



$2x + 3y = 12$

x	6	0
y	0	4

$x - y = 1$

x	1	0
y	0	–1

$x = 0$

$$\text{area of } \triangle ABC = \frac{1}{2} \times 5 \times 3$$

$$= \frac{15}{12}$$

$$= 7.5 \text{ sq. units}$$

4. (c) Let number of coins of ₹ 1 = x

number of coins of ₹ 2 = y

$$\therefore \quad x + y = 50$$

$$\underline{-x \; \underline{+} \; 2y = \underline{-}75}$$

$$-y = -25$$

$$y = 25$$

So, $x = 50 - y$

$= 50 - 25$

$= 25$

5. (b) Let x be the tens digit and y be the ones digit.

$$\therefore \quad x + y = 9 \quad \text{...(i)}$$

and $10x + y + 27 = 10y + x$

$$9x - 9y = -27$$

$$x - y = -3 \quad \text{....(ii)}$$

On solving (i) and (ii), we get

$$\therefore \quad x + y = 9$$
$$x - y = -3$$
$$2x = 6$$
$$x = 3$$

From (i), $y = 9 - x = 9 - 3 = 6$

So, number is $10x + y$

$$= 10(3) + 6 = 36$$

WORKSHEET - 1

SECTION-A

1. $3x - y + 8 = 0, \quad 6x - ky + 16 = 0$

   The equations represent coincident lines if

   $$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

   i.e. $\frac{3}{6} = \frac{-1}{-k} = \frac{8}{16}$

   $k = 2$

2. Let number of girls be x and number of boys be y.

   $$x + y = 15 \quad ...(i)$$
   $$x = 5 + y \Rightarrow x - y = 5 \quad ...(ii)$$

   On solving equations (i) and (ii), we get

   $$x + y = 15$$
   $$x - y = 5$$
   $$2x = 20$$
   $$x = 10$$
   $$y = 15 - x = 15 - 10 = 5$$

3. General form of a pair of linear equations in two variables x and y is

   $$a_1x + b_1y + c_1 = 0$$
   $$a_2x + b_2y + c_2 = 0$$

4. If a pair of linear equations in two variables is consistent, then the lines are either intersecting or coincident.

5. $2x + 3y = 7$

   $8x + (a + b)y = 28$

   Given pair of equations has infinitely many solutions if

   $$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$
   $$\frac{2}{8} = \frac{3}{a+b} = \frac{7}{28}$$
   $$a + b = \frac{3 \times 28^4}{7} = 12$$
   $$a + b = 12$$

6. The pair of linear equations has infinitely many solutions if

   $$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$
   $$\frac{10}{20} = \frac{5}{10} = \frac{k-5}{k}$$
   $$\frac{5}{10} = \frac{1}{2} = \frac{k-5}{k}$$
   $$k = 2k - 10$$
   $$k = 10$$

7. $2x + 3y = 7$

   $(a + b)x + (2a - b)y = 21$

   System of equations has infinitely many

   solutions if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

   i.e. $\frac{2}{a+b} = \frac{3}{2a-b} = \frac{7}{21}$

   $4a - 2b = 3a + 3b,$ $\quad 2a - b = \frac{3 \times 21}{7} = 9$

   $a = 5b$ $\quad 2a - b = 9$

   $2(5b) - b = 9$

   $9b = 9$

   $b = 1$

   But, $a = 5b$

   $a = 5(1)$

   $a = 5$

8. The given system of linear equation has unique solution if

   $$\frac{a}{l} \neq \frac{b}{m}$$

   i.e. $am \neq bl$

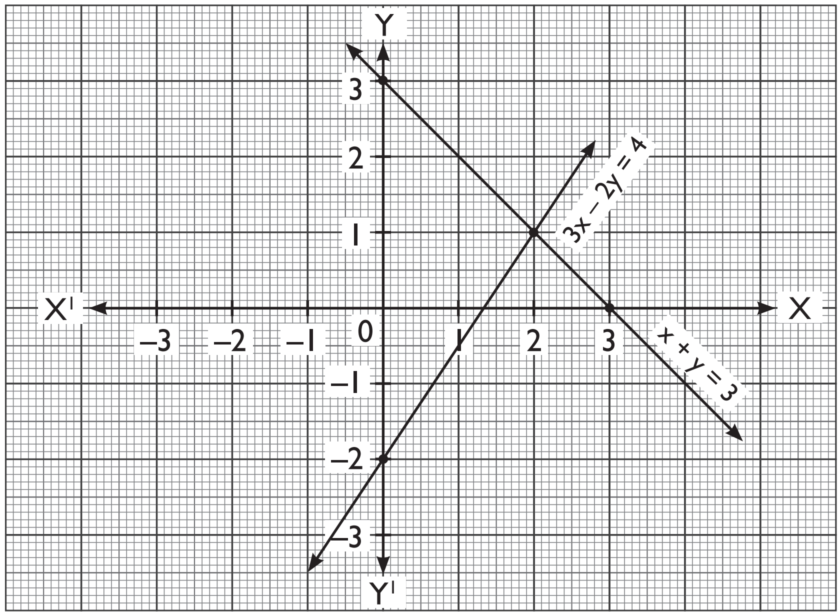
SECTION-B

9. $x + y = 3$

$3x - 2y = 4$

x	3	0
y	0	3

x	0	2
y	–2	1

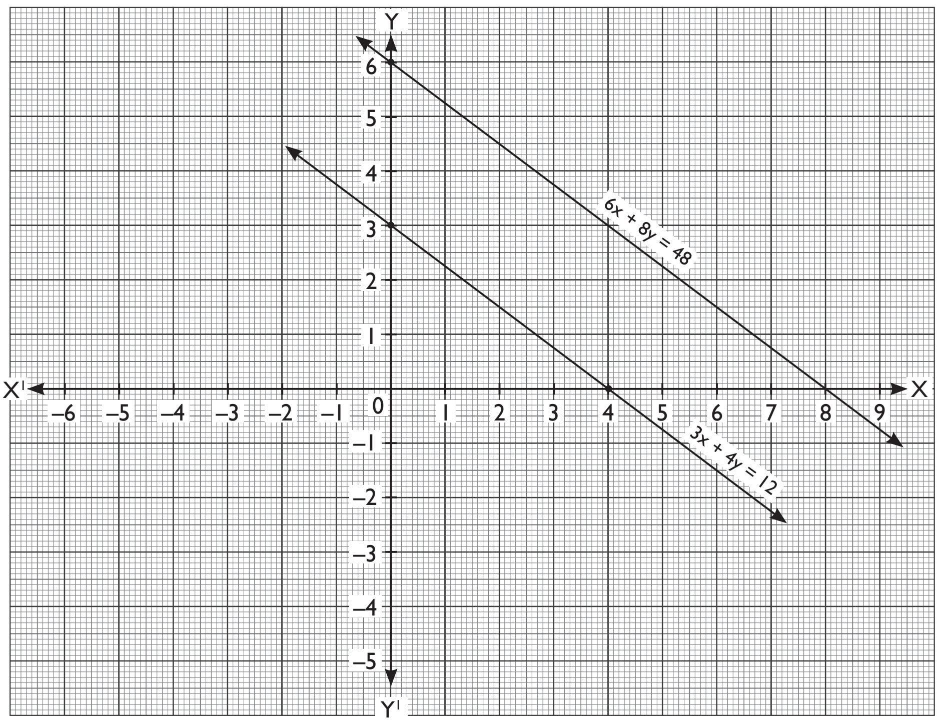


10. $3x + 4y = 12$

$6x + 8y = 48$

x	4	0
y	0	3

x	8	0
y	0	6



11. (a) $x + 2y = -1$

(b) $2x - 3y = 12$

From (a), $x = -1 - 2y$

$2(-1 - 2y) - 3y = 12$ [Put in (b)]

$-2 - 4y - 3y = 12$

$-7y = 14$

$y = -2$

So, $x = -1 - 2y$

$= -1 - 2(-2)$

$= -1 + 4$

$= 3$

12. $\frac{2x}{a} + \frac{y}{b} = 2$ ...(i)

$\frac{x}{a} - \frac{y}{b} = 4$ ...(ii)

From (ii), we have $\frac{x}{a} = 4 + \frac{y}{b}$

$x = a \; 4 + \frac{y}{b}$ ...(iii)

Putting this value of x in (i), we get

$$\frac{2a}{a}\left(4+\frac{y}{b}\right)+\frac{y}{b} = 2$$

$$2\ \ 4+\frac{y}{b}\ \ +\frac{y}{b} = 2$$

$$8+\frac{2y}{b}+\frac{y}{b} = 2$$

$$\frac{3y}{b} = -6$$

$$y = \frac{-6b}{3} = -2b$$

From (iii), $x = a\left(4+\frac{y}{b}\right)$

$$= a\left(4-\frac{2b}{b}\right)$$

$$= a(4-2)$$

$$= 2a$$

13. $28x + 5y = 9$ ...(i)

$3x + 2y = 4$ ...(ii)

On multiplying (i) by 2 and (ii) by 5, we get

$$\begin{aligned} 56x + 10y &= 18 \\ 15x + 10y &= 20 \\ - \quad - \quad &\ \ - \\ \hline 41x &= -2 \end{aligned}$$

$$x = -\frac{2}{41}$$

From (i), $28\left(-\frac{2}{41}\right)+5y = 9$

$$-\frac{56}{41}+5y = 9$$

$$5y = 9+\frac{56}{41} = \frac{425}{41}$$

$$y = \frac{85}{41}$$

14. Let $\frac{1}{x} = p,\ \frac{1}{y} = q$

$2p + \frac{2}{3}q = \frac{1}{6} \Rightarrow 12p + 4q = 1$

Other equation becomes $3p + 2q = 0$

On solving equation $12p + 4q = 1$ and $3p + 2q = 0$, we get

$$12p + 4q = 1$$

$$2(3p + 2q = 0)$$

$$\begin{aligned} 12p + 4q &= 1 \\ 6p + 4q &= 0 \\ - \quad - \quad &\ \ - \\ \hline 6p &= 1 \end{aligned}$$

$$p = \frac{1}{6} = \frac{1}{x}$$

$$x = \frac{1}{p} = 6$$

From equation $3p + 2q = 0$, we get

$$3\left(\frac{1}{6}\right) + 2q = 0$$

$$2q = -\frac{1}{2}$$

$$q = -\frac{1}{4} \Rightarrow y = -4$$

Now, we need to find a

$$y = ax - 4$$

$$-4 = 6a - 4$$

$$6a = 0$$

$$a = 0$$

15. $2x + y = 35$ ...(i),

$3x + 4y = 65$ ...(ii)

On multiplying equation (i) by 3 and equation (ii) by 2, we get

$$\begin{aligned} 6x + 3y &= 105 \\ 6x + 8y &= 130 \\ - \quad - \quad &\ \ - \\ \hline -5y &= -25 \end{aligned}$$

$$y = 5$$

From (i), $2x + 5 = 35$

$$2x = 30$$

$$x = 15$$

16. For unique solution: $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$$\frac{k}{3} \neq \frac{2}{1}$$
$$k \neq 6$$

For infinitely many solutions: $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$$\frac{k}{3} = \frac{2}{1} = \frac{5}{2.5}$$
$$k = 6$$

17. $2x + ky = 11$

$5x - 7y = 5$

For no solution: $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$$\frac{2}{5} = \frac{k}{-7} \neq \frac{11}{5}$$
$$5k = -14$$
$$k = \frac{-14}{5}$$

For unique solution: $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$$\frac{2}{5} \neq \frac{k}{-7}$$
$$k \neq -\frac{14}{5}$$

18. For infinitely many solutions: $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$$\frac{8}{k} = \frac{5}{10} = \frac{9}{18}$$
$$5k = 8 \times 10$$
$$k = 16$$

For unique solution: $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$$\frac{8}{k} \neq \frac{5}{10}$$

$5k \neq 8 \times 10;\quad k \neq 16$

SECTION-C

19. Let the numbers be 5x and 6x.

If 8 is subtracted from each of the numbers, they become

$5x - 8$ and $6x - 8$

According to the given condition,

$$\frac{5x-8}{6x-8} = \frac{4}{5}$$
$$25x - 40 = 24x - 32$$
$$x = 8$$

So, numbers are $5x = 5(8) = 40$

and $6x = 6(8) = 48$.

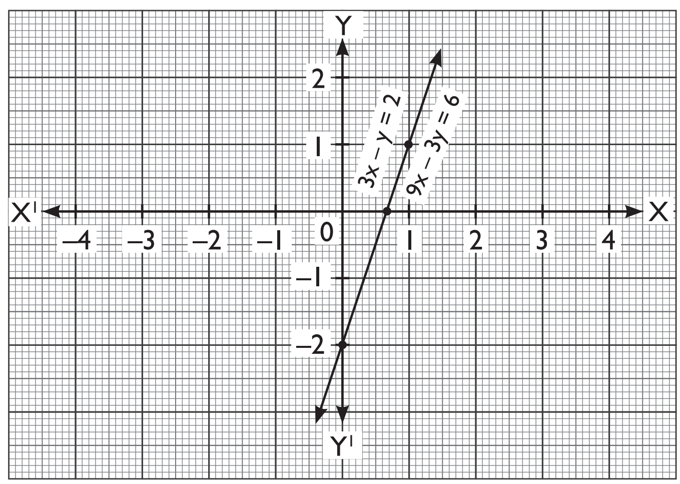
20. $3x - y = 2$

x	0	$\frac{2}{3}$
y	–2	0

$9x - 3y = 6$

x	1	0
y	1	–2

As lines are coincident, so, system of equations has infinitely many solutions.



21. $\frac{5}{x+y} - \frac{2}{x-y} = -1, \qquad \frac{15}{x+y} + \frac{7}{x-y} = 10$

Let $\frac{1}{x+y} = p$ and $\frac{1}{x-y} = q$

$5p - 2q = -1, \qquad 15p + 7q = 10$

Using elimination method, we get

$$3[(5p - 2q) = -1] \Rightarrow \quad 15p - 6q = -3$$
$$\underline{\underset{-}{}15p \underset{-}{+} 7q = \underset{-}{}10}$$
$$-13q = -13$$
$$q = 1$$

$$\boxed{x - y = 1}$$

From equation $5p - 2q = -1$, we get

$$5p - 2(1) = -1$$
$$5p = 1$$
$$p = \frac{1}{5}$$

$$\boxed{x + y = 5}$$

On solving equations $x + y = 5$ and $x - y = 1$, we get

$$x - y = 1$$
$$x + y = 5$$
$$2x = 6$$
$$x = 3 \Rightarrow y = 5 - 3 = 2$$

22. Let $p(x) = x^3 + ax^2 + 2bx - 24$

As $(x - 4)$ is a factor of $p(x)$,

$p(4) = 0$

$$4^3 + a(4)^2 + 2b(4) - 24 = 0$$
$$64 + 16a + 8b - 24 = 0$$
$$16a + 8b + 40 = 0$$
$$2a + b + 5 = 0$$

From $a - b = 8$, $\quad b = a - 8$

So, $\quad 2a + (a - 8) + 5 = 0$

$$3a = 3$$
$$a = 1$$

$\therefore \quad b = a - 8 = 1 - 8 = -7$

23. Let number of rows be x and number of students in each row be y. So, total number of students = xy

According to question,

$$(y + 3)(x - 1) = xy$$
$$xy + 3x - y - 3 = xy$$
$$3x - y = 3 \qquad ...(i)$$

Again, $\quad (y - 3)(x + 2) = xy$

$$xy + 2y - 3x - 6 = xy$$
$$-3x + 2y = 6 \qquad ...(ii)$$

On solving equations (i) and (ii), we get

$$2(3x - y = 3) \Rightarrow \quad 6x - 2y = 6$$
$$-3x + 2y = 6$$
$$3x = 12$$
$$x = 4$$

From (i), $\quad y = 3x - 3$

$$= 12 - 3$$
$$= 9$$

So, Total number of students = xy

$$= 4(9)$$
$$= 36$$

24. $\quad 3x + 2y = 5$

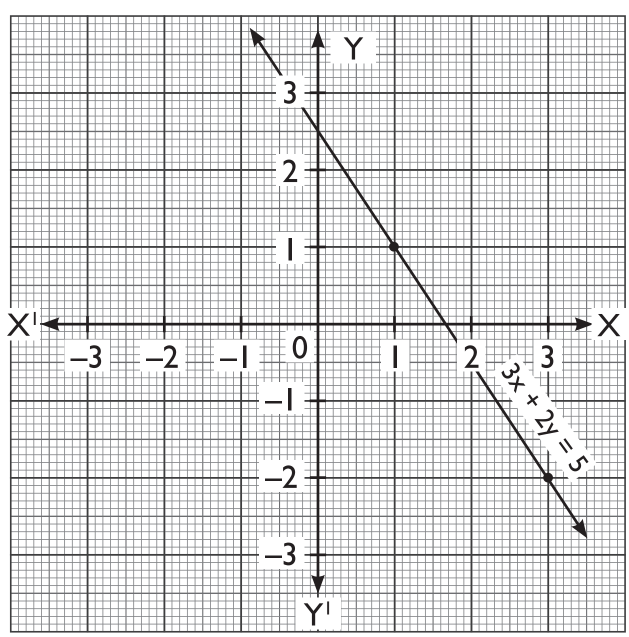
$$3x = 5 - 2y$$
$$x = \frac{5 - 2y}{3}$$

To check : (1, 1) is a point on the $3x + 2y = 5$

$$LHS = 3x + 2y$$
$$= 3(1) + 2(1)$$
$$= 5$$
$$= RHS$$

So, (1, 1) is a point on the line $3x + 2y = 5$.

x	1	3
y	1	–2



25. Let digit at ten's place be x and digit at unit's place be y

So, number = 10x + y

According to question,

$$x + y = 5 \quad ...(i)$$

$$10y + x = 10x + y + 9$$

$$0 = 9x - 9y + 9$$

$$x - y = -1 \quad ...(ii)$$

On solving (i) and (ii), we get

$$x + y = 5$$

$$x - y = -1$$

$$2x = 4$$

$$x = 2$$

From (i), $y = 5 - 2 = 3$

So, number = 10x + y

= 10(2) + 3

= 23

26. Let the adjacent angle be x.

$$\text{Other angle} = \frac{4}{5}x$$

As sum of adjacent angles of a parallelogram is 180°,

$$x + \frac{4}{5}x = 180$$

$$\frac{9x}{5} = 180$$

$$x = \frac{\cancel{180}^{20} \times 5}{\cancel{9}} = 100°$$

Angles are x, $\frac{4}{5}x$

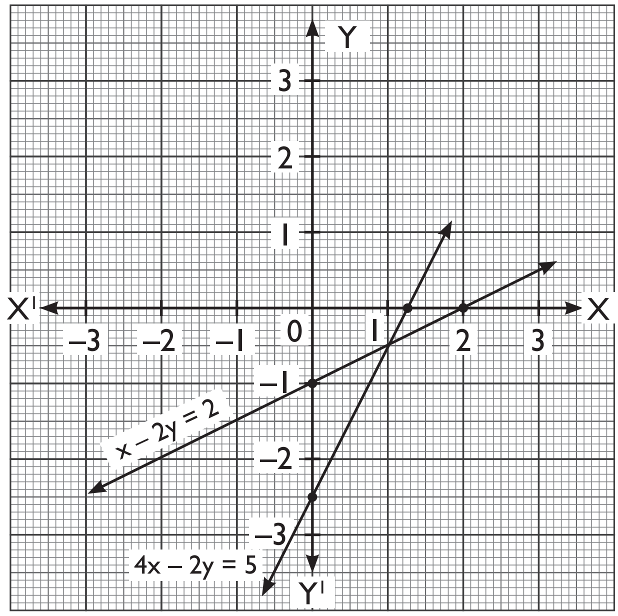
$$= 100, \frac{4}{5}(100)$$

$$= 100°, 80°$$

27. (a) x – 2y = 2, 4x – 2y = 5

x	0	2
y	–1	0

x	0	1.25
y	–2.5	0

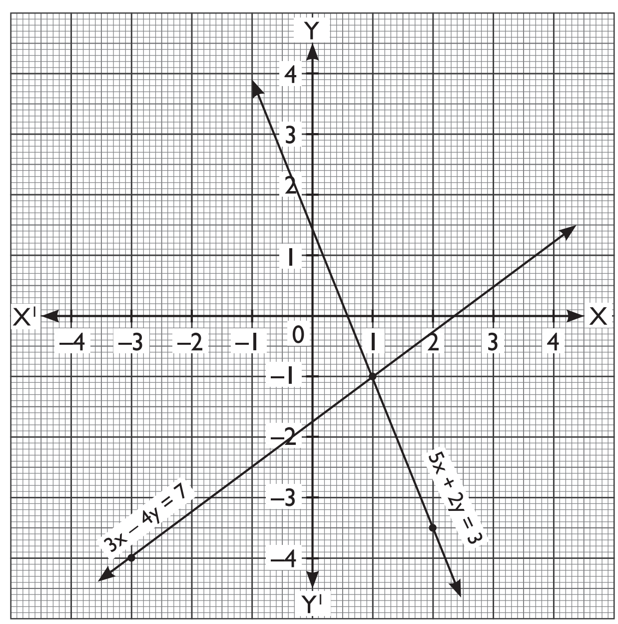


As the lines are intersecting, so the system of equations has unique solution and hence, consistent.

(b) 3x – 4y = 7, 5x + 2y = 3

x	–3	1
y	–4	–1

x	1	2
y	–1	–3.5



28. In ΔABC,

$$\angle A + \angle B + \angle C = 180° \text{ (Angle sum property)}$$

$$x + 3x + y = 180°$$

$$4x + y = 180° \quad ...(i)$$

Given: $3y - 5x = 30° \quad ...(ii)$

From (i), $y = 180 - 4x$

So, eq^n (ii) becomes $3(180 - 4x) - 5x = 30$

$$540 - 12x - 5x = 30$$
$$17x = 510$$
$$x = 30$$

From (i), $y = 180 - 4(30)$

$$= 180 - 120$$
$$= 60°$$

So, $\angle A = x = 30°$

$$\angle B = 3x = 90°$$
$$\angle C = y = 60°$$

In $\triangle ABC$, $\angle B = 90°$, so it is a right angled triangle.

SECTION-D

29. Let speed of boat in still water be x km/hr and that of stream be y km/hr.

So, speed of boat upstream = $(x - y)$ km/hr

Speed of boat downstream = $(x + y)$ km/hr

According to question,

$$\frac{32}{x-y} + \frac{36}{x+y} = 7$$
$$\frac{40}{x-y} + \frac{48}{x+y} = 9$$

Let $\frac{1}{x-y} = p$, $\frac{1}{x+y} = q$

So, we get equations as

$$32p + 36q = 7 \quad ...(i)$$
$$40p + 48q = 9 \quad ...(ii)$$

On multiplying (i) by 5 and (ii) by 4, we get

$$160p + 180q = 35$$
$$\underset{-}{160p} \underset{-}{+} 192q = \underset{-}{36}$$
$$-12q = -1$$
$$q = \frac{1}{12}$$

i.e. $x + y = 12 \quad ...(iii)$

From (i), $32p + 36\,\frac{1}{12} = 7$

$$32p = 7 - 3 = 4$$
$$p = \frac{1}{8}$$
$$x - y = 8 \quad ...(iv)$$

On solving (iii) and (iv), we get

$$x = 10$$
$$y = 2$$

Speed of boat in still water = 10 km/hr

Speed of stream = 2 km/hr

30. $ax + by = 1 \quad ...(i)$

$$bx + ay = \frac{(a+b)^2}{a^2+b^2} - 1 \quad ...(ii)$$
$$= \frac{a^2 + b^2 + 2ab - a^2 - b^2}{a^2+b^2}$$
$$bx + ay = \frac{2ab}{a^2+b^2} \quad ...(iii)$$

On multiplying (i) by b and (iii) by a, we get

$$abx + b^2y = b$$
$$\underset{-}{abx} + \underset{-}{a^2y} = \underset{-}{\frac{2a^2b}{a^2+b^2}}$$
$$y(b^2 - a^2) = b - \frac{2a^2b}{a^2+b^2}$$
$$y(b^2 - a^2) = \frac{a^2b + b^3 - 2a^2b}{a^2+b^2}$$
$$= \frac{b^3 - a^2b}{a^2+b^2}$$
$$= \frac{b(b^2 - a^2)}{a^2+b^2}$$
$$\therefore \quad y = \frac{b}{a^2+b^2}$$

From (i), $ax + b\,\frac{b}{a^2+b^2} = 1$

$$ax = 1 - \frac{b^2}{a^2+b^2}$$

$$= \frac{a^2}{a^2+b^2}$$

$$ax = \frac{a^2}{a^2+b^2}$$

$$x = \frac{a}{a^2+b^2}$$

31. Let speed of X be x km/hr and that of Y be y km/hr.

Time taken by X to walk 30 km

$$= \frac{30}{x} \text{ hours}$$

Time taken by Y to walk 30 km

$$= \frac{30}{y} \text{ hours}$$

According to question,

$$\frac{30}{x} = \frac{30}{y} + 3$$

$$\frac{30}{x} - \frac{30}{y} = 3$$

$$\frac{1}{x} - \frac{1}{y} = \frac{1}{10} \quad \text{...(i)}$$

Also, $\frac{30}{2x} = \frac{30}{y} - \frac{3}{2}$

$$\frac{15}{x} = \frac{30}{y} - \frac{3}{2}$$

$$\frac{15}{x} - \frac{30}{y} = \frac{-3}{2}$$

$$\frac{1}{x} - \frac{2}{y} = -\frac{1}{10} \quad \text{...(ii)}$$

Let $\frac{1}{x} = p$, $\frac{1}{y} = q$

So, equations (i) and (ii) become

$$p - q = \frac{1}{10} \Rightarrow 10p - 10q = 1 \quad \text{...(iii)}$$

and $p - 2q = -\frac{1}{10} \Rightarrow 10p - 20q = 1$ ...(iv)

On solving equations (iii) and (iv), we get

$$\begin{array}{rcl} 10p - 10q &=& 1 \\ \underset{-}{10p} \underset{+}{-} 20q &=& \underset{+}{-1} \\ \hline 10q &=& 2 \end{array}$$

$$q = \frac{1}{5} \Rightarrow y = 5$$

From (iii), we get $10p - 10\left(\frac{1}{5}\right) = 1$

$$10p = 1 + 2 = 3$$

$$p = \frac{3}{10}$$

$$x = \frac{10}{3}$$

So, Speed of X = $\frac{10}{3}$ km/hr

Speed of Y = 5 km/hr

32. $a(x+y) + b(x-y) = a^2 - ab + b^2$ ...(i)

$a(x+y) - b(x-y) = a^2 + ab + b^2$ ...(ii)

Let $x + y = p$ and $x - y = q$

So, equations (i) and (ii) becomes

$ap + bq = a^2 - ab + b^2$ ...(iii)

$ap - bq = a^2 + ab + b^2$ ...(iv)

On adding (iii) and (iv) we get,

$$2ap = 2(a^2 + b^2)$$

$$p = \frac{1}{a}(a^2 + b^2)$$

From equation (iii),

$$a\frac{1}{a}(a^2+b^2) + bq = a^2 - ab + b^2$$

$$a^2 + b^2 + bq = a^2 - ab + b^2$$

$$bq = -ab$$

$$q = -a$$

So,

$$\begin{array}{rcl} x + y &=& \frac{1}{a}(a^2+b^2) \\ x - y &=& -a \\ \hline 2x &=& \frac{1}{a}(a^2+b^2) - a \end{array}$$

$$2x = a + \frac{b^2}{a} - a = \frac{b^2}{a}$$

$$\boxed{x = \frac{b^2}{2a}}$$

So, $y = x + a = \frac{b^2}{2a} + a = \frac{b^2 + 2a^2}{2a}$

33. Let $\frac{1}{2x+3y} = p$ and $\frac{1}{3x-2y} = q$

So, equations become

$$\frac{1}{2}p + \frac{12}{7}q = \frac{1}{2}$$

and $\quad 7p + 4q = 2$

i.e. $\quad 7p + 24q = 7 \quad$ ....(i)

and $\quad 7p + 4q = 2 \quad$ .... (ii)

On subtracting (ii) from (i), we get

$$20q = 5 \Rightarrow q = \frac{1}{4}$$

From (i), $7p + 24\left(\frac{1}{4}\right) = 7$

$$7p = 1$$

$$p = \frac{1}{7}$$

So, we get $\quad 2x + 3y = 7 \quad$ ...(iii)

$\quad 3x - 2y = 4 \quad$ ...(iv)

On multiplying (iii) by 3 and (iv) by 2 and subtracting, we get

$$\begin{array}{r} 6x + 9y = 21 \\ \underset{-}{6x} \underset{+}{-} 4y = \underset{-}{8} \\ \hline 13y = 13 \end{array}$$

$$y = 1$$

From (iii),

$$2x + 3(1) = 7$$

$$2x = 4$$

$$x = 2$$

34. $\quad kx - y = 2$

$\quad 6x - 2y = 3$

(i) For unique solution : $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$$\frac{k}{6} \neq \frac{-1}{-2}$$

$$k \neq 3$$

(ii) For no solution : $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$$\frac{k}{6} = \frac{-1}{-2} \neq \frac{2}{3}$$

$$k = 3$$

The system has infinitely many

solutions, if $\quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

i.e. $\quad \frac{k}{6} = \frac{-1}{-2} = \frac{2}{3}$

Clearly, $\frac{-1}{-2} \neq \frac{2}{3}$,

So, there is no value of k for which the given system of equations has infinitely many solutions.

35. Let the expenditures of U and V be 19x and 16x and income of U and V be 8y and 7y.

According to question,

$8y - 19x = 1250 \quad$ ...(i)

$7y - 16x = 1250 \quad$ ...(ii)

On multiplying (i) by 7 and (ii) by 8, we get

$$\begin{array}{r} 56y - 133x = 8750 \\ \underset{-}{56y} \underset{+}{-} 128x = \underset{-}{10000} \\ \hline -5x = -1250 \\ x = 250 \end{array}$$

From (i), $8y - 19(250) = 1250$

$$8y - 4750 = 1250$$

$$8y = 6000$$

$$y = 750$$

Therefore,

Income of U $= 8y = 8\ (750)$

$= ₹\ 6000$

Income of V $= 7y = 7\ (750)$

$= ₹\ 5250$

36. The system of equations has infinite number of solutions if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

i.e. $\dfrac{2}{p+q} = \dfrac{3}{2p-q} = \dfrac{9}{3(p+q+1)}$

i.e. $\dfrac{2}{p+q} = \dfrac{3}{2p-q} = \dfrac{3}{p+q+1}$

$4p - 2q = 3p + 3q$

$p = 5q$ ...(i)

$3p + 3q + 3 = 6p - 3q$

$0 = 3p - 6q - 3$

$p - 2q = 1$ ...(ii)

On solving (i) and (ii), we get

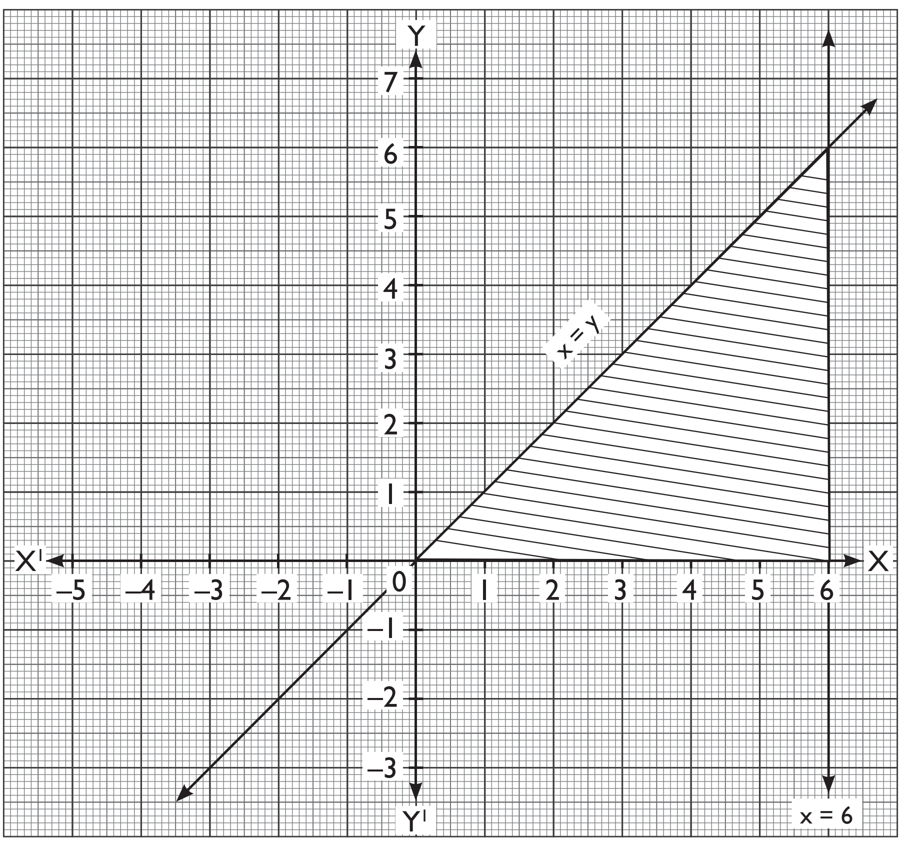
$$5q - 2q = 1 \Rightarrow q = \frac{1}{3}$$

So, $p = 5q = \dfrac{5}{3}$

WORKSHEET - 2

SECTION-A

1.



Area of $\Delta = \frac{1}{2} \times 6 \times 6$

$= 18$ sq. units

2. The system of equations has no solution if $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$

i.e. $\dfrac{1}{3} = \dfrac{2}{k} \neq \dfrac{5}{15}$

$k = 6$

3. $3x + y = 1$

$(2k - 1)x + (k - 1)y = 2k + 1$

System of equations is inconsistent if $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$

i.e. $\frac{3}{2k-1} = \frac{1}{k-1} \neq \frac{1}{2k+1}$

$$3k - 3 = 2k - 1$$

$$k = 2$$

4. The system of equations represent intersecting lines if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$$\frac{2}{k} \neq \frac{5}{7}$$

$$k \neq \frac{14}{5}$$

5. The system of equations has a unique solution if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$$\frac{k}{6} \neq \frac{-1}{-2}$$

$$k \neq 3$$

6. $x + ky = 0$

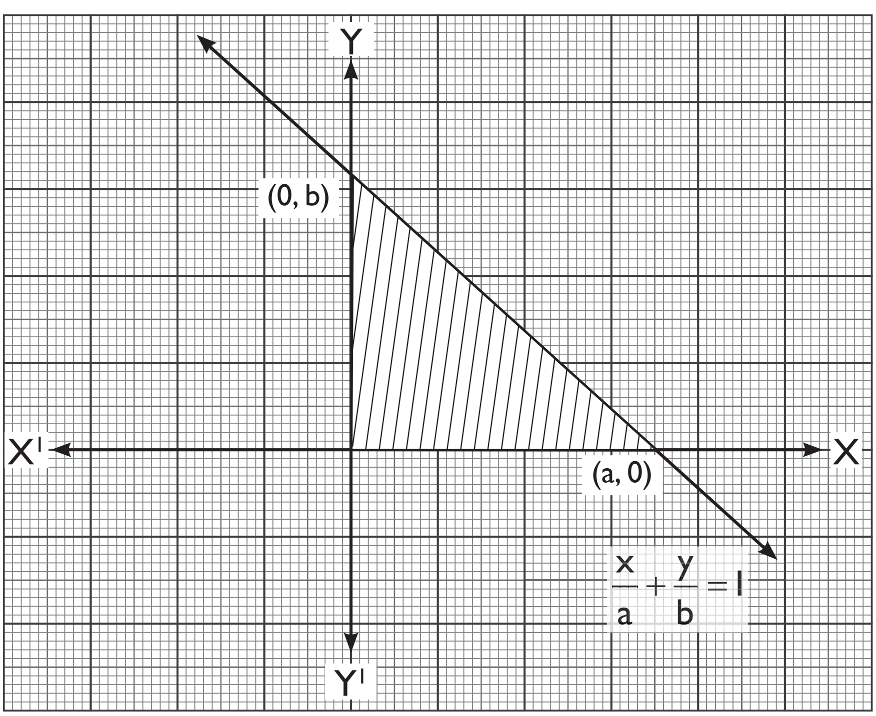
   $2x - y = 0$

   System of equations has a unique solution if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

   i.e. $\frac{1}{2} \neq \frac{k}{-1}$

   i.e. $k \neq \frac{-1}{2}$

7.



Area of triangle $= \frac{1}{2} \times a \times b = \frac{ab}{2}$.

8. As (3, a) lies on line $2x - 3y = 5$

$$2(3) - 3(a) = 5$$

$$6 - 3a = 5$$

$$3a = 1$$

$$a = \frac{1}{3}$$

9.

$$x + 2y = 8$$

$$2x + 4y = 16$$

Here, $\frac{a_1}{a_2} = \frac{1}{2}$

$$\frac{b_1}{b_2} = \frac{2}{4} = \frac{1}{2}$$

$$\frac{c_1}{c_2} = \frac{8}{16} = \frac{1}{2}$$

As $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

So, the system of equations has infinitely many solutions.

10. $x + y = 14$

    $x - y = 4$

Here, $\frac{a_1}{a_2} = \frac{1}{1}$, $\frac{b_1}{b_2} = \frac{1}{-1} = -1$, $\frac{c_1}{c_2} = \frac{14}{4} = \frac{7}{2}$

As $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$,

The system has a unique solution i.e. the system is consistent.

SECTION-B

11. $-4x + y = 1$ ...(i)

$6x - 5y = 9$ ...(ii)

On multiplying eq^n (i) by 5 and adding both the equations, we get

$$5(-4x + y) + 6x - 5y = 5 + 9$$
$$-20x + 5y + 6x - 5y = 14$$
$$-14x = 14$$
$$x = -1$$

From (i), $y = 1 + 4x = 1 - 4$

$$y = -3$$

12. The given pair of linear equations intersect if

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ $\left[\begin{array}{ll}\text{Here, } a_1 = 2, & b_1 = -3 \\ a_2 = 4, & b_2 = -5\end{array}\right]$

i.e. $\frac{2}{4} \neq \frac{-3}{-5}$

13. $2x = 5y + 4$ ...(i)

$3x - 2y + 16 = 0$ ...(ii)

From (i), we get $x = \frac{5y + 4}{2}$

From (ii), $3\ \frac{5y+4}{2} - 2y + 16 = 0$

$$15y + 12 - 4y + 32 = 0$$
$$11y + 44 = 0$$
$$y = -\frac{44}{11} = -4$$

So, $x = \frac{5y + 4}{2} = \frac{5(-4) + 4}{2}$

$$= \frac{-20 + 4}{2} = -8$$

14. For unique solution, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

i.e. $\frac{6}{k} \neq \frac{2}{1}$

$$k \neq 3$$

For infinitely many solutions, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$$\frac{6}{k} = \frac{2}{1} = \frac{3}{2}$$

Clearly $\frac{2}{1} \neq \frac{3}{2}$. So, there does not exist any values of k for which the system of equations has infinitely many solutions.

15. $99x + 101y = 499$ ...(i)

$101x + 99y = 501$ ...(ii)

On subtracting (i) from (ii), we get

$$2x - 2y = 2$$

$x - y = 1$ ...(iii)

On adding (i) and (ii), we get

$$200x + 200y = 1000$$

$x + y = 5$ ...(iv)

On adding (iii) and (iv), we get

$$2x = 6$$
$$x = 3$$

From (iv), $y = 5 - x$

$$= 5 - 3$$
$$= 2$$

16. The system of equations has infinite solutions if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

i.e. $\frac{1}{k+1} = \frac{k+1}{9} = \frac{5}{8k-1}$

$$\frac{1}{k+1} = \frac{k+1}{9}; \quad \frac{k+1}{9} = \frac{5}{8k-1}$$

$(k + 1)^2 = 9$

$k + 1 = \underline{+}3$

$k = -4, 2$

$8k^2 + 8k - k - 1 = 45$

$8k^2 + 7k - 46 = 0$

$$k = \frac{-7 \pm \sqrt{49+1472}}{16}$$

$$= \frac{-7 \pm \sqrt{1521}}{16}$$

$$= \frac{-7 \quad 39}{16}$$

$$= \frac{-46}{16}, \frac{32}{16}$$

$$= \frac{-23}{8}, 2$$

So, we get k = 2.

17. Let the numerator be x and denominator be y.

So, fraction $= \frac{x}{y}$

According to question,

$$\frac{x+1}{y+1} = \frac{7}{8}$$

$$8x + 8 = 7y + 7$$

$$8x - 7y = -1 \qquad \text{...(i)}$$

Again, $\frac{x-1}{y-1} = \frac{6}{7}$

$$7x - 7 = 6y - 6$$

$$7x - 6y = 1 \qquad \text{...(ii)}$$

On multiplying (i) by 7 and (ii) by 8, we get,

$$56x - 49y = -7$$

$$56x - 48y = 8$$

$$- \quad + \quad -$$

$$-y = -15$$

$$y = 15$$

From (i), $8x - 7(15) = -1$

$$8x = -1 + 105 = 104$$

$$x = \frac{104}{8} = 13$$

18. The system of equations has infinitely many solutions if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{3}{a+b} = \frac{4}{2(a-b)} = \frac{12}{5a-1}$$

$$\frac{3}{a+b} = \frac{4}{2(a-b)}$$

$$\frac{3}{a+b} = \frac{2}{a-b}$$

$3a - 3b = 2a + 2b$

$a = 5b \qquad$ ...(i)

$$\frac{4}{2(a-b)} = \frac{12}{5a-1}$$

$$\frac{4}{2(a-b)} = \frac{12}{5a-1}$$

$5a - 1 = 6(a - b)$

$5a - 1 = 6a - 6b$

$-a + 6b = 1 \qquad$ ...(ii)

From (i) and (ii),

$a = 5b$ and $-a + 6b = 1$

So, we get $-5b + 6b = 1, b = 1$

$\therefore \quad a = 5b = 5(1) = 5$

19. $2x - 3y + 6 = 0 \qquad$ ...(i)

$4x - 5y + 2 = 0 \qquad$ ...(ii)

Here, $\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$

$\frac{b_1}{b_2} = \frac{-3}{-5} = \frac{3}{5}$

$\frac{c_1}{c_2} = \frac{6}{2} = 3$

As $\frac{a_1}{a_2} \quad \frac{b_1}{b_2}$, so, the system has a unique solution.

On multiplying (i) by 2 and subtracting (ii) from (i), we get

$$4x - 6y + 12 = 0$$

$$4x - 5y + 2 = 0$$

$$- \quad + \quad -$$

$$-y = -10$$

$$y = 10$$

From (i), $2x - 3(10) + 6 = 0$

$$2x - 24 = 0$$

$$x = 12$$

20. $\frac{x}{10} + \frac{y}{5} + 1 = 15$

$\frac{x}{10} + \frac{y}{5} = 14$

$2x + y = 140 \qquad$ ...(i)

Again, $\frac{x}{8}+\frac{y}{6} = 15$

$$\frac{3x+4y}{24} = 15$$

$3x + 4y = 360$ ...(ii)

From (i), $y = 140 - 2x$

On putting this value of y in (ii), we get

$$3x + 4(140 - 2x) = 360$$

$$3x + 560 - 8x = 360$$

$$-5x = -200$$

$$x = 40$$

So, $y = 140 - 2x$

$$= 140 - 2(40)$$

$$= 140 - 80$$

$$= 60$$

SECTION-C

21. Let the fixed charge be ₹x and cost of food per day be ₹y.

According to question,

$x + 20y = 3000$ ...(i)

$x + 25y = 3500$ ...(ii)

$-\quad -\quad\quad -$

$-5y = -500$

$y = 100$

From (i), we get $x = 3000 - 20(100)$

$$= 3000 - 2000$$

$$= 1000$$

So, fixed charge = ₹1000

Cost of food per day= ₹100

22. $x + y = 1$

$2x - 3y = 11$

According to cross multiplication method,

$$\frac{x}{-11-3} = \frac{y}{-2+11} = \frac{1}{-3-2}$$

$$\frac{x}{-14} = \frac{y}{9} = \frac{1}{-5}$$

$$\frac{x}{-14} = \frac{1}{-5}, \quad \frac{y}{9} = \frac{1}{-5}$$

$$x = \frac{14}{5}, \quad y = \frac{-9}{5}$$

23. (i) $5x + 6y = 15$

As $\frac{4}{5} \neq \frac{-5}{6}\left(\frac{a_1}{a_2} \neq \frac{b_1}{b_2}\right)$

(ii) $8x - 10y = 30$

As $\frac{4}{8} = \frac{-5}{-10} \quad \frac{10}{30} \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \quad \frac{c_1}{c_2}$

(iii) $8x - 10y = 20$

As $\frac{4}{8} = \frac{-5}{-10} = \frac{10}{20} \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

24. $2x - (a - 4)y - (2b + 1) = 0$

$4x - (a - 1)y - (5b - 1) = 0$

The system of equations has infinite solutions if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

i.e. $\frac{2}{4} = \frac{a-4}{a-1} = \frac{2b+1}{5b-1}$

$$\frac{1}{2} = \frac{a-4}{a-1}, \quad \frac{1}{2} = \frac{2b+1}{5b-1}$$

$$a - 1 = 2a - 8, \quad 5b - 1 = 4b + 2$$

$a = 7, \quad b = 3$

So, $a = 7, \ b = 3$

25. Let $\frac{1}{x-1} = p$ and $\frac{1}{y-2} = q$

So, equations become

$5p + q = 2$ ...(i)

$6p - 3q = 1$ ...(ii)

On multiplying (i) by 3 and adding equation (i) and (ii), we get

$$15p + 3q = 6$$
$$6p - 3q = 1$$
$$21p = 7$$
$$p = \frac{1}{3}$$

$\therefore \quad x - 1 = 3 \Rightarrow x = 4$

From eq^n (i), $q = 2 - 5p$

$$= 2 - 5\ \frac{1}{3}$$
$$= \frac{1}{3}$$

$\therefore \quad y - 2 = 3 \Rightarrow y = 5$

26. Let $\frac{1}{x} = p$ and $\frac{1}{y} = q$.

So, equations become

$$p - 4q = 2$$
$$p + 3q = 9$$
$$- \quad - \quad -$$
$$-7q = -7$$
$$q = 1$$

From eq^n, $p - 4q = 2$, we get

$$p = 2 + 4(1)$$
$$= 6$$

So, $x = \frac{1}{6}$, $y = 1$

27. Let father's age be x years and son's age be y years.

According to question,

$$2y + x = 70 \quad \text{...(i)}$$
$$2x + y = 95 \quad \text{...(ii)}$$

From (i), $x = 70 - 2y$

On putting value of x in (ii), we get

$$2(70 - 2y) + y = 95$$
$$140 - 4y + y = 95$$
$$3y = 45$$
$$y = 15$$

So, $x = 70 - 2y$
$$= 70 - 2(15)$$
$$= 70 - 30$$
$$= 40$$

So, age of father = 40 years

age of son = 15 years

28. Let speed of train be x km/hr and speed of car be y km/hr.

According to question,

$$\frac{160}{x} + \frac{600}{y} = 8$$
$$\frac{240}{x} + \frac{520}{y} = \frac{41}{5}$$

(as 8 hours +12 min = 8 + (12/60) = 41/5)

Let $\frac{1}{x} = p$, $\frac{1}{y} = q$

So, we get equations as

$$160p + 600q = 8 \quad \text{...(i)}$$
$$1200p + 2600q = 41 \quad \text{...(ii)}$$

On multiplying (i) by 30 and (ii) by 4, we get

$$4800p + 18000q = 240$$
$$4800p + 10400q = 164$$
$$- \quad - \quad -$$
$$7600q = 76$$
$$q = \frac{76}{7600} = \frac{1}{100}$$

i.e. $y = 100$

From (i), we get

$$160p + 600\ \frac{1}{100} = 8$$
$$160p + 6 = 8$$
$$160p = 2$$
$$p = \frac{1}{80}$$

i.e. $x = 80$

So, $\quad$ Speed of train $= 80$ km/hr

Speed of car $= 100$ km/hr

29. Let time taken by one man alone be x days.
Let time taken by one boy alone be y days.
According to question,

$$\frac{8}{x}+\frac{12}{y} = \frac{1}{10}$$

$$\frac{6}{x}+\frac{8}{y} = \frac{1}{14}$$

Let $\frac{1}{x} = p$, and $\frac{1}{y} = q$

So, we get equations as

$$8p + 12q = \frac{1}{10}$$

$$80p + 120q = 1 \qquad ...(i)$$

Another equation becomes,

$$6p + 8q = \frac{1}{14}$$

$$84p + 112q = 1 \qquad ...(ii)$$

On multiplying (i) by 21 and (ii) by 20, we get

$$1680p + 2520q = 21$$

$$\underline{\underset{-}{}1680p \underset{-}{+} 2240q = \underset{-}{20}}$$

$$280q = 1$$

$$q = \frac{1}{280}$$

So, $\quad y = 280$

From (i),

$$80p + 120\left(\frac{1}{280}\right) = 1$$

$$80p + \frac{3}{7} = 1$$

$$80p = 1 - \frac{3}{7} = \frac{4}{7}$$

$$p = \frac{1}{140}$$

So, $\quad x = 140$

$\therefore$ A man can complete the work in 140 days and a boy can complete the work in 280 days.

30. Let father's age = x years
Sum of ages of 2 children = y years
According to question,

$$x = 2y \qquad ...(i)$$

and

$$x + 20 = y + 20 + 20$$

$$x - y = 20 \qquad ...(ii)$$

On putting (i) in (ii), we get

$$2y - y = 20$$

$$y = 20$$

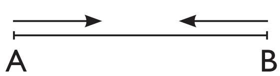
$$\therefore \quad x = 2y = 40$$

So, $\quad$ father's age $= 40$ years

SECTION-D

31. Let, Speed of car A = x km/hr
Speed of car B = y km/hr

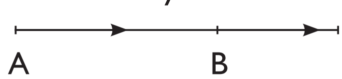
According to question,



$$\frac{4}{3}x + \frac{4}{3}y = 80$$

[1 hour 20 minutes = 1 + (20/60) = 4/3]

$$x + y = 60 \qquad ...(i)$$



$$8x - 8y = 80$$

$$x - y = 10 \qquad ...(ii)$$

On adding (i) and (ii), we get

$$2x = 70$$

$$x = 35$$

From (i), $\quad y = 60 - x$

$$= 60 - 35$$

$$= 25$$

So, Speed of car A = 35 km/hr

Speed of car B = 25 km/hr

32. Let cost of one chair be ₹ x and cost of one table be ₹ y.

According to question,

$$4x + 3y = 2100 \quad ...(i)$$

$$5x + 2y = 1750 \quad ...(ii)$$

On multiplying eqn (i) by 5 and (ii) by 4, we get

$$20x + 15y = 10500$$

$$\underset{-}{20x} \underset{-}{+ 8y} = \underset{-}{7000}$$

$$7y = 3500$$

$$y = \frac{3500}{7} = 500$$

From (i), $4x + 3(500) = 2100$

$$4x = 2100 - 1500$$

$$4x = 600$$

$$x = 150$$

Cost of one chair = ₹ 150

Cost of one table = ₹ 500

Therefore,

Cost of five chairs = 5 × 150

= ₹ 750

Cost of eight tables = 8 × 500

= ₹ 4000

33. Let father's age = x years

Son's age = y years

According to question,

$$x - 10 = 12(y - 10)$$

i.e. $x - 12y = -110 \quad ...(i)$

For another eqn,

$$x + 10 = 2(y + 10)$$

$$x - 2y = 10 \quad ...(ii)$$

On subtracting eqn (ii) from (i), we get

$$x - 12y = -110$$

$$\underset{-}{x} \underset{+}{- 2y} = \underset{-}{10}$$

$$-10y = -120$$

$$y = 12$$

From (ii), $x = 10 + 2y$

$= 10 + 24$

$= 34$

So, Father's age = 34 years

Son's age = 12 years

34. Perimeter of ABCDE = 21 cm

i.e. AB + BC + CD + DE + AE = 21

$$3 + x - y + x + y + x - y + 3 = 21$$

$$3x - y = 15 \quad ...(i)$$

As BE || CD and BC || DE,

BCDE is a parallelogram

∴ BE = CD (opposite sides of parallelogram)

i.e. $x + y = 5 \quad ...(ii)$

On adding equations (i) and (ii), we get

$$4x = 20$$

$$x = 5$$

from (i), $3(5) - y = 15$

$$y = 0$$

So, BC = x – y = 5 – 0 = 5 cm

CD = x + y = 5 + 0 = 5 cm

DE = x – y = 5 – 0 = 5 cm

BE = 5 cm

So, perimeter of quadrilateral BCDE

= 4 × 5 (perimeter = 4 × side)

= 20 cm

35. Let $\frac{1}{x} = p$ and $\frac{1}{y} = q$.

So, equations become

$$ap - bq = 0$$

$$ab^2p + a^2bq = a^2 + b^2$$

$$\frac{p}{a^2b + b^3 - 0} = \frac{q}{0 + a^3 + ab^2} = \frac{1}{a^3b + ab^3}$$

$$\frac{p}{b(a^2 + b^2)} = \frac{q}{a(a^2 + b^2)} = \frac{1}{ab(a^2 + b^2)}$$

$$\frac{p}{b(a^2 + b^2)} = \frac{1}{ab(a^2 + b^2)},$$

$$\frac{q}{a(a^2 + b^2)} = \frac{1}{ab(a^2 + b^2)}$$

$p = \frac{1}{a}$ $\qquad$ $q = \frac{1}{b}$

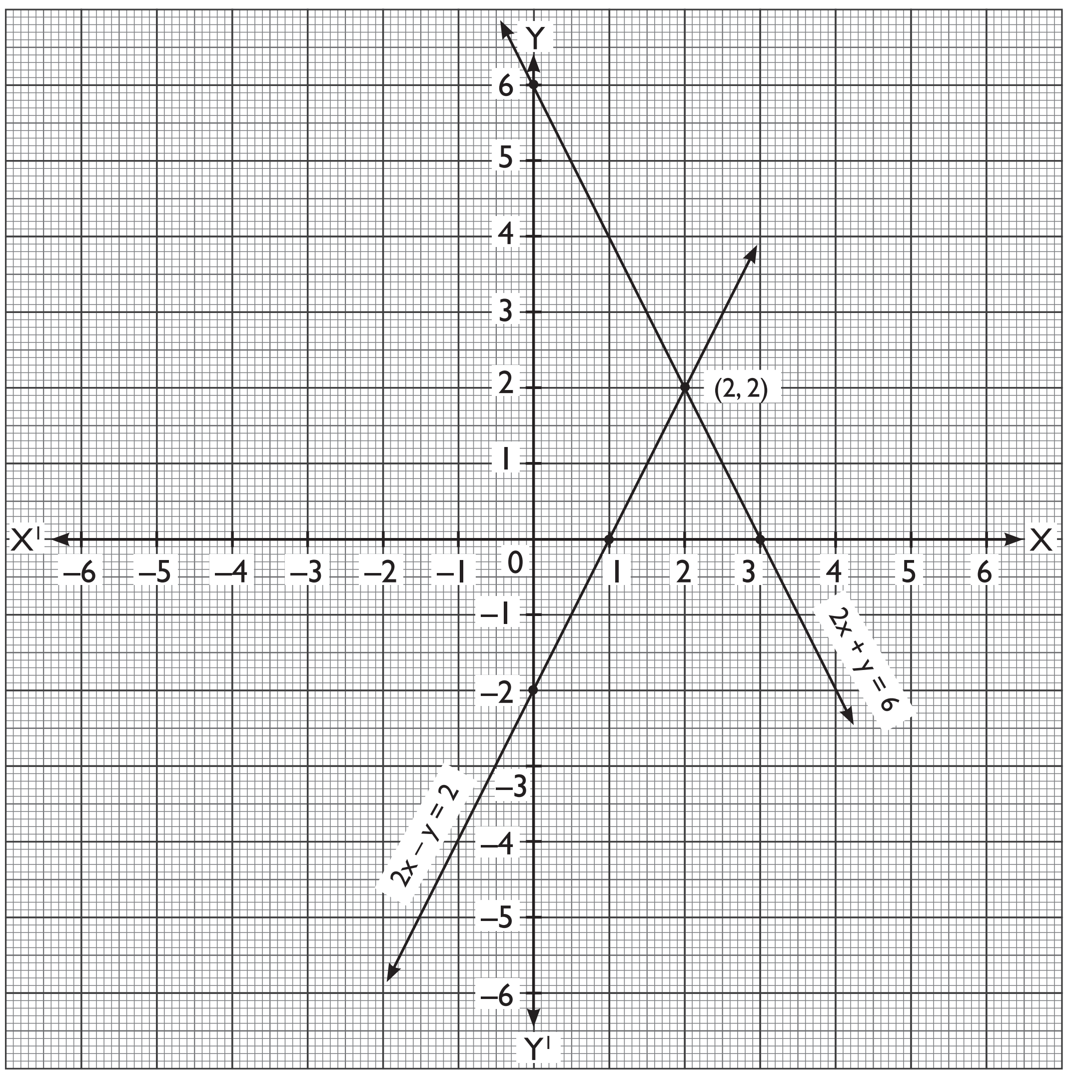
$\therefore\ x = a$ $\qquad$ $y = b$

36. $2x + y = 6$

x	3	0
y	0	6

$2x - y = 2$

x	0	1
y	–2	0



As the equations intersect at point (2, 2), so, (2, 2) is a solution of given set of equations.

Area of triangle formed by lines representing these equations with the x – axis $= \frac{1}{2} \times 2 \times 2$
= 2 sq units.

Area of triangle formed by lines representing these equations with the y – axis $= \frac{1}{2} \times 8 \times 2$
= 8 sq units.

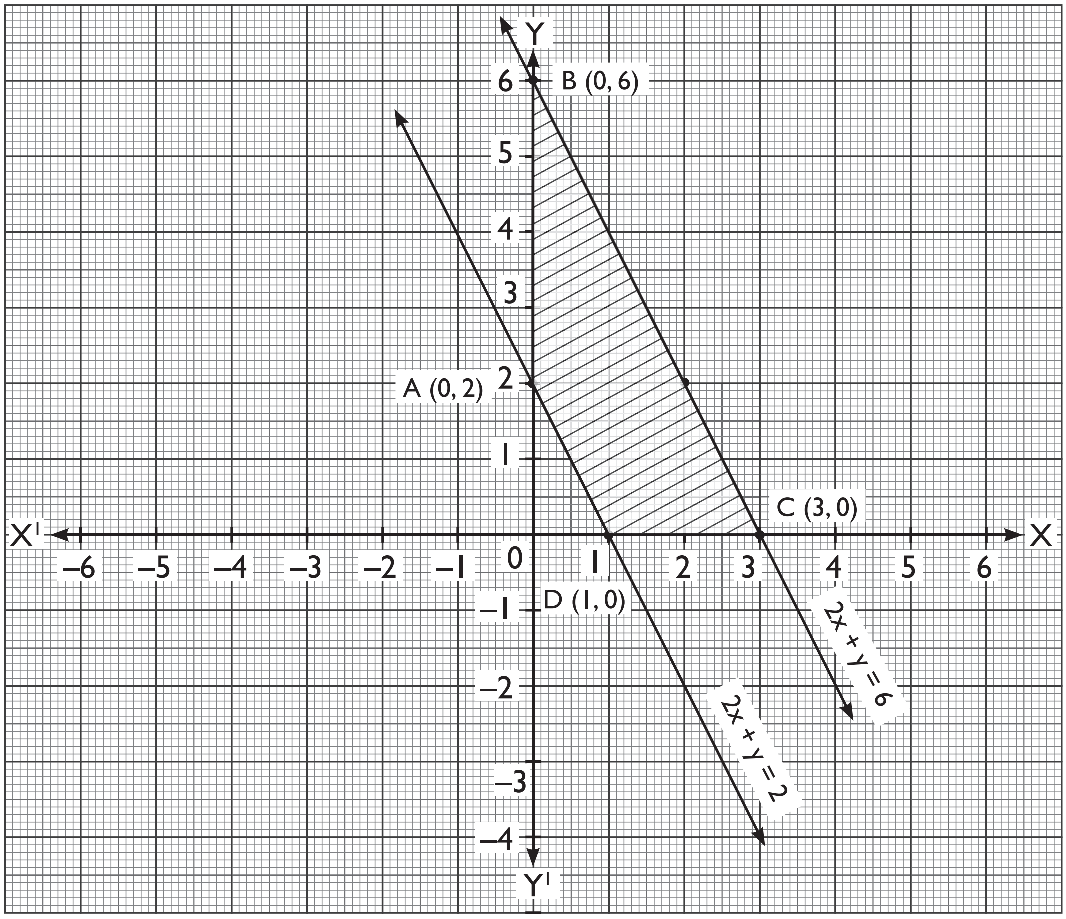
So, Ratio $= \frac{2}{8} = \frac{1}{4}$.

37. $2x + y = 2$

x	0	1
y	2	0

$2x + y = 6$

x	0	3
y	6	0



Vertices of trapezium are A(0, 2), B(0, 6), C(3, 0) and D(1, 0).

Area of trapezium ABCD = area of ΔBOC – area of ΔAOD

$$= \frac{1}{2} \times 3 \times 6 - \frac{1}{2} \times 1 \times 2$$
$$= 9 - 1$$
$$= 8 \text{ sq. units}$$

38. Let the numerator be x and denominator be y.
According to question,

$$y = 5 + 2x$$
$$-2x + y = 5 \qquad \text{...(i)}$$

For the other equation,

$$\frac{x-1}{y-1} = \frac{3}{8}$$
$$8x - 8 = 3y - 3$$
$$8x - 3y = 5 \qquad \text{...(ii)}$$

From (i), $y = 5 + 2x$

On putting this value of y in (ii), we get

$$8x - 3(5 + 2x) = 5$$
$$8x - 15 - 6x = 5$$
$$2x = 20$$
$$x = 10$$

So, $y = 5 + 2(10)$
$= 25$

So, Fraction $= \frac{x}{y} = \frac{10}{25}$

39. $mx - ny = m^2 + n^2$

$x + y = 2m$

$$\frac{x}{2mn + m^2 + n^2} = \frac{y}{-m^2 - n^2 + 2m^2} = \frac{1}{m+n}$$

$$\frac{x}{(m+n)^2} = \frac{y}{m^2 - n^2} = \frac{1}{m+n}$$

$$\frac{x}{(m+n)^2} = \frac{1}{m+n} \qquad \frac{y}{m^2 - n^2} = \frac{1}{m+n}$$

$$x = \frac{(m+n)^2}{m+n} = m + n \qquad y = \frac{m^2 - n^2}{m+n} = m - n$$

40. $(a - b)x + (a + b)y = a^2 - 2ab - b^2$

$(a + b)x + (a + b)y = a^2 + b^2$

$$\frac{x}{(a+b)(-a^2-b^2)-(a+b)(-a^2+2ab+b^2)} = \frac{y}{(a+b)(-a^2+2ab+b^2)-(a-b)(-a^2-b^2)} = \frac{1}{(a-b)(a+b)-(a+b)^2}$$

$$\frac{x}{-a^3-ab^2-a^2b-b^3+a^3-2a^2b-ab^2+a^2b-2ab^2-b^3} = \frac{y}{-a^3+2a^2b+ab^2-a^2b+2ab^2+b^3+a^3+ab^2-a^2b-b^3} = \frac{1}{a^2-b^2-a^2-b^2-2ab}$$

$$\frac{x}{-2b^3-2a^2b-4ab^2} = \frac{y}{4ab^2} = \frac{1}{-2b^2-2ab}$$

$$x = \frac{-2b^3-2a^2b-4ab^2}{-2b^2-2ab}$$

$$= \frac{-2b(b^2+a^2+2ab)}{-2b(b+a)}$$

$$= a + b$$

Also, $\frac{y}{4ab^2} = \frac{1}{-2b(a+b)}$

$$y = \frac{-2ab}{a+b}$$

CASE STUDY-1

(i) (b) The sum of number of students who took part in Quiz is b. The number of boys are represented by x and number of girls are represented by y.
$x + y = 10$

(ii) (c) The difference between the number of girls and number of boys is 4.
$x - y = 4$

(iii) (d)
$$\begin{aligned} x + y &= 10 \\ x - y &= 4 \\ \hline 2x &= 14 \\ x &= 7 \\ 7 + y &= 10 \\ y &= 3 \end{aligned}$$

∴ The solution of given pair of equation is (7, 3).

(iv) (a) Linear equations have unique solutions.

(v) (c) Area of $\Delta ABC = \frac{1}{2}$ (base (height)

The base is AB which is having lengths of 6 units.

The height is measured from x axis to point C. The ordinate of point C represents the height of ΔABC.

Height = 3 units

Area of $\Delta ABC = \frac{1}{2}$ (6) (3) = 9 sq units.

CASE STUDY-2

(i) (b) $x - 10 = y + 10$

$x - y = 20$

(ii) (b) $y + 20 = x$

(iii) (c)

$$\begin{aligned} x - y &= 20 \\ x + y &= 220 \\ \hline 2x &= 240 \\ x &= 120 \\ x - y &= 20 \\ 120 - y &= 20 \\ y &= 100 \end{aligned}$$

(iv) (c)

$$\begin{aligned} x - y &= 20 \\ x + y &= 220 \\ \hline 2x &= 240 \\ x &= 120 \\ y &= 100 \end{aligned}$$

The solution are x = 0 and y = b

a = 120

b = 100

$\therefore$ a > b

(v) (d) The line x = 120 lies parallel to y axis and line x = 100 lies parallel to x axis.

Thus both lines are intersecting.

Chapter 4

Quadratic Equations

Multiple Choice Questions

1. (b) As $x = -\frac{1}{2}$ is a solution of $3x^2 + 2kx - 3 = 0$

$$\therefore \quad 3\left(-\frac{1}{2}\right)^2 + 2k\left(-\frac{1}{2}\right) - 3 = 0$$

$$\frac{3}{4} - k - 3 = 0$$

$$k = \frac{3}{4} - 3$$

$$= \frac{3 - 12}{4}$$

$$= \frac{-9}{4}$$

2. (b) Equation has no real roots if $D < 0$

i.e. $b^2 - 4ac < 0$

i.e. $b^2 - 4(1)(1) < 0$

i.e. $b^2 - 4 < 0$

i.e. $(b + 2)(b - 2) < 0$

i.e. $-2 < b < 2$

3. (d) let α, β be the roots then $\alpha\beta = 3$

$\alpha\beta = 3$

$(1)\beta = 3 \quad (\because \alpha = 1)$

$\beta = 3$

4. (a) $3\sqrt{3}x^2 + 10x + \sqrt{3} = 0$

$D = b^2 - 4ac$

$= (10)^2 - 4(3\sqrt{3} \times \sqrt{3})$

$= 100 - 36$

$= 64$

5. (a) $x^2 - px + q = 0$

As p is the root

$\therefore \quad p^2 - p^2 + q = 0$

$\Rightarrow \quad q = 0$

Also, q is a root

$\therefore \quad q^2 - pq + q = 0$

$q(q - p + 1) = 0$

$q = 0$ or $q = p - 1$

$\therefore \quad q = p - 1$

$\Rightarrow \quad p = q + 1$

$= 0 + 1$

$= 1$

So, $p = 1, q = 0$

WORKSHEET - 1

SECTION-A

1. $x^2 - 7x + 12$

$x^2 - 3x - 4x + 12$

$x(x - 3) - 4(x - 3)$

$(x - 3)(x - 4)$

2. $2x^2 + 3x - 4 = 0$

$b^2 - 4ac = 9 - 4(2)(-4)$

$= 9 + 32$

$= 41 > 0$

As $b^2 - 4ac > 0$

$\Rightarrow$ The equation has real and distinct roots.

3. $3x^2 + 13x + 14 = 0$

LHS $= 3x^2 + 13x + 14$

$= 3(-2)^2 + 13(-2) + 14 \quad$ (Put $x = -2$)

$= 12 - 26 + 14$

$= 0$

$= \text{RHS}$

So, $x = -2$ is a root of $3x^2 + 13x + 4 = 0$

4. $x^2 - 3x - 1 = 0$

$\text{LHS} = x^2 - 3x - 1$

$= 1^2 - 3(1) - 1$ (Put $x = 1$)

$= 1 - 3 - 1$

$= -3 \neq \text{RHS} (= 0)$

So, $x = 1$ is not a solution of equation $x^2 - 3x - 1 = 0$

5. $x^2 - 3x - 10 = 0$

$D = b^2 - 4ac$

$= (-3)^2 - 4(1)(-10)$

$= 9 + 40$

$= 49$

6. Let $\sqrt{6+\sqrt{6+\sqrt{6+\ldots}}} = x$

$\sqrt{6+x} = x$

On squaring both sides, we get

$$6 + x = x^2$$

$$x^2 - x - 6 = 0$$

$$x^2 - 3x + 2x - 6 = 0$$

$$x(x - 3) + 2(x - 3) = 0$$

$$(x - 3)(x + 2) = 0$$

$$x = 3, x = -2$$

As value of $\sqrt{6+\sqrt{6+\sqrt{6+\ldots}}}$ cannot be negative, so, $x = 3$

7. $3x^2 - kx + 38 = 0$

The quadratic equation has equal roots

if $D = 0$

i.e. $b^2 - 4ac = 0$

i.e. $k^2 - 4(3)(38) = 0$

$k^2 - 456 = 0$

$k^2 = 456$

$k = \pm 2\sqrt{114}$

8. $bx^2 - 2\sqrt{ac}\,x + b = 0$

The equation has equal roots if discriminant $= 0$

i.e. $\left(2\sqrt{ac}\right)^2 - 4(b)(b) = 0$

$4ac - 4b^2 = 0$

$b^2 = ac$

SECTION-B

9. $16x^2 - 24x - 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-24) \pm \sqrt{(-24)^2 - 4(16)(-1)}}{2(16)}$$

$$= \frac{24 \pm \sqrt{576 + 64}}{32}$$

$$= \frac{24 \pm \sqrt{640}}{32}$$

$$= \frac{24 \pm 8\sqrt{10}}{32}$$

$$= \frac{3 \pm \sqrt{10}}{4}$$

10. $\frac{1}{x-2} + \frac{2}{x-1} = \frac{6}{x}$

$$\frac{x - 1 + 2x - 4}{(x - 1)(x - 2)} = \frac{6}{x}$$

$$x(3x - 5) = 6(x - 1)(x - 2)$$

$$3x^2 - 5x = 6(x^2 - 3x + 2)$$

$$3x^2 - 5x = 6x^2 - 18x + 12$$

$$0 = 3x^2 - 13x + 12$$

$$0 = 3x^2 - 9x - 4x + 12$$

$$0 = 3x(x - 3) - 4(x - 3)$$

$$0 = (3x - 4)(x - 3)$$

$3x - 4 = 0, \quad \text{or} \quad x - 3 = 0$

$x = \frac{4}{3}, \quad \text{or} \quad x = 3$

11. $x^2 - 2ax + a^2 - b^2 = 0$

$x^2 + [(-a - b) + (-a + b)]x + (a + b)(a - b) = 0$

$x^2 - (a + b)x - (a - b)x + (a + b)(a - b) = 0$

$x[x - (a + b)] - (a - b)[x - (a + b)] = 0$

$[x - (a - b)][x - (a + b)] = 0$

$x - (a - b) = 0 \quad \text{or} \quad x - (a + b) = 0$

$x = a - b \quad \text{or} \quad x = a + b$

12. $4x^2 - 4a^2x + (a^4 - b^4) = 0$

$4x^2 + [-2(a^2 - b^2) - 2(a^2 + b^2)]x + (a^4 - b^4) = 0$

$4x^2 - 2(a^2 - b^2)x - 2(a^2 + b^2)x + (a^2 - b^2)(a^2 + b^2) = 0$

$2x[2x - a^2 - b^2] - (a^2 + b^2)(2x - a^2 - b^2) = 0$

$[2x - (a^2 - b^2)][2x - (a^2 + b^2)] = 0$

$2x - (a^2 - b^2) = 0 \quad \text{or} \quad 2x - (a^2 + b^2) = 0$

$x = \frac{-b^2 + a^2}{2} \quad \text{or} \quad x = \frac{a^2 + b^2}{2}$

13. $(k - 12)x^2 + 2(k - 12)x + 2 = 0$

The equation has equal roots if discriminant D = 0

i.e. $b^2 - 4ac = 0$

$4(k - 12)^2 - 4(k - 12)(2) = 0$

$(k - 12)[4(k - 12) - (4)(2)] = 0$

$(k - 12)(4k - 48 - 8) = 0$

$(k - 12)(4k - 56) = 0$

$(k - 12)4(k - 14) = 0$

$k = 12, 14$

14. $12abx^2 - (9a^2 - 8b^2)x - 6ab = 0$

$D = (9a^2 - 8b^2)^2 - 4(12ab)(-6ab)$

$= (9a^2 - 8b^2)^2 + 288a^2b^2$

$= 81a^4 + 64b^4 - 144a^2b^2 + 288a^2b^2$

$= 81a^4 + 64b^4 + 144a^2b^2$

$= (9a^2 + 8b^2)^2$

$x = \frac{-b \pm \sqrt{D}}{2a}$

$= \frac{(9a^2 - 8b^2) \pm (9a^2 + 8b^2)}{24ab}$

$x = \frac{18a^2}{24ab} \quad \text{or} \quad x = \frac{-16b^2}{24ab}$

$x = \frac{3a}{4b} \quad \text{or} \quad x = \frac{-2b}{3a}$

15. Let the two numbers be x and 16 – x.

According to question,

$\frac{1}{x} + \frac{1}{16 - x} = \frac{1}{3}$

$\frac{16 - x + x}{x(16 - x)} = \frac{1}{3}$

$48 = 16x - x^2$

$x^2 - 16x + 48 = 0$

$x^2 - 12x - 4x + 48 = 0$

$x(x - 12) - 4(x - 12) = 0$

$(x - 4)(x - 12) = 0$

$x = 4, 12$

If $x = 4$, Other number = 16 – 4 = 12

if $x = 12$, Other number = 16 – 12 = 4

16. $x + \frac{1}{x} = 11\frac{1}{11}$

$\frac{x^2 + 1}{x} = \frac{122}{11}$

$11(x^2 + 1) = 122x$

$11x^2 - 122x + 11 = 0$

$11x^2 - x - 121x + 11 = 0$

$x(11x - 1) - 11(11x - 1) = 0$

$(11x - 1)(x - 11) = 0$

$11x - 1 = 0 \quad \text{or} \quad x - 11 = 0$

$x = \frac{1}{11} \quad \text{or} \quad x = 11$

SECTION-C

17. Let D_1 and D_2 be the discriminants of equations $x^2 + 2cx + ab = 0$ and $x^2 - 2(a + b) x + a^2 + b^2 + 2c^2 = 0$ respectively.

$x^2 + 2cx + ab = 0$

$D_1 = (2c)^2 - 4(1)(ab)$

$= 4c^2 - 4ab$

$= 4(c^2 - ab)$

As roots are real and unequal,

so $D_1 > 0$

$c^2 - ab > 0$ ...(i)

$x^2 - 2(a + b)x + a^2 + b^2 + 2c^2 = 0$

$D_2 = 4(a + b)^2 - 4(1)(a^2 + b^2 + 2c^2)$

$= 8ab - 8c^2$

$= -8(c^2 - ab) < 0$ [From (i)]

So, the given equation has no real roots.

18. $$\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$$

$$\frac{x-7-x-4}{(x+4)(x-7)} = \frac{11}{30}$$

$$\frac{-11}{x^2-3x-28} = \frac{11}{30}$$

$$\frac{-1}{x^2-3x-28} = \frac{1}{30}$$

$$x^2 - 3x - 28 + 30 = 0$$

$$x^2 - 3x + 2 = 0$$

$$x^2 - 2x - x + 2 = 0$$

$$x(x - 2) - 1(x - 2) = 0$$

$$(x - 1)(x - 2) = 0$$

$$x = 1, 2$$

19. Let the smaller side and larger side be x cm and y cm respectively.

Hypotenuse $= 3\sqrt{5}$ cm

So, $x^2 + y^2 = \left(3\sqrt{5}\right)^2$

$x^2 + y^2 = 45$ ...(i)

If smaller side is tripled and the larger side is doubled,

$$(3x)^2 + (2y)^2 = (15)^2$$

$9x^2 + 4y^2 = 225$ ...(ii)

From (i), $x^2 = 45 - y^2$

So, we get $9(45 - y^2) + 4y^2 = 225$

$$405 - 9y^2 + 4y^2 = 225$$

$$5y^2 = 180$$

$$y^2 = \frac{180}{5} = 36$$

$$y = \pm 6$$

For $y = -6$, $x^2 = 45 - 36 = 9$

$x = \pm 3$

For $y = 6$, $x^2 = 45 - 36 = 9$

$x = \pm 3$

As length cannot be negative,

So, $y = -6$, $x = -3$ rejected

$\therefore$ $x = 3$, $y = 6$

Length of smaller side = 3 cm

Length of larger side = 6 cm

20. As $x = -2$ is a root of equation $3x^2 + 7x + p = 0$, we have

$$3(-2)^2 + 7(-2) + p = 0$$

$$12 - 14 + p = 0$$

$$p = 2$$

$$x^2 + k(4x + k - 1) + p = 0$$

$x^2 + k(4x + k - 1) + 2 = 0$ (Put p = 2)

$$x^2 + (4k)x + k^2 - k + 2 = 0$$

As roots are equal,

Discriminant (D) = 0

$$(4k)^2 - 4(k^2 - k + 2) = 0$$

$$16k^2 - 4k^2 + 4k - 8 = 0$$

$$12k^2 + 4k - 8 = 0$$

$$3k^2 + k - 2 = 0$$
$$3k^2 + 3k - 2k - 2 = 0$$
$$3k(k + 1) - 2(k + 1) = 0$$
$$(3k - 2)(k + 1) = 0$$
$$3k - 2 = 0 \quad \text{or} \quad k + 1 = 0$$
$$k = \frac{2}{3} \quad \text{or} \quad k = -1$$

21. $x^2(a^2 + b^2) + 2(ac + bd)x + (c^2 + d^2) = 0$

Consider, Discriminant (D)

$= 4(ac + bd)^2 - 4(a^2 + b^2)(c^2 + d^2)$

$= 4(a^2c^2 + b^2d^2 + 2abcd) - 4(a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2)$

$= 8abcd - 4a^2d^2 - 4b^2c^2$

$= -4[(ad)^2 + (bc)^2 - 2abcd)$

$= -4(ad - bc)^2$

< 0

For no real roots, $D < 0$

i.e. $D \neq 0$ i.e. $ad \neq bc$

22. As 2 is a root of the quadratic equation $3x^2 + px - 8 = 0$,

$$3(2)^2 + p(2) - 8 = 0$$
$$12 + 2p - 8 = 0$$
$$2p = -4$$
$$p = -2$$

$\therefore$ Other equation becomes

$$4x^2 - 2(-2)x + k = 0$$
$$4x^2 + 4x + k = 0$$

As roots are equal,

Discriminant (D) $= 0$

i.e. $16 - 4(4)(k) = 0$

$$16 - 16k = 0$$
$$k = 1$$

23. $(x - a)(x - b) + (x - b)(x - c) + (x - c)(x - a) = 0$

$\Rightarrow x^2 - bx - ax + ab + x^2 - cx - bx + bc + x^2 - ax - cx + ac = 0$

$\Rightarrow 3x^2 - 2bx - 2ax - 2cx + ab + bc + ca = 0$

$\Rightarrow 3x^2 - 2(a + b + c)x + (ab + bc + ca) = 0$

Discriminant (D)

$= 4(a + b + c)^2 - 12(ab + bc + ca)$

$= 4(a^2 + b^2 + c^2 + 2ab + 2bc + 2ac - 3ab - 3bc - 3ca)$

$= 4(a^2 + b^2 + c^2 - ab - bc - ca)$

$= 2(2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ac)$

$= 2[(a - b)^2 + (b - c)^2 + (a - c)^2]$

$D = 2[(a - b)^2 + (b - c)^2 + (a - c)^2] \geq 0$

As $D \geq 0$, so roots are real.

Roots are equal if $D = 0$

i.e. $2[(a - b)^2 + (b - c)^2 + (a - c)^2] = 0$

i.e. $a - b = 0$, $b - c = 0$, $a - c = 0$,
$a = b$, $b = c$, $a = c$

i.e. $a = b = c$.

24. Let the two numbers be x and y such that $x > y$.

$$x - y = 3 \quad \text{...(i)}$$

Also, $$\frac{1}{y} - \frac{1}{x} = \frac{3}{28} \quad \text{...(ii)}$$

From (i), $x = 3 + y$

Putting in (ii), we get

$$\frac{1}{y} - \frac{1}{3 + y} = \frac{3}{28}$$
$$\frac{3 + y - y}{y(3 + y)} = \frac{3}{28}$$
$$\frac{3}{y(3 + y)} = \frac{3}{28}$$
$$28 = y^2 + 3y$$
$$y^2 + 3y - 28 = 0$$

$$y^2 + 7y - 4y - 28 = 0$$

$$y(y + 7) - 4(y + 7) = 0$$

$$(y - 4)(y + 7) = 0$$

$$y = 4, -7$$

As y is a natural number,

$y = -7$ is rejected

So, $y = 4$

$\therefore \quad x = 3 + y = 7$

SECTION-D

25. $$\frac{x-2}{x-3} + \frac{x-4}{x-5} = \frac{10}{3}$$

$$\frac{(x-2)(x-5)+(x-3)(x-4)}{(x-3)(x-5)} = \frac{10}{3}$$

$$\frac{x^2-7x+10+x^2-7x+12}{x^2-8x+15} = \frac{10}{3}$$

$$\frac{2x^2-14x+22}{x^2-8x+15} = \frac{10}{3}$$

$$\frac{x^2-7x+11}{x^2-8x+15} = \frac{5}{3}$$

$$3x^2 - 21x + 33 = 5x^2 - 40x + 75$$

$$0 = 2x^2 - 19x + 42$$

$$0 = 2x^2 - 12x - 7x + 42$$

$$0 = 2x(x - 6) - 7(x - 6)$$

$$0 = (2x - 7)(x - 6)$$

$$(2x - 7)(x - 6) = 0$$

$$2x - 7 = 0 \quad \text{or} \quad x - 6 = 0$$

$$x = \frac{7}{2} \quad \text{or} \quad x = 6$$

26. Let speed of stream be x km/hr

Speed of boat in still water = 18 km/hr

So, Speed of boat downstream = (18 + x) km/hr

Speed of boat upstream = (18 – x) km/hr

According to equation,

$$\frac{24}{18-x} = \frac{24}{18+x} + 1 \qquad (\because \text{up} = \text{D} + 1)$$

$$\frac{24}{18-x} - \frac{24}{18+x} = 1$$

$$\frac{18+x-18+x}{(18-x)(18+x)} = \frac{1}{24}$$

$$\frac{2x}{324-x^2} = \frac{1}{24}$$

$$324 - x^2 = 48x$$

$$x^2 + 48x - 324 = 0$$

$$x^2 + 54x - 6x - 324 = 0$$

$$x(x + 54) - 6(x + 54) = 0$$

$$(x - 6)(x + 54) = 0$$

$$x = 6, -54$$

As speed cannot be negative,

$x = -54$ is rejected.

So, $x = 6$

$\therefore$ Speed of stream = 6 km/hr

27. $$3\left(\frac{3x-1}{2x+3}\right) - 2\left(\frac{2x+3}{3x-1}\right) = 5$$

Let $\frac{3x-1}{2x+3} = y$

So, equation becomes

$$3y - \frac{2}{y} = 5$$

$$3y^2 - 2 = 5y$$

$$3y^2 - 5y - 2 = 0$$

$$3y^2 - 6y + y - 2 = 0$$

$$3y(y - 2) + 1(y - 2) = 0$$

$$(3y + 1)(y - 2) = 0$$

$$3y + 1 = 0 \quad \text{or} \quad y - 2 = 0$$

$$y = -\frac{1}{3} \quad \text{or} \quad y = 2$$

$$y = -\frac{1}{3} \qquad\qquad y = 2$$

$$\frac{3x-1}{2x+3} = -\frac{1}{3}$$

$$9x - 3 = -2x - 3$$

$$11x = 0$$

$$x = 0$$

$$\frac{3x-1}{2x+3} = 2$$

$$3x - 1 = 4x + 6$$

$$x = -7$$

28. Let original speed of the aircraft be x km/hr.

∴ New speed = (x – 200) km/hr.

Duration of flight at original speed

$$= \frac{600}{x} \text{ hours}$$

Duration of flight at reduced speed

$$= \frac{600}{x-200} \text{ hours}$$

According to question,

$$\frac{600}{x-200} = \frac{1}{2} + \frac{600}{x}$$

$$\frac{600}{x-200} - \frac{600}{x} = \frac{1}{2}$$

$$\frac{1}{x-200} - \frac{1}{x} = \frac{1}{1200}$$

$$\frac{x-x+200}{x(x-200)} = \frac{1}{1200}$$

$$x^2 - 200x = 240000$$

$$x^2 - 200x - 240000 = 0$$

$$x^2 - 600x + 400x - 240000 = 0$$

$$x(x-600) + 400(x-600) = 0$$

$$(x+400)(x-600) = 0$$

$$x = -400 \text{ or } x = 600$$

As x, being speed of aircraft can't be negative.

So, x = 600

∴ Original speed of aircraft = 600 km/hr

$$\text{Duration of flight} = \frac{600}{600} = 1 \text{ hour}$$

29. Let the usual speed of plane be x km / hr

$$\therefore \quad \text{Time taken} = \frac{1500}{x} \text{ hours}$$

New speed = x + 250 km / hr

$$\therefore \quad \text{Time taken} = \frac{1500}{x+250} \text{ hours}$$

According to question,

$$\frac{1500}{x+250} = \frac{1500}{x} - \frac{1}{2}$$

$$\frac{1500}{x} - \frac{1500}{x+250} = \frac{1}{2}$$

$$\frac{1}{x} - \frac{1}{x+250} = \frac{1}{3000}$$

$$\frac{x+250-x}{x(x+250)} = \frac{1}{3000}$$

$$x^2 + 250x = 750000$$

$$x^2 + 250x - 750000 = 0$$

$$x^2 + 1000x - 750x - 750000 = 0$$

$$x(x+1000) - 750(x+1000) = 0$$

$$(x-750)(x+1000) = 0$$

$$x = 750 \text{ or } x = -1000$$

Now, x being the speed of plane cannot be negative,

x = –1000 is rejected

So, x = 750

∴ Speed of plane = 750 km/hr

30. Let total number of camels be x.

According to question,

$$\frac{1}{4}x + 2\sqrt{x} + 15 = x$$

$$2\sqrt{x} + 15 = x - \frac{x}{4}$$

$$2\sqrt{x} + 15 = \frac{3x}{4}$$

$$8\sqrt{x} + 60 = 3x$$

$$3x - 8\sqrt{x} - 60 = 0$$

$$3\left(\sqrt{x}\right)^2 - 8\sqrt{x} - 60 = 0$$

$$\text{Let } \sqrt{x} = y$$

$$3y^2 - 8y - 60 = 0$$

$$3y^2 - 18y + 10y - 60 = 0$$

$$3y(y - 6) + 10(y - 6) = 0$$

$$(3y + 10)(y - 6) = 0$$

$$y = -\frac{10}{3} \quad \text{or} \quad y = 6$$

Now, $y = -\frac{10}{3}$ is rejected as number of camels can not be negative.

So, $y = 6$

i.e. $\sqrt{x} = 6$

$\therefore$ $x = 36$

So, total number of camels = 36

31. Let Varun's age be x years and Nihal's age be y years.

According to question,

$$x - 7 = 5(y - 7)^2$$

$$x - 7 = 5(y - 7)^2 \qquad \text{...(i)}$$

For second equation,

$$y + 3 = \frac{2}{5}(x + 3)$$

$$5y + 15 = 2x + 6$$

$$2x - 5y = 9 \qquad \text{...(ii)}$$

From (ii), $x = \frac{9 + 5y}{2}$

Putting in (i), we get

$$\frac{9 + 5y}{2} - 7 = 5(y - 7)^2$$

$$9 + 5y - 14 = 10(y^2 + 49 - 14y)$$

$$5y - 5 = 10(y^2 + 49 - 14y)$$

$$y - 1 = 2(y^2 + 49 - 14y)$$

$$y - 1 = 2y^2 + 98 - 28y$$

$$2y^2 - 29y + 99 = 0$$

$$y = \frac{29 \pm \sqrt{841 - 8(99)}}{4}$$

$$y = \frac{29 \pm \sqrt{49}}{4}$$

$$y = \frac{29 \pm 7}{4}$$

$$y = \frac{29 + 7}{4}, \qquad y = \frac{29 - 7}{4}$$

$$y = 9, \qquad y = \frac{11}{2}$$

Now, $y = \frac{11}{2}$ is rejected

So, $y = 9$

$\therefore$ Nihal's age = 9 years

$$\text{Varun's age} = \frac{9 + 5y}{2} = \frac{9 + 45}{2} = 27 \text{ years}$$

32. $$\frac{1}{a + b + x} - \frac{1}{x} = \frac{1}{a} + \frac{1}{b}$$

$$\frac{x - (a + b + x)}{x(a + b + x)} = \frac{a + b}{ab}$$

$$\frac{-(a + b)}{x(a + b + x)} = \frac{a + b}{ab}$$

$$\frac{-1}{x(a + b + x)} = \frac{1}{ab}$$

$$x(a + b + x) + ab = 0$$

$$xa + xb + x^2 + ab = 0$$

$$x^2 + xa + xb + ab = 0$$

$$x(x + a) + b(x + a) = 0$$

$$(x + a)(x + b) = 0$$

$$x = -a \quad \text{or} \quad x = -b$$

WORKSHEET - 2

SECTION-A

1. $$\text{LHS} = x^2 - 3\sqrt{3}\,x + 6$$

$$= (-2\sqrt{3})^2 - 3\sqrt{3}\,(-2\sqrt{3}) + 6$$

$$= 12 + 18 + 6$$
$$= 36$$
$$\neq \text{RHS } (= 0)$$

So, $x = -2\sqrt{3}$ is not a solution of the given equation.

2. As $x = -\frac{1}{2}$ is a solution of $3x^2 + 2kx - 3 = 0$,

$$3\left(-\frac{1}{2}\right)^2 + 2k\left(-\frac{1}{2}\right) - 3 = 0$$
$$\frac{3}{4} - k - 3 = 0$$
$$k = \frac{3}{4} - 3 = \frac{-9}{4}$$

3. Let the two consecutive positive integers be x and x + 1.

According to question,

$$x(x + 1) = 240$$
$$x^2 + x - 240 = 0$$

4.
$$x^2 + 6x + 5 = 0$$
$$x^2 + 5x + x + 5 = 0$$
$$x(x + 5) + 1(x + 5) = 0$$
$$(x + 1)(x + 5) = 0$$
$$x + 1 = 0 \quad \text{or} \quad x + 5 = 0$$
$$x = -1 \quad \text{or} \quad x = -5$$

5.
$$x + \frac{2}{x} = 3$$
$$x^2 + 2 = 3x$$
$$x^2 - 3x + 2 = 0$$
$$x^2 - 2x - x + 2 = 0$$
$$x(x - 2) - 1(x - 2) = 0$$
$$(x - 1)(x - 2) = 0$$
$$x = 1, 2$$

6. $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$

$$\text{Discriminant} = (-2\sqrt{2})^2 - 4(\sqrt{3})(-2\sqrt{3})$$
$$= 8 + 24$$
$$= 32$$

7. $2x^2 + 5\sqrt{3}x + 6 = 0$

$$\text{Discriminant (D)} = (5\sqrt{3})^2 - 4(2)(6)$$
$$= 75 - 48$$
$$= 27 > 0$$

So, the given equation has real roots.

8.
$$abx^2 + (b^2 - ac)x - bc = 0$$
$$abx^2 + b^2x - acx - bc = 0$$
$$bx(ax + b) - c(ax + b) = 0$$
$$(bx - c)(ax + b) = 0$$
$$bx - c = 0 \quad \text{or} \quad ax + b = 0$$
$$x = \frac{c}{b} \quad \text{or} \quad x = -\frac{b}{a}$$

9. $2x^2 - kx + 1 = 0$

As the equation has real and equal roots,

$$\text{Discriminant (D)} = 0$$
$$k^2 - 4(2)(1) = 0$$
$$k^2 = 8$$
$$k = \pm 2\sqrt{2}$$

10.
$$x^2 + \left(a + \frac{1}{a}\right)x + 1 = 0$$
$$ax^2 + (a^2 + 1)x + a = 0$$
$$ax^2 + a^2x + x + a = 0$$
$$ax(x + a) + 1(x + a) = 0$$
$$(ax + 1)(x + a) = 0$$
$$ax + 1 = 0 \quad \text{or} \quad x + a = 0$$
$$x = -\frac{1}{a} \quad \text{or} \quad x = -a$$

SECTION-B

11. As $x = \frac{2}{3}$ is a root of equation

$$ax^2 + 7x + b = 0$$

$$a\left(\frac{2}{3}\right)^2 + 7\left(\frac{2}{3}\right) + b = 0$$

$$\frac{4}{9}a + \frac{14}{3} + b = 0$$

$$\frac{4a + 42 + 9b}{9} = 0$$

$$4a + 9b = -42 \quad ...(i)$$

As $x = -3$ is a root of equation

$$ax^2 + 7x + b = 0$$

$$9a - 21 + b = 0$$

$$9a + b = 21 \quad ...(ii)$$

From (ii), $b = 21 - 9a$

Putting in (i), we get

$$4a + 9(21 - 9a) = -42$$

$$4a + 189 - 81a = -42$$

$$189 + 42 = 81a - 4a$$

$$231 = 77a$$

$$a = 3$$

So, $b = 21 - 9(3)$

$$= 21 - 27$$

$$= -6$$

12. As -5 is a root of equation

$$px^2 + px + k = 0$$

$$p(-5)^2 + p(-5) + k = 0$$

$$25p - 5p + k = 0$$

$$20p + k = 0 \quad ...(i)$$

Also, as equation has equal roots,

$$\text{Discriminant} = 0$$

$$p^2 - 4pk = 0$$

$$p(p - 4k) = 0$$

$$p = 0 \quad \text{or} \quad p = 4k$$

if $p = 0$, $20(0) + k = 0$

$$k = 0$$

if $p = 4k$, $20(4k) + k = 0$

$$k = 0$$

13. $$\sqrt{2x + 9} + x = 13$$

$$\sqrt{2x + 9} = 13 - x$$

Squaring both sides,

$$2x + 9 = 169 + x^2 - 26x$$

$$x^2 - 28x + 160 = 0$$

$$x^2 - 20x - 8x + 160 = 0$$

$$x(x - 20) - 8(x - 20) = 0$$

$$(x - 8)(x - 20) = 0$$

$$x = 8 \quad \text{or} \quad x = 20$$

If $x = 20$

LHS = $\sqrt{40 + 9} + 20 = 27 \neq$ RHS (= 13)

So, $x = 20$ is rejected

If $x = 8$,

$$\text{LHS} = \sqrt{16 + 9} + 8$$

$$= 5 + 8$$

$$= 13$$

$$= \text{RHS}$$

Therefore, $x = 8$

14. $9x^2 - 6b^2x - (a^4 - b^4) = 0$

$9x^2 + [-3(b^2 - a^2) - 3(b^2 + a^2)]x + (-a^4 + b^4) = 0$

$9x^2 - 3(b^2 - a^2)x - 3(b^2 + a^2)x + (a^2 + b^2)(-a^2 + b^2) = 0$

$3x[3x - (b^2 - a^2)] - (a^2 + b^2)[3x - (b^2 - a^2)] = 0$

$[3x - (a^2 + b^2)][3x - (b^2 - a^2)] = 0$

$$x = \frac{a^2 + b^2}{3} \quad \text{or} \quad x = \frac{b^2 - a^2}{3}$$

15.
$$\frac{4}{x} - 3 = \frac{5}{2x+3}$$
$$\frac{4 - 3x}{x} = \frac{5}{2x+3}$$
$$(4 - 3x)(2x + 3) = 5x$$
$$8x + 12 - 6x^2 - 9x = 5x$$
$$6x^2 + 6x - 12 = 0$$
$$x^2 + x - 2 = 0$$
$$x^2 + 2x - x - 2 = 0$$
$$x(x + 2) - 1(x + 2) = 0$$
$$(x - 1)(x + 2) = 0$$
$$x = 1, -2$$

16.
$$\sqrt{2}y^2 + 7y + 5\sqrt{2} = 0$$
$$\sqrt{2}\,y^2 + 2y + 5y + 5\sqrt{2} = 0$$
$$\sqrt{2}\,y(y + \sqrt{2}) + 5(y + \sqrt{2}) = 0$$
$$(y + \sqrt{2})(\sqrt{2}\,y + 5) = 0$$
$$y + \sqrt{\ } = 0 \quad \text{or} \quad \sqrt{2}y + 5 = 0$$
$$y = -\sqrt{2} \quad \text{or} \quad y = -\frac{5}{\sqrt{2}}$$

17. Roots of the equation are equal if Discriminant (D) = 0

$$mx(6x + 10) + 25 = 0$$
$$6mx^2 + 10mx + 25 = 0$$
$$D = 0$$
$$(10m)^2 - 4(6m)(25) = 0$$
$$100m^2 - 600m = 0$$
$$100m(m - 6) = 0$$
$$m = 0, 6$$

For m = 0, equation will become 25 = 0, which is not possible.

So, m = 6

18.
$$2\sqrt{3}x^2 - 5x + \sqrt{3} = 0$$
$$2\sqrt{3}\,x^2 - 2x - 3x + \sqrt{3} = 0$$
$$2x(\sqrt{3}x - 1) - \sqrt{3}(\sqrt{3}x - 1) = 0$$
$$(2x - \sqrt{3})(\sqrt{3}x - 1) = 0$$
$$2x - \sqrt{3} = 0 \quad \text{or} \quad \sqrt{3}x - 1 = 0$$
$$x = \frac{\sqrt{3}}{2} \quad \text{or} \quad x = \frac{1}{\sqrt{3}}$$

19. Let x be the side of square.

So, area of square = x^2

Number of students = $x^2 + 24$

If side of a square is increased by one student, side = x + 1

So, number of students = $(x + 1)^2 - 25$

According to question,

$$x^2 + 24 = (x + 1)^2 - 25$$
$$x^2 + 24 = x^2 + 1 + 2x - 25$$
$$48 = 2x$$
$$x = 24$$

∴ Number of students = $x^2 + 24$

$$= (24)^2 + 24$$
$$= 576 + 24$$
$$= 600$$

20.
$$4x^2 + 3x + 5 = 0$$
$$(2x)^2 + 2\left(\frac{3}{2}\right)x + 5 = 0$$
$$(2x)^2 + 2\left(\frac{3}{4}\right)2x + 5 = 0$$
$$(2x)^2 + 2\left(\frac{3}{4}\right)2x + \left(\frac{3}{4}\right)^2 - \frac{3}{4}^2 + 5 = 0$$
$$\left(2x + \frac{3}{4}\right)^2 - \frac{9}{16} + 5 = 0$$
$$\left(2x + \frac{3}{4}\right)^2 = \frac{9}{16} - 5$$
$$\left(2x + \frac{3}{4}\right)^2 = -\frac{71}{16}$$

As square of a number can't be negative.
So, the given equation has no real roots.

SECTION-C

21. $$(x-5)(x-6) = \frac{25}{(24)^2}$$

$$x^2 - 11x + 30 = \frac{25}{576}$$

$$x^2 - 11x + 30 - \frac{25}{576} = 0$$

$$x^2 - 11x + \frac{17255}{576} = 0$$

$$576x^2 - 6336x + 17255 = 0$$

$$576x^2 - 2856x - 3480x + 17255 = 0$$

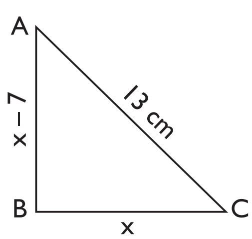
$$24x(24x - 119) - 145(24x - 119) = 0$$

$$(24x - 145)(24x - 119) = 0$$

$$24x - 145 = 0 \text{ or } 24x - 119 = 0$$

$$x = \frac{145}{24} \text{ or } x = \frac{119}{24}$$

22.



Let the base of Δ ABC = x cm

$\therefore$ Altitude of Δ ABC = (x – 7) cm

We know that,

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

$$AC^2 = AB^2 + BC^2$$

$$(13)^2 = (x-7)^2 + x^2$$

$$169 = x^2 + 49 - 14x + x^2$$

$$2x^2 - 14x - 120 = 0$$

$$x^2 - 7x - 60 = 0$$

$$x^2 - 12x + 5x - 60 = 0$$

$$x(x-12) + 5(x-12) = 0$$

$$(x+5)(x-12) = 0$$

$$x = -5, 12$$

Since, side cannot be negative,

So, $x = -5$ is rejected

$$\therefore \quad x = 12$$

$$BC = x = 12 \text{ cm}$$

$$AB = x - 7 = 12 - 7 = 5 \text{ cm}$$

23. $(a-b)x^2 + (b-c)x + (c-a) = 0$

As roots of equation are equal,

Discriminant (D) = 0

$(b-c)^2 - 4(a-b)(c-a) = 0$

$(b^2 + c^2 - 2bc) - 4(ac - a^2 - bc + ab) = 0$

$\Rightarrow$ $4a^2 + b^2 + c^2 - 4ac + 2bc - 4ab = 0$

$\Rightarrow$ $(2a)^2 + b^2 + c^2 - 4ac + 2bc - 4ab = 0$

$\Rightarrow$ $(-2a + b + c)^2 = 0$

$\Rightarrow$ $-2a + b + c = 0$

$\Rightarrow$ $2a = b + c$

24. Let the sides of two squares be x and y

Area of square with side x = x^2

Area of square with side y = y^2

Perimeter of square with side x = 4x

Perimeter of square with side y = 4y

According to question,

$$x^2 + y^2 = 468 \quad \text{...(i)}$$

$$4x - 4y = 24$$

$$\text{i.e. } x - y = 6 \quad \text{...(ii)}$$

From (ii), x = 6 + y

On putting in (i), we get

$$(6+y)^2 + y^2 = 468$$

$$36 + y^2 + 12y + y^2 = 468$$

$$2y^2 + 12y - 432 = 0$$

$$y^2 + 6y - 216 = 0$$

$$y^2 - 12y + 18y - 216 = 0$$

$$y(y-12) + 18(y-12) = 0$$

$$(y-12)(y+18) = 0$$

$$y = 12, -18$$

As side cannot be negative,

$$y = -18 \text{ is rejected}$$

$$\therefore \quad y = 12$$

So, $x = 6 + y$

$= 6 + 12$

$= 18$

So, sides of two squares are 12m and 18m respectively.

25.
$$a^2 x^2 - 3abx + 2b^2 = 0$$

$$(ax)^2 - 2\left(\frac{3}{2}\right)abx + 2b^2 = 0$$

$$(ax)^2 - 2ax\left(\frac{3\,b}{2}\right) + 2b^2 = 0$$

$$(ax)^2 - 2ax\left(\frac{3\,b}{2}\right) + \left(\frac{3\,b}{2}\right)^2 + 2b^2 - \left(\frac{3\,b}{2}\right)^2 = 0$$

$$\left(ax - \frac{3\,b}{2}\right)^2 + 2b^2 - \frac{9}{4}b^2 = 0$$

$$\left(ax - \frac{3\,b}{2}\right)^2 - \frac{b^2}{4} = 0$$

$$\left(ax - \frac{3\,b}{2}\right)^2 = \frac{b^2}{4}$$

$$ax - \frac{3b}{2} = \frac{b}{2}$$

$ax \; \frac{3b}{} = \frac{b}{2}$ $\qquad$ $ax - \frac{3b}{2} = -\frac{b}{2}$

$ax = \frac{4\,b}{2} = 2b$ $\qquad$ $ax = -\frac{b}{2} + \frac{3b}{2} = b$

$x = \frac{2b}{a}$ $\qquad$ $x = \frac{b}{a}$

26.
$$\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0$$

$$\Rightarrow \frac{2x(2x+3)+(x-3)+3x+9}{(x-3)(2x+3)} = 0$$

$$\Rightarrow 2x\,(2x + 3) + (x - 3) + 3x + 9 = 0$$

$$\Rightarrow 4x^2 + 6x + x - 3 + 3x + 9 = 0$$

$$\Rightarrow 4x^2 + 10x + 6 = 0$$

$$\Rightarrow 4x^2 + 4x + 6x + 6 = 0$$

$$\Rightarrow 4x\,(x + 1) + 6\,(x + 1) = 0$$

$$\Rightarrow (4x + 6)\,(x + 1) = 0$$

$$\Rightarrow 4x + 6 = 0 \quad \text{or} \quad x + 1 = 0$$

$$\Rightarrow x = -\frac{3}{2} \quad \text{or} \quad x = -1$$

27. Let the three consecutive natural numbers be $x - 1$, x and $x + 1$.

According to question,

$x^2 = [(x + 1)^2 - (x - 1)^2] + 60$

$x^2 = x^2 + 1 + 2x - x^2 - 1 + 2x + 60$

$x^2 = 4x + 60$

$$x^2 - 4x - 60 = 0$$

$$x^2 - 10x + 6x - 60 = 0$$

$$x\,(x - 10) + 6\,(x - 10) = 0$$

$$(x + 6)\,(x - 10) = 0$$

$$x = -6 \text{ or } 10$$

As x is a natural number,

$$x = -6 \text{ is rejected}$$

So, $x = 10$

$\therefore$ The three numbers 9, 10, 11.

28. Let the time taken by smaller tap to fill tank completely = x hours

Time taken by larger tap to fill tank completely = $x - 8$ hours

According to question,

$$\frac{1}{x} + \frac{1}{x-8} = \frac{5}{48}$$

$$\frac{x-8+x}{x(x-8)} = \frac{5}{48}$$

$$\frac{2x-8}{x(x-8)} = \frac{5}{48}$$

$$48\,(2x - 8) = 5x\,(x - 8)$$

$$96x - 384 = 5x^2 - 40x$$

$$5x^2 - 136x + 384 = 0$$

$$5x^2 - 16x - 120x + 384 = 0$$

$$x\,(5x - 16) - 24\,(5x - 16) = 0$$

$$(x - 24)\,(5x - 16) = 0$$

$$x = 24 \quad \text{or} \quad \frac{16}{5}$$

For x = 24,

Time taken by smaller tap = 24 hours

Time taken by larger tap = x – 8

$= 24 - 8$

$= 16$ hours

For $x = \frac{16}{5}$,

Time taken by larger pipe = x – 8

$$= \frac{16}{5} - 8$$

$$= -\frac{24}{5}$$

Since time cannot be negative,

$$x = \frac{16}{5} \text{ is rejected.}$$

∴ Time taken by smaller tap = 24 hours

Time taken by larger tap = 16 hours

29. $9x^2 - 63x - 162 = 0$

Discriminant (D) $= (-63)^2 - 4(9)(-162)$

$= 3969 + 5832$

$= 9801$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{63 \pm \sqrt{9801}}{18}$$

$$= \frac{63 \pm 99}{18}$$

$$x = \frac{63+99}{18} \quad \text{or} \quad x = \frac{63-99}{18}$$

$$x = 9 \quad \text{or} \quad x = -2$$

30. Let the larger part be x.

∴ Smaller part = 16 – x

According to question,

$$2(x)^2 = (16 - x)^2 + 164$$

$$2x^2 = 256 + x^2 - 32x + 164$$

$$x^2 + 32x - 420 = 0$$

$$x^2 + 42x - 10x - 420 = 0$$

$$x(x + 42) - 10(x + 42) = 0$$

$$(x - 10)(x + 42) = 0$$

$$x = 10 \quad \text{or} \quad -42$$

x = – 42 is rejected as x < 0.

$$\therefore \quad x = 10$$

So, the required parts are 10 and 6.

SECTION-D

31. $$\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$$

$$\frac{1}{2a+b+2x} - \frac{1}{2x} = \frac{1}{b} + \frac{1}{2a}$$

$$\frac{2x - 2a - b - 2x}{2x(2a+b+2x)} = \frac{2a+b}{2ab}$$

$$\frac{-2a-b}{2x(2a+b+2x)} = \frac{2a+b}{2ab}$$

$$\frac{-1}{2x(2a+b+2x)} = \frac{1}{2ab}$$

$$\frac{-1}{x(2a+b+2x)} = \frac{1}{ab}$$

$$x(2a + b + 2x) + ab = 0$$

$$2x^2 + 2ax + bx + ab = 0$$

$$2x(x + a) + b(x + a) = 0$$

$$(2x + b)(x + a) = 0$$

$$x = \frac{-b}{2}, -a$$

32. Let number of books = x

∴ Cost of each book $= \frac{80}{x}$

According to question,

$$\frac{80}{x+4} = \frac{80}{x} - 1$$

$$\frac{80}{x} - \frac{80}{x+4} = 1$$

$$\frac{1}{x} - \frac{1}{x+4} = \frac{1}{80}$$

$$\frac{x+4-x}{x(x+4)} = \frac{1}{80}$$

$$\frac{1}{x(x+4)} = \frac{1}{320}$$

$$x^2 + 4x - 320 = 0$$

$$x^2 - 16x + 20x - 320 = 0$$

$$x(x-16) + 20(x-16) = 0$$

$$(x-16)(x+20) = 0$$

$$x = 16 \text{ or } x = -20$$

Since, number of books cannot be negative, $x = 16$

So, number of books = 16

33. Let original duration of flight = x hours

Speed of an aircraft = $\frac{2800}{x}$ km/hr

If time increased by 30 minutes

i.e. $\frac{1}{2}$ hour, speed = $\frac{2800}{x+\frac{1}{2}}$

According to question,

$$\frac{2800}{x+\frac{1}{2}} = \frac{2800}{x} - 100$$

$$\frac{2800}{x} - \frac{2800}{\frac{2x+1}{2}} = 100$$

$$\frac{2800}{x} - \frac{5600}{2x+1} = 100$$

$$\frac{28}{x} - \frac{56}{2x+1} = 1$$

$$\frac{1}{x} - \frac{2}{2x+1} = \frac{1}{28}$$

$$\frac{2x+1-2x}{x(2x+1)} = \frac{1}{28}$$

$$\frac{1}{x(2x+1)} = \frac{1}{28}$$

$$2x^2 + x - 28 = 0$$

$$2x^2 + 8x - 7x - 28 = 0$$

$$2x(x+4) - 7(x+4) = 0$$

$$(x+4)(2x-7) = 0$$

$$x = -4, \frac{7}{2}$$

Since, time cannot be negative,

$$x = \frac{7}{2} = 3.5 \text{ hours}$$

34. Let speed of stream = x km/hr

Speed of boat in still water = 20 km/hr

Speed of boat upstream = (20 – x) km/hr

Speed of boat downstream = (20 + x) km/hr

According to question,

$$\frac{48}{20-x} = \frac{48}{20+x} + 1$$

$$\frac{1}{20-x} - \frac{1}{20+x} = \frac{1}{48}$$

$$\frac{20+x-20+x}{(20-x)(20+x)} = \frac{1}{48}$$

$$\frac{2x}{(20-x)(20+x)} = \frac{1}{48}$$

$$96x = 400 - x^2$$

$$x^2 + 96x - 400 = 0$$

$$x^2 + 100x - 4x - 400 = 0$$

$$x(x+100) - 4(x+100) = 0$$

$$(x-4)(x+100) = 0$$

$$x = 4, -100$$

Being the speed, x can not be negative.

So, x = –100 is rejected

$\therefore$ x = 4

Speed of stream = 4 km/hr

35. $$\frac{1}{2x-3} + \frac{1}{x-5} = 1$$

$$\frac{x-5+2x-3}{(2x-3)(x-5)} = 1$$

$$\frac{3x - 8}{2x^2 - 10x - 3x + 15} = 1$$

$$2x^2 - 13x + 15 = 3x - 8$$

$$2x^2 - 16x + 23 = 0$$

$$\text{Discriminant (D)} = (-16)^2 - 4\,(2)\,(23)$$

$$= 256 - 184$$

$$= 72$$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{16 \pm \sqrt{72}}{4}$$

$$= \frac{16 \pm 6\sqrt{2}}{4}$$

$$x = \frac{8 \pm 3\sqrt{2}}{2}$$

Roots are $\frac{8 + 3\sqrt{2}}{2}$ and $\frac{8 - 3\sqrt{2}}{2}$.

36. Let present age of sister be x years.

$\therefore$ Age of girl = 2x years

According to question,

$$(x + 4)\,(2x + 4) = 160$$

$$2x^2 + 12x + 16 - 160 = 0$$

$$2x^2 + 12x - 144 = 0$$

$$x^2 + 6x - 72 = 0$$

$$x^2 + 12x - 6x - 72 = 0$$

$$x\,(x + 12) - 6\,(x + 12) = 0$$

$$(x - 6)\,(x + 12) = 0$$

$$x = 6, -12$$

As age cannot be negative,

$x = -12$ is rejected

$\therefore$ $x = 6$

Age of sister = 6 years

Age of girl = 2x

= 2 (6)

= 12 years

37. Let number of articles be x

$\therefore$ Cost of production of each article = 2x + 3

According to question,

$$x\,(2x + 3) = 90$$

$$2x^2 + 3x - 90 = 0$$

$$2x^2 - 12x + 15x - 90 = 0$$

$$2x\,(x - 6) + 15\,(x - 6) = 0$$

$$(2x + 15)\,(x - 6) = 0$$

$x = \frac{-15}{2}$ or $x = 6$

Being number of articles, x cannot be negative.

$\therefore$ $x = 6$

Number of articles = 6

Cost of production of each article

= 2x + 3

= 12 + 3

= ₹ 15

38. Let Shefali's marks in English be x.

$\therefore$ Shefali's marks in Mathematics = 30 – x

According to question,

$$(30 - x + 2)\,(x - 3) = 210$$

$$(32 - x)\,(x - 3) = 210$$

$$32x - 96 - x^2 + 3x = 210$$

$$x^2 - 35x + 306 = 0$$

$$x^2 - 17x - 18x + 306 = 0$$

$$x\,(x - 17) - 18\,(x - 17) = 0$$

$$(x - 17)\,(x - 18) = 0$$

$x = 17$ or $x = 18$

If x = 17

Shefali's marks in English = 17

Shefali's marks in Mathematics = 30 – 17 = 13

If $x = 18$

Shefali's marks in English = 18

Shefali's marks in Mathematics = 30 – 18

= 12

39. Let speed of train = x km/hr

Distance covered = 360 km

So, time taken = $\frac{360}{x}$ hours

According to question,

$$\frac{360}{x+5} = \frac{360}{x} - 1$$

$$\frac{360}{x} - \frac{360}{x+5} = 1$$

$$\frac{x+5-x}{x(x+5)} = \frac{1}{360}$$

$$\frac{5}{x(x+5)} = \frac{1}{360}$$

$$x^2 + 5x - 1800 = 0$$

$$x^2 - 40x + 45x - 1800 = 0$$

$$x(x-40) + 45(x-40) = 0$$

$$(x-40)(x+45) = 0$$

$$x = 40, -45$$

Being speed of train, $x = -45$ is rejected.

∴ Speed of train = 40 km/hr

40. Let breadth of rectangular mango grove = x m

∴ Length = 2x m

According to question,

$$2x(x) = 800$$

$$2x^2 = 800$$

$$x^2 = 400$$

$$x = \pm 20$$

Being a dimension, x cannot be negative.

∴ $x = 20$

So, Breadth = 20 m

Length = 2x = 40 m

CASE STUDY-1

(i) (b) Area of rectangle = xy

100 – xy

$$x = \frac{100}{y}$$

(ii) (d) $x + y + y = 30$

$x + 2y = 30$

(iii) (d) $x = \frac{100}{y}$ ...(i)

$x + 2y = 30$ ...(ii)

Put equation (i) in equation (ii)

$$\frac{100}{y} + 2y = 30$$

$$100 + 2y^2 = 30y$$

$$2y^2 - 30y + 100 = 0$$

$$y^2 - 15y + 50 = 0$$

(iv) (a) $x = \frac{100}{y}$

$y = \frac{100}{x}$

Substitute $y = \frac{100}{x}$ in equation $y^2 - 15y + 50 = 0$

$$\left(\frac{100}{x}\right)^2 - 15\left(\frac{100}{x}\right) + 50 = 0$$

$$\frac{10000}{x^2} - \frac{1500}{x} + 50 = 0$$

$$\frac{200}{x^2} - \frac{30}{x} + 1 = 0$$

$$200 - 30x + x^2 = 0$$

$$x^2 - 30x + 200 = 0$$

(v) (c) $x^2 - 30x + 200 = 0$

$$x = \frac{30 \pm \sqrt{(30)^2 - 4(1)(200)}}{2}$$

$$= \frac{30 \pm \sqrt{(30)^2 - 4(1)(200)}}{2}$$

$$= \frac{30 \quad \sqrt{900 - 800}}{2}$$

$$= \frac{30 \pm 10}{2}$$

$x = \frac{30 + 10}{2}$, $x = 20$ | $x = \frac{30 - 10}{2}$, $x = 10$

when $x = 10$

$$y = \frac{100}{x} = \frac{100}{10}$$

$= 10$ m

When $x = 20$

$$y = \frac{100}{20}$$

$= 5$ m

As $x > y$

$\therefore$ $x = 20$ x, $y = 5$ m is the correct dimension.

CASE STUDY-2

(i) (c) Twice the number of articles produced is represented by 2x.

ATQ

Three more than twice the number of articles produced is $2x + 3$.

(ii) (b) The articles produced are represented by x.

$x \times$ (cost of I articles) = ₹ 90

cost of I article = ₹ $\frac{90}{x}$

(iii) (a) Cost of production of I article = $2x + 3$

$$2x + 3 = \frac{90}{x}$$

$$2x^2 + 3x = 90$$

$$2x^2 + 3x - 90 = 0$$

(iv) (d) $2x^2 + 3x - 90 = 0$

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(2)(-90)}}{2(2)}$$

$$= \frac{-3 \pm \sqrt{9 + 720}}{4}$$

$$= \frac{-3 \quad 27}{4}$$

$x = \frac{-3 + 27}{4} = \frac{24}{4} = 6$ | $x = \frac{-3 - 27}{4}$, $x = \frac{-30}{4} = -7.5$

As the number of articles cannot be negative 4 m fractions hence discarding the value $x = -7.5$

(v) (a) Cost of per article = $\frac{90}{x}$

$$= \frac{90}{6} = ₹15$$

Chapter 5 Arithmetic Progressions

Multiple Choice Questions

1. (b) $a_n = 3n + 7$

$a_{n+1} = 3(n + 1) + 7 = 3n + 10$

So, $d = a_{n+1} - a_n$

$= 3n + 10 - 3n - 7$

$= 3$

2. (c) $a = 1, a_n = 11$

$S_n = 36$

We know that $S_n = \frac{n}{2}(a + a_n)$

$36 = \frac{n}{2}(1 + 11)$

$n = \frac{36 \times 2}{12}$

$= 6$

3. (b) $S_n = 2n^2 + 5n$

$a_n = S_n - (S_{n-1})$

$= (2n^2 + 5n) - [2(n - 1)^2 + 5(n - 1)]$

$= 2n^2 + 5n - 2n^2 - 2 + 4n - 5n + 5$

$= 4n + 3$

4. (d) We can write reverse AP as

185, ..., 13, 9, 5

Such that a = 185, d = –4

So, $a_9 = 185 + (9 - 1)(-4)$

$= 185 - 32$

$= 153$

5. (a) 18, a, b, – 3 are in AP.

$\therefore \quad a - 18 = b - a = -3 - b$

$a - 18 = b - a$

$2a - b = 18$ ...(i)

$b - a = -3 - b$

$a - 2b = 3$ ...(ii)

Solving (i) and (ii), we get

$a = 11, b = 4$

So, $a + b = 11 + 4$

$= 15$

WORKSHEET - 1

SECTION-A

1. k + 9, 2k – 1 and 2k + 7 are in A.P. if

$(2k - 1) - (k + 9) = (2k + 7) - (2k - 1)$

$k - 10 = 8$

$k = 18$

2. $S_n = 3n^2 + 5n$

$S_{20} = 3(20)^2 + 5(20)$

$= 3(400) + 100$

$= 1200 + 100$

$= 1300$

3. Consider A.P. : 2, 4, 6, 8, ..., n

Here a = 2, d = 2

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$= \frac{n}{2}[4 + (n - 1) \times 2]$

$= \frac{n}{2}[2n + 2]$

$= n(n + 1)$

4. A.P.: $-5, -\frac{5}{2}, 0, \frac{5}{2}, \dots$.

$a_n = a + (n-1)d, d = -\frac{5}{2} + 5 = \frac{5}{2}$

$\therefore a_{25} = -5 + (25-1)\frac{5}{2}$

$= -5 + 24 \frac{5}{2}$

$= -5 + 60$

$= 55$

5. $S_p = ap^2 + bp$

$a_p = S_p - S_{p-1}$

$= (ap^2 + bp) - [a(p-1)^2 + b(p-1)]$

$= ap^2 + bp - [ap^2 + a - 2ap + bp - b]$

$= ap^2 + bp - ap^2 - a + 2ap - bp + b$

$= 2ap - a + b$

$\therefore a_{p+1} = 2a(p+1) - a + b$

$= 2ap + 2a - a + b$

$= 2ap + a + b$

So, $d = a_{p+1} - a_p$

$= 2ap + a + b - 2ap + a - b$

$= 2a$

6. $a_n = n^2 + 1$

$a_1 = 1 + 1 = 2$

$a_2 = 2^2 + 1 = 5$

7. $a_n = \frac{3n - 2}{4n + 5}$

$a_1 = \frac{3 - 2}{4 + 5} = \frac{1}{9}$, $a_2 = \frac{6 - 2}{8 + 5} = \frac{4}{13}$

$a_3 = \frac{9 - 2}{17} = \frac{7}{17}$

So, sequence is $\frac{1}{9}, \frac{4}{13}, \frac{7}{17}, \dots$

8. $a_n = 3n - 2$

$a_1 = 3 - 2 = 1$

$a_2 = 6 - 2 = 4$

$a_3 = 9 - 2 = 7$ so on

So, sequence is 1, 4, 7, ...

$a_2 - a_1 = 4 - 1 = 3$

$a_3 - a_2 = 7 - 4 = 3$

As difference between the terms is same, so, the given sequence is in A.P.

$a_{10} = 30 - 2 = 28$

SECTION-B

9. A.P : 18, 16, 14, ...

$S_n = 0$

$\frac{n}{2}[2a + (n-1)d] = 0$

$\frac{n}{2}[36 + (n-1)(-2)] = 0$

$n(36 - 2n + 2) = 0$

$n(38 - 2n) = 0$

$n = \frac{38}{2} = 19$

10. $a_4 = 0 \Rightarrow a + 3d = 0 \Rightarrow a = -3d$

To prove : $a_{25} = 3a_{11}$

Consider, $a_{25} = a + (25-1)d$

$= a + 24d$

$= -3d + 24d$

$= 21d$

$a_{11} = a + 10d$

$= -3d + 10d$

$= 7d$

So, $a_{25} = 3a_{11}$

11. A. P: 6, 13, 20, ..., 216

$a_n = a + (n-1)d$

$216 = 6 + (n-1)7$

$210 = 7(n-1)$

$30 = n - 1$

$n = 31$

So, 216 is 31st term of an A.P.

So, 16^{th} term is the middle term

$$\begin{aligned} a_{16} &= 6 + (16 - 1)\,7 \\ &= 6 + 7\,(15) \\ &= 6 + 105 \\ &= 111 \end{aligned}$$

12. Consider 9, 12, 15, 18, ...

$$\begin{aligned} a_2 - a_1 &= 12 - 9 = 3 \\ a_3 - a_2 &= 15 - 12 = 3 \\ a_4 - a_3 &= 18 - 15 = 3 \end{aligned}$$

As difference between the terms is same,

So, the terms are in A.P.

$$\begin{aligned} a_{16} &= a + 15d \\ &= 9 + 15\,(3) \\ &= 9 + 45 \\ &= 54 \end{aligned}$$

$$\begin{aligned} a_n &= a + (n - 1)\,d \\ &= 9 + (n - 1)\,3 \\ &= 9 + 3n - 3 \\ &= 3n + 6 \end{aligned}$$

13. $S_5 + S_7 = 167$

$$\frac{5}{2}\,[2a + (5 - 1)\,d] + \frac{7}{2}\,[2a + (7 - 1)\,d] = 167$$

$$\Rightarrow \quad 5a + \frac{5}{2}\,(4d) + 7a + \frac{7}{2}\,(6d) = 167$$

$$\Rightarrow \quad 5a + 10d + 7a + 21d = 167$$

$$\Rightarrow \quad 12a + 31d = 167 \qquad ...(i)$$

$$S_{10} = 235$$

$$\frac{10}{2}\,[2a + (10 - 1)\,d] = 235$$

$$2a + 9d = 47 \qquad ...(ii)$$

Multiplying equation (ii) by 6 and subtracting (i) from (ii), we get

$$\begin{aligned} (12a + 54d) - (12a + 31d) &= 282 - 167 \\ 23d &= 115 \end{aligned}$$

$$d = \frac{115}{23} = 5$$

From (ii), $2a + 9(5) = 47$

$$\begin{aligned} 2a + 45 &= 47 \\ a &= 1 \end{aligned}$$

So, A.P. is a, a + d, a + 2d, ...

i.e. 1, 6, 11, ...

14. A.P. : 5, 15, 25, ...

Let n^{th} term of an AP be 130 more than its 31^{st} term

i.e.

$$\begin{aligned} a_n &= 130 + a_{31} \\ 5 + (n - 1)\,10 &= 130 + 5 + (31 - 1)\,10 \\ 5 + 10n - 10 &= 135 + 300 \\ 10n &= 435 + 5 \\ 10n &= 440 \\ n &= 44 \end{aligned}$$

So, 44^{th} term of an AP is 130 more than its 31^{st} term.

15.

$$\begin{aligned} a_5 + a_9 &= 72 \\ a + 4d + a + 8d &= 72 \\ 2a + 12d &= 72 \\ a + 6d &= 36 \qquad ...(i) \\ a_7 + a_{12} &= 97 \\ a + 6d + a + 11d &= 97 \\ 2a + 17d &= 97 \qquad ...(ii) \end{aligned}$$

On multiplying (i) by 2 and subtracting (i) from (ii), we get

$$\begin{aligned} (2a + 17d) - (2a + 12d) &= 97 - 72 \\ 5d &= 25 \\ d &= 5 \end{aligned}$$

From (i),

$$\begin{aligned} a &= 36 - 6\,(5) \\ &= 36 - 30 = 6 \\ a &= 6 \end{aligned}$$

So, A.P. is a, a + d, a + 2d, ...

i.e. 6, 11, 16, ...

16. Consider AP : 7, 14, 21, ..., 497

$$a_n = a + (n - 1)d$$
$$497 = 7 + (n - 1)7$$
$$497 - 7 = 7(n - 1)$$
$$\frac{490}{7} = 70 = n - 1$$
$$n = 71$$

SECTION-C

17. $S_7 = 49$

$$\frac{7}{2}[2a + 6d] = 49$$
$$2a + 6d = 14$$
$$a + 3d = 7 \quad ...(i)$$

Also, $S_{17} = 289$

$$\frac{17}{2}[2a + 16d] = 289$$
$$2a + 16d = 34$$
$$a + 8d = 17 \quad ...(ii)$$

On subtracting (ii) from (i), we get

$$(a + 3d) - (a + 8d) = 7 - 17$$
$$-5d = -10$$
$$d = 2$$

From (i), $a = 7 - 3d$

$$= 7 - 6$$
$$= 1$$

So, $S_n = \frac{n}{2}[2a + (n - 1)d]$

$$= \frac{n}{2}[2 + (n - 1)2]$$
$$= n[1 + (n - 1)]$$
$$= n^2$$

18. Let S_n and S_n^I be sum of a terms of two A.P.

$$\frac{S_n}{S_n^I} = \frac{7n+1}{4n+27}$$

$$\frac{\frac{n}{2}\left[2a+(n-1)d\right]}{\frac{n}{2}\left[2a^I+(n-1)d^I\right]} = \frac{7n+1}{4n+27}$$

$$\frac{2a+(n-1)d}{2a^I+(n-1)d^I} = \frac{7n+1}{4n+27}$$

$$\frac{a+\left(\frac{n-1}{2}\right)d}{a^I+\left(\frac{n-1}{2}\right)d^I} = \frac{7n+1}{4n+27}$$

$$\frac{a+(m-1)d}{a^I+(m-1)d^I} = \frac{7(2m-1)+1}{4(2m-1)+27}$$

$$\frac{a_m}{a_m^I} = \frac{14m-6}{8m+23}$$

19. Let the digits be a – d, a, a + d

$$a - d + a + a + d = 15$$
$$3a = 15$$
$$a = 5$$

Also, $100(a - d) + 10a + a + d$

$$= [100(a + d) + 10a + a - d] - 594$$

$\therefore$ $100a - 100d + 11a + d$

$$= 100a + 100d + 11a - d - 594$$
$$0 = 200d - 2d - 594$$
$$198d = 594$$
$$d = 3$$

So, number $= 100(a + d) + 10a + a - d$

$$= 100(8) + 50 + 2$$
$$= 852$$

20. $a_p = q \Rightarrow a + (p - 1)d = q \quad ...(i)$

$a_q = p \Rightarrow a + (q - 1)d = p \quad ...(ii)$

On subtracting (ii) from (i), we get

$$(p - 1)d - (q - 1)d = q - p$$
$$d[p - 1 - q + 1] = q - p$$
$$d = \frac{q-p}{p-q} = -1$$

From (i), $a + (p-1)(-1) = q$

$$a = p - 1 + q$$

So, $a_n = a + (n-1)d$

$= (p - 1 + q) + (n-1)(-1)$

$= p - 1 + q - n + 1$

$= p + q - n$

21. Here, $a_2 - a_1 = 19\frac{1}{4} - 20$

$= \frac{77}{4} - 20$

$= \frac{77-80}{4} = -\frac{3}{4}$

$a_3 - a_2 = 18\frac{1}{2} - 19\frac{1}{4}$

$= \frac{37}{2} - \frac{77}{4}$

$= \frac{74-77}{4}$

$= \frac{-3}{4}$

as $a_3 - a_2 = a_2 - a_1$

i.e. difference between the terms is same, so, the given sequence forms an A.P.

Here, $a = 20, d = \frac{-3}{4}$

$a_n < 0$

$a + (n-1)d < 0$

$20 + (n-1)\left(\frac{-3}{4}\right) < 0$

$\frac{-3}{4}(n-1) < -20$

$n - 1 > -20\left(\frac{-4}{3}\right)$

$n - 1 > \frac{80}{3}$

$n > \frac{80}{3} + 1 = \frac{83}{3} = 27.67$

So, $n = 28$

So, a_{28} is the first negative term.

22. For S_1, $a = 1$, $d = 1$

So, $S_1 = \frac{n}{2}[2 + (n-1)(1)]$

$= \frac{n}{2}(n+1)$

For S_2, $a = 1$, $d = 2$

So, $S_2 = \frac{n}{2}[2 + (n-1)(2)]$

$= n[1 + n - 1]$

$= n^2$

For S_3, $a = 1$, $d = 3$

$S_3 = \frac{n}{2}[2 + (n-1)3]$

$= \frac{n}{2}[3n - 1]$

Consider, $S_1 + S_3 = \frac{n}{2}(n+1) + \frac{n}{2}(3n-1)$

$= \frac{n^2}{2} + \frac{n}{2} + \frac{3n^2}{2} - \frac{n}{2}$

$= \frac{4n^2}{2}$

$= 2n^2$

$= 2S_2$

23. A.P.: a, 7, b, 23, c

As the terms are in A.P.,

$7 - a = b - 7 = 23 - b = c - 23$

As $7 - a = b - 7$

$a + b = 14$ ...(i)

As $b - 7 = 23 - b$

$2b = 30$

$b = 15$

From (i), $a = 14 - b = 14 - 15$

$= -1$

As $23 - b = c - 23$

$23 - 15 = c - 23$

$c = 31$

24. Let the four parts be

$a - 3d, a - d, a + d, a + 3d$ such that

$a - 3d + a - d + a + d + a + 3d = 32$

$$4a = 32$$

$$a = 8$$

Also, $\dfrac{(a-3d)(a+3d)}{(a-d)(a+d)} = \dfrac{7}{15}$

i.e. $\dfrac{(8-3d)(8+3d)}{(8-d)(8+d)} = \dfrac{7}{15}$

$$\frac{64-9d^2}{64-d^2} = \frac{7}{15}$$

$$960 - 135d^2 = 448 - 7d^2$$

$$512 = 128\,d^2$$

$$d^2 = 4$$

$$d = \pm 2$$

For a = 8, d = 2

Four parts are

a – 3d, a – d, a + d, a + 3d

i.e. 8 – 6, 8 – 2, 8 + 2, 8 + 6

i.e. 2, 6, 10, 14

For a = 8, d = – 2

Four parts are

a – 3d, a – d, a + d, a + 3d

i.e. 8 + 6, 8 + 2, 8 – 2, 8 – 6

i.e. 14, 10, 6 and 2.

SECTION-D

25. Let the time in which policeman catches the thief is n minutes.

Uniform speed of thief = 100 m/min

As after one minute a policeman runs after the thief to catch him.

So, distance travelled by thief

= 100 (n + 1) minutes

Given that speed of policeman increases by 10m/min.

speed of policeman forms an AP:

100 m/min, 110 m/min, 120 m/min, ...

So, distance travelled by policeman

$= S_n$

$= \dfrac{n}{2}[2a + (n-1)d]$

$= \dfrac{n}{2}[200 + (n-1)10]$

$= n[100 + 5n - 5]$

$= n(95 + 5n)$

Distance travelled by thief

= Distance travelled by policeman

$$100(n+1) = n(95+5n)$$

$$100n + 100 = 95n + 5n^2$$

$$5n^2 - 5n - 100 = 0$$

$$n^2 - n - 20 = 0$$

$$n^2 - 5n + 4n - 20 = 0$$

$$n(n-5) + 4(n-5) = 0$$

$$(n+4)(n-5) = 0$$

$$n = -4 \text{ or } n = 5$$

As n cannot be negative, n = 5

26. Consider the sequence formed by all three digit numbers which leaves a remainder 3, when divided by 4: 103, 107, 111, 115, ..., 999.

The above sequence forms an A.P. with a = 103 and common difference d = 4

$$a_n = a + (n-1)d$$

$$999 = 103 + (n-1)4$$

$$4(n-1) = 999 - 103$$

$$4(n-1) = 896$$

$$n - 1 = 224$$

$$n = 225$$

The middle term is $\left(\dfrac{n+1}{2}\right)^{th}$ term

i.e. $\dfrac{225+1}{2} = 113^{th}$ term

$$a_{113} = 103 + (113-1)4$$

$= 103 + 112 (4)$

$= 103 + 448$

$= 551$

Sum of all terms before middle term

$= S_{112}$

$= \frac{112}{2} [2 (103) + (112 - 1) 4]$

$= 56 [206 + 444]$

$= 56 (650)$

$= 36,400$

$S_{225} = \frac{225}{2} [2 (103) + (225 - 1) 4]$

$= \frac{225}{2} [206 + 896]$

$= \frac{225}{2} (1102)$

$= 123975$

So, sum of terms after the middle term

$= 123975 - (S_{112} + 551)$

$= 123975 - 36400 - 551$

$= 87024$

27. Given: $S_m = S_n$

To prove: $S_{m+n} = 0$

$$S_m = \frac{m}{2} [2a + (m - 1)d]$$

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

As $S_m = S_n$

$$\therefore \quad \frac{m}{2} [2a + (m - 1)d] = \frac{n}{2} [2a + (n - 1)d]$$

$$m [2a + (m - 1)d] = n [2a + (n - 1)d]$$

$$2am + md (m - 1)] = 2na + nd (n - 1)$$

$$2am + m^2d - md = 2an + n^2d - nd$$

$$2am - 2an + m^2d - n^2d - md + nd = 0$$

$$2a (m - n) +d (m^2 - n^2) - d (m - n) = 0$$

$$(m - n) [2a + (m + n) d - d) = 0$$

$$(m - n) [2a + (m + n - 1) d] = 0$$

As $m \neq n, 2a + (m + n - 1) d = 0$

Consider

$$S_{m+n} = \frac{m+n}{2} [2a + (m + n - 1) d]$$

$$= \frac{m+n}{2} (0)$$

$$= 0$$

So, sum of its (m + n) terms is zero.

28. AP = –12, –9, –6, ... , 21

If 1 is added to each term,

A.P. becomes –12 + 1, –9 + 1, –6 + 1, ..., 21 + 1

i.e. – 11, –8, –5, ..., 22

We know that

$$a_n = a + (n - 1) d$$

$$22 = -11 + (n - 1) (3)$$

$$\frac{33}{3} = n - 1$$

$$n = 12$$

$$S_{12} = \frac{12}{2} [2 (-11) + (12 - 1) 3]$$

$$= 6 [-22 + 33]$$

$$= 6 (11)$$

$$= 66$$

29. Let the prizes be a, a – 20, a – 40, ...

$$S_{10} = 1600$$

$$\frac{10}{2} [2a + (10 - 1) (- 20)] = 1600$$

$$5 (2a - 180) = 1600$$

$$2a - 180 = 320$$

$$2a = 500$$

$$a = 250$$

So, the prize are 250, 230, 210, 190, 170, 150, 130, 110, 90.

30. First term = a

Second term = b

last term (a_n) = c

To prove: $S_n = \frac{(a+c)(b+c-2a)}{2(b-a)}$

Here, $d = b - a$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

or

$$= \frac{n}{2}[a + a_n]$$

$$= \frac{n}{2}[a + c] \qquad ...(i)$$

We know that $a_n = c$

i.e. $a + (n-1)(b-a) = c$

$$(n-1) = \frac{c-a}{b-a}$$

$$n = \frac{c-a}{b-a} + 1$$

$$= \frac{c+b-2a}{b-a} \qquad ...(ii)$$

On putting (ii) in (i), we get

$$S_n = \frac{1}{2}(a+c)\left(\frac{c+b-2a}{b-a}\right)$$

$$= \frac{(a+c)(b+c-2a)}{2(b-a)}$$

31. Amount paid in cash = ₹ 60,000

Amount pending = ₹ 120000 – ₹ 60000

= ₹ 60000

Amount of the first installment

$$= 5000 + \frac{12}{100}(60,000)$$

$$= 5000 + 7200$$

$$= 12200$$

Amount of second installment

$$= 5000 + \frac{12}{100}(60000 - 5000)$$

$$= 5000 + 6600$$

$$= 11600$$

So, amount paid for installments:

12200, 11600, ... forms an AP

First term (a) = 12200

Common difference (d) = 11600 – 12200

= –600

$n = 12$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{12}{2}[2(12200) + (12-1)(-600)]$$

$$= 6[24400 + 11(-600)]$$

$$= 6(24400 - 6600)$$

$$= 6(17800)$$

= ₹ 106,800

32. $a_4 + a_8 = 24$

$$a + 3d + a + 7d = 24$$

$$2a + 10d = 24$$

$$a + 5d = 12 \qquad ...(i)$$

Again,

$$a_6 + a_{10} = 44$$

$$a + 5d + a + 9d = 44$$

$$2a + 14d = 44$$

$$a + 7d = 22 \qquad ...(ii)$$

On subtracting eq^n (i) from (ii), we get

$$a + 7d - a - 5d = 22 - 12$$

$$2d = 10$$

$$d = 5$$

From (i), $a = 12 - 5d$

$= 12 - 25$

$= -13$

$$S_{10} = \frac{10}{2}[2(-13) + (10-1)5]$$

$$= 5(-26 + 45)$$

$$= 5(19) = 95$$

WORKSHEET - 2

SECTION-A

1. Consider an AP: 12, 18, 24, ..., 96

$$a_n = a + (n-1)d$$
$$96 = 12 + (n-1)6$$
$$96 - 12 = 6(n-1)$$
$$n - 1 = \frac{84}{6}$$
$$n - 1 = 14$$
$$n = 15$$

2. $$S_q = 2q + 3q^2$$
$$S_{q-1} = 2(q-1) + 3(q-1)^2$$
$$= 2q - 2 + 3q^2 + 3 - 6q$$
$$= 3q^2 - 4q + 1$$
$$a_q = s_q - s_{q-1}$$
$$= 2q + 3q^2 - 3q^2 + 4q - 1$$
$$= 6q - 1$$
$$a_{q+1} = 6q + 6 - 1 = 6q + 5$$
$$\therefore\ d = a_{q+1} - a_q = 6q + 5 - 6q + 1$$
$$= 6$$

3. Consider AP: 1, 3, 5, 7, ..., n

with a = 1, d = 2

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$= \frac{n}{2}[2 + (n-1)2]$$
$$= n[1 + n - 1]$$
$$= n^2$$

4. As terms are in AP,

$$13 - (2p + 1) = (5p - 3) - 13$$
$$13 - 2p - 1 = 5p - 3 - 13$$
$$12 + 16 = 7p$$
$$7p = 28$$
$$p = 4$$

5. First term = a

Second term = b

Last term (a_n) = 2a

Common difference (d) = b – a

$$S_n = \frac{n}{2}[2a + (n-1)d] \text{ or } \frac{n}{2}[a + a_n]$$
$$S_n = \frac{n}{2}[a + 2a]$$
$$= \frac{3a}{2}n \qquad \text{...(i)}$$

As $a_n = 2a$

$$a + (n-1)d = 2a$$
$$a + (n-1)(b-a) = 2a$$
$$(n-1)(b-a) = a$$
$$n - 1 = \frac{a}{b-a}$$
$$n = \frac{a}{b-a} + 1$$
$$n = \frac{b}{b-a} \qquad \text{...(ii)}$$

On putting (ii) in (i), we get

$$S_n = \frac{3a}{2}\frac{b}{(b-a)}$$
$$= \frac{3ab}{2(b-a)}$$

6. $\frac{1}{m}, \frac{1+m}{m}, \frac{1+2m}{m}, ...$

Here, $a = \frac{1}{m}$

$$d = \frac{1+m}{m} - \frac{1}{m} = \frac{1}{m} + 1 - \frac{1}{m} = 1$$
$$a_n = a + (n-1)d$$
$$= \frac{1}{m} + (n-1)1$$
$$= \frac{1}{m} + n - 1$$

7. Let 184 be the n^{th} term of

AP = 3, 7, 11, ... where

$$a = 3$$
$$d = 7 - 3 = 4$$

$$a_n = a + (n-1)d$$
$$184 = 3 + (n-1)4$$
$$184 - 3 = 4(n-1)$$
$$n - 1 = \frac{181}{4}$$
$$n = \frac{185}{4}$$ which is not a natural number.

So, 184 is not a term of AP: 3, 7, 11, ...

8. Consider AP: 254, ..., 14, 9, 4

where $a = 254$

$d = 9 - 14 = -5$

So, $a_{10} = 254 + (10-1)(-5)$

$= 254 - 45$

$= 209$

9. $a_1 = 4$

$a_n = 4a_{n-1} + 3, n > 1$

$a_2 = 4a_1 + 3 = 16 + 3 = 19$

$a_3 = 4a_2 + 3 = 4(19) + 3 = 79$

$a_4 = 4a_3 + 3 = 4(79) + 3 = 319$

$a_5 = 4a_4 + 3 = 4(319) + 3 = 1279$

$a_6 = 4a_5 + 3 = 4(1279) + 3$

$= 5119.$

10. $a_n = 4n + 5$

$a_1 = 4 + 5 = 9$

$a_2 = 4(2) + 5 = 13$

$a_3 = 4(3) + 5 = 17$

$a_4 = 4(4) + 5 = 21$

$a_2 - a_1 = 13 - 9 = 4$

$a_3 - a_2 = 17 - 13 = 4$

$a_4 - a_3 = 21 - 17 = 4$

As difference between the terms is same, the sequence defined by $a_n = 4n + 5$ is an A.P. such that d = 4.

SECTION-B

11. A.P : 27, 24, 21, ...

Let sum of n terms of the A.P. be 0.

Here, first term (a) = 27

Common difference (d) = 24 – 27

= –3

$S_n = 0$

$$\frac{n}{2}[2a + (n-1)d] = 0$$
$$\frac{n}{2}[54 + (n-1)(-3)] = 0$$
$$n(54 - 3n + 3) = 0$$
$$n(18 - n + 1) = 0$$
$$18 - n + 1 = 0$$
$$n = 19$$

So, sum of 19 terms is 0.

12. $\frac{S_m}{S_n} = \frac{m^2}{n^2}$

To prove : $\frac{a_m}{a_n} = \frac{2m-1}{2n-1}$

As $\frac{S_m}{S_n} = \frac{m^2}{n^2}$

$$\therefore \frac{\frac{m}{2}\left[2a+(m-1)d\right]}{\frac{n}{2}\left[2a+(n-1)d\right]} = \frac{m^2}{n^2}$$
$$\frac{2a+(m-1)d}{2a+(n-1)d} = \frac{m}{n}$$
$$\frac{a+\frac{(m-1)}{2}d}{a+\frac{(n-1)}{2}d} = \frac{m}{n}$$

On replacing m by 2m – 1 and n by 2n – 1 on both sides of equation, we get

$$\frac{a+(m-1)d}{a+(n-1)d} = \frac{2m-1}{2n-1}$$

13. $S_n = \frac{3n^2}{2} + \frac{5n}{2}$

We know that $a_n = S_n - S_{n-1}$

So, $a_{25} = S_{25} - S_{24}$

$$= \left[\frac{3}{2}(625)+\frac{5}{2}(25)\right]-\left[\frac{3}{2}(576)+\frac{5}{2}(24)\right]$$

$$= \frac{1875}{2}+\frac{125}{2}-\frac{1728}{2}-\frac{120}{2}$$

$$= \frac{1875+125-1728-120}{2}$$

$$= 76$$

14. Number of terms = 111

So, middle term must be the 56^{th} term

$$a_{56} = 30$$

$$a + 55d = 30$$

Consider $S_n = \frac{n}{2}[2a + (n-1)d]$

$$\therefore \quad S_{111} = \frac{111}{2}[2a + 110d] = 111[a + 55d]$$

$$= 111(30) = 3330$$

15. First term (a) = – 7

Common difference (d) = 5

We know that $a_n = a + (n-1)d$

$$= -7 + (n-1)5$$

$$= -7 + 5n - 5$$

$$= 5n - 12$$

So, $a_{18} = 5(18) - 12$

$$= 90 - 12$$

$$= 78$$

16. $a_{10} = 52$

$\therefore \quad a + 9d = 52$ ...(i)

$$a_{17} = 20 + a_{13}$$

$$a + 16d = 20 + a + 12d$$

$$4d = 20$$

$$d = 5$$

From (i), $a + 9(5) = 52$

$$a + 45 = 52$$

$$a = 7$$

So, AP is a, a + d, a + 2d, ...

i.e. 7, 12, 17, ...

17. $a_9 = -32$

$a + 8d = -32$ ...(i)

Also, $a_{11} + a_{13} = -94$

$$a + 10d + a + 12d = -94$$

$$2a + 22d = -94$$

$a + 11d = -47$ ...(ii)

On subtracting (i) from (ii), we get

$$a + 11d - a - 8d = -47 + 32$$

$$3d = -15$$

$$d = -5$$

18. To prove: $S_{30} = 3(S_{20} - S_{10})$

Consider, $3(S_{20} - S_{10})$

$$= 3\left\{\frac{20}{2}\left[2a+(20-1)d\right]-\frac{10}{2}\left[2a+(10-1)d\right]\right\}$$

$$= 3\left\{10\left[2a+(19)d\right]-5\left[2a+9d\right]\right\}$$

$$= 30(2a + 19d) - 15(2a + 9d)$$

$$= 60a + 570d - 30a - 135d$$

$$= 30a + 435d$$

Also, $S_{30} = \frac{30}{2}[2a + (30-1)d)$

$$= 15[2a + 29d)$$

$$= 30a + 435d$$

$$\therefore \quad S_{30} = 3[S_{20} - S_{10}]$$

19. $a_{14} = 2a_8$

$$a + 13d = 2[a + 7d]$$

$$a + 13d = 2a + 14d$$

$$-d = a$$

$$a_6 = -8$$

$$a + 5d = -8$$

$-d + 5d = -8$ (As $a = -d$)

$$4d = -8$$

$$d = -2$$

So, $a = -d = 2$

We know that $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_{20} = \frac{20}{2}[2(2) + (20-1)(-2)]$

$= 10[4 - 38]$

$= 10(-34)$

$= -340$

20. First term (a) = 7

Last term $(a_n) = 49$

$S_n = 420$

We know that $S_n = \frac{n}{2}[a + a_n]$

$420 = \frac{n}{2}[7 + 49]$

$n = \frac{420}{28} = 15$

Now, $a_n = 49$

$a + (n-1)d = 49$

$7 + (15-1)d = 49$

$14d = 42$

$d = 3$

SECTION-C

21. $a_2 + a_7 = 30$

$a + d + a + 6d = 30$

$2a + 7d = 30 \quad \text{...(i)}$

Also, $a_{15} = 2a_8 - 1$

$a + 14d = 2[a + 7d] - 1$

$a + 14d = 2a + 14d - 1$

$0 = a - 1$

$a = 1$

From (i), $2(1) + 7d = 30$

$7d = 28$

$d = 4$

So, A.P. is a, a + d, a + 2d, ...

i.e. 1, 5, 9, ...

22. AP : $18, 15\frac{1}{2}, 13, ..., -49\frac{1}{2}$

i.e. $18, \frac{31}{2}, 13, ..., -\frac{99}{2}$

Here, first term (a) = 18

Common difference (d) $= \frac{31}{2} - 18$

$= \frac{31 - 36}{2} = -\frac{5}{2}$

Last term $(a_n) = -\frac{99}{2}$

$a + (n-1)d = -\frac{99}{2}$

$18 + (n-1)\left(-\frac{5}{2}\right) = -\frac{99}{2}$

$-\frac{5}{2}(n-1) = -\frac{99}{2} - 18$

$-\frac{5}{2}(n-1) = \frac{-99 - 36}{2}$

$-\frac{5}{2}(n-1) = -\frac{135}{2}$

$n - 1 = -\frac{135}{2} \times \frac{2}{-5} = 27$

$n = 28$

So, number of terms (n) = 28

We know that $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{28} = \frac{28}{2}\left[36 + (28-1)\left(\frac{-5}{2}\right)\right]$

$= 14\left[36 - \frac{135}{2}\right]$

$= 14\left(\frac{72 - 135}{2}\right)$

$= 7(-63)$

$= -441$

23. $a_n = -4n + 15$

$a_1 = -4 + 15 = 11$

$a_2 = -4\ (2) + 15 = -8 + 15 = 7$

$a_3 = -12 + 15 = 3.$

So, First term (a) = 11

Common difference (d) = $7 - 11 = -4$

We know that

$$S_n = \frac{n}{2}[2a + (n-1)\,d]$$

$$S_{20} = \frac{20}{2}[2(11) + (20-1)(-4)]$$

$$= 10\,[22 - 76]$$

$$= 10\,(-54)$$

$$= -540$$

24.
$$a_8 = 31$$

$$a + 7d = 31 \quad ...(i)$$

$$a_{15} = a_{11} + 16$$

$$a + 14d = a + 10d + 16$$

$$4d = 16$$

$$d = 4 \quad ...(ii)$$

From (i), $a + 28 = 31$

$$a = 3$$

So, A.P. is a, a + d, a + 2d, ...

i.e. 3, 7, 11, ...

25.
$$a_{15} = 3 + 2a_7$$

$$a + 14d = 3 + 2\,(a + 6d)$$

$$a + 14d = 3 + 2a + 12d$$

$$0 = a - 2d + 3 \quad ...(i)$$

Also, $a_{10} = 41$

$$a + 9d = 41 \quad ...(ii)$$

On subtracting (i) from (ii), we get

$$a + 9d - a + 2d = 41 + 3$$

$$11d = 44$$

$$d = 4$$

From (ii), $a + 9(4) = 41$

$$a = 41 - 36$$

$$= 5$$

We know that $a_n = a + (n-1)d$

$$= 5 + (n-1)\,4$$

$$= 4n + 1$$

26. Consider an AP = 504, 511, 518, ..., 896

Here, first term (a) = 504

Common difference (d) = 511 – 504 = 7

Last term (a_n) = 896

As $a_n = 896$

$$a + (n-1)d = 896$$

$$504 + (n-1)\,7 = 896$$

$$7\,(n-1) = 392$$

$$n - 1 = 56$$

$$n = 57$$

We know that $S_n = \frac{n}{2}[2a + (n-1)\,d]$

$$\therefore S_{57} = \frac{57}{2}[2 \times 504 + (57-1)7]$$

$$= \frac{57}{2}[1008 + 392]$$

$$= 39900$$

27. First term (a) = 5

Let d be the common difference.

$$S_4 = \frac{1}{2}[S_8 - S_4]$$

i.e. $\frac{4}{2}[2a + (4-1)\,d]$

$$= \frac{1}{2}\left\{\frac{8}{2}\left[2a + (8-1)\,d\right] - \frac{4}{2}\left[2a + (4-1)d\right]\right\}$$

i.e. $2\,(2a + 3d) = 2\,(2a + 7d) - (2a + 3d)$

$$4a + 6d = 4a + 14d - 2a - 3d$$

$$4a + 6d = 2a + 11d$$

$$2a = 5d$$

$$d = \frac{2a}{5} = \frac{2}{5}(5) = 2$$

So, common difference (d) = 2

28. A.P: 3, 9, 15, ..., 99

Here, first term (a) = 3

Common difference (d) = 9 – 3 = 6

Last term (a_n) = 99

$$a + (n-1)d = 99$$
$$3 + (n-1)6 = 99$$
$$6(n-1) = 96$$
$$n - 1 = 16$$
$$n = 17$$

We know that $S_n = \frac{n}{2}[2a + (n-1)d]$

$$S_{17} = \frac{17}{2}[6 + (17-1)6]$$
$$= \frac{17}{2}[6 + 96]$$
$$= 867$$

29. First term (a) = 8

Last term (a_n) = 350

Common difference (d) = 9

As $a_n = 350$

$$a + (n-1)d = 350$$
$$8 + (n-1)9 = 350$$
$$9(n-1) = 342$$
$$n - 1 = \frac{342}{9} = \frac{114}{3} = 38$$
$$n - 1 = 38$$
$$n = 39$$

We know that

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$S_{39} = \frac{39}{2}[16 + (39-1)9]$$
$$= \frac{39}{2}[16 + 342]$$
$$= 6981$$

30. Let the first term of an AP be 'a' and common difference be 'd'.

$$S_{10} = -150$$
$$\frac{10}{2}[2a + (10-1)d] = -150$$
$$5(2a + 9d) = -150$$
$$2a + 9d = -30 \quad \text{...(i)}$$

Also, $S_{20} - S_{10} = -550$

$$\frac{20}{2}(2a + 19d) - \frac{10}{2}(2a + 9d) = -550$$
$$10(2a + 19d) - 5(2a + 9d) = -550$$
$$2(2a + 19d) - (2a + 9d) = -110$$
$$4a + 38d - 2a - 9d = -110$$
$$2a + 29d = -110 \quad \text{...(ii)}$$

On subtracting (ii) from (i), we get

$$2a + 9d - 2a - 29d = -30 + 110$$
$$-20d = 80$$
$$d = -4$$

From (i), $2a + 9(-4) = -30$

$$2a = -30 + 36$$
$$2a = 6$$
$$a = 3$$

So, A.P. is a, a + d, a + 2d, ...

i.e. 3, 3 – 4, 3 – 8, ...

i.e. 3, –1, –5, ...

Section D

31. Let A, D be first term and common difference respectively.

$$S_p = a \Rightarrow \frac{p}{2}[2A + (p-1)D] = a$$

$$S_q = b \Rightarrow \frac{q}{2}[2A + (q-1)D] = b$$

$S_r = c \Rightarrow \frac{r}{2}[2A + (r-1)D] = c$

Consider,

$\frac{a}{p}(q-r) + \frac{b}{q}(r-p) + \frac{c}{r}(p-q)$

$= \frac{1}{p}\frac{p}{2}[2A + (p-1)D](q-r) + \frac{1}{q}\frac{q}{2}[2A + (q-1)D](r-p) + \frac{1}{r}\frac{r}{2}[2A + (r-1)D](p-q)$

$= \frac{1}{2}[2A + (p-1)D](q-r) + \frac{1}{2}[2A + (q-1)D](r-p) + \frac{1}{2}[2A + (r-1)D](p-q)$

$= [A(q-r) + A(r-p) + A(p-q)] + \frac{D}{2}[(p-1)(q-r) + (q-1)(r-p) + (r-1)(p-q)]$

$= A(q-r+r-p+p-r) + \frac{D}{2}[(p-1)(q-r) + (q-1)(r-p) + (r-1)(p-q)]$

$= 0 + \frac{D}{2}[pq - pr - q + r + qr - qp - r + p + rp - rq - p + q]$

$= 0 + 0$

$= 0$

32. Let a and d be the first term and common term of an A.P.

$$a_m = \frac{1}{n}$$

$$a + (m-1)d = \frac{1}{n} \quad ...(i)$$

Also, $$a_n = \frac{1}{m}$$

$$a + (n-1)d = \frac{1}{m} \quad ...(ii)$$

On subtracting (i) from (ii), we get

$$a + (n-1)d - a - (m-1)d = \frac{1}{m} - \frac{1}{n}$$

$$d(n-1-m+1) = \frac{n-m}{mn}$$

$$d(n-m) = \frac{n-m}{mn}$$

$$d = \frac{1}{mn}$$

From (i), $a + (m-1)\frac{1}{mn} = \frac{1}{n}$

$$a + (m-1)\frac{1}{mn} = \frac{1}{n}$$

$$a + \frac{1}{n} - \frac{1}{mn} = \frac{1}{n}$$

$$a = \frac{1}{mn}$$

Consider,

$$S_{mn} = \frac{mn}{2}\left[\frac{2}{mn} + (mn-1)\frac{1}{mn}\right]$$

$$= \frac{mn}{2}\left[\frac{2}{mn} + 1 - \frac{1}{mn}\right]$$

$$= \frac{mn}{2}\left[\frac{1}{mn} + 1\right]$$

$$= \frac{mn}{2}\left[\frac{mn+1}{mn}\right]$$

$$= \frac{1}{2}(mn+1)$$

33. Length of each step = 50 m

Width of each step = $\frac{1}{2}$ m

Height of first step = $\frac{1}{4}$ m

Height of second step = $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$ m

Height of third step = $\frac{3}{4}$ m so on.

Volume of concrete required to build the first step (V_1) = $50 \times \frac{1}{2} \times \frac{1}{4}\ m^3$

Volume of concrete required to build the second step (V_2) = $50 \times \frac{1}{2} \times \left(2 \times \frac{1}{4}\right)$

(V_3) = $50 \times \frac{1}{2} \times \frac{3}{4}\ m^3$ and so on.

Total volume of concrete

$$= V_1 + V_2 + V_3 + \ldots + V_{15}$$

$$= \left(50 \times \frac{1}{2} \times \frac{1}{4}\right) + \left[50 \times \frac{1}{2} \times \left(2 \times \frac{1}{4}\right)\right]$$

$$+ \left(50 \times \frac{1}{2} \times 3 \times \frac{1}{4}\right) + \ldots. + \left[50 \times \frac{1}{2} \times \left(15 \times \frac{1}{4}\right)\right]$$

$$= \left(50 \times \frac{1}{2}\right)\left[\frac{1}{4} + \frac{2}{4} + \frac{3}{4} + \ldots. + \frac{15}{4}\right] m^3$$

$$= 25 \left[\frac{1}{4} + \frac{2}{4} + \frac{3}{4} + \ldots. + \frac{15}{4}\right] m^3$$

$$= \frac{25}{4} (1 + 2 + \ldots. + 15) \ m^3$$

$$= \frac{25}{4} \times \frac{15}{2} (1 + 15) = 750 \ m^3$$

34. Let the first term and common difference of an A.P. be a and d respectively.

Let S and S^I be the sum of odd terms and even terms of A.P.

$$S = a_1 + a_3 + a_5 + \ldots. + a_{2n+1}$$

$$= \frac{n+1}{2} (a_1 + a_{2n+1})$$

$$= \frac{n+1}{2} [a + a + (2n + 1 - 1) d]$$

$$= (n + 1) (a + nd)$$

$$S^I = a_2 + a_4 + a_6 + \ldots. + a_{2n}$$

$$S^I = \frac{n}{2} [2a + 2nd]$$

$$= n (a + nd)$$

Consider $\dfrac{S}{S^I} = \dfrac{(n+1)(a+nd)}{n(a+nd)}$

$$= \frac{n+1}{n}$$

35. Consider 1, 2, 3, ..., 999, 1000

This sequence forms an AP with first term

(a) = 1 and common difference (d) = 1

We know that

$$S_n = \frac{n}{2} [2a + (n - 1) d]$$

$$S_{1000} = \frac{1000}{2} [2 + (1000 - 1) 1]$$

$$= 500 (2 + 999)$$

$$= 500 (1001)$$

$$= 500500$$

Now consider list of numbers divisible by 2: 2, 4, 6, 8, ..., 1000

This sequence also forms an AP with a = 2, d = 2, $n = \dfrac{1000}{2} = 500$

$$S_{500} = \frac{500}{2} [2 (2) + (500 - 1) 2]$$

$$= 250 (4 + 499 (2)]$$

$$= 250500$$

Again, consider list of numbers divisible by 5: 5, 10, 15, ..., 1000

Here, a = 5, d = 5, $n = \dfrac{1000}{5} = 200$

$$S_{200} = \frac{200}{2} [10 + (200 - 1) 5]$$

$$= 100 [10 + 5 (199)]$$

$$= 100500$$

Now, we will consider list of numbers divisible by both 2 and 5 i.e. 2 × 5 = 10

10, 20, 30, ..., 1000

This list of numbers form an AP with

a = 10, d = 10, $n = \dfrac{1000}{10} = 100$

$$S_{100} = \frac{100}{2} [20 + (100 - 1) 10]$$

$$= 50 (20 + 990)$$

$$= 50500$$

Therefore, sum of numbers which are either divisible by 2 or 5

$$= S_{200} + S_{500} - S_{100}$$

$$= 100500 + 250500 - 50500$$

$$= 300500$$

So, sum of numbers from 1 to 1000 that are neither divisible by 2 nor by 5

$$= S_{1000} - 300500$$
$$= 500500 - 300500$$
$$= 200000$$

36. Suppose the work is completed in n days.

Consider an AP: 150, 146, 142, ...

Here, First term (a) = 150

Common difference (d) = – 4

Total number of workers who worked all the n days = S_n

$$= \frac{n}{2}[2(150) + (n-1)(-4)]$$
$$= \frac{n}{2}(300 - 4n + 4)$$
$$= \frac{n}{2}[304 - 4n]$$
$$= n(152 - 2n)$$

If the workers did not drop, work would have been finished in (n – 8) days such that 150 workers work on each day.

∴ Total number of workers who worked all the n days = 150 (n – 8)

$$\therefore \quad n(152 - 2n) = 150(n - 8)$$
$$152n - 2n^2 = 150n - 1200$$
$$152n - 150n = 2n^2 - 1200$$
$$2n^2 - 2n - 1200 = 0$$
$$n^2 - n - 600 = 0$$
$$n^2 - 25n + 24n - 600 = 0$$
$$n(n - 25) + 24(n - 25) = 0$$
$$(n + 24)(n - 25) = 0$$
$$n = -24, n = 25$$

Being the number of days, n cannot be negative, so, n = 25

∴ Work was completed in 25 days.

37. Consider the sequence: 200, 250, 300, ...

This sequence form an AP with first term (a) = 200 and common difference (d) = 50

We know that

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$S_{30} = \frac{30}{2}[2(200) + (30-1)50]$$
$$= 15[400 + 1450]$$
$$= 27,750$$

∴ The contractor has to pay ₹ 27,750 as penalty, if he has delayed the work by 30 days.

38. Consider AP: 20, 19, 18, ...

Here, First term (a) = 20

Common difference (d) = – 1

Let 200 logs be placed in n rows

$$\therefore S_n = 200$$
$$\frac{n}{2}[2(20) + (n-1)(-1)] = 200$$
$$\frac{n}{2}[40 - n + 1] = 200$$
$$n(41 - n) = 400$$
$$-n^2 + 41n - 400 = 0$$
$$n^2 - 41n + 400 = 0$$
$$n^2 - 16n - 25n + 400 = 0$$
$$n(n - 16) - 25(n - 16) = 0$$
$$(n - 16)(n - 25) = 0$$
$$n = 16 \text{ or } 25$$

If n = 25,

$$a_{25} = 20 + (25 - 1)(-1)$$
$$= 20 - 24$$
$$= -4 \quad \text{not possible}$$

So, n = 16

So, 200 logs are placed in 16 rows.

$$a_{16} = 20 + (16 - 1)(-1)$$
$$= 20 - 15 = 5$$

So, there are 5 logs in the top row.

39. Given : a^2, b^2, c^2 are in A.P.

To prove : $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P.

$\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in AP if

$$\frac{1}{c+a} - \frac{1}{b+c} = \frac{1}{a+b} - \frac{1}{c+a}$$

i.e. $$\frac{(b+c)-(c+a)}{(b+c)(a+c)} = \frac{(a+c)-(a+b)}{(a+b)(a+c)}$$

i.e. $$\frac{b+c-c-a}{(b+c)(a+c)} = \frac{a+c-a-b}{(a+b)(a+c)}$$

i.e. $$\frac{b-a}{(b+c)(a+c)} = \frac{c-b}{(a+b)(a+c)}$$

i.e. $$\frac{b-a}{b+c} = \frac{c-b}{a+b}$$

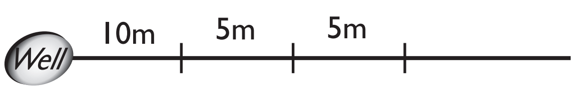
i.e. $(b - a)(a + b) = (c - b)(b + c)$

i.e. $ab + b^2 - a^2 - ab = bc + c^2 - b^2 - bc$

i.e. $b^2 - a^2 = c^2 - b^2$

∴ a^2, b^2, c^2 are in A.P.

40.



Distance covered by gardener to water 1st tree and return to the initial position

= 10 m + 10 m = 20 m

Distance covered by gardener to water 2nd tree and return to initial position

= 15 m + 15 m = 30 m

Distance covered by gardener to water 3rd tree and return to initial position

= 20 m + 20 m = 40 m

So, we get an AP: 20, 30, 40, ...

Where, first term (a) = 20

difference (d) = 10

Total distance covered by the gardener

$= S_{25}$

$= \frac{25}{2}[2(20) + (25 - 1)10]$

$= \frac{25}{2}[40 + 240]$

$= \frac{25}{2} \times 280$

$= 25 \times 140$

= 3500 m

∴ Total distance covered by the gardener to water all trees = 3500 m

CASE STUDY-1

(i) (a) $A_{16} = a + 15l$

$A_9 = a + d$

$A_{16} = A_9 + 7$

$A_{16} - A_9 = 7$

$a + 15d - (a + 8d) = 7$

$15d - 8d = 7$

$d = 1$

(ii) (c) $A_4 = a + 3d$

$A_8 = a + 7d$

$A_4 + A_8 = 30$

$a + 3d + a + 7d = 30$

$2a + 10d = 30$

$a + 5d = 15$

a = 0 ∵ the origin is occupied by the teacher.

∴ $5d = 15$

$d = 3$

(iii) (c) $A20 - A6 = 84$

$A20 = a + 19d$

$A6 = a + 5d$

$a + 19d - (a + 5d) = 84$

$14d = 84$

$d = 6$

(iv) (d) There are total 21 persons standing in the queue the 10th person is the middle one.

(v) (d) The positions of the teacher and students are in AP hence the distance. $A_9 A_{10}$ is equal to the distance between any two consecutive persons.

$\therefore$ Distance $A_9 A_{10}$ = Distance $A_{19} A_{20}$

CASE STUDY-2

(i) (b) The 20^{th} row is the bottom row. Hence an is 20.

(ii) (b) a = Number of logs in first row

n = total number of logs

a = 1

n = 20

a + n = 21

(iii) (c) The sum of logs is 200.

$\frac{n}{2}$ using the formula for the sum of AP whose common difference is –1 and first term is 20.

$S_n = \frac{n}{2}[2a + (n-1)d]$

$200 = \frac{n}{2}[2(20) + (n-1)(-1)]$

$400 = n[40 + 1 - n]$

$n^2 - 414 + 400 = 0$

(iv) (c) The number of rows can be calculated by solving the below given linear equation.

$n^2 - 41n + 400 = 0$

$$n = \frac{-(-41) \pm \sqrt{(-41)^2 - 4(1)(400)}}{2}$$

$$= \frac{-41 \pm \sqrt{1681 - 1600}}{2}$$

$$= \frac{41 \pm 9}{2}$$

$n = \frac{41+9}{2} = 25$ | $n = \frac{41-9}{2} = 16$

For n = 16, the number of logs in the 16th row is:

$a_{16} = a + (n-1)d$

$a = 20, d = -1, n = 16$

$a_{16} = 20 + 15(-1)$

$= 5$

For n = 25, the number of logs in 25^{th} row is:

$a_{25} = 20 + (25-1)(-1)$

$= 20 - 24$

$= -4$

As the number of logs cannot be negative hence n = 16.

(v) (b) The 16th row from the bottom is the top row. Number of logs in the 16th row is

$a^{16} = a + (16-1)d$

$= 20 + 15(-1)$

$= 20 + 5$

$= 5$

Chapter 6

Triangles

Multiple Choice Questions

1. (b) Δ ABC ~ Δ PQR

$$\therefore \frac{ar\Delta ABC}{ar\Delta PQR} = \left(\frac{BC}{QR}\right)^2$$

$$\Rightarrow \frac{9}{16} = \frac{(4.5)^2}{(QR)^2}$$

$$\Rightarrow (QR)^2 = \frac{4.5\times 4.5\times 16}{9}$$

$$\Rightarrow QR = \frac{4.5\times 4}{3}$$

$$= 1.5 \times 4$$

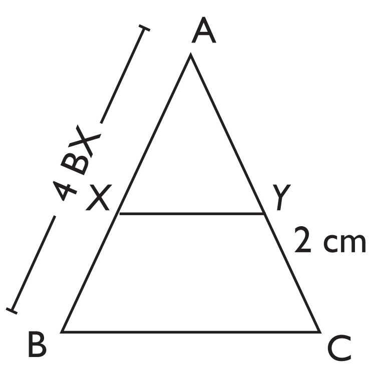
$$= 6 \text{ cm}$$

2. (a) We know that ratio of area of two similar triangles is equal to square of ratio of their corresponding sides (say x and y)

$$\Rightarrow \frac{9}{16} = \left(\frac{x}{y}\right)^2$$

$$\Rightarrow \frac{x}{y} = \frac{3}{4}$$

3. (d)



AS $XY \parallel BC$, so by basic proportionality theorem

$$\frac{AX}{BX} = \frac{AY}{YC}$$

$$\frac{AX}{BX} + 1 = \frac{AY}{YC} + 1$$

$$\frac{AB}{BX} = \frac{AC}{CY}$$

$$\frac{4\ BX}{BX} = \frac{AC}{2} \quad (\because AB = 4\ BX)$$

$$4 = \frac{AC}{2}$$

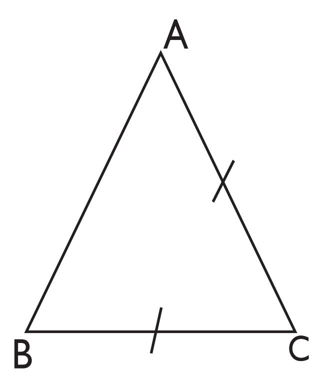
$$AC = 8 \text{ cm}$$

So, $AY = AC - CY$

$= 8 - 2$

$= 6$ cm

4. (c)



$AB^2 = 2\,AC^2$

$= AC^2 + AC^2$

$= AC^2 + BC^2$

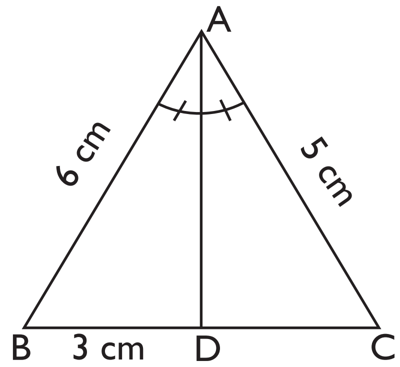
$[\because AC = BC]$

$\therefore AB^2 = AC^2 + BC^2$

$\therefore$ ΔABC is a triangle right angled at C

i.e. $\angle C = 90^\circ$

5.(b)



As AD bisects $\angle BAC$

$$\therefore \quad \frac{AB}{BD} = \frac{AC}{CD}$$

[By internal angle bisector theorem]

$$\Rightarrow \quad \frac{6}{3} = \frac{5}{CD}$$

$$\Rightarrow \quad CD = \frac{3 \times 5}{6} = 2.5 \text{ cm}$$

WORKSHEET - 1

SECTION-A

1. $\Delta ABC \sim \Delta DEF$

In ΔABC, $\angle A + \angle B + \angle C = 180^\circ$

$57^\circ + \angle B + 73^\circ = 180^\circ$

$\angle B + 130^\circ = 180^\circ$

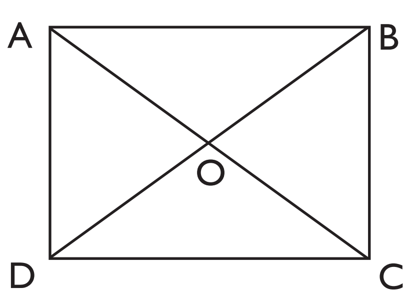
$\angle B = 108^\circ - 130^\circ$

$= 50^\circ$

$\therefore \quad \angle E = \angle B = 50^\circ$

[Corresponding angles of similar triangles are equal.]

2.



AC = 30 cm

BD = 40 cm

$OA = OC = \frac{1}{2} AC = 15$ cm

$OB = OD = \frac{1}{2} BD = 20$ cm

In ΔAOB, $\angle AOB = 90^\circ$

(Diagonals of rhombus bisect each other at 90°)

$AB^2 = AO^2 + OB^2$ (Pythagoras theorem)

$= (15^2) + (20^2)$

$= 225 + 400$

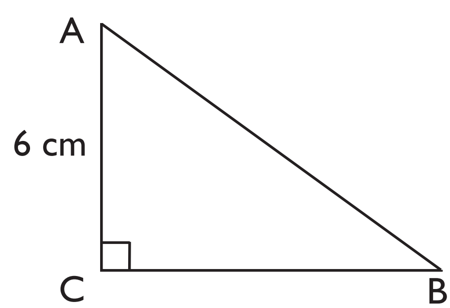
$= 625$

AB = 25 cm

$\therefore$ AB = BC = CD = AD = 25 cm

(All sides of rhombus are equal)

3.



In ΔABC,

AC = BC = 6 cm (As ΔABC is isosceles)

Also, $\angle C = 90^\circ$

$\therefore \quad AB^2 = AC^2 + BC^2$ (Pythagoras theorem)

$= 6^2 + 6^2$

$= 36 + 36$

$AB^2 = 72$

$AB = 6\sqrt{2}$ cm

4. As, $\Delta DEF \sim \Delta ABC$

$$\frac{DE}{AB} = \frac{EF}{BC} = \frac{DF}{AC}$$

$$\frac{DE}{3} = \frac{4}{2} = \frac{DF}{2.5}$$

$\frac{DE}{3} = \frac{4}{2}$

$DE = \frac{12}{2}$

$= 6$ cm

$\frac{4}{2} = \frac{DF}{2.5}$

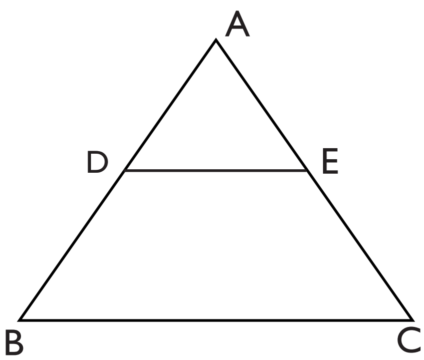
$DF = \frac{4 \times 2.5}{2}$

$= 5$ cm

Perimeter of Δ DEF = DE + EF + DF

= 6 + 4 + 5 = 15 cm

5.



Let AE = x cm

$\therefore$ CE = AC – AE = 5.6 – x cm

As DE II BC,

$$\frac{AD}{DB} = \frac{AE}{CE}$$

(By Basic proportionality theorem)

$$\frac{3}{5} = \frac{x}{5.6 - x}$$

5x = 3 (5.6 – x)

5x = 16.8 – 3x

8x = 16.8

x = 2.1 cm

$\therefore$ AE = x =2.1 cm

6. We know that ratio of the areas of two similar triangles is equal to the square of their altitudes.

$\therefore$ Ratio of areas $= \left(\frac{2}{3}\right)^2 = 4:9$

7. $\Delta ABC \sim \Delta PQR$

$$\frac{ar\Delta ABC}{ar\Delta PQR} = \left(\frac{BC}{QR}\right)^2$$

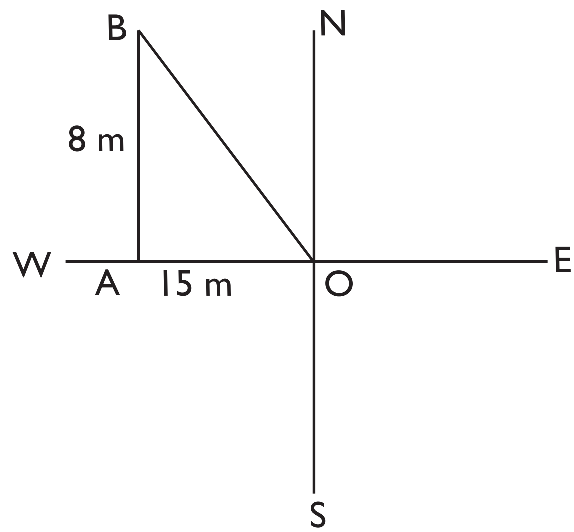
(Ratio of areas of two similar triangles is equal to square of their corresponding sides)

$$\frac{54}{ar\Delta PQR} = \frac{1^2}{3^2}$$

$$\frac{54}{ar\Delta PQR} = \frac{1}{9}$$

$$ar\ \Delta PQR = \frac{54 \times 9}{1} = 486 \text{ cm}^2$$

8.



In ΔBAO, $\angle BAO = 90°$

$OB^2 = AB^2 + AO^2$ (Pythagoras theorem)

$= 8^2 + 15^2$

$= 64 + 225$

$= 289$

$\therefore$ OB = 17 m

SECTION-B

9. $\Delta ABC \sim \Delta DEF$,

$$\frac{ar\Delta ABC}{ar\Delta DEF} = \frac{BC^2}{EF^2}$$

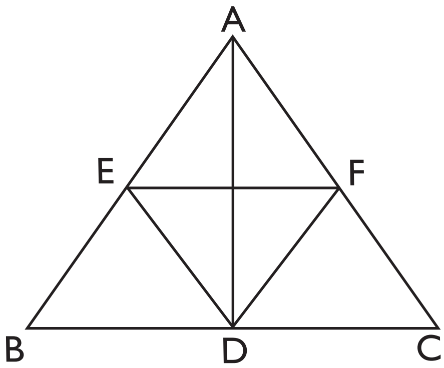
(In two similar triangles, the ratio of their areas is the square of ratio of their sides)

$$\frac{64}{121} = \frac{BC^2}{(15.4)^2}$$

$$(BC)^2 = \frac{64}{121} \times 15.4 \times 15.4$$

$\therefore$ BC = 11.2 cm

10.



In $\triangle ADB$, DE is bisector of $\angle ADB$

$$\frac{BD}{BE} = \frac{AD}{AE}$$

i.e. $$\frac{BD}{AD} = \frac{BE}{AE} \quad \text{(i)}$$

In $\triangle ADC$, DF is bisector of $\angle ADC$

$$\frac{CD}{CF} = \frac{AD}{AF}$$

i.e. $$\frac{CD}{AD} = \frac{CF}{AF}$$

$$\frac{BD}{AD} = \frac{CF}{AF} \quad \text{(ii)}$$

(As AD is median $\therefore$ BD = CD)

From (i) and (ii), we get

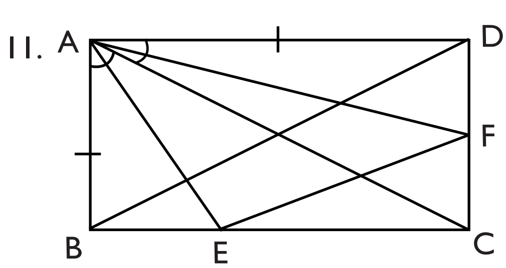
$$\frac{BE}{AE} = \frac{CF}{AF}$$

$$\frac{AE}{BE} = \frac{AF}{CF}$$

So, by converse of Basic proportionality theorem

EF II BC

11.



In Δ ADC, AF bisects $\angle$ DAC

$$\therefore \frac{CF}{DF} = \frac{AC}{AD}$$

$$= \frac{AC}{AB} \quad \text{(As AB = AD)} \quad ...(i)$$

In $\triangle ABC$, AE bisects $\angle$ BAC

$$\frac{CE}{BE} = \frac{AC}{AB} \quad ...(ii)$$

From (i) and (ii), we get

$$\frac{CF}{DF} = \frac{CE}{BE}$$

$\therefore$ EF II BD

(By converse of Basic proportionaly theorem)

12. In $\triangle AOB \sim \triangle COD$

$\angle AOB = \angle COD$ (Vertically opposite angles)

$$\frac{AO}{OC} = \frac{BO}{DO} \quad \text{(Given)}$$

$\therefore$ $\triangle AOB \sim \triangle AOB$ (SAS)

So, $$\frac{AO}{OC} = \frac{BO}{OD} = \frac{AB}{DC}$$

(Corresponding sides of similar triangles are proportional)

$$\frac{1}{2} = \frac{5}{CD}$$

CD = 10 cm

13. In Δ KPN and Δ KLM,

$\angle K = \angle K$ (Common)

$\angle KNP = \angle KML = 46°$ (Given)

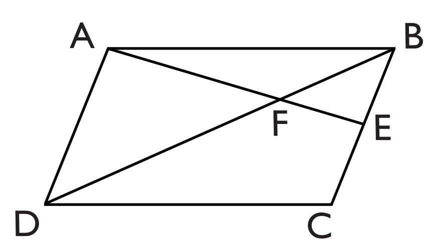
$\therefore$ $\triangle KPN \sim \triangle KLM$ (AA similarity criterion)

$$\frac{KP}{KL} = \frac{PN}{LM} = \frac{KN}{KM}$$

$$\frac{x}{a} = \frac{c}{b + c}$$

$$x = \frac{ac}{b + c}$$

14.



In $\triangle AFD$ and $\triangle BEF$

$\angle DAF = \angle FEB$ (Alternate interior angles)

$\angle AFD = \angle BFE$ (Vertically opposite angles)

$\therefore \Delta AFD \sim \Delta EFB$

So, $\frac{EF}{FA} = \frac{FB}{DF}$

(Corresponding sides of similar triangles are proportional)

DF x EF = FB x FA

15. As DE II AC, so in ΔABC,

$\frac{BD}{AD} = \frac{BE}{EC}$ (i)

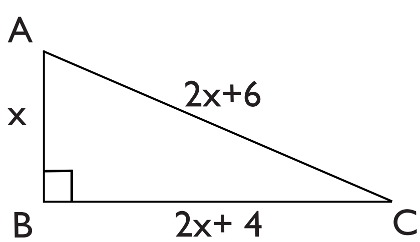
(Basic proportionality theorem)

Also, $\frac{BE}{EC} = \frac{BC}{CP}$ (ii) (Given)

From (i) and (ii), we have $\frac{BD}{AD} = \frac{BC}{CP}$

$\therefore$ DC II AP

(By converse of Basic proportionality theorem.)

16. 

Let the shorter side be x m

$\therefore$ Hypotenuse $= 2x + 6$ m

Also, Third side $= 2x + 6 - 2$

$= 2x + 4$ m

In ΔABC, $AC^2 = AB^2 + BC^2$

$(2x + 6)^2 = x^2 + (2x + 4)^2$

$4x^2 + 36 + 24x = x^2 + 4x^2 + 16 + 16x$

$0 = x^2 - 20 - 8x$

$x^2 - 8x - 20 = 0$

$x^2 - 10x + 2(x^2 - 10) = 0$

$(x^2 - 10)(x - 2) = 0$

$x = 10, -2$

Being a side, $x = -2$ is rejected

$\therefore x = 10$

So, AB $= 10$ m

BC $= 2x + 4 = 24$ m

AC $= 2x + 6 = 26$ m

17. We know that diagonals of rhombus bisect each other at 90°.

Let AC = 24 cm

BD = 10 cm

$AO = OC = \frac{1}{2} AC = 12$ cm

$BO = OD = \frac{1}{2} BD = 5$ cm

In ΔAOB,

$AB^2 = BO^2 + AO^2$

$= 5^2 + 12^2$

$= 25 + 144$

$= 169$

$\therefore$ AB $= 13$ cm

As all sides of rhombus are equal,

AB = BC = CD = AD = 13 cm

18. In ΔABC, DE II BC

$\frac{AD}{DB} = \frac{AE}{EC}$

[Basic proportionality theorem]

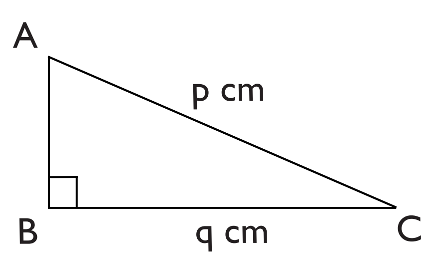
$\frac{x}{x-2} = \frac{x+2}{x-1}$

$x(x-1) = (x-2)(x+2)$

$x^2 - x = x^2 - 4$

$x = 4$

SECTION-C

19. 

In Δ ABC, $AC^2 = AB^2 + BC^2$

$p^2 = AB^2 + q^2$

$AB^2 = p^2 - q^2$

$= (p - q)\ (p + q)$

$= 1\ (p + q)$

$AB^2 = p + q$

$AB = \sqrt{p+q}$

20. $\frac{QT}{PR} = \frac{QR}{QS}$ (Given)

In ΔPQR, $\angle 1 = \angle 2$

$\therefore$ PQ = PR

[sides opposite to equal angles are equal]

So, $\frac{QT}{PR} = \frac{QR}{QS}$

Also, $\angle Q = \angle Q$ (common)

$\therefore$ $\Delta PQS \sim \Delta TQR$

[By SAS Similarity criterion]

21. In ΔCBQ and ΔCAP,

$\angle BCQ = \angle ACP$ (common)

$\angle QBC = \angle PAC = 90°$

(PA and QB are perpendicular)

$\therefore$ $\Delta CBQ \sim \Delta CAP$ (AA Similarity criterion)

$\frac{BC}{AC} = \frac{BQ}{AP} = \frac{CQ}{CP}$

[Corresponding sides of similar triangles are proportional]

$\frac{BC}{AC} = \frac{z}{x}$ ...(i)

In ΔABQ and ΔACR,

$\angle BAQ = \angle CAR$ (common)

$\angle ABQ = \angle ACR = 90°$

(BQ and RC are perpendicular)

$\therefore$ $\Delta ABQ \sim \Delta ACR$ (AA Similarity criterion)

$\frac{AB}{AC} = \frac{BQ}{CR} = \frac{AQ}{AR}$

$\frac{AB}{AC} = \frac{z}{y}$ ...(ii)

From (i),

$1 - \frac{BC}{AC} = 1 - \frac{z}{x}$

$\frac{AC - BC}{AC} = \frac{x - z}{x}$

$\frac{AB}{AC} = \frac{x - z}{x}$ ...(iii)

From (ii) and (iii)

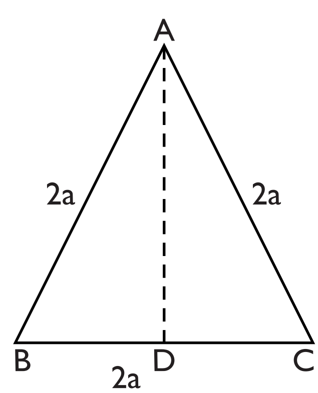
$\frac{AB}{AC} = \frac{z}{y} = \frac{x - z}{x}$

$\frac{z}{y} = 1 - \frac{z}{x}$

$\frac{z}{x} + \frac{z}{y} = 1$

$\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$

22.



Draw AD $\perp$ BC

In ΔADB and ΔADC

AB = AC = 2a (Given)

AD = AD (Common)

$\angle ADB = \angle ADC$

= 90° (By Construction)

$\therefore\ \Delta ADB \cong \Delta ADC$ (RHS)

$\therefore\ BD = DC = \frac{1}{2} BC$

$= a$ (CPCT)

In ΔADC, right angled at D

$AC^2 = AD^2 + DC^2$

$(2a)^2 = AD^2 + a^2$

$AD^2 = 4a^2 - a^2 = 3a^2$

$AD = \sqrt{3}\,a$

So, length of the altitude of an equilateral triangle $= \sqrt{3}\,a$ cm

23. In ΔAOB, $XY \parallel AB$

$\therefore\ \frac{OX}{AX} = \frac{OY}{BY}$ ...(i)

[Basic proportionality theorem]

In ΔAOC, $XZ \parallel AC$

$\therefore\ \frac{OZ}{ZC} = \frac{OX}{AX}$ ...(ii)

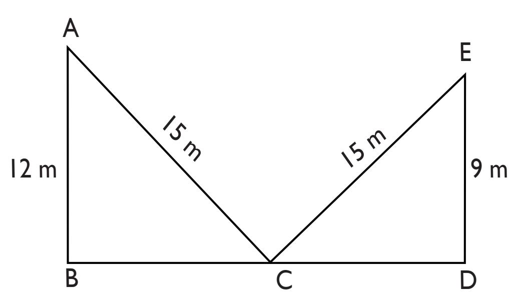
[Basic proportionality theorem]

By (i) and (ii), $\frac{OY}{BY} = \frac{OZ}{ZC}$

$\therefore\ YZ \parallel BC$

[By Converse of Basic proportionality theorem]

24.



Let AC = CE denotes the ladder

In ΔABC, $AC^2 = AB^2 + BC^2$

$15^2 = 12^2 + BC^2$

$225 - 144 = BC^2$

$BC^2 = 81$

$BC = 9$ m

In ΔCDE, $CE^2 = DE^2 + CD^2$

$15^2 = 9^2 + CD^2$

$225 - 81 = CD^2$

$144 = CD^2$

$12 = CD$

So, $BD = BC + CD$

$= 9 + 12 = 21$ m

SECTION-D

25. In ΔXPQ and ΔXYZ,

$\frac{XP}{PY} = \frac{XQ}{QZ} = 3$ (Given)

$\angle X = \angle X$ (Common)

$\therefore\ \Delta XPQ \sim \Delta XYZ$

(SAS Similarity criterion)

So, $\frac{ar\,\Delta XPQ}{ar\,\Delta XYZ} = \left(\frac{XP}{XY}\right)^2 = \left(\frac{PQ}{YZ}\right)^2 = \left(\frac{XQ}{XZ}\right)^2$

[Ratio of area of two similar triangles is equal to square of their corresponding sides]

$\frac{ar\,\Delta XPQ}{32} = \left(\frac{XQ}{XZ}\right)^2 = \left(\frac{3}{4}\right)^2$

$ar\,\Delta XPQ = \frac{9}{16} \times 32 \quad \left[\begin{array}{l} \frac{XP}{PY} = 3 \\ \frac{PY}{XP} = \frac{1}{3} \\ \frac{PY}{XP} + 1 = \frac{1}{3} + 1 \\ \frac{XY}{XP} = \frac{4}{3} \end{array}\right]$

$= 18$ cm²

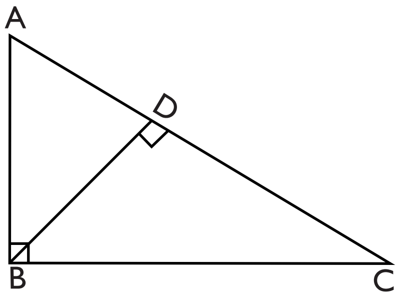
area of quadrilateral PYZQ

$= ar\,\Delta XYZ - ar\,\Delta XPQ$

$= 32 - 18$ cm²

$= 14$ cm²

26.



In ΔABC, right angled at B,

We need to prove $AC^2 = AB^2 + BC^2$

Draw $BD \perp AC$

We know that if a perpendicular drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.

So, ΔCBA and ΔCDB

$\frac{CB}{CD} = \frac{CA}{CB}$ [Corresponding sides of similar triangles are proportional]

$CB^2 = CA \times CD$ ...(i)

Also, ΔABC and ΔADB

$\frac{AB}{AD} = \frac{BC}{BD} = \frac{AC}{AB}$

$\frac{AB}{AD} = \frac{AC}{AB}$

$AB^2 = AC \times AD$ ...(ii)

From (i) and (ii),

$$AB^2 + BC^2 = AC \times AD + AC \times CD$$
$$= AC (AD + CD)$$
$$= AC \times AC$$
$$= AC^2$$

$\therefore AB^2 + BC^2 = AC^2$

27. As $XY \parallel AC$

$\angle BXY = \angle A$ (Corresponding angles)

$\angle BYX = \angle C$ (Corresponding angles)

$\therefore \Delta ABC \sim \Delta XBY$ (AA Similarity criterion)

So, $\frac{ar\,\Delta ABC}{ar\,\Delta XBY} = \left(\frac{AB}{XB}\right)^2$ ...(i)

[Ratio of areas of two similar triangles is equal to square of ratio of their corresponding sides]

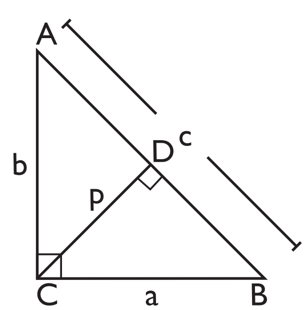
Also, ar ΔABC = 2 ar ΔXBY

i.e. $\frac{ar\,\Delta ABC}{ar\,\Delta XBY} = \frac{2}{1}$ ...(ii)

From (i) and (ii),

$$\left(\frac{AB}{XB}\right)^2 = \frac{2}{1}$$
$$\frac{AB}{XB} = \frac{\sqrt{2}}{1}$$
$$\frac{AX}{AB} = \frac{1}{\sqrt{2}}$$
$$\therefore 1 - \frac{AX}{AB} = 1 - \frac{1}{\sqrt{2}}$$
$$\frac{AB - XB}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}}$$
$$\frac{AX}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}}$$
$$= \frac{2-\sqrt{2}}{2}$$

28.



In ΔACB, right angled at C such that $CD \perp AB$.

We know that if a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.

So, $\Delta BDC \sim \Delta BCA$

$$\therefore \frac{BD}{BC} = \frac{DC}{CA} = \frac{BC}{BA}$$

i.e. $\frac{p}{b} = \frac{a}{c}$

$pc = ab$

$\Rightarrow p = \frac{ab}{c}$

$\Rightarrow p^2 = \frac{a^2b^2}{c^2}$

$\Rightarrow \frac{1}{p^2} = \frac{c^2}{a^2b^2}$

In $\Delta ACB, AC^2 + BC^2 = AB^2$

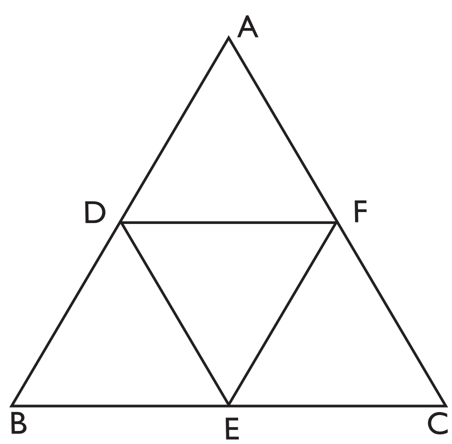
$b^2 + a^2 = c^2$

$$\frac{1}{p^2} = \frac{a^2 + b^2}{a^2 b^2}$$

$$\frac{1}{p^2} = \frac{a^2}{a^2 b^2} + \frac{b^2}{a^2 b^2}$$

$$\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

29.



Given : In ΔABC D, E and F are midpoints of sides AB, BC and AC respectively.

As D and E are midpoints of sides AB and BC respectively.

$\therefore$ DE || AC (Midpoint Theorem)

In ΔBDE and ΔBAC,

$\angle BDE = \angle BAC$ (Corresponding angles)

$\angle BED = \angle BCA$ (Corresponding angles)

$\therefore \Delta BDE \sim \Delta BAC$ (AA Similarity criterion)

Also, E and F are midpoints of sides BC and AC respectively.

$\therefore$ EF || AB (Midpoint Theorem)

In ΔCFE and ΔCAB,

$\angle CFE = \angle CAB$ (Corresponding angles)

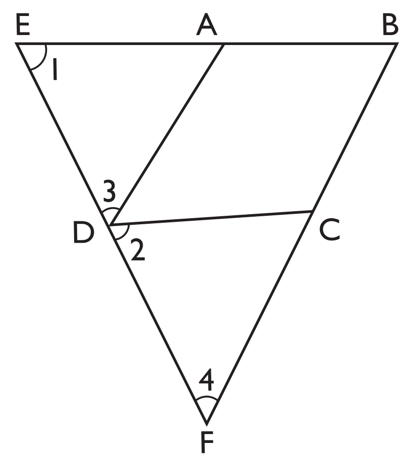
$\angle CEF = \angle CBA$ (Corresponding angles)

$\therefore \Delta CFE \sim \Delta CAB$ (AA similarity criterion)

Similarly, we can prove $\Delta AFD \sim \Delta ACB$

So, the line segment joining the midpoints of the sides of a triangle form four triangles, each of which is similar to the original triangle.

30.



Consider ΔEDA and ΔEFB

$\angle 1 = \angle 1$ (Common)

$\angle 3 = \angle 4$

[Corresponding angles as AD || BF]

$\therefore \Delta EDA \sim \Delta EFB$ (AA Similarity criterion)

$$\therefore \frac{DA}{FB} = \frac{EA}{EB}$$

[Corresponding sides of similar triangles are proportional]

$$\Rightarrow \frac{DA}{AE} = \frac{FB}{BE} \quad ...(i)$$

Consider ΔEDA and ΔDFC

$\angle 1 = \angle 2$ (Corresponding angles as BE || CD)

$\angle 3 = \angle 4$ (Corresponding angles as AD || BF)

$\therefore$ $\Delta EDA \sim \Delta DFC$ (AA Similarity criterion)

$$\therefore \frac{ED}{DF} = \frac{DA}{FC} = \frac{EA}{DC}$$

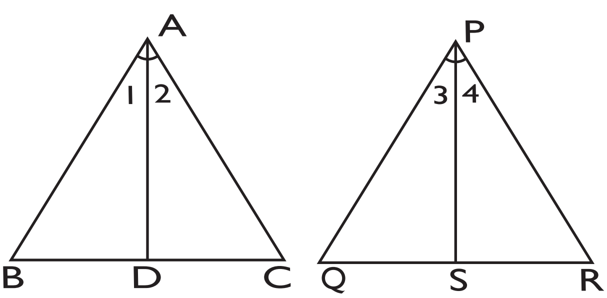
[Corresponding sides of similar triangles are proportional]

i.e. $\frac{DA}{FC} = \frac{EA}{DC}$

$\Rightarrow \frac{DA}{AE} = \frac{FC}{CD}$ ...(ii)

From (i) and (ii),

$$\frac{DA}{AE} = \frac{FB}{BE} = \frac{FC}{CD}$$

31. 

Given: AD and PS are bisectors of $\angle A$ and $\angle P$ respectively. Such that

$$\frac{BD}{DC} = \frac{QS}{SR}$$

To prove = $\Delta ABC \sim \Delta PQR$

Proof: In ΔABC, AD is bisector of $\angle A$

$\therefore \frac{AB}{BD} = \frac{AC}{CD}$

i.e. $\frac{AB}{AC} = \frac{BD}{CD}$ ...(i)

In ΔPQR, PS is bisector of $\angle P$

$\therefore \frac{PQ}{QS} = \frac{PR}{RS}$

i.e. $\frac{PQ}{PR} = \frac{QS}{RS}$ ...(ii)

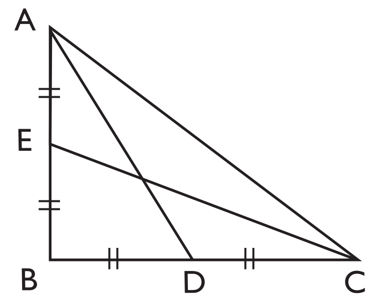
Also, $\frac{BD}{DC} = \frac{QS}{SR}$ ...(iii)

From (i), (ii), (iii), we get

$$\frac{AB}{AC} = \frac{PQ}{PR} \Rightarrow \frac{AB}{PQ} = \frac{AC}{PR}$$

Also, $\angle A = \angle P$ (Given)

$\therefore$ $\Delta ABC \sim \Delta PQR$

32. 

ΔABC is a right triangle, right-angled at B.

$\therefore$ $AD^2 = AB^2 + BD^2$

(By Pythagoras theorem)

$\Rightarrow AD^2 = AB^2 + \left(\frac{BC}{2}\right)^2$ [$\because$ BD = DC]

$\Rightarrow AD^2 = AB^2 + \frac{1}{4} BC^2$ ...(i)

Also, ΔBCE is a right triangle, right angled at B.

$\therefore$ $CE^2 = BC^2 + BE^2$

$\Rightarrow CE^2 = BC^2 + \left(\frac{AB}{2}\right)^2$ [$\because$ BE = EA]

$\Rightarrow CE^2 = BC^2 + \frac{1}{4} AB^2$ ...(ii)

On adding (i) and (ii), we get

$$AD^2 + CE^2 = \frac{5}{4}(AB^2 + BC^2)$$

$\Rightarrow AD^2 + CE^2 = \frac{5}{4} AC^2$

[As ΔABC is right triangle $\therefore AC^2 = AB^2 + BC^2$]

$\Rightarrow \left(\frac{3\sqrt{5}}{2}\right)^2 + CE^2 = \frac{5}{4}(25)$

$\Rightarrow CE^2 = \frac{125}{4} - \frac{45}{} = 20$

$\therefore$ $CE = \sqrt{20}$ cm $= 2\sqrt{5}$ cm

WORKSHEET - 2

SECTION-A

1. $\Delta ABC \sim \Delta RPQ$

$$\therefore \frac{AB}{RP} = \frac{BC}{PQ} = \frac{AC}{RQ}$$

[Corresponding sides of similar triangles are proportional]

$$\therefore \frac{3}{6} = \frac{5}{10} = \frac{6}{RQ}$$

$$RQ = \frac{6 \times 10}{5} = 12 \text{ cm}$$

2. $\Delta ABC \sim \Delta DEF$

$$\therefore \frac{ar\Delta ABC}{ar\Delta DEF} = \left(\frac{AB}{DE}\right)^2$$

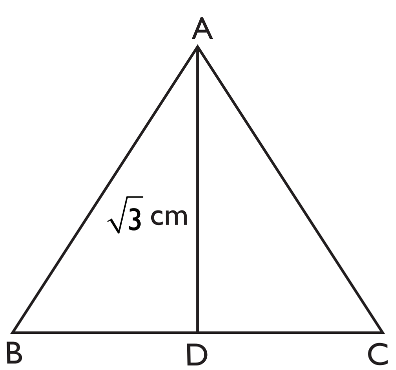
[Ratio of areas of similar triangles is proportional to the square of ratio of their corresponding sides]

$$\Rightarrow \frac{169}{121} = \frac{(26)^2}{(DE)^2}$$

$$\Rightarrow DE^2 = \frac{(26)^2 \times 121}{169}$$

$$\Rightarrow DE = \frac{26 \times 11}{13} = 22 \text{ cm}$$

3.



ΔABC is equilateral and AD is the median such that $AD = \sqrt{3}$ cm.

In an equilateral triangle, median and altitude are same.

$\therefore$ $AD \perp BC$

Also, $DC = \frac{1}{2} AC$

[As AD is the Median]

$$AC^2 = AD^2 + CD^2$$

$$AC^2 = \left(\sqrt{3}\right)^2 + \left(\frac{1}{2}AC\right)^2$$

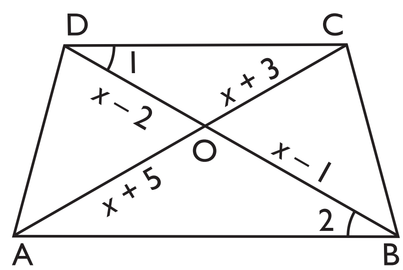
$$AC^2 = 3 + \frac{1}{4}AC^2$$

$$\frac{3}{4}AC^2 = 3$$

$$AC^2 = 4$$

$$AC = 2 \text{ cm}$$

4.



In ΔCOD and ΔAOB,

$$\angle 1 = \angle 2$$

[Alternate interior angles as AB || CD]

$\angle AOB = \angle COD$

[Vertically opposite angles]

$\therefore$ $\Delta COD \sim \Delta AOB$

$$\therefore \frac{CO}{AO} = \frac{OD}{OB} = \frac{CD}{AB}$$

[Corresponding sides of similar triangles are proportional]

$$\frac{x+3}{x+5} = \frac{x-2}{x-1}$$

$$\Rightarrow (x + 3)(x - 1) = (x - 2)(x + 5)$$

$$\Rightarrow x^2 + 2x - 3 = x^2 + 3x - 10$$

$$\Rightarrow 7 = x$$

5. In ΔSPT and ΔQPR,

$\angle PST = \angle PQR$

[Corresponding angles as ST || QR]

$\angle PTS = \angle PRQ$

$\therefore \quad \Delta SPT \sim \Delta QPR$

[AA Similarity criterion]

$$\therefore \quad \frac{ar\Delta PST}{ar\Delta PQR} = \left(\frac{PT}{PR}\right)^2$$

[Ratio of areas of two similar triangles is equal to square of ratio of their corresponding sides]

$$= \left(\frac{PT}{PT+TR}\right)^2$$

$$= \left(\frac{2}{2+4}\right)^2$$

$$= \left(\frac{2}{6}\right)^2$$

$$= \frac{1}{9}$$

6. DE || BC

$$\therefore \frac{AD}{BD} = \frac{AC}{CE}$$

(Basic proportionality theorem)

$$\Rightarrow \frac{BD}{AD} = \frac{CE}{AE}$$

$$\Rightarrow \frac{BD}{AD} + 1 = \frac{CE}{AE} + 1$$

$$\Rightarrow \frac{BD+AD}{AD} = \frac{CE+AE}{AE}$$

$$\frac{AB}{AD} = \frac{AC}{AE}$$

Also, $\angle A = \angle A$ (Common)

$\therefore \Delta ADE \sim \Delta ABC$

$$\Rightarrow \frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC}$$

[Corresponding sides of similar triangles are proportional]

$$\Rightarrow \frac{AD}{AB} = \frac{DE}{BC}$$

$$\Rightarrow \frac{1.5}{6} = \frac{DE}{8}$$

$$\Rightarrow DE = \frac{1.5\times 8}{6} = 2 \text{ cm}$$

7. As MN || AB,

$$\frac{CM}{AM} = \frac{CN}{BN}$$

[Basic proportionality theorem]

$$\frac{2}{4} = \frac{BC - BN}{BN}$$

$$\frac{1}{2} = \frac{7.5 - BN}{BN}$$

$\therefore \quad BN = 15 - 2BN$

$\Rightarrow 3BN = 15$

$BN = 5$ cm

8. We know that ratio of area of two similar triangles is equal to square of ratio of their corresponding sides.

So, Ratio of corresponding sides

$$= \sqrt{\frac{25}{64}} = \frac{5}{8}$$

9. DE || BC

$$\Rightarrow \frac{AD}{DB} = \frac{AE}{CE}$$

[Basic proportionality theorem]

$$\Rightarrow \frac{DB}{AD} + 1 = \frac{CE}{AE} + 1$$

$$\Rightarrow \frac{AB}{AD} = \frac{AC}{AE}$$

Also, $\angle A = \angle A$ (Common)

$\therefore \quad \Delta ADE \sim \Delta ABC$

(SAS Similarity criterion)

$$\therefore \quad \frac{ar\Delta ADE}{ar\Delta ABC} = \left(\frac{DE}{BC}\right)^2$$

$$\frac{ar\Delta ADE}{81} = \frac{\left(\frac{2}{3}BC\right)^2}{BC^2}$$

$$\frac{ar\Delta ADE}{81} = \frac{4}{9}$$

$$\text{ar } \Delta ADE = \frac{4}{9} \times 81 = 36 \text{ cm}^2$$

10. Consider $AC^2 + BC^2$

$= AC^2 + AC^2 \quad (\because AC = BC)$

$= 2AC^2$

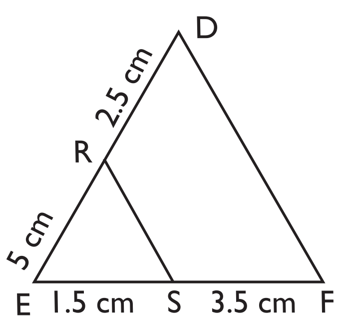
$= AB^2$

$\therefore$ ΔABC is right angled triangle

[As we know that in a triangle, if square of one side is equal to sum of the squares of other two sides then the angle opposite the first side is a right angle.]

SECTION-B

11.



Consider, $\frac{ER}{RD} = \frac{5}{2.5}$

$= 2$

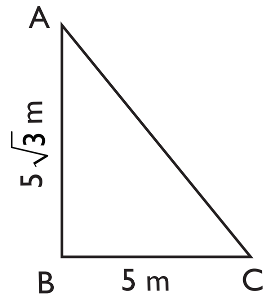
Also, $\frac{ES}{SF} = \frac{1.5}{3.5} = \frac{3}{7}$

As, $\frac{ER}{RD} \neq \frac{ES}{SF}$

$\therefore$ RS is not parallel to DF

[As we know that if a line divides any two sides of a triangle in the same ratio then the line is parallel to the third side]

12.



In ΔABC, right angled at B

$AC^2 = AB^2 + BC^2$

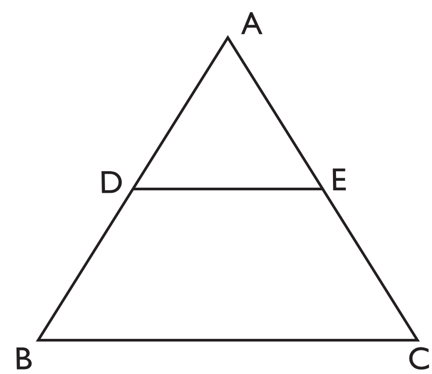
$= \left(5\sqrt{3}\right)^2 + (5)^2$

$= 75 + 25$

$= 100$

$\therefore$ AC = 10 m

13.



As DE || BC,

$$\frac{AD}{DB} = \frac{AE}{CE}$$

$$\frac{BD}{AD} = \frac{CE}{AE}$$

$$\Rightarrow \frac{BD}{AD} + 1 = \frac{CE}{AE} + 1$$

$$\Rightarrow \frac{AB}{AD} = \frac{AC}{AE}$$

$$\Rightarrow \frac{AD}{AB} = \frac{AE}{AC} \quad ...(i)$$

Also, $\angle A = \angle A$ (Common)

$\therefore$ $\Delta ADE \sim \Delta ABC$ (SAS Similarity criterion)

$\therefore \dfrac{AD}{AB} = \dfrac{DE}{BC} = \dfrac{AE}{AC}$

[Corresponding sides of similar triangles are proportional]

$\Rightarrow \dfrac{AD}{AD+BD} = \dfrac{AE}{AC}$

$\Rightarrow \dfrac{AD}{AD+3AD} = \dfrac{4.5}{AC}$

$\Rightarrow \dfrac{AD}{4AD} = \dfrac{4.5}{AC}$

$\Rightarrow$ AC $= 4.5 \times 4$

$= 18$ cm

Also, $\dfrac{AD}{AB} = \dfrac{AE}{AC}$ (From (i))

$\dfrac{AD}{AD+BD} = \dfrac{AE}{18}$

$\dfrac{AD}{AD+3AD} = \dfrac{AE}{18}$

$\dfrac{1}{4} = \dfrac{AE}{18}$

AE $= \dfrac{18}{4} = \dfrac{9}{2} = 4.5$ cm

14. Consider ΔABC with sides as

AB = (a – 1) cm

BC = $\left(2\sqrt{a}\right)$ cm

AC = (a + 1) cm

Consider $AB^2 + BC^2$

$= \left(a-1\right)^2 + \left(2\sqrt{a}\right)^2$

$= a^2 + 1 - 2a + 4a$

$= a^2 + 2a + 1$

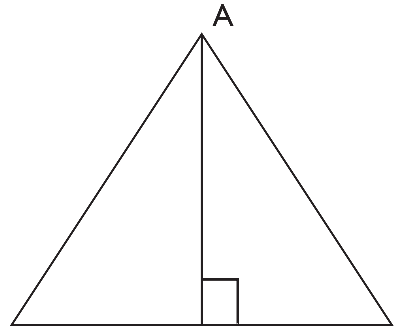
$= \left(a+1\right)^2$

$= AC^2$

$\therefore$ ΔABC is a right angled triangle

[As we know that in a triangle if square of one side is equal to the sum of squares of other two sides, then the angle opposite the first side is a right angle i.e. triangle is right angled]

15.



Draw AD $\perp$ BC

In ΔADB and ΔADC

AD = AD (Common)

AB = AC (ΔABC is equilateral)

$\angle$ADB = $\angle$ADC = 90° (By Construction)

$\therefore$ $\Delta ADB \cong \Delta ADC$ (RHS)

$\Rightarrow$ CD $= \dfrac{1}{2}$ BC $= \dfrac{1}{2} 3\sqrt{3}$ cm [CPCT]

In ΔADC,

$AC^2 = AD^2 + CD^2$

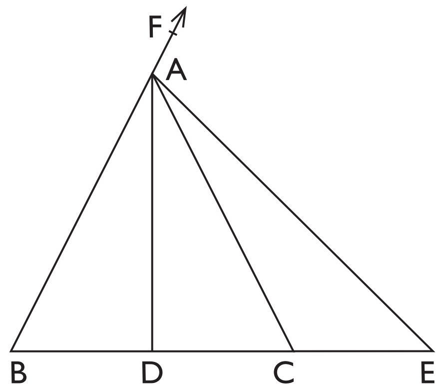
$\left(3\sqrt{3}\right)^2 = AD^2 + \dfrac{3\sqrt{3}}{2}^{2}$

$AD^2 = 27 - \dfrac{27}{4}$

$= \dfrac{108-27}{4}$

$= \dfrac{81}{4}$

$\therefore$ AC $= \dfrac{9}{2} = 4.5$ cm

16. 

To prove = $\frac{BD}{BE} = \frac{CD}{CE}$

As AD bisects ∠BAC,

$\frac{AB}{BD} = \frac{AC}{CD}$ [Interior angle bisector theorem]

$\therefore \quad \frac{CD}{BD} = \frac{AC}{AB}$ ...(i)

Also, AE bisects ∠CAF

$\therefore \quad \frac{BE}{AB} = \frac{CE}{AC}$

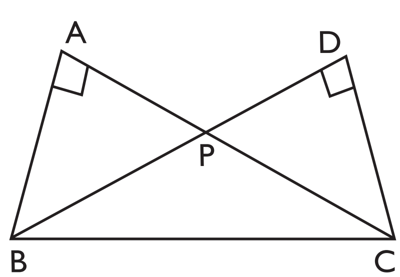
$\Rightarrow \quad \frac{BE}{CE} = \frac{AB}{AC}$

$\Rightarrow \quad \frac{CE}{BE} = \frac{AC}{AB}$ ...(ii)

From (i) and (ii)

$\frac{CD}{BD} = \frac{CE}{BE}$

$\Rightarrow \quad \frac{BD}{BE} = \frac{CD}{CE}$

17. 

To prove: AP × PC = BP × PD

Consider, ΔAPB and ΔDPC

∠BAP = ∠CDP = 90° (Given)

∠APB = ∠DPC (Vertically opposite angles)

∴ ΔAPB ~ ΔDPC (AA similarity criterion)

$\therefore \quad \frac{AP}{DP} = \frac{PB}{PC} = \frac{AB}{DC}$

(Corresponding sides of similar triangles are proportional)

⇒ AP × PC = BP × PD

18. Consider ΔQPM and ΔRSM

∠QPM = ∠RSM = 90°

∠QMP = ∠RMS (Vertically opposite angles)

∴ ΔQPM ~ ΔRSM (AA similarity criterion)

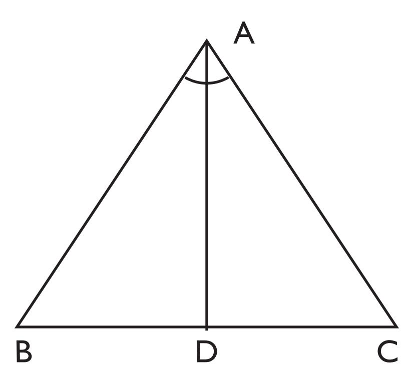
$\therefore \quad \frac{QP}{RS} = \frac{PM}{SM} = \frac{QM}{RM}$

[Corresponding sides of similar triangles are proportional]

i.e. $\frac{PM}{SM} = \frac{QM}{RM}$

$\frac{3}{4} = \frac{QM}{6}$

QM = $\frac{3\times6}{4} = \frac{3\times3}{2}$ = 4.5 cm

19. 

AD bisects ∠A. So, by Interior angle bisector theorem,

$\frac{AB}{AC} = \frac{BD}{DC}$

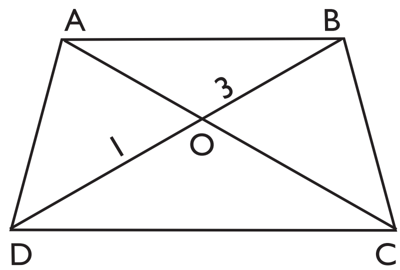
$$\frac{AB}{AC} = \frac{BD}{DC} = 1$$

[∵ BD = DC as D is a midpoint of BC]

AB = AC

∴ ΔABC is isosceles.

20.



Here, AC divides the diagonal BD in the ratio 1 : 3.

Consider ΔAOB and ΔCOD

∠BAO = ∠DCO

(Alternate interior angles as AB || CD)

∠AOB = ∠COD (Vertically opposite angles)

∴ ΔAOB ~ ΔCOD (AA similarity criterion)

$$\therefore \frac{AO}{CO} = \frac{OB}{OD} = \frac{AB}{CD}$$

[Corresponding sides of similar triangles are proportional]

$$\Rightarrow \frac{OB}{OD} = \frac{AB}{CD}$$

$$\Rightarrow \frac{3}{1} = \frac{AB}{CD}$$

⇒ AB = 3CD

SECTION-C

21. In ΔADE and ΔABC,

∠A = ∠A (Common)

∠ADE = ∠ABC (Given)

∴ ΔADE ~ ΔABC (AA similarity criterion)

$$\Rightarrow \frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC}$$

[Corresponding sides of similar triangles are proportional]

$$\therefore \frac{AD}{AB} = \frac{DE}{BC}$$

$$\Rightarrow \frac{7.6}{AE + BE} = \frac{DE}{8.4}$$

$$\Rightarrow \frac{7.6}{7.2 + 4.2} = \frac{DE}{8.4}$$

$$\Rightarrow \frac{7.6}{11.4} = \frac{DE}{8.4}$$

$$\Rightarrow DE = \frac{7.6 \times 8.4}{11.4}$$

$$= 5.6 \text{ cm}$$

22. In ΔABC, LM || CB,

$$\therefore \frac{AM}{BM} = \frac{AL}{CL} \quad ...(i)$$

[Basic proportionality theorem]

In ΔADC, LN || CD.

$$\therefore \frac{AN}{DN} = \frac{AL}{CL} \quad ...(ii)$$

[Basic proportionality theorem]

From (i) and (ii), $\frac{AM}{BM} = \frac{AN}{DN}$

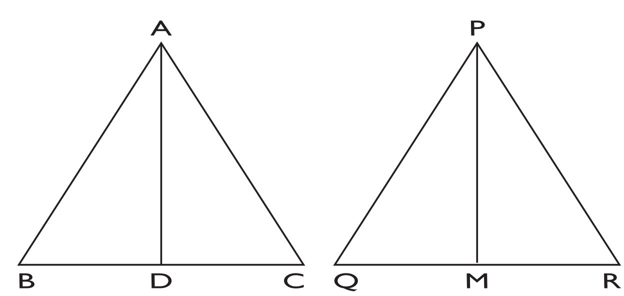
$$\Rightarrow \frac{BM}{AM} = \frac{DN}{AN}$$

$$\Rightarrow \frac{BM}{AM} + 1 = \frac{DN}{AN} + 1$$

$$\Rightarrow \frac{AB}{AM} = \frac{AD}{AN}$$

⇒ AM × AD = AB × AN

23.



In ΔABD and ΔPQM,

$$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{BC}{QR} \quad \text{(Given)}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{AD}{PM} = \frac{BD}{QM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{AD}{PM} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR}$$

(As AD and PM are the medians)

$\therefore \quad \Delta ABD \sim \Delta PQM$

(SSS similarity criterion)

$\therefore \quad \angle B = \angle Q$

[Corresponding angles of similar triangles are equal]

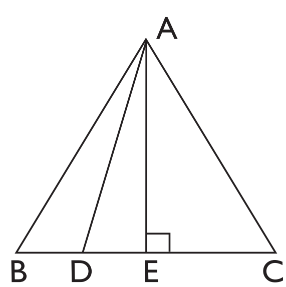
Now, in ΔABC and ΔPQR,

$$\frac{AB}{PQ} = \frac{BC}{QR} \quad \text{(Given)}$$

$\angle B = \angle Q$ (Proved)

$\therefore \Delta ABC \sim \Delta PQR$ (SSS similarity criterion)

24.



Let AB = BC = AC = a

$$\therefore BD = \frac{BC}{4} = \frac{a}{4}$$

Draw $AE \perp BC$

$$\therefore BE = EC = \frac{a}{2}$$

[In equilateral triangle altitude is same as median]

In right angled triangle ΔAED,

$AD^2 = DE^2 + AE^2$ ...(i)

Now, DE = BE – BD

$$= \frac{a}{2} - \frac{a}{4} \quad [\because BD = \frac{1}{4} = \frac{a}{4}]$$

$$= \frac{a}{4} \quad \text{...(ii)}$$

In ΔAEC,

$AC^2 = AE^2 + CE^2$

$$a^2 = AE^2 + \left(\frac{a}{2}\right)^2$$

$$AE^2 = a^2 - \frac{a^2}{4} = \frac{3a^2}{4} \quad \text{...(iii)}$$

On putting (ii), (iii) in (i), we get

$$AD^2 = \left(\frac{a}{4}\right)^2 + \frac{3a^2}{4}$$

$$= \frac{a^2}{16} + \frac{3a^2}{4}$$

$$= \frac{a^2 + 12a^2}{16}$$

$$= \frac{13a^2}{16}$$

$16 AD^2 = 13 a^2$

$16AD^2 = 13 BC^2$

25. As ΔABC is isosceles,

AB = AC

$\therefore \quad \angle B = \angle C$

(Angles opposite to equal sides are equal)

In ΔADB and ΔEFC,

$\angle ADB = \angle EFC$

(As $EF \perp AC$ and $AD \perp CB$)

$\angle B = \angle C$ (Proved)

$\therefore \quad \Delta ADB \sim \Delta EFC$ (AA similarity criterion)

$$\therefore \quad \frac{AD}{EF} = \frac{BD}{FC} = \frac{AB}{EC}$$

i.e. $\dfrac{AD}{EF} = \dfrac{AB}{EC}$

$\Rightarrow$ AD × EC = AB × EF

26. In ΔABC and ΔADE,

$\angle A = \angle A$ (Common)

$\angle ACB = \angle AED = 90°$

(As DE $\perp$ AB and ΔABC is right angled at C)

$\therefore$ $\Delta ABC \sim \Delta ADE$

(By AA similarity criterion)

$\Rightarrow \dfrac{AB}{AD} = \dfrac{BC}{DE} = \dfrac{AC}{AE}$

[Corresponding sides of similar triangles are proportional]

In ΔABC, $\angle C = 90°$

$\therefore$ $AB^2 = AC^2 + BC^2$ [By Pythagoras theorem]

$= (3 + 2)^2 + 12^2$

$= 25 + 144$

$= 169$

$\therefore$ AB = 13 cm

As $\dfrac{AB}{AD} = \dfrac{BC}{DE}$

$\therefore \dfrac{13}{3} = \dfrac{12}{DE}$

$\therefore$ $DE = \dfrac{12 \times 3}{13} = \dfrac{36}{13}$ cm

Also, $\dfrac{BC}{DE} = \dfrac{AC}{AE}$

$\Rightarrow \dfrac{12}{\frac{36}{13}} = \dfrac{5}{AE}$

$\Rightarrow \dfrac{12 \times 13}{36} = \dfrac{5}{AE}$

$\Rightarrow$ $AE = \dfrac{5 \times 36}{12 \times 13}$

$= \dfrac{15}{13}$ cm

27. As $\Delta NSQ \cong \Delta MTR$,

$\angle NQS = \angle MRT$ (CPCT)

$\Rightarrow$ PQ = PR ...(i)

(Sides opposite to equal angles are equal)

Also, as $\angle 1 = \angle 2$

$\therefore$ PS = PT ...(ii)

(Sides opposite to equal angles are equal)

On subtracting (ii) from (i), we get

PQ – PS = PR – PT

QS = TR ...(iii)

From (ii) and (iii),

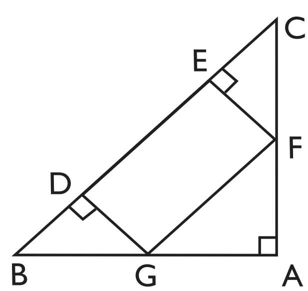
$\dfrac{PS}{QS} = \dfrac{PT}{TR} \Rightarrow \dfrac{PS}{PQ} = \dfrac{PT}{PR}$

Also, $\angle P = \angle P$ (Common)

$\therefore$ $\Delta PST \sim \Delta PQR$ (SAS similarity criterion)

SECTION-D

28.



In ΔAFG and ΔDBG,

$\angle AGF = \angle DBG$

(Corresponding angles as GF || BC)

$\angle GAF = \angle BDG = 90°$ ($\because$ DEFG is a square)

$\therefore$ $\Delta AFG \sim \Delta DBG$ ...(i)

(AA similarity criterion)

In ΔAGF and ΔEFC,

$\angle FAG = \angle CEF = 90°$

$\angle AFG = \angle ECF$

(Corresponding angles as GF || BC)

$\therefore \quad \Delta AGF \sim \Delta EFC$ ...(ii)

(AA similarity criterion)

From (i) and (ii), we get,

$\Delta DBG \sim \Delta EFC$

$\Rightarrow \quad \frac{BD}{EF} = \frac{DG}{EC}$

[Corresponding sides of similar triangles are proportional]

$\Rightarrow \quad \frac{BD}{DE} = \frac{DE}{EC}$

[As DEFG is a square, EF = DE and DG = DE]

$\Rightarrow \quad DE^2 = BD \times EC$

29. In ΔAOD, MO bisects $\angle AOD$,

So, by interior angle bisector theorem,

$\frac{AO}{OD} = \frac{AM}{DM}$ ...(i)

In ΔBOC, NO bisects $\angle BOC$,

So, by interior angle bisector theorem,

$\frac{BO}{CO} = \frac{BN}{CN}$

$\Rightarrow \quad \frac{CO}{BO} = \frac{CN}{BN}$ ...(ii)

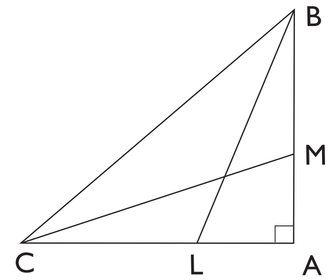
We know that AO = OD $\Rightarrow \frac{AO}{OD} = 1$

and CO = BO $\Rightarrow \frac{CO}{BO} = 1$

(Radii of same circle)

So, from (i) and (ii), we get

$\frac{AM}{DM} = \frac{CN}{BN}$

30. 

In ΔABC, $BC^2 = AB^2 + AC^2$

(By Pythagoras theorem)

In ΔABL, $BL^2 = AB^2 + AL^2$

$= AB^2 + \frac{1}{2}AC\ ^{2}$

[As L is a midpoint of AC $\therefore AL = \frac{1}{2}AC$]

$BL^2 = AB^2 + \frac{AC^2}{4}$

$4BL^2 = 4AB^2 + AC^2$ ...(i)

In ΔCMA, $CM^2 = AC^2 + AM^2$

$= AC^2 + \left(\frac{1}{2}AB\right)^2$

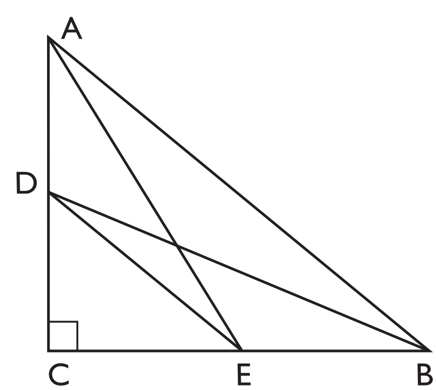
$= AC^2 + \frac{AB^2}{4}$

[As M is a midpoint of AB $\therefore AM = \frac{1}{2}AB$]

$\Rightarrow \quad 4\,CM^2 = 4AC^2 + AB^2$ ...(ii)

From (i) and (ii), we get

$4(BL^2 + CM^2) = 5\,AB^2 + 5\,AC^2 = 5\,BC^2$

31. 

To prove : $AE^2 + BD^2 = AB^2 + DE^2$

Proof: In ΔACE, $AE^2 = AC^2 + CE^2$ ...(i)

(By Pythagoras theorem)

In ΔDCB, $BD^2 = DC^2 + BC^2$ ...(ii)

(By Pythagoras theorem)

In ΔABC, $AB^2 = AC^2 + BC^2$ ...(iii)

(By Pythagoras theorem)

In ΔDCE, $DE^2 = DC^2 + CE^2$ ...(iv)

(By Pythagoras theorem)

Consider $AE^2 + BD^2$

$= AC^2 + CE^2 + DC^2 + BC^2$ [from (i) and (ii)]

$= (AC^2 + BC^2) + (CE^2 + DC^2)$

$= AB^2 + DE^2$ [from (iii) and (iv)]

CASE STUDY-1

(i) (d) ΔABC is $\frac{1}{10}^{th}$ time of ΔPQR as the scale factor of ΔABC to ΔPQR is 1:10.

$\therefore$ The sides PQ is 10(AB) or 30 cm

Similarly QR is 40 cm and PR is 50 cm.

(ii) (d) $PQ^2 + QR^2 = PR^2$ [Pythagores theoram]

(iii) (d) Perimeter of ΔPQR is the sum of all sides.

$\therefore$ Perimeter = 30 cm + 40 cm to 50 cm

= 120 cm

(iv) (b) Area of ΔPQR is $\frac{1}{2}$(PQ) (QR)

$\frac{1}{2}$(PQ) (QR) = $\frac{1}{2}$(40 (30)

= 600 cm^2

(v) (d) ΔABC and ΔPQR are similar,

$$\frac{QC}{AC} = \frac{QR}{RP}$$

$$\frac{BC}{QR} = \frac{AC}{PR}$$

CASE STUDY-2

(i) (d) In ΔBSP, $\angle SBP = 60$, SP = 6 cm

$$\frac{SP}{BP} = \tan 60$$

$$\frac{6}{BP} = \sqrt{3}$$

$$BP = \frac{6}{\sqrt{3}} \text{ cm}$$

(ii) (a) In ΔRQC

$$\frac{RQ}{QC} = \tan 60$$

$$\frac{6}{QC} = \tan 60$$

$$QC = \frac{6}{\sqrt{3}} \text{ cm}$$

$$BC = BP + PQ + QC$$

$$= \frac{6}{\sqrt{3}} + 6 + \frac{6}{\sqrt{3}}$$

$$= \left(\frac{12}{\sqrt{3}} + 6\right) \text{ cm}$$

$$= \frac{12 \times \sqrt{3}}{3} + 6$$

$$= 4\sqrt{3} + 6 \text{ cm}$$

(iii) (b) Area of equilateral triangle is $\frac{\sqrt{3}}{4}a^2$, where a is the side of triangle.

$$\text{Area} = \frac{\sqrt{3}}{4}\left(4\sqrt{3} + 6\right)^2$$

$$= \frac{\sqrt{3}}{4}\left(4 + 36 + 48\sqrt{3}\right)$$

$$= \frac{\sqrt{3}}{4}\left(84 + 48\sqrt{3}\right) \text{ cm}^2$$

$$= \sqrt{3}\left(21 + 12\sqrt{3}\right) \text{ cm}^2$$

$$= \left(36 + 21\sqrt{3}\right) \text{ cm}^2$$

(iv) (c) Area of square = $(\text{side})^2$

Side = 6 cm

Area of square = $(6)^2$

= 36 cm^2

(v) (c)

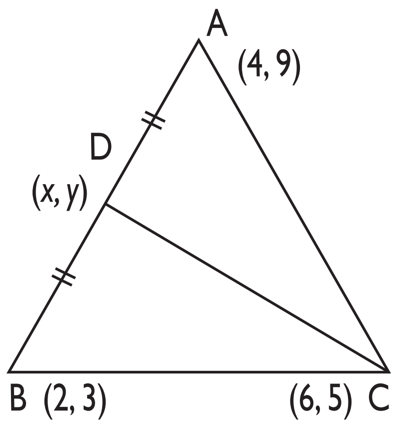
$$\frac{\text{Area of } \Delta ABC}{\text{Area of square } PQRS} = \frac{36 + 21\sqrt{3}}{36}$$

$$= \frac{12 + 7\sqrt{3}}{12}$$

Chapter 7 Coordinate Geometry

Multiple Choice Questions

1. (b)



$$D(x, y) = \left(\frac{4+2}{2}, \frac{9+3}{2}\right) = (3, 6)$$

$$\text{So, } CD = \sqrt{(6-3)^2 + (5-6)^2}$$

$$= \sqrt{9+1}$$

$$= \sqrt{10} \text{ units}$$

2. (c) As A, B and C are collinear

$$\therefore x(-4+5) - 3(-5-2) + 7(2+4) = 0$$

$$x + 21 + 42 = 0$$

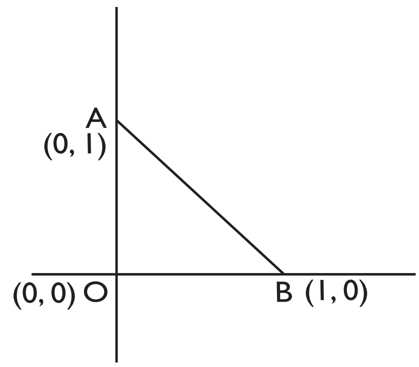
$$x = -63$$

3. (b) $(2, p) = \left(\frac{6-2}{2}, \frac{-5+11}{2}\right)$

$$= (2, 3)$$

$$\Rightarrow p = 3$$

4. (d)



In $\triangle AOB$,

$$AB^2 = AO^2 + OB^2$$

$$= 1^2 + 1^2$$

$$= 2$$

$$AB = \sqrt{2}$$

$$\text{Perimeter} = AO + OB + AB$$

$$= 1 + 1 + \sqrt{2}$$

WORKSHEET - 1

SECTION-A

1. $\text{Centroid} = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

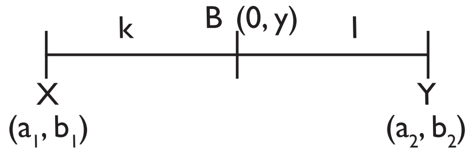
$$(1, 4) = \frac{4 - 9 + x_3}{3}, \frac{-3 + 7 + y_3}{3}$$

$$(1, 4) = \left(\frac{-5 + x_3}{3}, \frac{4 + y_3}{3}\right)$$

$\frac{-5 + x_3}{3} = 1$	$\frac{4 + y_3}{3} = 4$
$x_3 - 5 = 3$	$y_3 + 4 = 12$
$x_3 = 8$	$y_3 = 8$

So, third vertex is (8, 8).

2.



Let the ratio be k : l

So, $(0, y) = \left(\dfrac{ka_2 + a_1}{k+1}, \dfrac{kb_2 + b_1}{k+1}\right)$

$\dfrac{ka_2 + a_1}{k+1} = 0$

$ka_2 + a_1 = 0$

$ka_2 = -a_1$

$k = \dfrac{-a_1}{a_2}$

3. Distance $= \sqrt{\left(\dfrac{2}{5} + \dfrac{8}{5}\right)^2 + (2-2)^2}$

$= \sqrt{\left(\dfrac{2}{5} + \dfrac{8}{5}\right)^2 + 0}$

$= \sqrt{\left(\dfrac{2+8}{5}\right)^2}$

= 2 sq. units

4. Let point on y - axis be (0, y).

$\sqrt{(6-0)^2 + (5-y)^2} = \sqrt{(-4-0)^2 + (3-y)^2}$

$\sqrt{36 + 25 + y^2 - 10y} = \sqrt{16 + 9 + y^2 - 6y}$

$\sqrt{61 + y^2 - 10y} = \sqrt{25 + y^2 - 6y}$

$61 + y^2 - 10y = 25 + y^2 - 6y$

$36 = 4y$

$y = 9$

So, point on y - axis which is equidistant from point A (6, 5) and B (– 4, 3) is (0, 9).

5. As point A (0, 2) is equidistant from the points B (3, P) and C (P, 5). So,

$\sqrt{(3-0)^2 + (P-2)^2} = \sqrt{(P-0)^2 + (5-2)^2}$

$\sqrt{9 + (P-2)^2} = \sqrt{P^2 + 9}$

$(P-2)^2 = P^2$

$P^2 + 4 - 4P = P^2$

$4P = 4$

$P = 1$

6. $\sqrt{(4-1)^2 + (K-0)^2} = 5$

$\sqrt{3^2 + K^2} = 5$

On squaring both sides, we get

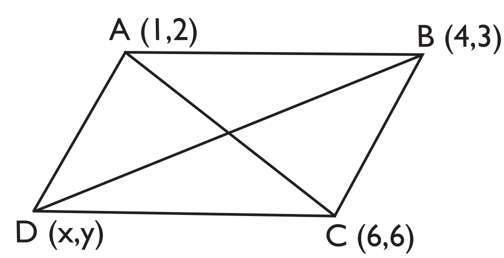
$9 + K^2 = 25$

$K^2 = 25 - 9 = 16$

$K^2 = 16$

$K = \pm 4$

7.



We know that diagonals of a parallelogram bisect each other

$\therefore \left(\dfrac{1+6}{2}, \dfrac{2+6}{2}\right) = \left(\dfrac{4+x}{2}, \dfrac{3+y}{2}\right)$

$\left(\dfrac{7}{2}, 4\right) = \left(\dfrac{4+x}{2}, \dfrac{3+y}{2}\right)$

$\therefore \dfrac{7}{2} = \dfrac{4+x}{2}$ and $4 = \dfrac{3+y}{2}$

$7 = 4 + x$ and $8 = 3 + y$

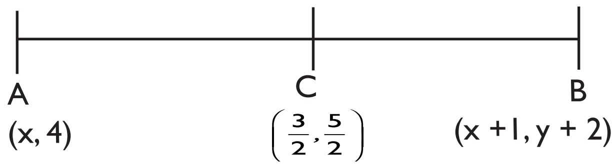
$x = 3$ and $y = 5$

So, coordinates of fourth vertex

$= (x, y)$

$= (3, 5)$

8.



As C is a midpoint of AB,

$$\left(\frac{x+x+1}{2},\frac{4+y+2}{2}\right)=\left(\frac{3}{2},\frac{5}{2}\right)$$

$$\left(\frac{2x+1}{2},\frac{y+6}{2}\right)=\left(\frac{3}{2},\frac{5}{2}\right)$$

$$\therefore \quad \frac{2x+1}{2}=\frac{3}{2} \text{ and } \frac{y+6}{2}=\frac{5}{2}$$

$2x + 1 = 3$ and $y + 6 = 5$

$2x = 2$ and $y = 5 - 6$

$x = 1$ and $y - 1$

SECTION-B

9. Let y – coordinate be v.

$\therefore$ x – coordinate = 2v

So, point P is (2v, v).

PQ = PR

$$\sqrt{(2-2v)^2+(-5-v)^2}=\sqrt{(-3-2v)^2+(6-v)^2}$$

On squaring both sides, we get

$$(2-2v)^2+(-5-v)^2=(-3-2v)^2+(6-v)^2$$

$$\therefore 4+4v^2-8v+25+v^2+10v$$

$$=9+4v^2+12v+36+v^2-12v$$

$$\Rightarrow \quad 5v^2+2v+29=5v^2+45$$

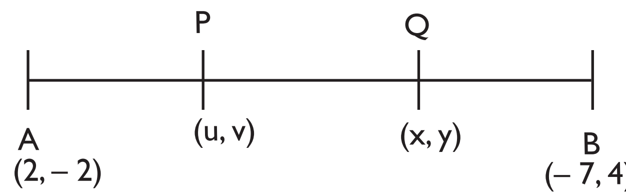
$$\Rightarrow \quad 2v=45-29$$

$$2v=16$$

$$v=8$$

So, point P is (2v, v) i.e. (16, 8).

10.



Point P divides AB in ratio 1 : 2

$$\text{So, P (u,v)} = \left(\frac{1(-7)+2(2)}{3},\frac{1(4)+2(-2)}{3}\right)$$

$$=\left(\frac{-7+4}{3},\frac{4-4}{3}\right)$$

$$=(-1, 0)$$

Point Q divides AB in ratio 2 : 1

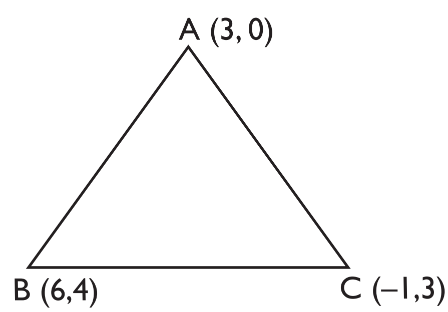
$$\text{So, Q (x, y)} = \left(\frac{2(-7)+1(2)}{3},\frac{2(4)+1(-2)}{3}\right)$$

$$=\left(\frac{-14+2}{3},\frac{8-2}{3}\right)$$

$$=\left(\frac{-12}{3},\frac{6}{3}\right)$$

$$=(-4, 2)$$

11.



$$AB=\sqrt{(6-3)^2+(4-0)^2}$$

$$=\sqrt{9+16}$$

$$=\sqrt{25}$$

$$=5$$

$$BC=\sqrt{(-1-6)^2+(3-4)^2}$$

$= \sqrt{49+1}$

$= \sqrt{50} = 5\sqrt{2}$

$AC = \sqrt{(-1-3)^2+(3-0)^2}$

$= \sqrt{16+9}$

$= \sqrt{25}$

$= 5$

As $AB = AC$, ΔABC is isosceles

Also,

$AB^2 + AC^2 = 5^2 + 5^2$

$= 25 + 25$

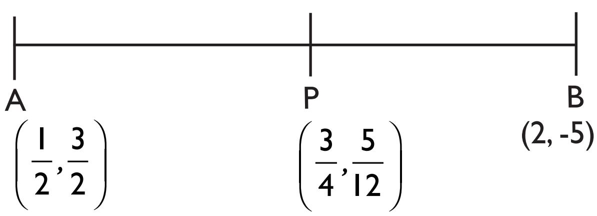
$= 50$

$= BC^2$

$\therefore \angle A = 90^\circ$

$\therefore$ ΔABC is an isosceles triangle right angled at A.

12.



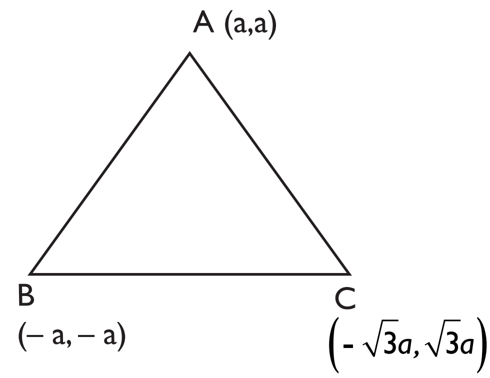
Let point P divides AB in ratio k : 1

So, $\left(\frac{3}{4},\frac{5}{12}\right) = \left(\frac{2k+\frac{1}{2}}{k+1}, \frac{-5k+\frac{3}{2}}{k+1}\right)$

$\frac{3}{4} = \frac{2k+\frac{1}{2}}{k+1}$	$\frac{5}{12} = \frac{-5k+\frac{3}{2}}{k+1}$
$\Rightarrow 3k + 3 = 8k + 2$	$5k + 5 = -60k + 18$
$\Rightarrow 1 = 5k$	$65k = 13$
$k = \frac{1}{5}$	$k = \frac{1}{5}$

So, point P divides AB in ratio 1 : 5

13.



$AB = \sqrt{(-a-a)^2+(-a-a)^2}$

$= \sqrt{4a^2+4a^2}$

$= \sqrt{8a^2}$

$= 2\sqrt{2}a$ units

$BC = \sqrt{(-\sqrt{3}a+a)^2+(\sqrt{3}a+a)^2}$

$= \sqrt{3a^2+a^2-2\sqrt{3}a^2+3a^2+2\sqrt{3}a^2+a^2}$

$= \sqrt{8a^2}$

$= 2\sqrt{2}a$ units

$AC = \sqrt{(-\sqrt{3}a-a)^2+(\sqrt{3}a-a)^2}$

$= \sqrt{3a^2+a^2+2\sqrt{3}a^2+3a^2+a^2-2\sqrt{3}a^2}$

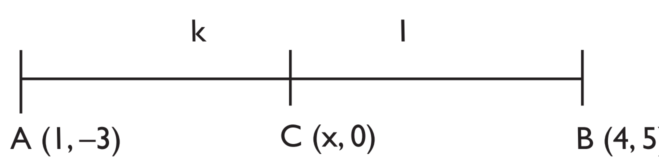
$= \sqrt{3a^2+a^2+3a^2+a^2}$

$= \sqrt{8a^2}$

$= 2\sqrt{2}a$ units

As $AB = BC = CA$, ΔABC is an equilateral triangle.

14.



Let point C (x, 0) divides AB in ratio k : 1

So,

$$(x, 0) = \left(\frac{4k+1}{k+1}, \frac{5k-3}{k+1}\right)$$

$$\therefore \quad \frac{5k-3}{k+1} = 0$$

$$5k - 3 = 0$$

$$k = \frac{3}{5}$$

So, x – axis divides the line segment joining the points (1, – 3) and (4, 5) in ratio 3 : 5.

15. $$\sqrt{(9-x)^2+(10-4)^2} = 10$$

$$81 + x^2 - 18x + 36 = 100$$

$$x^2 - 18x + 17 = 0$$

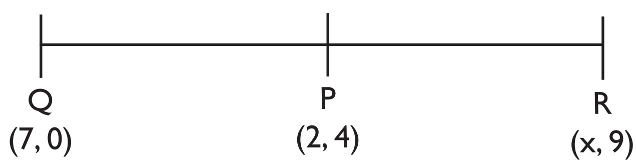
$$x^2 - 17x - x + 17 = 0$$

$$x(x - 17) - 1(x - 17) = 0$$

$$(x - 1)(x - 17) = 0$$

$$x = 1, 17$$

16.



PQ = PR

$$\Rightarrow \sqrt{(2-7)^2+(4-0)^2} = \sqrt{(x-2)^2+(9-4)^2}$$

$$\Rightarrow \sqrt{25+16} = \sqrt{x^2+4-4x+25}$$

$$\Rightarrow \sqrt{41} = \sqrt{x^2-4x+29}$$

On squaring both sides, we get

$$41 = x^2 - 4x + 29$$

$$0 = x^2 - 4x - 12$$

$$0 = x^2 - 6x + 2x - 12$$

$$0 = x(x - 6) + 2(x - 6)$$

$$0 = (x + 2)(x - 6)$$

$$x = -2 \text{ or } 6$$

$$PQ = \sqrt{(7-2)^2+(0-4)^2}$$

$$= \sqrt{5^2+(-4)^2}$$

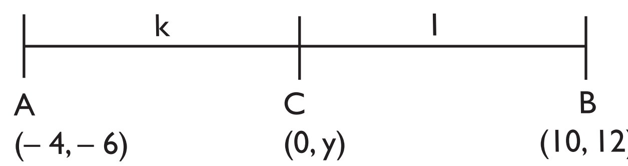
$$= \sqrt{25+16}$$

$$= \sqrt{41}$$

SECTION-C

17. Let y – axis divides the line segment joining the points (– 4, – 6) and (10, 12) in ratio k : 1

Point on y – axis must be of form (0, y)



$$(0, y) = \left(\frac{10k+(-4)}{k+1}, \frac{12k-6}{k+1}\right)$$

$$(0, y) = \left(\frac{10k-4}{k+1}, \frac{12k-6}{k+1}\right)$$

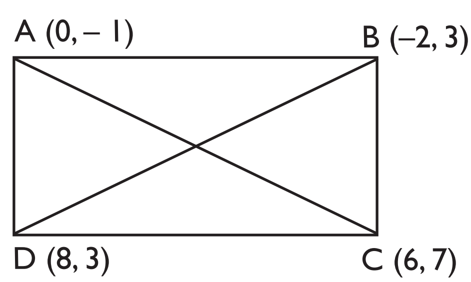
$$\Rightarrow \frac{10k-4}{k+1} = 0$$

$$\Rightarrow 10k = 4$$

$$k = \frac{2}{5}$$

So, ratio is 2 : 5.

18.



$$AB = \sqrt{(-2-0)^2+(3+1)^2}$$

$$= \sqrt{4+16}$$

$$= \sqrt{20}$$

$$= 2\sqrt{5} \text{ units}$$

$$CD = \sqrt{(6-8)^2 + (7-3)^2}$$

$$= \sqrt{4+16}$$

$$= \sqrt{20}$$

$$= 2\sqrt{5} \text{ units}$$

$\therefore$ AB = CD

$$AD = \sqrt{(-8-0)^2 + (3+1)^2}$$

$$= \sqrt{64+16} = \sqrt{80} = 4\sqrt{5} \text{ units}$$

$$BC = \sqrt{(6+2)^2 + (7-3)^2}$$

$$= \sqrt{64+16}$$

$$= \sqrt{80}$$

$$= 4\sqrt{5} \text{ units}$$

$\therefore$ AD = BC

As AB = CD and AD = BC,

So, ABCD is a parallelogram

$$AC = \sqrt{(6-0)^2 + (7+1)^2}$$

$$= \sqrt{36+64}$$

$$= \sqrt{100}$$

$$= 10 \text{ units}$$

$$BD = \sqrt{(8+2)^2 + (3-3)^2}$$

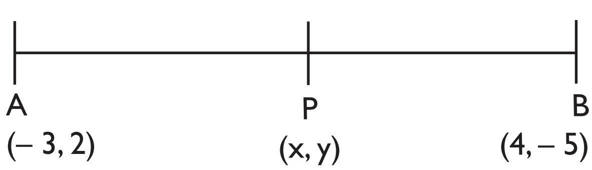
$$= \sqrt{100}$$

$$= 10 \text{ units}$$

So, AC = BD

$\therefore$ ABCD is a parallelogram in which both diagonals are equal.

So, ABCD is a rectangle.

19. 

As point P is equidistant from A and B,

AP = BP

$$\sqrt{(x+3)^2 + (y-2)^2} = \sqrt{(4-x)^2 + (-5-y)^2}$$

On squaring both sides, we get

$$(x + 3)^2 + (y - 2)^2 = (4 - x)^2 + (-5 - y)^2$$

$$x^2 + 9 + 6x + y^2 + 4 - 4y$$

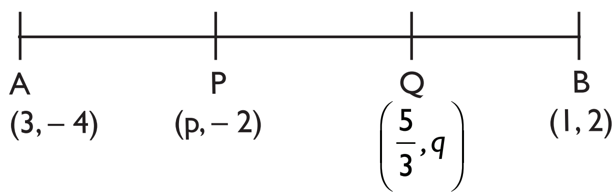
$$= 16 + x^2 - 8x + 25 + y^2 + 10y$$

$$14x - 14y + 13 = 41$$

$$14x - 14y - 28 = 0$$

$$x - y = 2$$

$$\therefore \quad y = x - 2$$

20. 

Point P divides AB in ratio 1 : 2.

So,

$$P\ (p, -2) = \left(\frac{1(1)+2(3)}{3}, \frac{1(2)+2(-4)}{3}\right)$$

$$P\ (p, -2) = \left(\frac{7}{3}, -2\right)$$

$$\therefore\ p = \frac{7}{3}$$

Point Q divides AB in ratio 2 : 1.

So,

$$Q\left(\frac{5}{3}, q\right) = \left(\frac{2(1)+1(3)}{3}, \frac{2(2)+1(-4)}{3}\right)$$

$$Q\left(\frac{5}{3}, q\right) = \left(\frac{5}{3}, 0\right)$$

$$\therefore \quad q = 0$$

21. As the points A (3p + 1, p), B (p + 2, p – 5) and C (p + 1, – p) are collinear,

area of ΔABC = 0

i. e. $\frac{1}{2}$ [(3p + 1) (p – 5 + p) + (p + 2) (– p – p) + (p + 1) (p – p + 5)] = 0

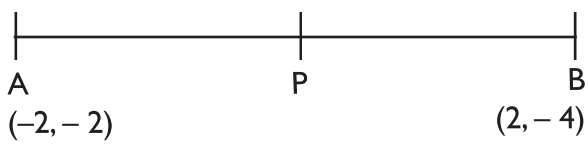
$\Rightarrow$ [(3p + 1) (2p – 5) – 2p (p + 2) + 5 (p + 1)] = 0

$\Rightarrow$ [$6p^2$ – 15p + 2p – 5 – $2p^2$ – 4p + 5p + 5] = 0

$\Rightarrow$ [$4p^2$ – 12p] = 0

$\therefore$ p = 0, 3

22.



$$AP = \frac{3}{7} AB$$

$$\Rightarrow AP = \frac{3}{7}(AP + BP)$$

$$\Rightarrow 7\,AP = 3\,AP + 3\,BP$$

$$\Rightarrow 4\,AP = 3\,BP$$

$$\Rightarrow \frac{AP}{PB} = \frac{3}{4}$$

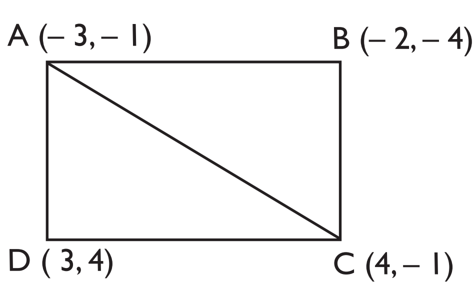
Let point P be (x, y), using section formula,

$$(x, y) = \left(\frac{3(2)+4(-2)}{7}, \frac{3(-4)+4(-2)}{7}\right)$$

$$= \left(\frac{6-8}{7}, \frac{-12-8}{7}\right)$$

$$= \left(\frac{-2}{7}, \frac{-20}{7}\right)$$

23.



Join AC

Area of ΔACD

$= \frac{1}{2}$ [–3 (– 1 – 4) + 4 (4 + 1) + 3 (–1+1)]

$= \frac{1}{2}$ [–3 (– 5) + 20]

$= \frac{1}{2}$ [15 + 20]

$= \frac{35}{2}$ sq. units

Area of ΔABC

$= \frac{1}{2}$ [(–3) (– 4 + 1) – 2 (– 1 + 1) + 4 (–1+4)]

$= \frac{1}{2}$ [–3 (–3) – 2 (0) + 4 (3)]

$= \frac{1}{2}$ [9 + 12] $= \frac{21}{2}$ sq. units

So, area of quadrilateral ABCD

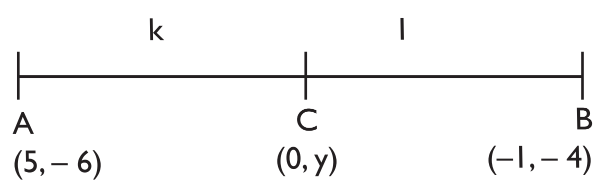
= area of ΔACD + area of ΔABC

$= \frac{35}{2} + \frac{21}{2}$

$= \frac{56}{2}$

= 28 sq. units

24. Let y – axis divides the line segment joining points A (5, – 6), B (– 1, – 4) in ratio k : 1. Point C on y – axis is of form (0, y).



By section formula,

$$(0, y) = \left(\frac{-k+5}{k+1}, \frac{-4k-6}{k+1}\right)$$

$$0 = \frac{-k+5}{k+1}$$

k = 5

So, y – axis divides AB in ratio 5 : 1

Also, $y = \frac{-4k-6}{k+1}$

$$= \frac{-20-6}{5+1}$$

$$= \frac{-26}{6}$$

$$= \frac{-13}{3}$$

So, $C(0, y) = (0, \frac{-13}{3})$

SECTION-D

25. Consider points $(x_1, y_1) = (t, t-2)$,

$(x_2, y_2) = (t+2, t-2)$ and

$(x_3, y_3) = (t+3, t)$

Area of triangle

$$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}[t(t-2-t) + (t+2)(t-t+2) + (t+3)(t-2-t+2)]$$

$$= \frac{1}{2}[t(-2) + (t+2)(2)]$$

$$= \frac{1}{2}[-2t + 2t + 4]$$

$$= \frac{1}{2}(4)$$

= 2 sq. units

So, area of triangle is independent of t.

26. $$\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{3}$$

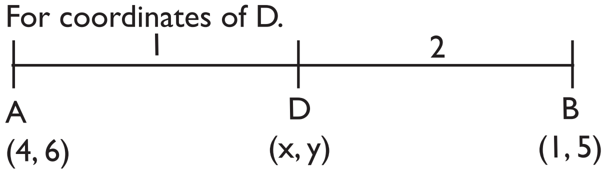
$$\Rightarrow \frac{AB}{AD} = \frac{AC}{AE} = \frac{3}{1}$$

$$\Rightarrow \frac{AB}{AD} - 1 = \frac{AC}{AE} - 1 = 3 - 1$$

$$\Rightarrow \frac{BD}{AD} = \frac{CE}{AE} = 2$$

$$\therefore \frac{AD}{BD} = \frac{AE}{CE} = \frac{1}{2}$$

For coordinates of D.



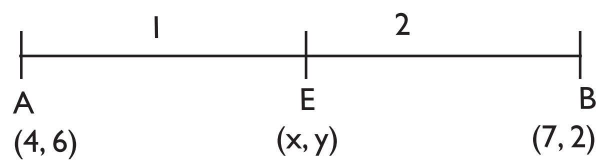
By using section formula,

$$(x, y) = \left(\frac{1(1)+2(4)}{3}, \frac{1(5)+2(6)}{3}\right)$$

$$(x, y) = \left(\frac{9}{3}\ \frac{17}{3}\right)$$

$$= \left(3, \frac{17}{3}\right)$$

For coordinates of E

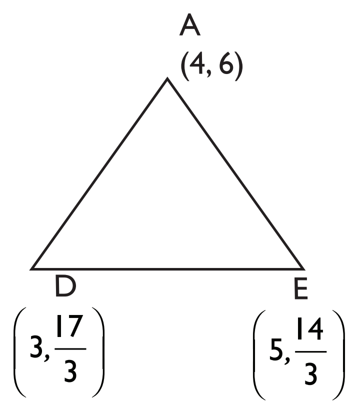


By using section formula,

$$(x, y) = \left(\frac{1(7)+2(4)}{3}, \frac{1(2)+2(6)}{3}\right)$$

$$= \left(\frac{7+8}{3}, \frac{2+12}{3}\right)$$

$$= \left(5, \frac{14}{3}\right)$$



ar ΔADE

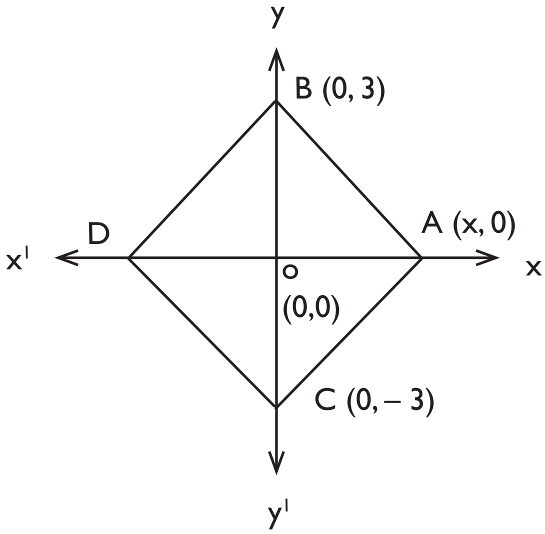
$$= \frac{1}{2}\left[4\left(\frac{17}{3} - \frac{14}{3}\right) + 3\left(\frac{14}{3} - 6\right) + 5\left(6 - \frac{17}{3}\right)\right]$$

$$= \frac{1}{2}\left[4\left(\frac{3}{3}\right) + 3\left(\frac{14-18}{3}\right) + 5\left(\frac{18-17}{3}\right)\right]$$

$$= \frac{1}{2}\left[4 + (-4) + \frac{5}{3}\right]$$

$$= \frac{5}{6} \text{ sq. units}$$

27.



Let coordinates of B be (0, y).

As (0, 0) is a midpoint of BC,

$\therefore (0, 0) = \left(\dfrac{0+0}{2}, \dfrac{y-3}{2}\right)$

$(0, 0) = \left(\dfrac{0}{2}, \dfrac{y-3}{2}\right)$

$(0, 0) = \left(0, \dfrac{y-3}{2}\right)$

$\dfrac{y-3}{2} = 0$

$y = 3$

So, point B is (0, 3).

Let coordinates of point A be (x, 0).

Using distance formula,

$AB = \sqrt{(x-0)^2+(0-3)^2}$

$= \sqrt{x^2+9}$

$BC = \sqrt{(0-0)^2+(-3-3)^2}$

$= \sqrt{36}$

$= 6$

As ΔABC is equilateral,

AB = BC

i.e. $\sqrt{x^2+9} = 6$

$x^2 + 9 = 36$

$x^2 = 27$

$x = \pm 3\sqrt{3}$

$\therefore$ Coordinates of point A are $(-3\sqrt{3}, 0)$.

As BACD is a rhombus and diagonals of rhombus bisect each other. So, OD = OA $= 3\sqrt{3}$ units

$\therefore$ Point D is $(-3\sqrt{3}, 0)$

28. Area of triangle = 5 sq. units

As third vertex lies on y = x + 3, so, it must be of form (x, x + 3).

Let $(x_1, y_1) = (2, 1)$

$(x_2, y_2) = (3, -2)$

$(x_3, y_3) = (x, x + 3)$

Area of triangle

$= \dfrac{1}{2}\,[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$5 = \dfrac{1}{2}[2(-2-x-3) + 3(x+3-1) + x(1+2)]$

$10 = [2(-5-x) + 3(x+2) + 3x]$

$10 = [-10 - 2x + 3x + 6 + 3x]$

$10 = [4x - 4]$

$\therefore \pm 10 = 4x - 4$

$4x - 4 = 10$	$4x - 4 = -10$
$4x = 14$	$4x = -6$
$x = \dfrac{7}{2}$	$x = \dfrac{-3}{2}$
So, third vertex is	So, third vertex is
(x, x + 3)	(x, x + 3)
$= \left(\dfrac{7}{2}, \dfrac{7}{2}+3\right)$	$= \left(\dfrac{-3}{2}, \dfrac{-3}{2}+3\right)$
$= \left(\dfrac{7}{2}, \dfrac{13}{2}\right)$	$= \left(\dfrac{-3}{2}, \dfrac{3}{2}\right)$

29. Let $(x_1, y_1) = (a, a^2)$

$(x_2, y_2) = (b, b^2)$

$(x_3, y_3) = (c, c^2)$

Consider, area of triangle

$= \frac{1}{2} [x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)]$

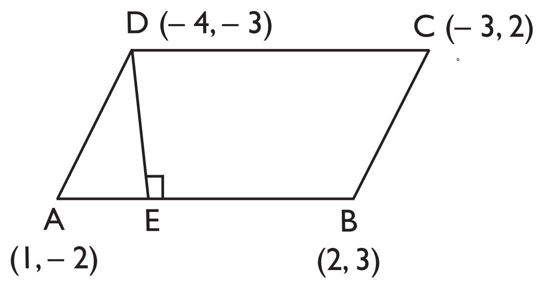
$= \frac{1}{2} [a (b^2 - c^2) + b (c^2 - a^2) + c (a^2 - b^2)]$

$= \frac{1}{2} [ab^2 - ac^2 + bc^2 - a^2b + a^2c - b^2c]$

$= \frac{1}{2} [ab (b - a) + ac (a - c) + bc (c - b)]$

Here, it is clear that area of triangle is 0 if $a = b = c$ but it is given that $a \neq b \neq c$.

30.



Let DE be the height of parallelogram ABCD.

For ΔABD,

Let $(x_1, y_1) = (1, -2)$

$(x_2, y_2) = (2, 3)$

$(x_3, y_3) = (-4, -3)$

area of ΔABD

$= \frac{1}{2} [x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)]$

$= \frac{1}{2} [1 (3 + 3) + 2 (-3 + 2) - 4 (-2 - 3)]$

$= \frac{1}{2} [6 - 2 + 20]$

= 12 sq. units

For ΔBCD,

Let $(x_1, y_1) = (2, 3)$

$(x_2, y_2) = (-3, 2)$

$(x_3, y_3) = (-4, -3)$

area of ΔBCD

$= \frac{1}{2} [x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)]$

$= \frac{1}{2} [2 (2 + 3) + (-3) (-3 - 3) - 4 (3 - 2)]$

$= \frac{1}{2} [10 + 18 - 4]$

= 12 sq. units

Area of parallelogram ABCD

= area of ΔABC + area of ΔBCD

= 12 + 12

= 24 sq. units

We know that area of parallelogram

= base x height

24 = AB x height

By using distance formula,

$AB = \sqrt{(2-1)^2 + (3+2)^2}$

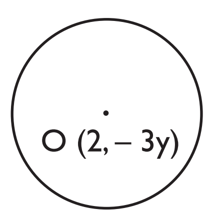
$= \sqrt{1 + 25}$

$= \sqrt{26}$ units

$\therefore 24 = \sqrt{26} \times$ height

height $= \frac{24}{\sqrt{26}}$ units $= \frac{12\sqrt{26}}{13}$ units

31.



Let the center be O (2, – 3y).

As points A and B lie on a circle,

AO = BO

$$\sqrt{(2+1)^2+(-3y-y)^2} = \sqrt{(2-5)^2+(-3y-7)^2}$$

$$\sqrt{9+16y^2} = \sqrt{9+9y^2+49+42y}$$

On squaring both sides, we get

$9 + 16y^2 = 9y^2 + 42y + 58$

$7y^2 - 42y - 49 = 0$

$y^2 - 6y - 7 = 0$

$y^2 - 7y + y - 7 = 0$

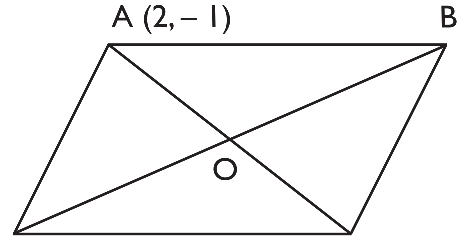
$y(y - 7) + (y - 7) = 0$

$(y + 1)(y - 7) = 0$

$y = -1, 7$

When $y = -1$	When $y = 7$
$A = (-1, y) = (-1, -1)$	$A = (-1, y)$
$O = (2, 3)$	$= (-1, 7)$
So,	$O = (2, -3y)$
radius = AO	$= (2, -21)$
$= \sqrt{(2+1)^2+(3+1)^2}$	So,
$= \sqrt{9+16}$	radius = AO
$= \sqrt{25}$	$= \sqrt{(2+1)^2+(-21-7)^2}$
= 5 units	$= \sqrt{9+784}$
	$= \sqrt{793}$ units

32.



By using distance formula,

$$AC = \sqrt{(5-2)^2+(6+1)^2}$$

$$= \sqrt{9+49} = \sqrt{58} \text{ units}$$

$$BD = \sqrt{(2-5)^2+(6+1)^2}$$

$$= \sqrt{9+49}$$

$$= \sqrt{58}$$

So, AC = BD

Also, by using midpoint formula,

$$\text{Midpoint of AC} = \left(\frac{2+5}{2}, \frac{-1+6}{2}\right)$$

$$= \left(\frac{7}{2}, \frac{5}{2}\right)$$

$$\text{Midpoint of BD} = \left(\frac{5+2}{2}, \frac{-1+6}{2}\right)$$

$$= \left(\frac{7}{2}, \frac{5}{2}\right)$$

So, Midpoint of AC = Midpoint of BD.

So, AC and BD bisect each other.

WORKSHEET - 2

SECTION-A

1. Let P (x, y) be the point equidistant from the pointly A (5, 1), B (–3, – 7) and C (7, – 1)

$\therefore$ PA = PB = PC

PA = PB

$$\Rightarrow \sqrt{(5-x)^2+(1-y)^2}$$

$$= \sqrt{(-3-x)^2+(-7-y)^2}$$

$$\Rightarrow \sqrt{25+x^2-10x+1+y^2-2y}$$

$$= \sqrt{9+x^2+6x+49+y^2+14y}$$

On squaring both sides, we get,

$x^2 + y^2 - 10x - 2y + 26 = x^2 + y^2 + 6x + 14y + 58$

$0 = 16x + 16y + 32$

$x + y = -2$ ...(i)

PB = PC

$\Rightarrow \quad \sqrt{(-3-x)^2+(-7-y)^2}$

$= \sqrt{(7-x)^2+(-1-y)^2}$

$\Rightarrow \quad \sqrt{9+x^2+6x+49+y^2+14y}$

$= \sqrt{49+x^2-14x+1+y^2+2y}$

On squaring both sides, we get

$x^2 + y^2 + 6x + 14y + 58$

$= x^2 + y^2 - 14x + 2y + 50$

$20x + 12y + 8 = 0$

$5x + 3y = -2$ ...(ii)

From (i), we get

$x = -2 - y$

On putting in (ii), we get

$5(-2-y) + 3y = -2$

$-10 - 5y + 3y = -2$

$-2y = 8$

$y = -4$

So, $x = -2 - y$

$= -2 + 4$

$= 2$

So, point P (2, – 4) is equidistant from point A (5, 1), B (–3, – 7) and C (7, – 1).

2. Reflection of (–3, 4) in X – axis (Q) = (– 3, – 4)

Reflection of (– 3, 4) in Y – axis (R) = (3, 4)

So, by using distance formula,

$QR = \sqrt{(3+3)^2+(4+4)^2}$

$= \sqrt{36+64}$

$= \sqrt{100}$

$= 10$ units

3. As point (3, a) lies on line $2x - 3y + 5 = 0$

$\therefore \quad 6 - 3a + 5 = 0$

$3a = 11$

$a = \frac{11}{3}$

4. By Distance formula,

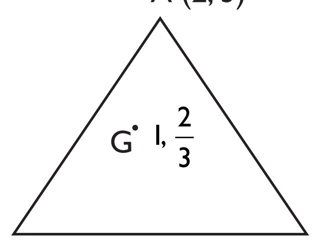
Distance $= \sqrt{(0+6)^2+(0-8)^2}$

$= \sqrt{36+64}$

$= \sqrt{100}$

$= 10$ units

5.



Let $(x_1, y_1) = (2, 3)$

$(x_2, y_2) = (-2, 1)$

$(x_3, y_3) = (x, y)$

Centroid (G) = $\left(1, \frac{2}{3}\right)$

We know that

Centroid $= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

$\left(1, \frac{2}{3}\right) = \left(\frac{2-2+x}{3}, \frac{3+1+y}{3}\right)$

$\left(1, \frac{2}{3}\right) = \left(\frac{x}{3}, \frac{4+y}{3}\right)$

$\Rightarrow \quad 1 = \frac{x}{3}$ and $\frac{2}{3} = \frac{4+y}{3}$

$\Rightarrow \quad x = 3$ and $y = -2$

6. Let $(x_1, y_1) = (k, 2k)$

$(x_2, y_2) = (3k, 3k)$

$(x_3, y_3) = (3, 1)$

Since the points are collinear, area of triangle is zero.

$$\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]=0$$

$$[k\ (3k-1)+3k\ (1-2k)+3\ (2k-3k)]=0$$

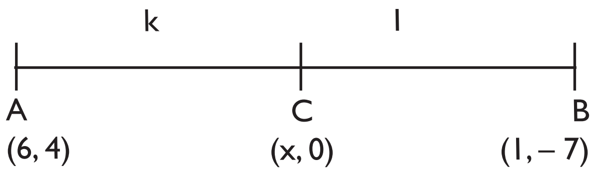
$$[3k^2-k+3k-6k^2-3k]=0$$

$$[-3k^2-k]=0$$

$$k\ (3k+1)=0$$

$$k=\frac{-1}{3}, 0$$

7.



Let the ratio in which x – axis divides AB be k : 1.

Point on x – axis must be of form (x, 0), so, by using section formula,

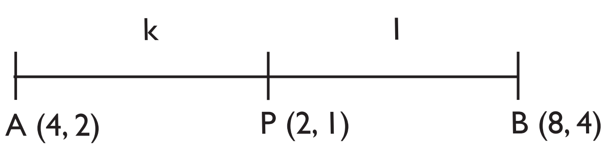
$$(x, 0) = \left(\frac{k+6}{k+1}, \frac{-7k+4}{k+1}\right)$$

$$\therefore \quad \frac{-7k+4}{k+1} = 0$$

$$k = \frac{4}{7}$$

So, x – axis divides line AB in ratio 4 : 7.

8.



Let AP : PB = k : 1

By section formula,

$$P\ (2, 1) = \left(\frac{8k+4}{k+1}, \frac{4k+2}{k+1}\right)$$

$$\therefore \quad 2 = \frac{8k+4}{k+1}, \quad 1 = \frac{4k+2}{k+1}$$

$$8k + 4 = 2k + 2$$

$$6k = -2$$

$$k = \frac{-2}{6}$$

$$= \frac{-1}{3}$$

$$\therefore \quad \frac{AB}{PB} = \frac{-1}{3}$$

$$\frac{PB}{AP} = -3$$

$$\frac{PB}{AP} + 1 = -3 + 1$$

$$\frac{AP+PB}{AP} = -2$$

$$\frac{AB}{AP} = -2$$

$$\frac{AP}{AB} = \frac{-1}{2}$$

$$AP = \frac{-1}{2}AB$$

9. By using distance formula,

$$\sqrt{(a\sin\alpha + a\cos\alpha)^2 + (-b\cos\alpha - b\sin\alpha)^2}$$

$$= \sqrt{a^2(\sin\alpha+\cos\alpha)^2 + b^2(\sin\alpha+\cos\alpha)^2}$$

$$= \sqrt{(a^2+b^2)(\sin\alpha+\cos\alpha)^2}$$

$$= (\sin\alpha + \cos\alpha)\sqrt{(a^2+b^2)}$$

10. Point A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) are collinear if

(i) AB + BC = AC

or (ii) AB + AC = BC

or (iii) BC + AC = AB

SECTION-B

11. Let the vertices of triangle be $(x_1, y_1) = (-3, 1)$, $(x_2, y_2) = (0, -2)$ and (x_3, y_3).

Centroid of triangle (x, y) = (0, 0)

We know that, Centroid of triangle

$$(x, y) = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

i.e. $$(0, 0) = \left(\frac{-3 + 0 + x_3}{3}, \frac{1 - 2 + y_3}{3}\right)$$

$$\Rightarrow \quad \frac{-3 + x_3}{3} = 0, \quad \frac{-1 + y_3}{3} = 0$$

$$\Rightarrow \quad x_3 = 3, \quad y_3 = 1$$

So, third vertex is $(x_3, y_3) = (3, 1)$

12. Let y – axis divide the line segment joining the point P (– 4, 5) and Q (3, – 7) in ratio k : 1.

Point on y – axis must be of form (0, y).

By using section formula,

$$(0, y) = \left(\frac{3k - 4}{k + 1}, \frac{-7 + 5}{k + 1}\right)$$

$$\frac{3k - 4}{k + 1} = 0$$

$$k = \frac{4}{3}$$

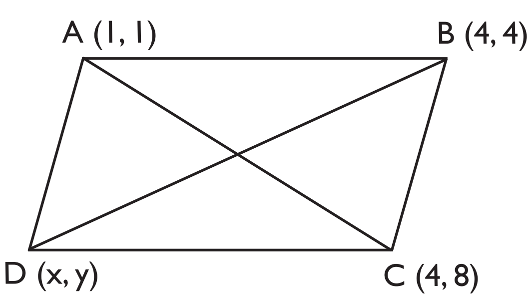
$$\therefore \quad y = \frac{-7k + 5}{k + 1}$$

$$= \frac{-7\left(\frac{4}{3}\right) + 5}{\frac{4}{3} + 1}$$

$$= \frac{\frac{-28}{3} + 5}{\frac{7}{3}}$$

$$= \frac{-13}{7}$$

13.



We know that diagonals of parallelogram bisect each other,

$$\therefore \quad \left(\frac{1+4}{2}, \frac{1+8}{2}\right) = \left(\frac{x+4}{2}, \frac{y+4}{2}\right)$$

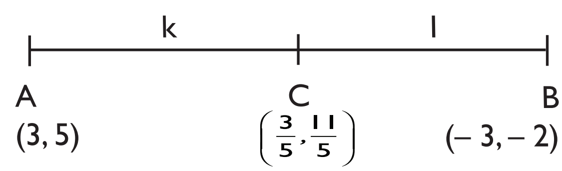
$$(5, 9) = (x + 4, y + 4)$$

$$x + 4 = 5, \quad y + 4 = 9$$

$$x = 1, \quad y = 5$$

So, fourth vertex is (1, 5).

14. Let the point C $\left(\frac{3}{5}, \frac{11}{5}\right)$ divide the line segment joining point A (3, 5) and B (–3, –2) in ratio k : 1.



By using section formula,

$$\left(\frac{3}{5}, \frac{11}{5}\right) = \left(\frac{-3k + 3}{k + 1}, \frac{-2k + 5}{k + 1}\right)$$

$$\therefore \quad \frac{-3k + 3}{k + 1} = \frac{3}{5}$$

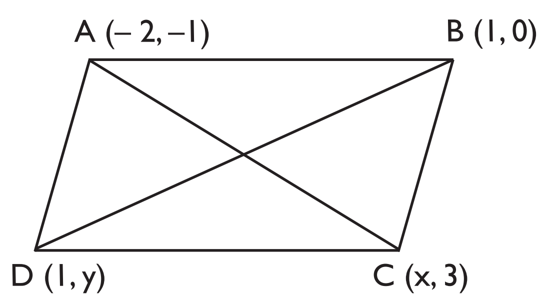
$$5(-3k + 3) = 3(k + 1)$$

$$-15k + 15 = 3k + 3$$

$$12 = 18k$$

$$k = \frac{2}{3}$$

15.



We know that diagonals of parallelogram bisect each other,

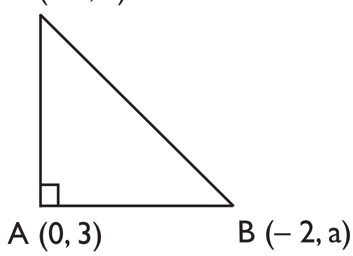
$$\therefore \quad \left(\frac{-2+x}{2}, \frac{-1+3}{2}\right) = \left(\frac{1+1}{2}, \frac{y+0}{2}\right)$$

$$\Rightarrow \quad \left(\frac{-2+x}{2}, 1\right) = \left(1, \frac{y}{2}\right)$$

$$\therefore \quad \frac{-2+x}{2} = 1, 1 = \frac{y}{2}$$

$x = 4,\ y = 2$

16. C (– 1, 4)



ΔABC is a right triangle, right angled at A.

So, by Pythagoras theorem,

$BC^2 = AB^2 + AC^2$

$$\left(\sqrt{(-2+1)^2+(a-4)^2}\right)^2 = \left(\sqrt{(-2-0)^2+(a-3)^2}\right)^2 + \left(\sqrt{(-1-0)^2+(4-3)^2}\right)^2$$

$$\Rightarrow \left(\sqrt{1+(a-4)^2}\right)^2 = \left(\sqrt{4+(a-3)^2}\right)^2 = \left(\sqrt{1+1}\right)^2$$

$\Rightarrow 1 + (a - 4)^2 = 4 + (a - 3)^2 + 2$

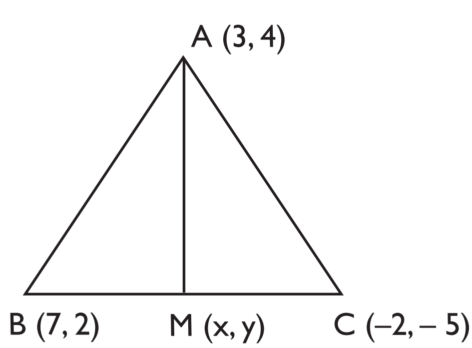
$\Rightarrow 1 + a^2 + 16 - 8a = 4 + a^2 + 9 - 6a + 2$

$\Rightarrow -8a + 17 = -6a + 15$

$\Rightarrow 2 = 2a$

$\Rightarrow a = 1$

17.



By midpoint formula,

$$M(x, y) = \left(\frac{7-2}{2}, \frac{2-5}{2}\right) = \left(\frac{5}{2}, \frac{-3}{2}\right)$$

By Distance formula,

$$AM = \sqrt{\left(\frac{5}{2}-3\right)^2+\left(\frac{-3}{2}-4\right)^2}$$

$$= \sqrt{\left(\frac{-1}{2}\right)^2+\left(\frac{-3-8}{2}\right)^2}$$

$$= \sqrt{\frac{1}{4}+\frac{121}{4}}$$

$$= \sqrt{\frac{122}{4}} = \sqrt{\frac{61}{2}}$$

18. As point A (x, y) is equidistant from B (6, – 1) and C (2, 3)

$\therefore$ AB = AC

$$\sqrt{(6-x)^2+(-1-y)^2} = \sqrt{(2-x)^2+(3-y)^2}$$

On squaring both sides, we get

$(6 - x)^2 + (-1 - y)^2 = (2 - x)^2 + (3 - y)^2$

$36 + x^2 - 12x + 1 + y^2 + 2y = 4 + x^2 - 4x + 9 + y^2 - 6y$

$\therefore -12x + 2y + 37 = -4x - 6y + 13$

$\Rightarrow \quad 0 = 8x - 8y - 24$

$\Rightarrow \quad 8x - 8y = 24$

$\Rightarrow \quad x - y = 3$

$\Rightarrow \quad x = y + 3$

19. As the points A (2, 1) and B (1, 2) are equidistant from the point C (x, y),

BC = AC

$$\sqrt{(x-1)^2+(y-2)^2} = \sqrt{(x-2)^2+(y-1)^2}$$

On squaring both sides, we get

$$(x-1)^2 + (y-2)^2 = (x-2)^2 + (y-1)^2$$

$$x^2 + 1 - 2x + y^2 + 4 + 4y = x^2 + 4 - 4x + y^2 + 1 - 2y$$

$$-2x + 4y + 5 = -4x - 2y + 5$$

$$2x + 6y = 0$$

$$x + 3y = 0$$

20. Let the vertices of triangle be

$$(x_1, y_1) = (5, 2)$$

$$(x_2, y_2) = (4, 7)$$

$$(x_3, y_3) = (7, -4)$$

Area of triangle

$$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}[5(7 + 4) + 4(-4 - 2) + 7(2 - 7)]$$

$$= \frac{1}{2}[5(11) + 4(-6) + 7(-5)]$$

$$= \frac{1}{2}[55 - 24 - 35]$$

$$= \frac{1}{2}[55 - 59]$$

$$= \frac{4}{2} = 2 \text{ sq. units}$$

SECTION-C

21. Given : AP = AQ

To prove : $ay = bx$

Proof : By using distance formula,

$$AP = \sqrt{(a+b-x)^2 + (b-a-y)^2}$$

$$AQ = \sqrt{(a-b-x)^2 + (a+b-y)^2}$$

$$AP = AQ$$

$$\sqrt{(a+b-x)^2 + (b-a-y)^2} = \sqrt{(a-b-x)^2 + (a+b-y)^2}$$

On squaring both sides, we get

$$\Rightarrow (a + b - x)^2 + (b - a - y)^2 = (a - b - x)^2 + (a + b - y)^2$$

$$\Rightarrow a^2 + b^2 + x^2 + 2ab - 2bx - 2ax + a^2 + b^2 + y^2 - 2ab + 2ay - 2by = a^2 + b^2 + x^2 - 2ab + 2bx - 2ax + a^2 + b^2 + y^2 + 2ab - 2by - 2ay$$

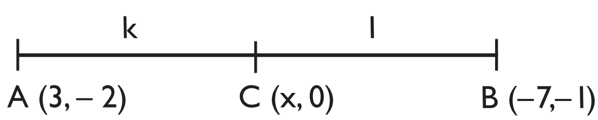
$$\Rightarrow 2bx - 2ax + 2ay - 2by = 2bx - 2by - 2ax - 2ay$$

$$4ay = 4bx$$

$$ay = bx$$

22. Internal ratio :

Let x – axis divides line segment joining the points A (3, –2) and B (– 7, – 1) in ratio k : 1. Point on x – axis is of form (x, 0).



By using section formula,

$$(x, 0) = \left(\frac{-7k+3}{k+1}, \frac{-k-2}{k+1}\right)$$

$$\therefore\ 0 = \frac{-k-2}{k+1}$$

$$k = -2$$

External ratio :

By using section formula,

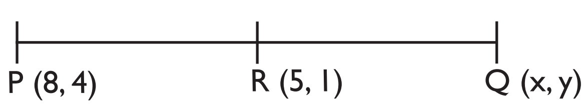
$$(x, 0) = \left(\frac{-7k-3}{k-1}, \frac{-k+2}{k-1}\right)$$

$$\therefore\ 0 = \frac{-k+2}{k-1}$$

$$-k + 2 = 0$$

$$k = 2$$

23.



By mid-point formula,

$$(5, 1) = \left(\frac{8+x}{2}, \frac{4+y}{2}\right)$$

$$5 = \frac{8+x}{2}, 1 = \frac{4+y}{2}$$

$x + 8 = 10,\ y + 4 = 2$

$x = 2,\ y = -2$

So, Coordinates of $Q = (x, y)$

$= (2, -2)$

24. Let points be

$(x_1, y_1) = (c, a+b)$

$(x_2, y_2) = (a, b+c)$

$(x_3, y_3) = (b, a+c)$

Area of triangle

$$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}[c(b + c - a - c) + a(a + c - a - b) + b(a + b - b - c)]$$

$$= \frac{1}{2}[c(b - a) + a(c - b) + b(a - c)]$$

$$= \frac{1}{2}[bc - ac + ac - ab + ab - bc]$$

$= 0$

As area of triangle = 0

So, points A, B and C are collinear.

25. Let the point be

$(x_1, y_1) = (a, 0)$

$(x_2, y_2) = (0, b)$

$(x_3, y_3) = (1, 1)$

Points are collinear, if area of triangle = 0

i.e. $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

$\Rightarrow \frac{1}{2}[a(b - 1) + 0(1 - 0) + 1(0 - b)] = 0$

$\Rightarrow \frac{1}{2}[ab - a - b] = 0$

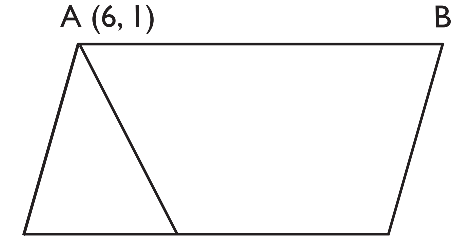
$\Rightarrow ab - a - b = 0$

$\Rightarrow ab = a + b$

$\Rightarrow 1 = \frac{a}{ab} + \frac{b}{ab}$

$\Rightarrow 1 = \frac{1}{a} + \frac{1}{b}$

26.



We know that diagonals of parallelogram bisect each other.

$\therefore$ Midpoint of AC = midpoint of BD

So, by midpoint formula,

$$\left(\frac{7+9}{2}, \frac{3+4}{2}\right) = \left(\frac{x+8}{2}, \frac{y+2}{2}\right)$$

i.e. $\left(\frac{15}{2}, \frac{5}{2}\right) = \left(\frac{x+8}{2}, \frac{y+2}{2}\right)$

i.e. $x + 8 = 15,\ y + 2 = 5$

$x = 7,\ y = 3$

So, point $D = (7, 3)$

Again, by midpoint formula,

$$E(u, v) = \left(\frac{7+9}{2}, \frac{3+4}{2}\right)$$

$$= \left(\frac{16}{2}, \frac{7}{2}\right)$$

$$= \left(8, \frac{7}{2}\right)$$

For area of ΔADE

Let $(x_1, y_1) = (6, 1)$

$(x_2, y_2) = (7, 3)$

$(x_3, y_3) = \left(8, \frac{7}{2}\right)$

area of ΔADE

$$= \frac{1}{2}\ [x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)]$$

$$= \left[6\left(3 - \frac{7}{2}\right) + 7\left(\frac{7}{2} - 1\right) + 8(1 - 3)\right]$$

$$= \frac{1}{2}\left[6\left(\frac{-1}{2}\right) + 7\left(\frac{5}{2}\right) + 8(-2)\right]$$

$$= \frac{1}{2}\left[-3 + \frac{35}{2} - 16\right]$$

$$= \frac{1}{2}\left[\frac{35}{2} - 19\right]$$

$$= \frac{1}{2}\left[\frac{35 - 38}{2}\right]$$

$$= \frac{3}{4} \text{ sq. units}$$

27. Let the points be

$(x_1, y_1) = (p + 1, 2p - 2)$

$(x_2, y_2) = (p - 1, p)$

$(x_3, y_3) = (p - 6, 2p - 6)$

Points are collinear if area of triangle is zero.

$$\frac{1}{2}\ [x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)] = 0$$

$$[(p + 1)(p - 2p + 6) + (p - 1)(2p - 6 - 2p + 2) + (p - 6)(2p - 2 - p)] = 0$$

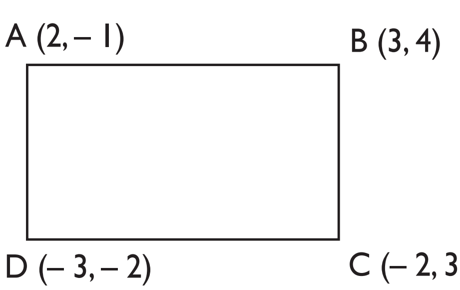
$$[(p + 1)(6 - p) + (p - 1)(-4) + (p - 6)(p - 2)] = 0$$

$$[6p - p^2 + 6 - p - 4p + 4 + p^2 - 2p - 6p + 12] = 0$$

$$[7p + 22] = 0$$

$$p = \frac{-22}{7}$$

28.



By using distance formula,

$$AB = \sqrt{(3-2)^2 + (4+1)^2}$$

$$= \sqrt{1+25} = \sqrt{26} \text{ units}$$

$$BC = \sqrt{(-2-3)^2 + (3-4)^2}$$

$$= \sqrt{25+1} = \sqrt{26} \text{ units}$$

$$CD = \sqrt{(-3+2)^2 + (-2-3)^2}$$

$$= \sqrt{1+25}$$

$$= \sqrt{26} \text{ units}$$

$$AD = \sqrt{(-3-2)^2 + (-2+1)^2}$$

$$= \sqrt{26} \text{ units}$$

As AB = BC = CD = AD,

ABCD is a rhombus

Again, by distance formula,

$$AC = \sqrt{(-2-2)^2 + (3+1)^2}$$

$$= \sqrt{16+16}$$

$$= \sqrt{32}$$

$$= 4\sqrt{2} \text{ units}$$

$$BD = \sqrt{(-3-3)^2 + (-2-4)^2}$$

$$= \sqrt{(-6)^2 + (-6)^2}$$

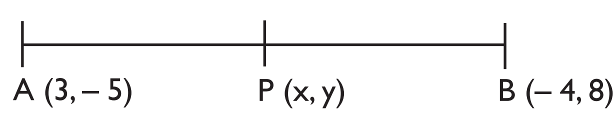
$$= \sqrt{72}$$

$$= 6\sqrt{2} \text{ units}$$

$\therefore$ AC $\neq$ BD

As diagonals are not equal, ABCD is a rhombus but not a square.

29.



$$\frac{AP}{PB} = \frac{k}{1}$$

Let point P be (x, y).

By using section formula,

$$(x, y) = \left(\frac{-4k+3}{k+1}, \frac{8k-5}{k+1}\right)$$

$$(x, y) = \left(\frac{-4k+3}{k+1}, \frac{8k-5}{k+1}\right)$$

$$\therefore \quad x = \frac{-4k+3}{k+1}, \quad y = \frac{8k-5}{k+1}$$

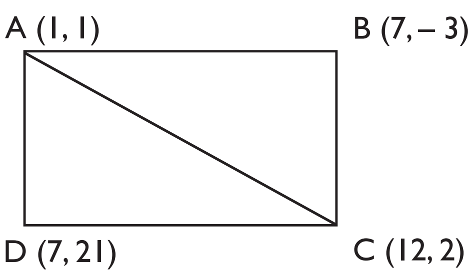
As point P lies on line $x + y = 0$

$$\therefore \quad \left(\frac{-4k+3}{k+1}\right) + \left(\frac{8k-5}{k+1}\right) = 0$$

$\Rightarrow \quad 4k - 2 = 0$

$\Rightarrow \quad k = \frac{1}{2}$

30.



Area of ΔABC

$= \frac{1}{2}$ [1 (– 3 – 2) + 7 (2 – 1) + 12 (1 + 3)]

$= \frac{1}{2}$ [– 5 + 7 + 48]

$= \frac{1}{2}$ [50]

= 25 sq. units

Area of ΔACD

$= \frac{1}{2}$ [1 (2 – 21) + 12 (21 – 1) + 7 (1 – 2)]

$= \frac{1}{2}$ [– 19 + 12 (20) – 7]

$= \frac{1}{2}$ [– 26 + 240]

$= \frac{1}{2}$ [214]

= 107 sq. units

So, area of quadrilateral ABCD

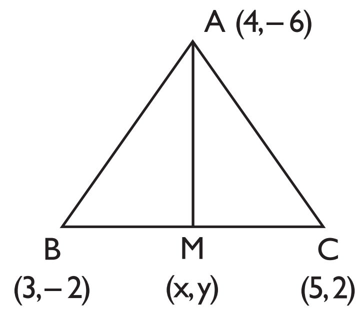
= area of ΔABC + area of ΔACD

= 25 + 107

= 132 sq. units

SECTION-D

31.



Let AM be the median such that point M is (x, y).

$$(x, y) = \left(\frac{3+5}{2}, \frac{-2+2}{2}\right)$$

$(x, y) = (4, 0)$

So, point M $(x, y) = (4, 0)$

Area of ΔAMB

$$= \frac{1}{2}\left[4(-2-0)+3(0+6)+4(-6+2)\right]$$

$$= \left|\frac{1}{2}\left[-8+18-16\right]\right| = \left|\frac{-6}{2}\right| = |-3| = 3 \text{ sq. units}$$

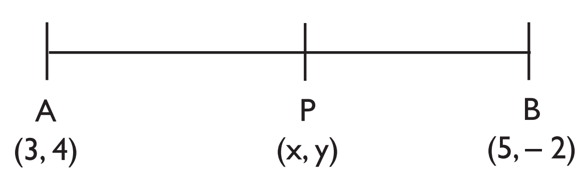
Area of ΔAMC

$$= \frac{1}{2}\left[4(0-2)+4(2+6)+5(-6-0)\right]$$

$$= \frac{1}{2}\left[-8+32-30\right] = \left|\frac{-6}{2}\right| = |-3| = 3 \text{ sq. units}$$

So, median divides the triangle into two triangle of equal area.

32.



PA = PB

$$\sqrt{(-3-x)^2+(4-y)^2} = \sqrt{(5-x)^2+(-2-y)^2}$$

On squaring both sides, we get

$$(3-x)^2 + (4-y)^2 = (5-x)^2 + (-2-y)^2$$

$\Rightarrow$ $9 + x^2 - 6x + 16 + y^2 - 8y = 25 + x^2 - 10x + 4 + y^2 + 4y$

$\Rightarrow$ $-6x - 8y + 25 = -10x + 4y + 29$

$\Rightarrow$ $4x - 12y - 4 = 0$

$\Rightarrow$ $x - 3y = 1$ ...(i)

Also, area of $\Delta PAB = 10$

$\therefore$ $\frac{1}{2}[x(4+2) + 3(-2-y) + 5(y-4)] = 10$

$\Rightarrow$ $[6x - 6 - 3y + 5y - 20] = 20$

$\Rightarrow$ $[6x + 2y - 26] = 20$

$\Rightarrow$ $[3x + y - 13] = 10$

$\Rightarrow$ $3x + y - 13 = \pm 10$

$\Rightarrow$ $3x + y = 23$ ...(ii)

or $3x + y = 3$ ...(iii)

From (i), $x = 1 + 3y$

So, eq. (ii) becomes $3 + 9y + y = 23$

$10y = 20$

$y = 2$

So, $x = 1 + 3y$

$= 1 + 6$

$= 7$

So, P (x, y) = (7, 2)

On putting $x = 1 + 3y$ in (iii), we get

$3(1 + 3y) + y = 3$

$3 + 9y + y = 3$

$10y = 0$

$y = 0$

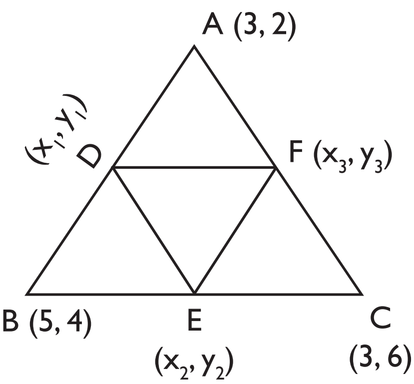
So, $x = 1 + 3y$

$= 1 + 0$

$= 1$

So, P (x, y) = (1, 0)

33.



By midpoint formula,

$D(x_1, y_1) = \left(\frac{3+5}{2}, \frac{2+4}{2}\right)$

$D(x_1, y_1) = (4, 3)$

Again, $E(x_2, y_2) = \left(\frac{5+3}{2}, \frac{4+6}{2}\right)$

$= (4, 5)$

$F(x_3, y_3) = \left(\frac{3+3}{2}, \frac{2+6}{2}\right)$

$= (3, 4)$

Area of ΔDEF

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[4(5-4) + 4(4-3) + 3(3-5)]$

$= \frac{1}{2}[4 + 4 - 6] = 1$ sq. unit

34. Let $(x_1, y_1) = (-2, 5)$

$(x_2, y_2) = (k, -4)$

$(x_3, y_3) = (2k + 1, 10)$

Area of triangle

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$$53 = \frac{1}{2}\,[-2(-4-10) + k(10-5) + (2k+1)(5+4)]$$

$$53 = \frac{1}{2}\,[28 + 5k + 18k + 9]$$

$$53 = \frac{1}{2}\,[23k + 37]$$

$\therefore\quad 23k + 37 = \pm 106$

if $\quad 23k + 37 = 106$

$\qquad 23k = 69$

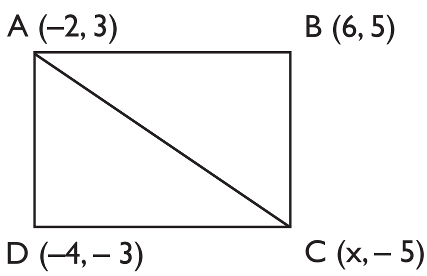
$\qquad k = 3$

if $\quad 23k + 37 = 106$

$\qquad 23k = -143$

$\qquad k = \frac{-143}{23}$, rejected $k < 0$.

$\therefore\quad k = 3.$

35. 

Area of quadrilateral ABCD = 80 sq. units

i.e. area of ΔABC + area of ΔACD = 80

i.e. $\frac{1}{2}\,[-2(5+5) + 6(-5-3) + x(3-5)] + \frac{1}{2}$
$[-2(-5+3) + x(-3-3) + (-4)(3+5)] = 80$

$\Rightarrow \frac{1}{2}\,[-20 - 48 - 2x + 4 - 6x - 32] = 80$

$\Rightarrow \frac{1}{2}\,[-80 - 96] = 80$

$\Rightarrow -8x - 96 = \pm 160$

$\Rightarrow -x - 12 = \pm 20$

$-x - 12 = 20$	$-x - 12 = -20$
$x = -32$	$x = 8$

$\therefore$ Positive value of x = 8.

36. Let D (x, y) be the Circumcentre.

We know that Circumcentre of a triangle is equidistant from each of the vertices.

Let the vertices be A (x_1, y_1) = (8, 6), B (x_2, y_2) = (8,– 2) and C (x_3, y_3) = (2,– 2).

So, AD = BD

$$\sqrt{(8-x)^2 + (6-y)^2} = \sqrt{(8-x)^2 + (-2-y)^2}$$

On squaring both sides, we get

$$(8-x)^2 + (6-y)^2 = (8-x)^2 + (-2-y)^2$$

$$(6-y)^2 = (-2-y)^2$$

$$36 + y^2 - 12y = 4 + y^2 + 4y$$

$$32 = 16y$$

$$y = 2$$

Also, BD = CD

$$\sqrt{(8-x)^2 + (-2-y)^2} = \sqrt{(2-x)^2 + (-2-y)^2}$$

$$(8-x)^2 + (-2-y)^2 = (2-x)^2 + (-2-y)^2$$

$$64 + x^2 - 16x + 4y^2 + 4y = 4 + x^2 - 4x + 4 + y^2 + 4y$$

$\Rightarrow -16x + 4y + 68 = -4x + 4y + 8$

$\Rightarrow 12x = 60$

$\Rightarrow \quad x = 5$

So, Circumcentre is (x, y) = (5, 2)

Circumradius = AD

$= \sqrt{(8-x)^2 + (6-y)^2}$

$= \sqrt{(8-5)^2 + (6-2)^2}$

$= \sqrt{9+16}$

$= \sqrt{25}$

$= 5$

37. By using midpoint formula,

$$C\ (x, y) = \left(\frac{0+2a}{2}, \frac{2b+0}{2}\right)$$

$= (a, b)$

Using distance formula, we have

$$BC = \sqrt{(a-0)^2+(b-2b)^2}$$

$$= \sqrt{a^2+b^2}$$

$$OC = \sqrt{(a-0)^2+(b-0)^2}$$

$$= \sqrt{a^2+b^2}$$

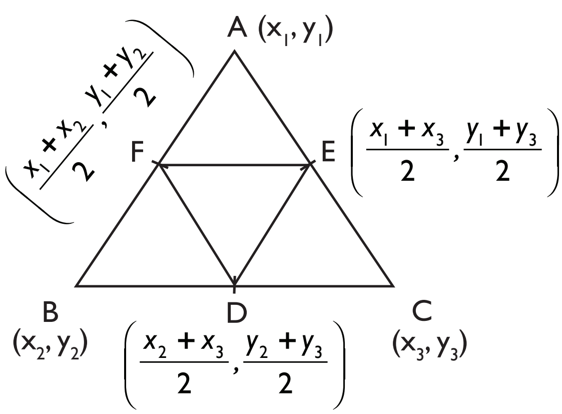
$$AC = \sqrt{(a-2a)^2+(b-0)^2}$$

$$= \sqrt{a^2+b^2}$$

So, BC = CO = AC

∴ Point C is equidistant from the vertices A, O and B.

38.



By midpoint formula,

D is $\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$

E is $\left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right)$

F is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Area of ΔABC

$$\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$$

Consider, ar ΔDEF

$$= \frac{1}{2}\left[\begin{array}{l}\left(\frac{x_1+x_2}{2}\right)\left[\left(\frac{y_2+y_3}{2}\right)-\left(\frac{y_1+y_3}{2}\right)\right] \\ +\left(\frac{x_2+x_3}{2}\right)\left[\left(\frac{y_1+y_3}{2}\right)-\left(\frac{y_1+y_2}{2}\right)\right] \\ +\left(\frac{x_1+x_3}{2}\right)\left[\left(\frac{y_1+y_2}{2}\right)-\left(\frac{y_2+y_3}{2}\right)\right]\end{array}\right]$$

$$= \frac{1}{8}\left[\begin{array}{l}(x_1+x_2)(y_2-y_1) \\ +(x_2+x_3)(y_3-y_2) \\ +(x_1+x_3)(y_1-y_3)\end{array}\right]$$

$$= \frac{1}{8}\left[\begin{array}{l}x_1\left[(y_2-y_1)+(y_1-y_3)\right] \\ +x_2\left[(y_2-y_1)+(y_3-y_2)\right] \\ +x_3\left[(y_3-y_2)+(y_1-y_3)\right]\end{array}\right]$$

$$= \frac{1}{8}\left[\begin{array}{l}x_1(y_2-y_3)+x_2(y_3-y_1) \\ +x_3(y_1-y_2)\end{array}\right]$$

$$= \left(\frac{1}{4}\right)\left(\frac{1}{2}\right)\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$$

$$= \frac{1}{4} \text{ area of } \Delta ABC$$

39. Using formula for area of triangle,

$$\text{ar }\Delta DBC = \frac{1}{2}\left[x(5+2)-3(-2-3x)+4(3x-5)\right]$$

$$= \frac{1}{2}\left[7x+6+9x+12x-20\right]$$

$$= \frac{1}{2}\left[28x-14\right]$$

$$= [14x-7] \quad \text{...(i)}$$

Using formula for area of triangle, or ΔABC

$$= \frac{1}{2}\left[6(5+2)-3(-2-3)+4(3-5)\right]$$

$$= \frac{1}{2}\left[42+15-8\right]$$

$$= \frac{1}{2}\left[49\right] \text{ sq. units}$$

As $\dfrac{ar\Delta DBC}{ar\Delta ABC} = \dfrac{1}{2}$

$\Rightarrow \dfrac{|14x-7|}{\dfrac{|49|}{2}} = \dfrac{1}{2}$

$\Rightarrow \dfrac{2|14x-7|}{49} = \dfrac{1}{2}$

$\Rightarrow 14x - 7 = \pm \dfrac{49}{4}$

If $14x - 7 = \dfrac{49}{4}$

$14x = \dfrac{49}{4} + 7 = \dfrac{49+28}{4} = \dfrac{77}{4}$

$\Rightarrow x = \dfrac{11}{8}$

If $14x - 7 = \dfrac{-49}{4}$

$14x = \dfrac{-49}{4} + 7 = \dfrac{-49+28}{4} = \dfrac{-21}{4}$

$\Rightarrow x = \dfrac{-3}{8}$

40. As the point (x_1, y_1), (x_2, y_2) and (x_3, y_3) lie on the same line, area of triangle formed by these points is 0.

i.e. $\dfrac{1}{2}\,[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)] = 0$

$[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)] = 0$

On dividing by $x_1\,x_2\,x_3$, we get

$\left[\dfrac{\text{üµüüµüüü}_3 + {}_2\;{}_3 - {}_1 + {}_3\;{}_1 - {}_2}{x_1\,x_2\,x_3}\right] = 0$

$\left[\dfrac{y_2-y_3}{x_2\,x_3} + \dfrac{y_3-y_1}{x_1\,x_3} + \dfrac{y_1-y_2}{x_1\,x_2}\right] = 0$

$\therefore \dfrac{y_2-y_3}{x_2\,x_3} + \dfrac{y_3-y_1}{x_1\,x_3} + \dfrac{y_1-y_2}{x_1\,x_2} = 0$

CASE STUDY-1

(i) (a) Distance covered on line AD = $\dfrac{1}{4} \times 100$

= 25 m

Distance covered on line AB = 2 m

$\therefore$ coordinates of green flag are (2, 25)

(ii) (c) Distance covered on line AD = $\dfrac{1}{5} \times 100$

= 20 m

Distance covered on line AB = 8 m

Coordinates of red flag are (8, 20)

(iii) (d) Coordinates of red flag are (8, 20)

Coordinates of green flag are (2, 25)

Distance between red and green flag

$= \sqrt{(8-2)^2 + (25)^2}$

$= \sqrt{36+25}$

$= \sqrt{61}$ m

(iv) (a) Coordinates of mid point of red flag and green flag are

$\dfrac{8+2}{2}, \dfrac{20+25}{2}$

= (5, 22.5)

(v) (d) Let the distance covered by Rohini is *x* m.

$\dfrac{1}{x} \times 100 = 22.5$

$x = \dfrac{22.5}{100}$

$x = \dfrac{9}{40}$

CASE STUDY-2

(i) (b) Point D is the mid point of AB.

$\therefore$ The coordinates of D would lie on halfway between A and B.

Let coordinates of B are (x, y) so that

$\dfrac{D+x}{2} = 1 \qquad \dfrac{-1+y}{2} = 0$

$x = 2 \qquad y = 1$

(ii) (a) Let coordinates of C are x' and y'

$\dfrac{x'+0}{2} = 0 \qquad \dfrac{y'-1}{2} = 1$

$x' = 0 \qquad y' = 3$

$\therefore$ coordinates of c are (0, 3)

(iii) (c) Let coordinates of F be (x'', y''). Using midpoint formula as F is the mid point of BC.

$x'' = \frac{2+0}{2}$

$= 1$

$y'' = \frac{1+3}{2}$

$= 2$

Coordinates of F are (1, 2)

(iv) (d) Area of DABC = $\frac{1}{2}$(base × height)

The base BC is 2 units 8 height AD is 2 unit

Area = $\frac{1}{2} \times 4 \times 2$

= 4 sq. units

(v) (d) Area of DDEF = $\frac{1}{2}$(base × height)

Base DE is 1 units

Height is 2 units.

Area = $\frac{1}{2} \times 2 \times 1 = 1$ sq. units

Chapter 8

Introduction to Trigonometry

Multiple Choice Questions

1. (a) $\text{Cot } x = \frac{12}{16} = \frac{3}{4}$

$$\frac{\sin x - \cos x}{\sin x + \cos x} = \frac{1 - \cot x}{1 + \cot x}$$

$$= \frac{1 - \frac{3}{4}}{1 + \frac{3}{4}}$$

$$= \frac{1}{4} \times \frac{4}{7}$$

$$= \frac{1}{7}$$

2. (a) $\frac{x(2)^2(\sqrt{2})^2}{8\left(\frac{1}{\sqrt{2}}\right)^2\left(\frac{\sqrt{3}}{2}\right)^2} = (\sqrt{3})^2 - \left(\frac{1}{\sqrt{3}}\right)^2$

$$\frac{8x}{3} = 3 - \frac{1}{3} = \frac{8}{3}$$

$$x = 1$$

3. (b) $A + B + C = 180°$

$\Rightarrow B + C = 180° - A$

$\therefore \quad = sin\left(\frac{B+C}{2}\right) = sin\left(\frac{180° - A}{2}\right)$

$$= sin\left(90° - \frac{A}{2}\right)$$

$$= cos\frac{A}{2}$$

4. (b) $\frac{tan 30°}{tan 0° - cot 30°}$

$$= \frac{\frac{1}{\sqrt{3}}}{0 - \sqrt{3}}$$

$$= \frac{-1}{3}$$

5. (b) Consider

$(a \sin\theta + b \cos\theta)^2$

$= a^2 \sin^2\theta + b^2 \cos^2\theta + 2ab \sin\theta \cos\theta$

$= a^2 (1 - \cos^2\theta) + b^2 (1 - \sin^2\theta) + 2ab \sin\theta \cos\theta$

$= a^2 + b^2 - a^2 \cos^2\theta - b^2\sin^2\theta + 2ab \sin\theta \cos\theta$

$= a^2 + b^2 - (a^2 \cos^2\theta + b^2 \sin^2\theta) + 2ab \sin\theta \cos\theta$ (i)

Also, $a \cos\theta - b \sin\theta = c$

$\Rightarrow (a \cos\theta - b \sin\theta)^2 = c^2$

$\Rightarrow a^2 \cos^2\theta + b^2 \sin^2\theta - 2ab \sin\theta \cos\theta = c^2$

$\Rightarrow a^2 \cos^2\theta + b^2 \sin^2\theta = c^2 + 2ab \sin\theta \cos\theta$ (ii)

So, $(a \sin\theta + b \cos\theta)^2$

$= a^2 + b^2 + 2ab \sin\theta \cos\theta - c^2 - 2ab \sin\theta \cos\theta$

[From (i) and (ii)]

$= a^2 + b^2 - c^2$

$\therefore a \sin + b \cos = \pm \sqrt{a^2 + b^2 - c^2}$

WORKSHEET - 1

SECTION-A

1. $$\frac{\cos(90-\theta)\sec(90-\theta)\tan\theta}{\mathrm{cosec}(90-\theta)\sin(90-\theta)\cot(90-\theta)}$$

$$+\frac{\tan(90-\theta)}{\cot\theta}$$

$$=\frac{\sin\theta\,\mathrm{cosec}\,\theta\tan\theta}{\sec\theta\cos\theta\tan\theta}$$

$$=\frac{\cos\theta}{\cot\theta}$$

$$=\frac{1}{1}+\frac{1}{1}=2$$

2. Consider

$$\frac{\tan A\tan B+\tan A\cot B}{Sin A\sec B}-\frac{\sin^2 B}{\cos^2 A}$$

$$=\frac{\tan(90^\circ-B)\tan B+\tan A\cot(90^\circ-A)}{\sin A\sec(90^\circ-A)}$$

$$-\frac{\sin^2 B}{\cos^2(90^\circ-B)}$$

$$=\frac{\cot B\tan B+\tan^2 A}{\sin A\,\mathrm{cosec}A}-\frac{\sin^2 B}{\sin^2 B}$$

$$=\frac{1+\tan^2 A}{1}-1=\tan^2 A$$

3. $$\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$$

$$=\sqrt{\frac{1+\sin\theta}{1-\sin\theta}\times\frac{1+\sin\theta}{1+\sin\theta}}$$

$$=\sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}}$$

$$=\sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}$$

$$=\frac{1+\sin\theta}{\cos\theta}$$

4. $\cos(a+b) = 0$

$a + b = 90^\circ$

$\therefore \quad a = 90^\circ - b$

Consider $\sin(a-b) = \sin[90^\circ - 2b] = \cos 2b$

The statement is true.

5. $$\frac{\tan^2-\sec^2\theta}{\cot^2-\mathrm{cosec}^2\theta}$$

$$=\frac{\sec^2\theta-\tan^2\theta}{\mathrm{cosec}^2\theta-\cot^2\theta}=\frac{1}{1}=1$$

6. $\mathrm{cosec}\theta = 3x \Rightarrow \quad x = \frac{1}{3}\,\mathrm{cosec}\theta$

$\cot\theta = \frac{3}{x} \Rightarrow \frac{1}{x} = \frac{1}{3}\cot\theta$

consider $x^2 - \frac{1}{x^2} = \frac{1}{9} \quad \mathrm{cosec}^2\theta - \frac{1}{9}\cot^2\theta = \frac{1}{9}$

7. $\tan A = \frac{5}{12}$

Consider $(\sin A + \cos A)\,\mathrm{Sec}\,A$

$= (\sin A + \cos A)\ \frac{1}{\cos A}$

$= \tan A + 1$

$=\frac{5}{12}+1=\frac{17}{12}$

8. Consider $6\tan^2\theta - \frac{6}{\cos^2\theta}$

$= 6(\tan^2\theta - \sec^2\theta)$

$= 6 \quad (1)$

$= 6$

SECTION-B

9. $2\sin^2 30^\circ - 3\cos^2 45^\circ + \tan^2 60^\circ$

$$=2\left(\frac{1}{2}\right)^2-3\left(\frac{1}{\sqrt{2}}\right)^2+\left(\sqrt{3}\right)^2$$

$$=\frac{1}{2}-\frac{3}{2}+3=2$$

10. (a) We know that $-1 \le \sin\theta \le 1$

$\therefore \quad 0 \le \sin^2\theta \le 1$

If $\sin\theta = x + \frac{1}{x}$,

On squaring both sides, we get

$\sin^2\theta = x^2 + \frac{1}{x^2} + 2$

Here, R H S $= x^2 + \frac{1}{x^2} + 2 > 2$

but maximum value of $\sin^2\theta$ is

$\therefore \quad \sin^2\theta$ is $\neq x + \frac{1}{x}$

(b) As $(a - b)^2 \ge 0$

$\Rightarrow \quad a^2 + b^2 - 2ab \ge 0$

$\Rightarrow \quad a^2 + b^2 \ge 2ab$

$\therefore \quad \cos\theta = \frac{a^2 + b^2}{2ab} \ge \frac{2ab}{2ab} = 1$

$\Rightarrow \quad \cos\theta \ge 1$

if $\cos = 1$

$\frac{a^2 + b^2}{2ab} = 1$

$a^2 + b^2 = 2ab$

$(a - b)^2 = 0$

$a = b$

but a and b are distinct numbers

$\therefore \quad \cos\theta > 1$

but $\quad -1 \le \cos\theta \le 1$

So, $\cos\theta \neq \frac{a^2 + b^2}{2ab}$

11. (a) $2 \sin 3x = \sqrt{3}$

$\sin 3x = \frac{\sqrt{3}}{2}$

$3x = 60°$

$x = 20°$

(b) $2 \sin \frac{x}{2} = 1$

$\sin \frac{x}{2} = \frac{1}{2} = \sin 30°$

$\frac{x}{2} = 30°$

$x = 60°$

12. $\sin\theta + \sin^2\theta = 1$

$\Rightarrow \quad \sin\theta = 1 - \sin^2\theta$

$\Rightarrow \quad \sin\theta = \cos^2\theta \qquad$ (i)

$\Rightarrow \quad \tan\theta = \cos\theta$

Consider $\quad \cos^2\theta + \cos^4\theta$

$= \tan^2\theta + \tan^4\theta$

$= \tan^2\theta\,(1 + \tan^2\theta)$

$= \tan^2\theta \sec^2\theta$

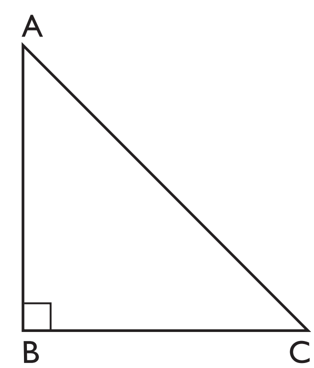
$= \tan^2\theta \frac{1}{\sin\theta} \qquad$ By (i)

$= \frac{\sin\theta}{\cos^2\theta}$

$= \frac{\sin\theta}{\sin\theta} \qquad$ By (i)

$= 1$

13.



$\tan A = \frac{1}{\sqrt{3}}$

$\frac{BC}{AB} = \frac{1}{\sqrt{3}}$

Let BC $= k$, AB $= k$ (3^0.5)

$\therefore \quad AC^2 = BC^2 + AB^2 = 4k^2$

AC $= 2k$

Consider

sinA cosC + cosA sinC

$= \left(\frac{BC}{AC}\right)\left(\frac{BC}{AC}\right) + \left(\frac{AB}{AC}\right)\left(\frac{AB}{AC}\right)$

$= \frac{BC^2}{AC^2} + \frac{AB^2}{AC^2}$

$= \frac{BC^2 + AB^2}{AC^2}$

$= \frac{AC^2}{AC^2} = 1$

14. Consider

$4\cot^2 45° - \sec^2 60° + \sin^2 60° - \cos^2 90°$

$= 4(1)^2 - (2)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - (0)^2$

$= 4 - 4 + \frac{3}{4}$

$= \frac{3}{4}$

15. Consider

$(\text{cosec}\theta - \cot\theta)^2 = \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$

$= \left(\frac{1-\cos\theta}{\sin\theta}\right)^2$

$= \frac{(1-\cos\theta)^2}{\sin^2\theta}$

$= \frac{(1-\cos\theta)^2}{1-\cos^2\theta}$

$= \frac{1-\cos\theta}{1+\cos\theta}$

16. Consider

$3\cos^2 30° + \sec^2 30° + 2\cos^2 0° + 3\sin^2 90° - \tan^2 60°$

$= 3\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{2}{\sqrt{3}}\right)^2 + 2(1)^2 + 3(1)^2 - \left(\sqrt{3}\right)^2$

$= \frac{9}{4} + \frac{4}{3} + 2 + 3 - 3$

$= \frac{9}{4} + \frac{4}{3} + 2$

$= \frac{27+16+24}{12} = \frac{67}{12}$

SECTION-C

17. $\tan\theta + \cot\theta = 2$

$\tan\theta + \frac{1}{\tan\theta} = 2$

$\tan^2\theta - 2\tan\theta + 1 = 0$

$(\tan\theta - 1)^2 = 0$

$\tan\theta - 1 = 0$

$\tan\theta = 1 = \tan 45°$

$\theta = 45°$

Consider $\tan^7\theta + \cot^7\theta$

$= \tan^7(45°) + \cot^7(45°)$

$= 1^7 + 1^7$

$= 1 + 1$

$= 2$

18. Consider

$\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta}$

$= \frac{\sin\theta(\sin\theta) + (1+\cos\theta)(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$

$= \frac{\sin^2\theta + 1 + \cos^2\theta + 2\cos\theta}{\sin\theta(1+\cos\theta)}$

$= \frac{2+2\cos\theta}{\sin\theta(1+\cos\theta)}$

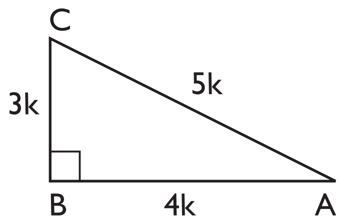
$= \frac{2+(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$

$$= \frac{2}{\sin\theta} = 2\operatorname{cosec}\theta$$

19. $\operatorname{Sec} A = \frac{5}{4} = \frac{AC}{AB}$

L H S

$$\frac{3\sin A - 4\sin^3 A}{4\cos^3 A - 3\cos A}$$



$BC^2 = AC^2 - AB^2$

$= 25k^2 - 16k^2$

$= 9k^2$

$\therefore$ BC = 3k

So, $\frac{3\sin A - 4\sin^3 A}{4\cos^3 A - 3\cos A}$

$$= \frac{3\left(\frac{3}{5}\right) - 4\left(\frac{3}{5}\right)^3}{4\left(\frac{4}{5}\right)^3 - 3\left(\frac{4}{5}\right)}$$

$$= \frac{\frac{9}{5} - \frac{108}{125}}{\frac{256}{125} - \frac{12}{5}}$$

$$\frac{\frac{225 - 108}{125}}{\frac{256 - 300}{125}} = \frac{177}{-44}$$

R H S

$$\frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$$

$$= \frac{3\left(\frac{3}{4}\right) - \left(\frac{3}{4}\right)^3}{1 - 3\left(\frac{3}{4}\right)^2}$$

$$= \frac{\frac{9}{4} - \frac{27}{64}}{1 - \frac{27}{16}}$$

$$= \frac{\frac{144 - 27}{64}}{\frac{16 - 24}{16}}$$

$$= \frac{117}{-44}$$

$$= \frac{-177}{44}$$

So, LHS = RHS

20. $a\cos\theta + b\sin\theta = m$

$a\sin\theta - b\cos\theta = n$

To prove : $a^2 + b^2 = m^2 + n^2$

Proof $a\cos\theta + b\sin\theta = m$

On squarring both sides, we get

$$a^2\cos^2\theta + b^2\sin^2\theta + 2ab\sin\theta\cos\theta = m^2 \quad (1)$$

$a\sin\theta - b\cos\theta = n$

On squarring both sides, we get

$$a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta = n^2 \quad (2)$$

On adding (1) and (2), we get

$$a^2(\sin^2\theta + \cos^2\theta) + b^2(\sin^2\theta + \cos^2\theta) = m^2 + n^2$$

$$\Rightarrow \quad a^2 + b^2 = m^2 + n^2$$

21. $x = a\cos^3\theta$

$y = b\sin^3\theta$

Consider $\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}}$

$$= \left(\frac{a\cos^3\theta}{a}\right)^{\frac{2}{3}} + \left(\frac{b\cos^3\theta}{b}\right)^{\frac{2}{3}}$$

$$= \cos^2\theta + \sin^2\theta$$

$$= 1$$

22. $\sin(A + B) = 1 = \sin 90^\circ$

$A + B = 90^\circ \quad (1)$

$\cos(A - B) = \frac{\sqrt{3}}{2} = \cos 30^\circ$

$A - B = 30^\circ \quad (2)$

On solving (1) and (2), we get

$A + B = 90^\circ$

$$\frac{A - B = 30}{2A = 120}$$

$A = 60^\circ$

From (1), $B = 90^\circ - A$

$= 90^\circ - 60^\circ$

$= 30^\circ$

23. Consider

$(1 - \sin\theta + \cos\theta)^2$

$= [(1 - \sin\theta) + \cos\theta]^2$

$= (1 - \sin\theta)^2 + \cos^2\theta + 2\cos\theta\,(1 - \sin\theta)$

$= (1 - \sin\theta)^2\,(1 - \sin^2\theta) + 2\cos\theta\,(1 - \sin\theta)$

$= (1 - \sin\theta)\,[1 - \sin\theta + 1 + \sin\theta + 2\cos\theta]$

$= (1 - \sin\theta)\,(2\cos\theta + 2)$

$= 2\,(1 + \cos\theta)\,(1 - \sin\theta)$

= R H S

24. Consider

$$\frac{\tan\theta + \sin\theta}{\tan\theta - \sin\theta}$$

$$= \frac{\frac{\sin\theta}{\cos\theta} + \sin\theta}{\frac{\sin\theta}{\cos\theta} - \sin\theta}$$

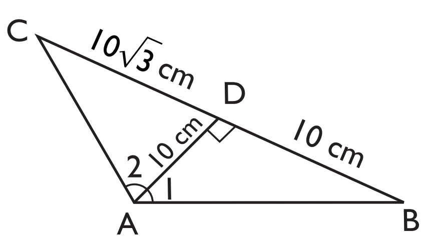
$$= \frac{\frac{1}{\cos\theta} + 1}{\frac{1}{\cos\theta} - 1}$$

$$= \frac{\sec\theta + 1}{\sec\theta - 1}$$

= R H S

SECTION-D

25.



In ΔADB,

$$\tan(\angle 1) = \frac{BD}{AD} = \frac{10}{10} = 1$$

$\therefore\ \angle 1 = 45°$

In ΔADC,

$$\tan(\angle 2) = \frac{10\sqrt{3}}{10} = \sqrt{3}$$

$\angle 2 = 60°$

So, $\angle A = \angle 1 + \angle 2$

$= 45° + 60°$

$= 105°$

26. Consider

$$\text{L H S} = \frac{\sin\theta}{1 - \cos\theta}$$

$$= \frac{\sin\theta(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$$

$$= \frac{\sin\theta(1+\cos\theta)}{1-\cos^2\theta}$$

$$= \frac{\sin\theta(1+\cos\theta)}{\sin^2\theta}$$

$$= \frac{1+\cos\theta}{\sin\theta}$$

$$= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$$

$= \text{cosec}\,\theta + \cot\theta$

= R H S

27. Consider

$\sin(2\theta + 45°) = \cos(30° - \theta)$

$\Rightarrow \cos[90° - (2\theta + 45°)] = \cos(30° - \theta)$

$\Rightarrow \cos(45° - 2\theta) = \cos(30° - \theta)$

$\Rightarrow 45° - 2\theta = 30° - \theta$

$\Rightarrow 15° = \theta$

$\therefore \tan\theta = \tan 15°$

$= \tan(45° - 30°)$

$$= \frac{\tan 45° - \tan 30°}{1 + 3\tan 45°\tan 30°}$$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{3}{\sqrt{3}}}$$

$$= \frac{\sqrt{3} - 1}{\sqrt{3}\left(1+\sqrt{3}\right)}$$

$$= \frac{\left(\sqrt{3} - 1\right)\left(\sqrt{3} - 1\right)}{\sqrt{3}\left(1+\sqrt{3}\right)\left(\sqrt{3} - 1\right)}$$

$$= \frac{3+1- 2\sqrt{3}}{\sqrt{3}(3 - 1)}$$

$$= \frac{4-2\sqrt{3}}{2\sqrt{3}}$$

$$= \frac{2-\sqrt{3}}{\sqrt{3}}$$

28. Consider

$$\text{L H S} = \frac{\tan A}{1- \cot A}+\frac{\cot A}{1- \tan A}$$

$$= \frac{\frac{\sin A}{\cos A}}{1-\frac{\cos A}{\sin A}}+\frac{\frac{\cos A}{\sin A}}{1-\frac{\sin A}{\cos A}}$$

$$= \frac{\sin^2 A}{\cos A\left(\sin A - \cos A\right)}+\frac{\cos^2 A}{\sin A\left(\cos A - \sin A\right)}$$

$$= \frac{\sin^2 A}{\cos A\left(\sin A - \cos A\right)}-\frac{\cos^2 A}{\sin A\left(\sin A - \cos A\right)}$$

$$= \frac{\sin^3 A-\cos^3 A}{\sin A\cos A\left(\sin A-\cos A\right)}$$

$$= \frac{\left(\sin A - \cos A\right)\left(\sin^2 A+\cos^2 A+\sin A\cos A\right)}{\sin A\cos A\left(\sin A \quad \cos A\right)}$$

$$= \frac{1+\sin A\cos A}{\sin A\cos A}$$

= 1 + cosec A sec A

= R H S

29. To prove :

$$\frac{1}{\operatorname{cosec} A - \cot A} - \frac{1}{\sin A}$$

$$= \frac{1}{\sin A}-\frac{1}{\operatorname{cosec} A+\cot A}$$

i.e. To prove

$$\frac{1}{\operatorname{cosec} A-\cot A}+\frac{1}{\operatorname{cosec} A+\cot A}=\frac{2}{\sin A}$$

Consider

$$\frac{1}{\operatorname{cosec} A - \cot A}+\frac{1}{\operatorname{cosec} A+\cot A}$$

$$= \frac{1}{\frac{1}{\sin A}-\frac{\cos A}{\sin A}}+\frac{1}{\frac{1}{\sin A}+\frac{\cos A}{\sin A}}$$

$$= \frac{\sin A}{1- \cos A}+\frac{\sin A}{1+\cos A}$$

$$= \left(\frac{1+\cos A+1-\cos A}{\left(1-\cos A\right)\left(1+\cos A\right)}\right)\sin A$$

$$= \left(\frac{2}{1-\cos^2 A}\right)\sin A$$

$$= \frac{2}{\sin^2 A}\sin A$$

$$= \frac{2}{\sin}$$

30. $\sin\theta + \cos\theta = p$, $\sec\theta + \operatorname{cosec}\theta = q$

Consider

$q\,(p^2 - 1)$

$= (\sec\theta + \operatorname{cosec}\theta)\ [\sin^2\theta + \cos\theta^2) - 2\ ]$

$= (\sec\theta + \operatorname{cosec}\theta)\ [\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1]$

$$= \frac{1}{\cos\theta}+\frac{1}{\sin\theta}\ (2\sin\theta\cos\theta)$$

$$= \left(\frac{\sin\theta+\cos\theta}{\sin\theta\cos\theta}\right)(2\sin\theta\cos\theta)$$

$= 2\,(\sin\theta + \cos\theta)$

$= 2p$

31. $\text{Sec}\theta + \tan\theta = p$ (i)

We know that $\text{Sec}^2\theta - \tan^2\theta = 1$

$\Rightarrow (\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$

$\Rightarrow (\sec\theta - \tan\theta)\, p = 1$

$\Rightarrow \sec\theta - \tan\theta = \dfrac{1}{p}$ (ii)

On adding (i) and (ii), we get

$2\sec\theta = p + \dfrac{1}{p}$

$\sec\theta = \dfrac{1}{2}\left(p + \dfrac{1}{p}\right)$

On subtracting (i) from (ii), we get

$-2\tan\theta = \dfrac{1}{p} - p$

$\tan\theta = \dfrac{1}{2}\left(p - \dfrac{1}{p}\right)$

Also,

$$\sin\theta = \frac{\tan\theta}{\sec\theta} = \frac{\frac{1}{2}\left(p - \frac{1}{p}\right)}{\frac{1}{2}\left(p + \frac{1}{p}\right)} = \frac{p^2 - 1}{p^2 + 1}$$

32. $\sin\theta + \cos\theta = \sqrt{2}$

$\Rightarrow \dfrac{1}{\sqrt{2}}\sin\theta + \dfrac{1}{\sqrt{2}}\cos\theta = 1$

$\Rightarrow \cos\dfrac{\pi}{4}\sin\theta + \sin\dfrac{\pi}{4}\cos\theta = 1$

$\Rightarrow \sin\ \dfrac{\pi}{4} + \theta\ = 1 = \sin\dfrac{\pi}{2}$

$\Rightarrow \dfrac{\pi}{4} + \theta = \dfrac{\pi}{2}$

$\Rightarrow \theta = \dfrac{\pi}{2} - \dfrac{\pi}{4}$

$\dfrac{\pi}{4}$

Consider

$\tan\theta + \cot\theta$

$= \tan\dfrac{\pi}{4} + \cot\dfrac{\pi}{4}$

$= 1 + 1$

$= 2$

WORKSHEET - 2

SECTION-A

1. Consider

$(1 + \cot^2\theta)\sin^2\theta$

$= \left(1 + \dfrac{\cos^2\theta}{\sin^2\theta}\right)\sin^2\theta$

$= \sin^2\theta + \cos^2\theta$

$= 1$

2. Consider

$\text{cosec}^2\theta\,(1 + \cos\theta)(1 - \cos\theta) = x$

$\Rightarrow \text{cosec}^2\theta\,(1 - \cos^2\theta) = x$

$\Rightarrow \text{cosec}^2\theta \sin^2\theta = x$

$\Rightarrow \dfrac{1}{\sin^2\theta}\sin^2\theta = x$

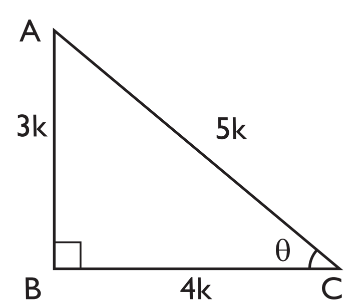
$\Rightarrow 1 = x$

3. $\cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots \cos 179^\circ \cos 188^\circ$

$= \cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots \cos 90^\circ \ldots \cos 179^\circ \cos 188^\circ$

$= 0$

4.



$\cot\theta = 4$

$\cot\theta = \dfrac{4}{3}$

$= \dfrac{BC}{AB}$

Let BC = 4k, AB = 3k

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$= (3K)^2 + (4K)^2$

$= 9k^2 + 16k^2$

$= 25k^2$

$\therefore \quad AC = 5k$

Consider $\dfrac{4\cos\theta - \sin\theta}{2\cos\theta + \sin\theta}$

$$= \frac{4\left(\frac{4}{5}\right) - \left(\frac{3}{5}\right)}{2\left(\frac{4}{5}\right) + \frac{3}{5}}$$

$$= \frac{\frac{16}{5} - \frac{3}{5}}{\frac{8}{5} + \frac{3}{5}}$$

$$= \frac{13}{5} \times \frac{5}{11}$$

$$= \frac{13}{11}$$

5. Consider (secA + tanA) (1 – sinA)

$$= \frac{1}{\cos A} \; \frac{\sin}{\cos A} \; (1 - \sin A)$$

$$= \frac{(1 + \sin A)(1 - \sin A)}{\cos A}$$

$$= \frac{1 - \sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A}$$

$= \cos A$

6. $3\cos\theta = 5\sin\theta$

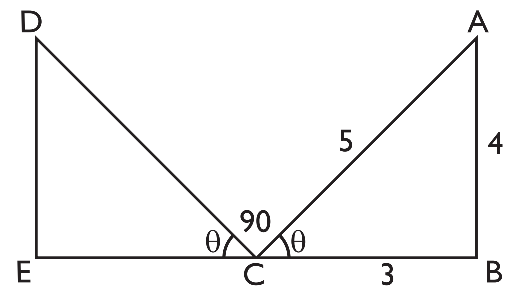
$$\frac{3}{5} = \frac{\sin\theta}{\cos\theta}$$

$$\tan\theta = \frac{3}{5}$$

$$\frac{5\sin\theta + 2\cos\theta}{5\sin\theta - 2\cos\theta} = \frac{\left[\frac{5\sin\theta}{\cos\theta} + 2\right]\cos\theta}{\left[\frac{5\sin\theta}{\cos\theta} - 2\right]\cos\theta}$$

$$\frac{5\tan\theta + 1}{5\tan\theta - 1} = \frac{5 \times \frac{3}{5} - 1}{5 \times \frac{3}{5} + 1}$$

$$= \frac{2}{4} = \frac{1}{2}$$

7. 

$$\cos\theta = \frac{Base}{hypotenuse}$$

$$\cos\theta = \frac{3}{5}$$

$\boxed{\theta \approx 53°}$

$90° + \theta + \phi = 180$

$\boxed{\phi \approx 37°}$

$\cos 37° = 0.8$

8. $\cos^2 17° - \sin^2 73°$

$= \cos^2 (90° - 73°) - \sin^2 73°$

$= \sin^2 73° - \sin^2 73°$

$= 0$

9. $$\frac{2\tan 30}{1 + \tan^2 30}$$

$$= \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1 + \left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{2}{\sqrt{3}} \times \frac{3}{4}$$

$= \frac{\sqrt{3}}{2}$

SECTION-B

10. $\tan 2\theta = \cot(\theta + 6^\circ)$

$\Rightarrow \quad \cot(90^\circ - 2\theta) = \cot(\theta + 6^\circ)$

$\Rightarrow \quad (90^\circ - 2\theta) = (\theta + 6^\circ)$

$\Rightarrow \quad 90^\circ - 6^\circ = 3\theta$

$\Rightarrow \quad \frac{84}{3} = \theta$

$\Rightarrow \quad 28^\circ = \theta$

11. $\frac{2\cos 67}{\sin 23} - \frac{\tan 40}{\cot 50} - \cos 0$

$= \frac{2\cos(90^\circ - 23^\circ)}{\sin 23^\circ} - \frac{\tan(90^\circ - 50^\circ)}{\cot 50^\circ} - \cos 0^\circ$

$= \frac{2\sin 23^\circ}{\sin 23^\circ} - \frac{\cot 50^\circ}{\cot 50^\circ} - \cos 0^\circ$

$= 2 - 1 - 1$

$= 0$

12. $\sec 4A = \operatorname{cosec}(A - 20^\circ)$

$\Rightarrow \quad \operatorname{cosec}(90^\circ - 4A) = \operatorname{cosec}(A - 20^\circ)$

$\Rightarrow \quad 90^\circ - 4A = A - 20^\circ$

$\Rightarrow \quad 110^\circ = 5A$

$\Rightarrow \quad A = 22^\circ$

13. $\frac{\cos 70^\circ}{\sin 20^\circ} + \frac{\cos 59^\circ}{\sin 31^\circ} - 8\sin^2 30^\circ$

$= \frac{\cos(90^\circ - 20^\circ)}{\sin 20^\circ} + \frac{\cos(90^\circ - 31^\circ)}{\sin 31^\circ} - 8\left(\frac{1}{2}\right)^2$

$= \frac{\sin 20^\circ}{\sin 20^\circ} + \frac{\sin 31^\circ}{\sin 31^\circ} - \frac{8}{4}$

$= 1 + 1 - 2$

$= 0$

14. $\sin 75^\circ = \sin(45^\circ + 30^\circ)$

$= \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$

$= \frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}}{2}\right) + \frac{1}{\sqrt{2}}\left(\frac{1}{2}\right)$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

15. $\sin\theta = \cos\theta$

$\frac{\sin\theta}{\cos\theta} = 1$

$\tan\theta = 1$

$\therefore \quad \theta = \frac{\pi}{4}$

Consider $2\tan^2\theta + \sin^2\theta - 1$

$= 2\tan^2\frac{\pi}{4} + \sin^2\frac{\pi}{4} - 1$

$= 2(1)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 - 1$

$= 2 - \frac{1}{2} - 1$

$= 1 - \frac{1}{2}$

$= \frac{1}{2}$

16. $\alpha + \beta = 90^\circ$

To prove: =

$\sqrt{\cos\alpha \operatorname{cosec}\beta - \cos\alpha \sin\beta} = \sin\alpha$

Consider $\sqrt{\cos\alpha \sec\alpha - \cos\alpha \cos\alpha}$

$= \sqrt{\cos\alpha \operatorname{cosec}(90 - \alpha) - \cos\alpha \sin(90 - \alpha)}$

$= \sqrt{\cos\alpha \sec\alpha - \cos\alpha \cos\alpha}$

$= \sqrt{1 - \cos^2\alpha}$

$= \sqrt{\sin^2 \alpha}$

$= \sin \alpha$

17. Consider

$$2\left(\frac{\cos 58^\circ}{\sin 32^\circ}\right) - \sqrt{3}\left(\frac{\cos 38^\circ \operatorname{cosec} 52^\circ}{\tan 15^\circ \tan 60^\circ \tan 75^\circ}\right)$$

$$= 2\left(\frac{\cos(90^\circ - 32^\circ)}{\sin 32^\circ}\right)$$

$$-\sqrt{3}\left(\frac{\cos(90^\circ - 52^\circ)\operatorname{cosec} 52^\circ}{\tan(90^\circ - 75^\circ)\tan 60^\circ \tan 75^\circ}\right)$$

$$= 2\left(\frac{\sin 32^\circ}{\sin 32^\circ}\right) - \sqrt{3}\left(\frac{\sin 52^\circ \operatorname{cosec} 52^\circ}{\cot 75^\circ \tan 60^\circ \tan 75^\circ}\right)$$

$$= 2 - \sqrt{3}\left(\frac{1}{\sqrt{3}}\right)$$

$= 2 - 1$

$= 1$

18. $\tan\theta + \dfrac{1}{\tan\theta} = 2$

On squarring both sides, we get

$$\tan^2\theta + \frac{1}{\tan^2\theta} + 2\tan\theta \frac{1}{\tan\theta} = 4$$

$$\tan^2\theta + \frac{1}{\tan^2\theta} + 2 = 4$$

$$\tan^2\theta + \frac{1}{\tan^2\theta} = 4 - 2$$

$$\frac{1}{\tan^2\theta} = 2$$

19. $\dfrac{2\tan 67^\circ}{\cot 23^\circ} - \dfrac{\sin 40^\circ}{\cos 50^\circ} - \tan 0^\circ$

$$= \frac{2\tan(90^\circ - 23^\circ)}{\cot 23^\circ} - \frac{\sin(90^\circ - 50^\circ)}{\cos 50^\circ} - \tan 0^\circ$$

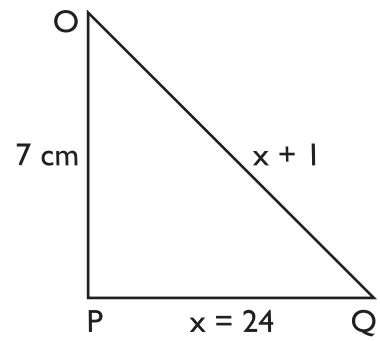
$$= \frac{2\cot 23^\circ}{\cot 23^\circ} - \frac{\cos 50^\circ}{\cos 50^\circ} - \tan 0^\circ$$

$= 2 - 1 = 0$

$= 1$

SECTION-C

20.



	$OQ - PQ$	$= 1$
	Let PQ	$= x$
∴	OQ	$= x + 1$

By Pythagoras theorem,

OQ	$= OP + PQ$
$(x + 1)^2$	$= 7^2 + x^2$
$x^2 + 1 + 2x$	$= 49 + x^2$
$2x$	$= 48$
x	$= 24$

∴ $PQ = 24$ cm and $OQ = 25$ cm

$$\sin Q = \frac{OP}{OQ} = \frac{7}{25}$$

$$\cos Q = \frac{PQ}{OQ} = \frac{24}{25}$$

21. Consider

$(\sec\theta - \tan\theta)^2$

$$= \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2$$

$$= \left(\frac{1 - \sin\theta}{\cos\theta}\right)^2$$

$$= \frac{(1 - \sin\theta)^2}{\cos^2\theta}$$

$$= \frac{(1 - \sin\theta)^2}{1 - \sin^2\theta}$$

$$= \frac{(1 - \sin\theta)(1 - \sin\theta)}{(1 - \sin\theta)(1 + \sin\theta)}$$

$$= \frac{1 - \sin\theta}{1 + \sin\theta}$$

22. Consider

$$\frac{\sec\theta - \tan\theta}{\sec\theta + \tan\theta}$$

$$= \frac{(\sec\theta - \tan\theta)(\sec\theta - \tan\theta)}{(\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}$$

$$= \frac{(\sec\theta - \tan\theta)^2}{\sec^2\theta - \tan^2\theta}$$

$$= \frac{\sec^2\theta + \tan^2\theta - 2\sec\theta\tan\theta}{1}$$

$= 1 + \tan^2\theta + \tan^2\theta - 2\sec\theta\tan\theta$

$= 1 - 2\sec\theta\tan\theta + 2\tan^2\theta$

23. $$\frac{\cos^2 20 + \cos^2 70}{\sec^2 50 - \cot^2 40} + 2\,\text{cosec}58°$$

$- 2\cot 58° \tan 32° - (4\tan 13° \tan 37° \tan 45° \tan 53° \tan 77°)$

$$= \frac{\cos^2(90 - 70) + \cos^2 70}{\sec^2(90 - 40) - \cot^2 40} + 2\,\text{cosec}58°$$

$- 2\tan 32° \cot(90° - 32°) - 4\tan(90° - 77°) \tan(90° - 53°)(1)\tan 53° \tan 77°$

$$= \frac{\sin^2 70 + \cos^2 70}{\text{cosec}^2 40 - \cot^2 40} + 2\,\text{cosec}\,58°$$

$- 2\tan 32°\,\text{cosec}32° - 4\cot 77° \cot 53° \tan 53° \tan 77°$

$= 1 + 2\,\text{cosec}58° - 2\sec 32° - 4$

$= 1 + 2\,\text{cosec}(90° - 32°) - 2\sec 32° - 4$

$= 1 + 2\sec 32° - 2\sec 32° - 4$

$= -3$

24. $\text{cosec}\theta + \cot\theta = p$ (i)

Consider

$\text{cosec}^2\theta - \cot^2\theta = 1$

$(\text{cosec}\theta - \cot\theta)(\text{cosec}\theta + \cot\theta) = 1$

$(\text{cosec}\theta - \cot\theta) = \frac{1}{p}$ (ii)

On adding (i) and (ii), we get

$2\,\text{cosec}\theta = p + \frac{1}{p}$

$\text{cosec}\theta = \frac{1}{2}\left(p - \frac{1}{p}\right)$

On subtracting (i) and (ii), we get

$-2\cot\theta = \frac{1}{p} - p$

$\cot\theta = \frac{1}{2}\ p - \frac{1}{p}$

$$\therefore \cos\theta = \frac{\cot\theta}{\text{cosec}\theta}$$

$$= \frac{\frac{1}{2}\ p - \frac{1}{p}}{\frac{1}{2}\ p + \frac{1}{p}}$$

$$= \frac{p^2 - 1}{p^2 + 1}$$

25. $\tan\theta = \frac{1}{\sqrt{7}}$

$$\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta}$$

$$= \frac{(1 + \cot^2\theta) - (1 + \tan^2\theta)}{(1 + \cot^2\theta) + (1 + \tan^2\theta)}$$

$$= \frac{\cot^2\theta - \tan^2\theta}{2 + \cot^2\theta - \tan^2\theta}$$

$$= \frac{(\sqrt{7})^2 - \left(\frac{1}{\sqrt{7}}\right)^2}{2 + (\sqrt{7})^2 + \left(\frac{1}{\sqrt{7}}\right)^2}$$

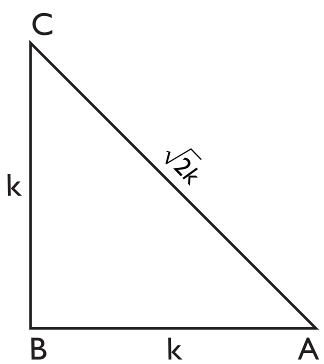
$$= \frac{7 - \frac{1}{7}}{2 + 7 + \frac{1}{7}}$$

$$= \frac{48}{7} \times \frac{7}{64} = \frac{48}{64}$$

$$= \frac{12}{16}$$

$$= \frac{3}{4}$$

26.



$$\text{cosec } A = \frac{\sqrt{2}}{1} = \frac{AC}{BC}$$

Let $AC = \sqrt{2}k$

$BC = k$

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$2k^2 = AB^2 + k^2$

$AB^2 = 2k^2 - k^2 = k^2$

$AB = k$

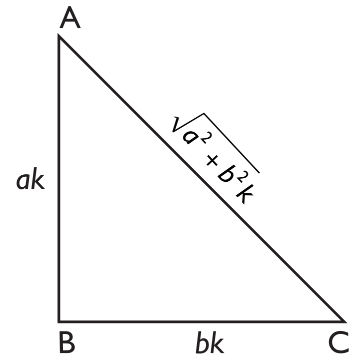
So, $\dfrac{2\sin^2 A + 3\cot^2 A}{4\tan^2 A - \cos^2 A}$

$$= \frac{2\left(\frac{1}{\sqrt{2}}\right)^2 + 3(1)^2}{4(1)^2 - \left(\sqrt{2}\right)^2}$$

$$= \frac{1+3}{4-2}$$

$= 2$

27.



$$\sin\theta = \frac{1}{\sqrt{a^2 + b^2}}$$

$$= \frac{AB}{AC}$$

Let $AB = ak$, $AC = \sqrt{a^2 + b^2}k$

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$(a^2 + b^2)k^2 = a^2k^2 + BC^2$

$BC^2 = b^2k^2$

$BC = bk$

$$\therefore \quad \cos\theta = \frac{BC}{AC}$$

$$= \frac{b}{\sqrt{a^2 + b^2}}$$

$$\tan\theta = \frac{AB}{BC} = \frac{a}{b}$$

28. Consider $= \sin^6A + \cos^6A$

$$= \left[\left(\sin^2 A\right)^3 + \left(\cos^2 A\right)^3\right]$$

$= (\sin^2A + \cos^2A)(\sin^4A + \cos^4A - \sin^2A\cos^2A)$

$= (\sin^4A + \cos^4A - \sin^2A\cos^2A)$

$$= \left[\left(\sin^2 A + \cos^2 A\right)^2 - 3\sin^2 A\cos^2 A\right]$$

$= 1 - 3\sin^2A\cos^2A$

29. $5 \tan x = 4$

$\tan x = \frac{4}{5}$

Consider $\frac{5\sin x - 3\cos x}{5\sin x + 2\cos x}$

$= \frac{\frac{5\sin x - 3\cos x}{\cos x}}{\frac{5\sin x + 2\cos x}{\cos x}}$

$= \frac{5\tan x - 3}{5\tan x + 2}$

$= \frac{5\left(\frac{4}{5}\right) - 3}{5\left(\frac{4}{5}\right) + 2}$

$= \frac{4-3}{4+2} = \frac{1}{6}$

SECTION-D

30. $\sec\theta = x + \frac{1}{4x}$

We know that $\sec^2\theta - \tan^2\theta = 1$

$\tan^2\theta = \sec^2\theta - 1$

$= \left(x + \frac{1}{4x}\right)^2 - 1$

$= x^2 + \left(\frac{1}{4x}\right)^2 + 2x\left(\frac{1}{4x}\right) - 1$

$= x^2 + \left(\frac{1}{4x}\right)^2 - \frac{1}{2}$

$= \left(x - \frac{1}{4x}\right)^2$

So, $\tan^2\theta = \left(x - \frac{1}{4x}\right)$

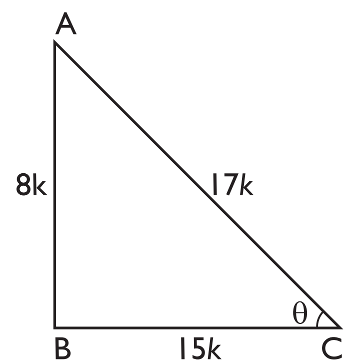
$\therefore \quad \tan\theta = \pm\left(x - \frac{1}{4x}\right)$

Case 1	**Case 2**
$\sec\theta = x + \frac{1}{4x}$	$\sec\theta = x + \frac{1}{4x}$
$\tan\theta = x - \frac{1}{4x}$	$\tan\theta = -\left(x - \frac{1}{4x}\right)$
So,	So,
$\sec\theta + \tan\theta$	$\sec\theta + \tan\theta$
$= 2x$	$= x + \frac{1}{4x} - x + \frac{1}{4x}$
	$= \frac{2}{4x}$
	$= \frac{1}{2x}$

$\therefore \quad \sec\theta + \tan\theta = 2x$ or $\frac{1}{2x}$

31. $\cot\theta = \frac{15}{8}$

$= \frac{BC}{AB}$



Let BC = 15k

AB = 8k

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$= 64k^2 + 225k^2$

$= 289k^2$

$\therefore \quad AC = 17k^2$

(a) Consider $\frac{(2+2\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(2-2\cos\theta)}$

$$= \frac{2(1+\sin\theta)(1-\sin\theta)}{2(1+\cos\theta)(1-\cos\theta)}$$

$$= \frac{1-\sin^2\theta}{1-\cos^2\theta}$$

$$= \frac{\cos^2\theta}{\sin^2\theta}$$

$$= \cot^2\theta$$

$$= \frac{225}{64}$$

(b) $$\frac{\text{cosec}^2\theta - \cot^2\theta}{\text{cosec}^2\theta + \cot^2\theta}$$

$$= \frac{1}{\text{cosec}^2\theta + \cot^2\theta}$$

$$= \frac{1}{\left(\frac{17}{8}\right)^2 + \left(\frac{15}{8}\right)^2}$$

$$= \frac{64}{289+225}$$

$$= \frac{64}{514}$$

$$= \frac{32}{257}$$

(c) $\sec^2\theta + \tan^2\theta$

$$= \left(\frac{17}{15}\right)^2 + \left(\frac{8}{15}\right)^2$$

$$= \frac{289}{225} + \frac{64}{225}$$

$$= \frac{353}{225}$$

32. $\tan A = n \tan B$

$$\Rightarrow \frac{\sin A}{\cos A} = n\frac{\sin B}{\cos B} \quad \text{(i)}$$

Also, $\sin A = m \sin B$

$$\Rightarrow \frac{\sin A}{\sin B} = m \quad \text{(ii)}$$

From (i), and (ii), we get

$$m = n\frac{\cos A}{\cos B}$$

$$\Rightarrow \cos B = \frac{n}{m}\cos A \quad \text{(iii)}$$

On putting value of sin B and cos B from (ii) and (iii) in $\cos^2 B + \sin^2 B = 1$, we get

$$\frac{n^2}{m^2}\cos^2 A + \frac{1}{m^2}\sin^2 A = 1$$

$$n^2\cos^2 A + \sin^2 A = m^2$$

$$n^2\cos^2 A + 1 - \cos^2 A = m^2$$

$$n^2\cos^2 A - \cos^2 A = m^2 - 1$$

$$(n^2 - 1)\cos^2 A = m^2 - 1$$

$$\therefore \quad \cos^2 A = \frac{m^2-1}{n^2-1}$$

33. $\text{cosec}\,\theta - \sin\theta = l$

$\sec\theta - \cos\theta = m$

Consider

$l^2 - m^2 \, (l^2 + m^2 + 3)$

$= (\text{cosec}\theta - \sin\theta)^2 (\sec\theta - \cos\theta)^2[(\text{cosec}\theta - \sin\theta)^2 (\sec\theta - \cos\theta)^2] + 3$

$$= \left(\frac{1}{\sin\theta} - \sin\theta\right)^2\left(\frac{1}{\cos\theta} - \cos\theta\right)^2 \left[\left(\frac{1}{\sin\theta} - \sin\theta\right)^2 + \left(\frac{1}{\cos\theta} - \cos\theta\right)^2\right] + 3$$

$$= \left(\frac{1-\sin^2\theta}{\sin\theta}\right)^2 = \left(\frac{1-\cos^2\theta}{\cos\theta}\right)^2 \left[\left(\frac{1-\sin^2\theta}{\sin\theta}\right)^2 + \left(\frac{1-\cos^2\theta}{\cos\theta}\right)^2 + 3\right]$$

$$= \left(\frac{\cos^2\theta}{\sin\theta}\right)^2\left(\frac{\sin^2\theta}{\cos\theta}\right)^2 \left[\left(\frac{\cos^2\theta}{\sin\theta}\right)^2 + \left(\frac{\sin^2\theta}{\cos\theta}\right)^2 + 3\right]$$

$$= \frac{\cos^4\theta}{\sin^2\theta}\ \frac{\sin^4\theta}{\cos^2\theta}\left[\frac{\cos^4\theta}{\sin^2\theta}+\frac{\sin^4\theta}{\cos^2\theta}+3\right]$$

$$= \sin^2\theta\cos^2\theta\left[\frac{\cos^6\theta+\sin^6\theta+3\sin^2\theta\cos^2\theta}{\sin^2\theta\cos^2\theta}\right]$$

$= (\sin^2\theta)^3 + (\cos^2\theta)^3 + 3\sin^2\theta\cos^2\theta$

$= (\sin^2\theta + \cos^2\theta)(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta) + 3\sin^2\theta\cos^2\theta$

$= \sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta$

$= (\sin^2\theta + \cos^2\theta)^2$

$= 1^2$

$= 1$

34. $\dfrac{\cos\alpha}{\cos\beta} = m$ and $\dfrac{\cos\alpha}{\sin\beta} = n$

Consider

$(m^2 + n^2)\cos^2\beta$

$$= \left(\frac{\cos^2\alpha}{\cos^2\beta}+\frac{\cos^2\alpha}{\sin^2\beta}\right)\cos^2\beta$$

$$= \left(\frac{\cos^2\alpha\sin^2\beta+\cos^2\beta\cos^2\alpha}{\cos^2\beta\sin^2\beta}\right)\cos^2\beta$$

$$= \frac{\cos^2\alpha\sin^2\beta+\cos^2\beta\cos^2\alpha}{\sin^2\beta}$$

$$= \left(\frac{\cos\alpha}{\sin\beta}\right)^2$$

$= n^2$

35. $(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C)$

$= (\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$ (i)

On multiplying both side of (i) by $(\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$, we get

$(\sec^2 A - \tan^2 A)(\sec^2 B - \tan^2 B)(\sec^2 C - \tan^2 C)$

$= (\sec A - \tan A)^2(\sec B - \tan B)^2(\sec C - \tan C)^2$

$\Rightarrow 1 = (\sec A - \tan A)^2(\sec B - \tan B)^2(\sec C - \tan C)^2$

$\Rightarrow (\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C) = \pm 1$

Again, Multiplying both sides of (i) by

$(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C)$

we get

$(\sec A + \tan A)^2(\sec B + \tan B)^2(\sec C + \tan C)^2$

$= (\sec^2 A - \tan^2 A)(\sec^2 B - \tan^2 B)(\sec^2 C - \tan^2 C)$

$= 1$

$\therefore (\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C) = \pm 1$

36. $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$

$\Rightarrow x\sin^3\theta + \cos^2\theta\,(y\cos\theta) = \sin\theta\cos\theta$

$\Rightarrow x\sin^3\theta + \cos^2\theta\, x\sin\theta = \sin\theta\cos\theta$

$[\because y\cos\theta = x\sin\theta]$

$\Rightarrow x\sin\theta + (\sin^2\theta + \cos^2\theta) = \sin\theta\cos\theta$

$\Rightarrow x\sin\theta = \sin\theta\cos\theta$

$\Rightarrow x = \cos\theta$

$\therefore\ y = \dfrac{x\sin\theta}{\cos\theta} = \dfrac{\cos\theta\sin\theta}{\cos\theta} = \sin\theta$

Also, we know that

$\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow y^2 + x^2 = 1$

$\Rightarrow x^2 + y^2 = 1$

37. $\operatorname{cosec}\theta - \sin\theta = m$

$\Rightarrow \dfrac{1}{\sin\theta} - \sin\theta = m$

$\Rightarrow \dfrac{1-\sin^2\theta}{\sin\theta} = m$

$\Rightarrow \dfrac{\cos^2\theta}{\sin\theta} = m$

Also, $\sec\theta - \cos\theta = n$

$\dfrac{1}{\cos\theta} - \cos\theta = n$

$\dfrac{1-\cos^2\theta}{\cos\theta} = n$

$\dfrac{\sin^2\theta}{\cos\theta} = n$

So,

$$\text{L H S} = \left(m^2n\right)^{\frac{2}{3}} + \left(mn^2\right)^{\frac{2}{3}}$$

$$= \left(\frac{\cos^4\theta}{\sin^2\theta}\frac{\sin^2\theta}{\cos\theta}\right)^{\frac{2}{3}} + \left(\frac{\cos^2\theta}{\sin\theta}\frac{\sin^4\theta}{\cos^2\theta}\right)^{\frac{2}{3}}$$

$$= \left(\cos^3\theta\right)^{\frac{2}{3}} + \left(\sin^3\theta\right)^{\frac{2}{3}}$$

$$= \cos^2\theta + \sin^2\theta$$

$$= 1$$

38. $a \sec\theta + b \tan\theta + c = 0$

$p \sec\theta + q \tan\theta + r = 0$

To prove:

$(br - qc)^2 - (pc - ar)^2 = (aq - pb)^2$

Consider

$(br - qc)^2 - (pc - ar)^2$

$= [b(-p\sec\theta - q\tan\theta) + q(a\sec\theta + b\tan\theta)]^2 - [p(-a\sec\theta - b\tan\theta) + a(p\sec\theta + q\tan\theta)]^2$

$= [-bp\sec\theta - bq\tan\theta + aq\sec\theta + bq\tan\theta]^2 - [-ap\sec\theta - bp\tan\theta + ap\sec\theta + aq\tan\theta]^2$

$= [\sec\theta(aq - bp)]^2 - [(aq - bp)\tan\theta]^2$

$= (aq - bp)^2(\sec^2\theta - \tan^2\theta)$

$= (aq - bp)^2$

39. $\tan^2\theta = 1 - a^2$

Consider

$(\sec\theta + \tan^3\theta \operatorname{cosec}\theta)$

$$= \sqrt{1+\tan^2\theta} + \tan^2\theta \tan\theta \operatorname{cosec}\theta$$

$$= \sqrt{1+\tan^2\theta} + \tan^2\theta \tan\theta \sqrt{1+\cot^2\theta}$$

$$= \sqrt{1+1-a^2} + \left(1-a^2\right)\sqrt{1-a^2}\sqrt{1+\frac{1}{\tan^2\theta}}$$

$$= \sqrt{2-a^2} + \left(1-a^2\right)\sqrt{1-a^2}\sqrt{1+\frac{1}{1-a^2}}$$

$$= \sqrt{2-a^2} + \left(1-a^2\right)\frac{\sqrt{1-a^2}}{\sqrt{1-a^2}}\sqrt{2-a^2}$$

$$= \sqrt{2-a^2} + \left(1-a^2\right)\sqrt{2-a^2}$$

$$= \sqrt{2-a^2} + \left(1+1-a^2\right)$$

$$= \sqrt{2-a^2} + \left(2-a^2\right)$$

$$= \left(2-a^2\right)^{\frac{3}{2}}$$

CASE STUDY-1

(i) (b)

$\cos(A + B) = 0$

$\cos(A + B) = \cos 90^\circ$

$A + B = 90$

$(A = 90 - B)$ ...(i)

$\sin(A - B) = \sin(90 - B - B)$

$= \sin(90 - 2B)$

As $\sin(90 - \theta) = \cos\theta$

$\therefore \sin(90 - 2B) = \cos 2B$

(ii) (c)

$\text{Cos } 9A = \text{Sin } A$

As $\cos(90 - 0) = \sin\theta$

$\therefore \cos(90 - A) = \sin A$

$\cos 9A = \cos(90 - A)$

$9A = 90 - A$

$A = 9$

$$\tan 5A = \frac{\sin 5A}{\cos 5A} = \frac{\sin 5(9)}{\cos 5(9)}$$

$$\tan 5A = \frac{\sin 45}{\cos 45} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1$$

(iii) (b) $\sin(45^\circ + A) - \cos(45^\circ - A)$

$\because \cos(90 - \theta) = \sin\theta$

$\therefore \sin(45 + A) = \cos[90 - (45 + A)]$

$= \cos[45 - A]$

Hence, $\sin(45 + A) - \cos(45 - A)$

$= \cos(45 - A) - \cos(45 - A) = 0$

(iv) (d) $A + B + C = 180$

$A + B = 180 - C$

$\therefore \text{cosec}\ \frac{A+B}{2} = \text{cosec}\ \frac{180 - c}{2}$

$= \text{cosec}\left(90 - \frac{c}{2}\right)$

$= \sec\frac{c}{2}$

(v) (d) tan10° tan75° tan15° tan80°

$\tan 10° = \cot(90 - 10)$

$= \cot 80°$

$\tan 75° = \cot(90 - 75)$

$= \cot 15°$

$\therefore$ tan10° tan75° tan15° tan80°

= cot80° cot15° tan15° tan80°

As, $\tan\theta \cot\theta = 1$

$\therefore$ cot80° cot15° tan15° tan80° = 1

CASE STUDY-2

(i) (c) $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$= \frac{\sin\theta}{\sqrt{1 - \sin^2\theta}}$

(ii) (b) $\tan A + \cot A = 4$

Squaring both sides

$\tan^2 A + \cot^2 A + 2\tan A \cot A = 16$

$\tan^2 A + \cot^2 A + 2 = 16$

$\tan^2 A + \cot^2 A = 14$

Squaring both sides

$\tan^4 A + \cot^4 A + 2\tan^4 A \cot^4 A = 196$

$\tan^4 A + \cot^4 A = 196 - 2$

$\tan^4 A + \cot^4 A = 194$

(iii) (d) $9x^2 = \text{cosec}^2\theta$

$\frac{9}{x^2} = \cot^2\theta$

$3\left(x^2 - \frac{1}{x^2}\right) = \left(3x^2 - \frac{3}{x^2}\right)$

$= \frac{\text{cosec}^2\theta}{3} - \frac{\cot^2\theta}{3}$

$= \frac{1}{3\sin^2\theta} - \frac{\cos^2\theta}{3\sin^2\theta}$

$= \frac{1}{3}\left[\frac{1 - \cos\theta^2}{\sin^2\theta}\right]$

$= \frac{1}{3}\left[\frac{\sin^2\theta}{\sin^2\theta}\right]$

$= \frac{1}{3}$

(iv) (c) $\sin A - \cos A = 0$

$\sin A = \cos A$ ...(i)

As $\sin(90 - \theta) = \cos\theta$

$\therefore \sin A = \sin(90 - A)$

$A = 90 - A$

$2A = 90$

$A = 45°$

$\sin^4 A + \cos^4 A = \sin^4 45 + \cos^4 45$

$= \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4$

$= \frac{1}{4} + \frac{1}{4}$

$= \frac{1}{2}$

(v) (c) $\sin\theta + \text{cosec}\theta = 2$

$\text{cosec}\theta = \frac{1}{\sin\theta}$

$\sin\theta + \frac{1}{\sin\theta} = 2$

$\sin^2\theta + 1 = 2\sin\theta$

Let $\sin\theta = x$

$x^2 + 1 = 2x$

$\Rightarrow x^2 - 2x + 1 = 0$

$\Rightarrow x^2 - x - x + 1 = 0$

$\Rightarrow x(x - 1) - 1(x - 1) = 0$

$\Rightarrow (x - 1)^2 = 0$

$x = 1$

$\sin\theta = 1$

$\theta = 90°$

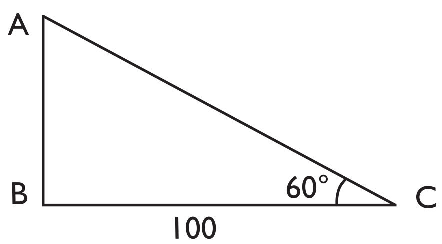
$\sin^{19}\theta + \text{cosec}^{19}\theta = (1)^{19} + (1)^{19}$

$= 1 + 1 = 2$

Chapter 9 Some Application of Trigonometry

Multiple Choice Questions

1. (d)

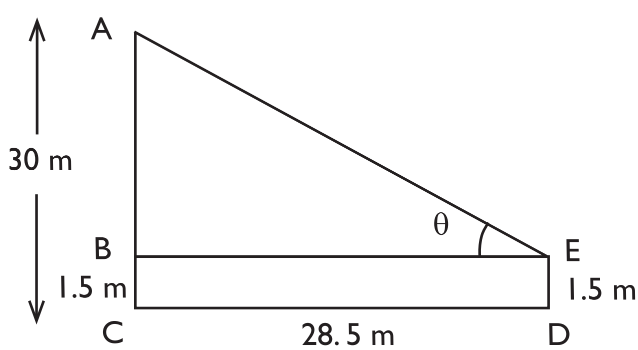


$\tan 60° = \dfrac{AB}{BC}$

$\sqrt{3} = \dfrac{AB}{100}$

$\therefore \quad AB = 100\sqrt{3}$ m

2. (c)



$AB = AC - BC$

$= 30 - 1.5$

$= 28.5$ m

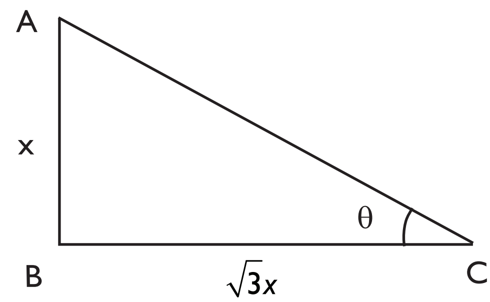
$BE = CD = 28.5$ m

In ΔABE,

$\tan\theta = \dfrac{AB}{BE} = \dfrac{28.5}{28.5} = 1$

$\Rightarrow \theta = 45°$

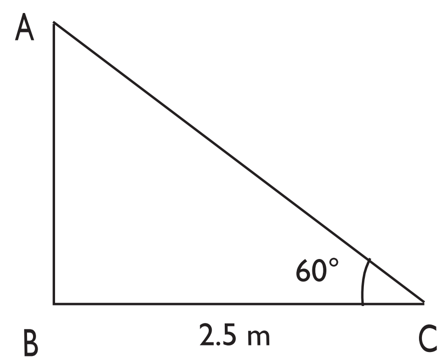
3. (d)



$\tan\theta = \dfrac{AB}{BC} = \dfrac{x}{\sqrt{3}x} = \dfrac{1}{\sqrt{3}}$

$\theta = 30°$

4. (b)

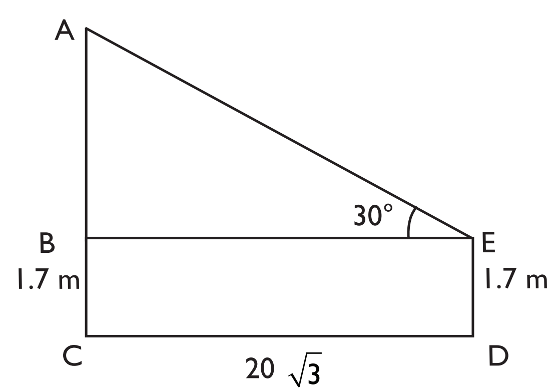


$\text{Cos } 60° = \dfrac{BC}{AC}$

$\dfrac{1}{2} = \dfrac{2.5}{AC}$

$AC = 5$ m

5. (a)



$$\text{In } \Delta ABE,\ BE = CD = 20\sqrt{3}\ \text{m}$$

$$\tan 30^\circ = \frac{AB}{BE}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{20\sqrt{3}}$$

$$\therefore \quad AB = 20\ \text{m}$$

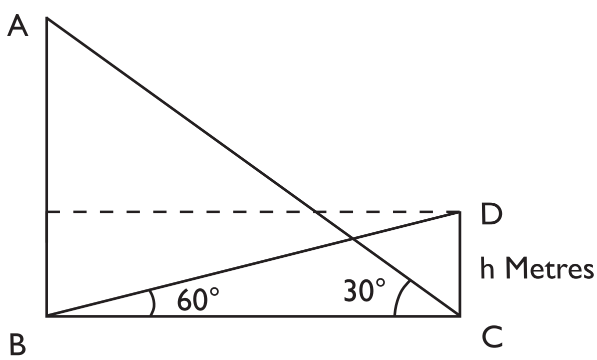
So, $AC = AB + BC = 20 + 1.7$

$= 21.7$ m

WORKSHEET - 1

SECTION-A

1.



Let AB denotes the tower.

In ΔBCD,

$$\tan 60^\circ = \frac{CD}{BC}$$

$$\sqrt{3} = \frac{h}{BC}$$

$$BC = \frac{h}{\sqrt{3}}\ \text{Metre}$$

In ΔABC,

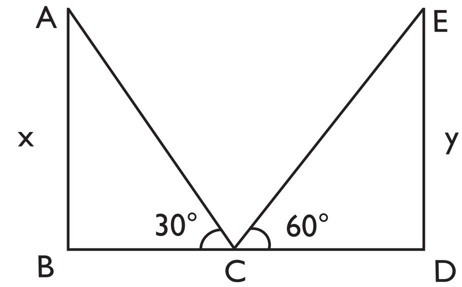
$$\tan 30^\circ = \frac{AB}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{h}$$

$$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{h}$$

$$AB = \frac{h}{3}\ \text{Metre}$$

2.



Let AB and DE denote two towers

In ΔABC,

$$\tan 30^\circ = \frac{AB}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{x}{BC}$$

$$\Rightarrow \quad BC = \sqrt{3}$$

In ΔCDE,

$$\tan 60^\circ = \frac{DE}{CD}$$

$$\sqrt{3} = \frac{y}{CD}$$

$$CD = \frac{y}{\sqrt{3}}$$

As $BC = CD$

$$\therefore \quad \sqrt{3}x = \frac{y}{\sqrt{3}}$$

$$3x = y$$

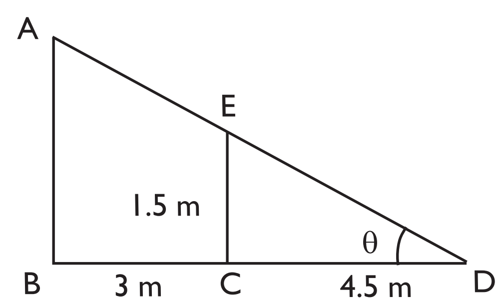
$$\Rightarrow \quad x : y = 1 : 3$$

3. In ΔABC,

$$\tan C = \frac{AB}{BC} - \frac{20}{20\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\therefore \quad C = 30^\circ$$

4.



Let CE denotes the boy and AB denotes a

lamp-post.

In ΔDCE,

$$\tan\theta = \frac{CE}{CD} = \frac{1.5}{4.5} = \frac{1}{3}$$

In ΔABC,

$$\tan\theta = \frac{AB}{BD}$$

$$\frac{1}{3} = \frac{AB}{7.5}$$

$\therefore$ AB = 2. 5 m

5. In ΔABC,

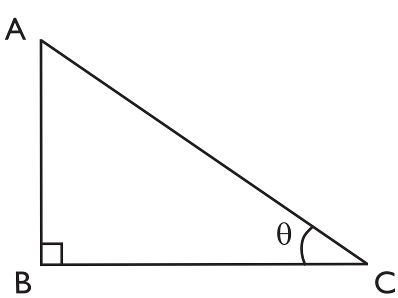
$$\sin 45° = \frac{BC}{AC}$$

$$\frac{1}{\sqrt{2}} = \frac{BC}{150}$$

$$BC = \frac{150}{\sqrt{2}}$$

$$= 75\sqrt{2} \text{ m}$$

6.



Let AB denotes the vertical pole and BC denotes the shadow of the pole.

Let AB = BC = x

In ΔABC,

$$\tan\theta = \frac{AB}{BC} = \frac{x}{x} = 1$$

$\therefore$ $\theta = 45°$

7. In ΔBAC,

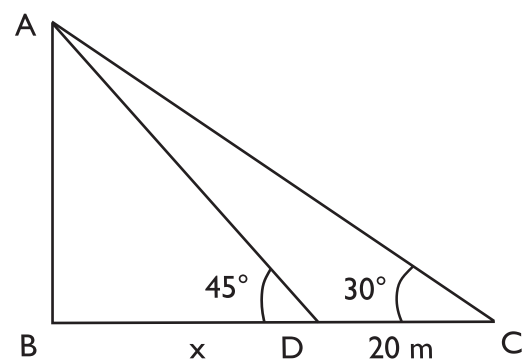
$$\tan\theta = \frac{AB}{AC}$$

$$= \frac{5a\sqrt{3}}{5a}$$

$$= \sqrt{3}$$

$\theta = 60°$

8.



Let AB denotes the chimney.

Let BD = x metre

In ΔABC,

$$\tan 30° = \frac{AB}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{x+20}$$

$$AB = \frac{1}{\sqrt{3}}(x + 20) \qquad \text{...(i)}$$

In ΔABC,

$$\tan 45° = \frac{AB}{BD}$$

$$1 = \frac{AB}{x}$$

$$AB = x \qquad \text{...(ii)}$$

From (i) and (ii),

$$AB = x = \frac{1}{\sqrt{3}}(x + 20)$$

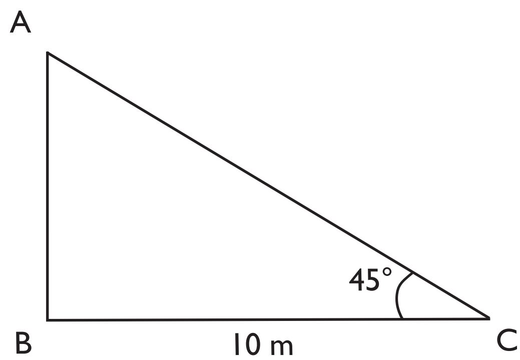
$$\sqrt{3}x - x = 20$$

$$x = \frac{20}{\sqrt{3} - 1}$$

$$= \frac{20}{2}\left(\sqrt{3}+1\right)$$

$$x = 10\left(\sqrt{3}+1\right)$$

$$\therefore \quad AB = x = 10\left(\sqrt{3}+1\right)$$

9. 

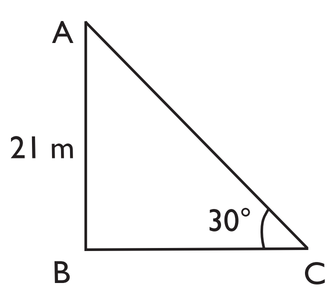
Let AB denotes the tower and BC denotes the shadow

In ΔABC,

$$\tan 45^\circ = \frac{AB}{BC}$$

$$1 = \frac{AB}{10}$$

$$AB = 10 \text{ m}$$

10. 

Let AC denotes the string of kite.

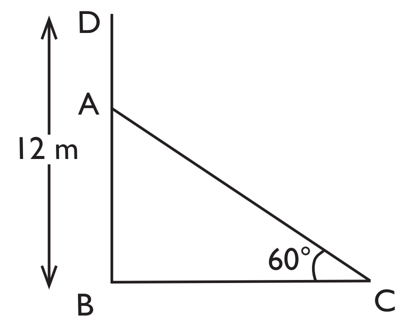
In ΔABC,

$$\sin 30^\circ = \frac{AB}{AC}$$

$$\frac{1}{2} = \frac{21}{AC}$$

$$AC = 42 \text{ m}$$

SECTION-B

11. 

Let BD denotes a tree broken by the wind at point A. Such that its top touches the ground at point C

So, AD = AC

Let AB = x metre

AD = 12 m

AB + AD = 12

x + AD = 12

AD = 12 – x

AC = 12 – x $(\because AD = AC)$

In ΔABC,

$$\sin 60^\circ = \frac{AB}{AC}$$

$$\frac{\sqrt{3}}{2} = \frac{x}{12 - x}$$

$$2x = 12\sqrt{3} - \sqrt{3}x$$

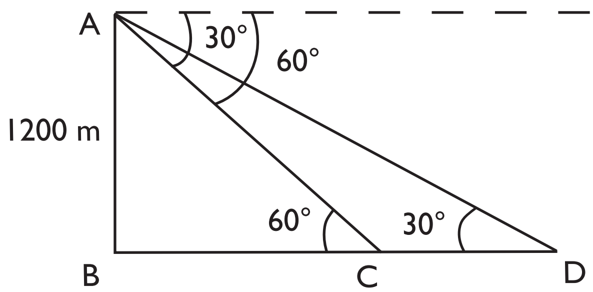
$$x\left(\sqrt{3}+2\right) = 12\sqrt{3}$$

$$x = \frac{12\sqrt{3}}{\sqrt{3}+2}$$

$$= \frac{12\sqrt{3}}{-1}\left(\sqrt{3}-2\right)$$

$$= 12\sqrt{3}\left(2-\sqrt{3}\right) \text{ m}$$

12.



Let the aeroplane be at point A and C and D denote two ships sailing towards the aeroplane.

To find : CD

In ΔABC,

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{1200}{BC}$$

$$BC = \frac{1200}{\sqrt{3}}$$

$$= 400\sqrt{3} \text{ m}$$

In ΔABD,

$$\tan 30^\circ = \frac{AB}{BD}$$

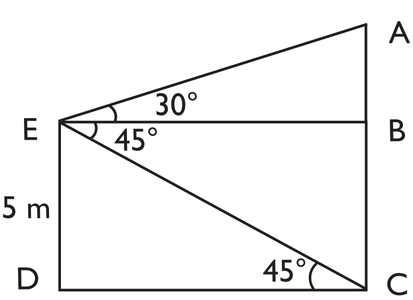
$$\frac{1}{\sqrt{3}} = \frac{1200}{BC + CD}$$

$$\frac{1}{\sqrt{3}} = \frac{1200}{400\sqrt{3} + CD}$$

$$CD = 1200\sqrt{3} - 400\sqrt{3}$$

$$= 800\sqrt{3}$$

13.



Let the window be at point E and AC be the house.

Let

To find : AC

In ΔCDE,

$$\tan 45^\circ = \frac{DE}{CD}$$

$$1 = \frac{15}{CD}$$

$$\therefore \quad CD = 15 \text{ m}$$

$$\therefore \quad BE = CD = 15 \text{ m}$$

In ΔABE,

$$\tan 30^\circ = \frac{AB}{BE}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{15}$$

$$\therefore \quad AB = \frac{15}{\sqrt{3}} = 5\sqrt{3} \text{ m}$$

Also, $BC = DE = 15$ m

$$\therefore \quad AC = AB = BC$$

$$AC = 5\sqrt{3} + 15$$

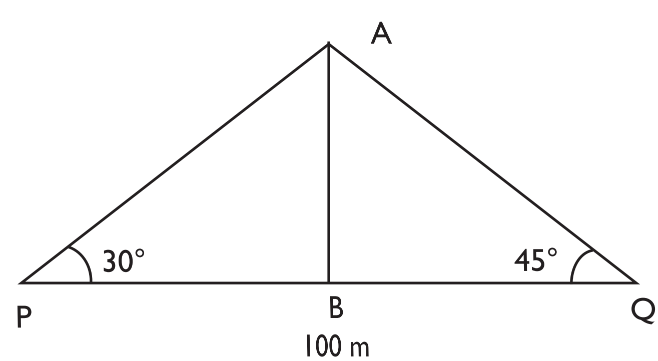
$$= 5(\sqrt{3} + 3) \text{ m}$$

$$= 5\,(1.732 + 3) \text{ m}$$

$$= 5 \times 4.732$$

$$= 23.66 \text{ m}$$

14.



Let AB denotes the tree.

Let BP = x metre such that

$BQ = PQ - BP$

$= 100 - x$

To find AB

In ΔABQ,

$\tan 45° = \frac{AB}{BQ}$

$1 = \frac{AB}{100 - x}$

$\therefore AB = 100 - x$ (i)

In ΔABP,

$\tan 30° = \frac{AB}{PB}$

$\frac{1}{\sqrt{3}} = \frac{AB}{x}$

$AB = \frac{x}{\sqrt{3}}$ (ii)

From (i) and (ii),

$AB = 100 - x = \frac{x}{\sqrt{3}}$

$100 = x + \frac{x}{\sqrt{3}}$

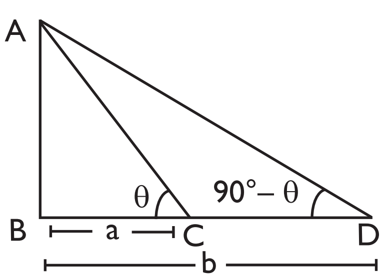
$100\sqrt{3} = x(\sqrt{3} + 1)$

$x = \frac{100\sqrt{3}}{\sqrt{3}+1}$

$= \frac{100\sqrt{3}}{2}\left(\sqrt{3}-1\right)$

$= 50\sqrt{3}\left(\sqrt{3}-1\right)$

15.



Let AB denotes the tree.

To prove : $AB = \sqrt{ab}$ metres

In ΔABC,

$\tan\theta = \frac{AB}{BC}$

$\tan\theta = \frac{AB}{a}$

$\therefore AB = a\tan\theta$ (i)

In ΔABD,

$\tan(90 - \theta) = \frac{AB}{BD}$

$\cot\theta = \frac{AB}{b}$

$\therefore AB = b\cot\theta$ (ii)

From (i) and (ii)

$AB = a\tan\theta = b\cot\theta$

$\tan^2\theta = \frac{b}{a}$

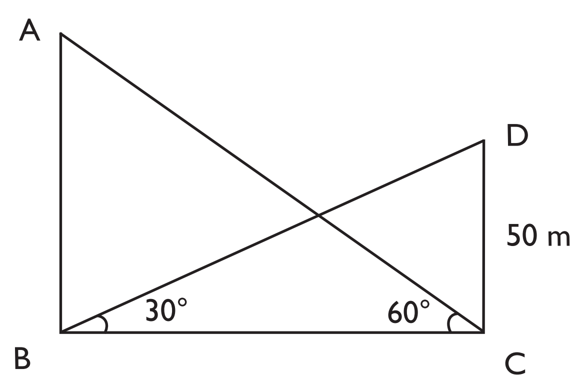
$\therefore \tan\theta = \sqrt{\frac{b}{a}}$

So, $AB = a\tan\theta$

$= a\sqrt{\frac{b}{a}}$

$= \sqrt{ab}$ metres

16.



Let AB denotes the hill and CD denotes the tower such that CD = 50 m

In ΔBCD,

$\tan 30° = \dfrac{CD}{BC}$

$\dfrac{1}{\sqrt{3}} = \dfrac{50}{BC}$

$\therefore$ BC $= 50\sqrt{3}$ m

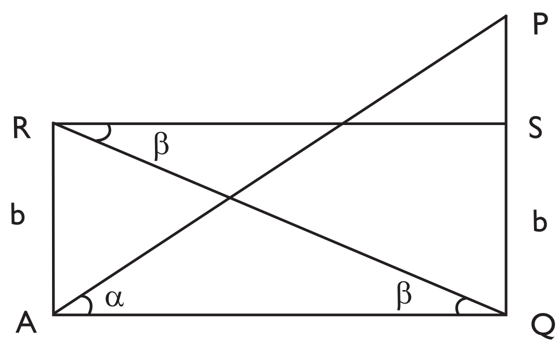
In ΔABC,

$\tan 60° = \dfrac{AB}{BC}$

$\sqrt{3} = \dfrac{AB}{50\sqrt{3}}$

$\therefore$ AB $= 150$ m

17.



Let PQ denotes the tower

To prove : PQ = b tanα cotβ

QS = AR = b ft

In ΔRAQ,

$\tan\beta = \dfrac{AR}{AQ}$

$\tan\beta = \dfrac{b}{AQ}$

$AQ = b\cot\beta$

In ΔPQA,

$\tan\alpha = \dfrac{PQ}{AQ}$

$\tan\alpha = \dfrac{PS+QS}{\cot\beta}$

$\tan\alpha = \dfrac{PS+b}{b\cot\beta}$

$\Rightarrow \quad PS + b = b\tan\alpha\cot\beta$

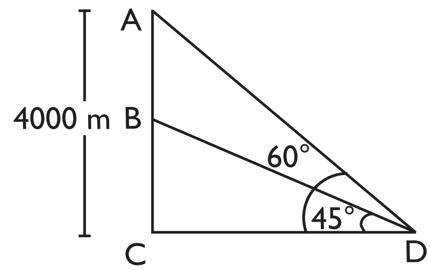
$\Rightarrow \quad PS = b\tan\alpha\cot\beta - b$

So, $PQ = PS + SQ$

$= b\tan\alpha\cot\beta - b + b$

$= b\tan\alpha\cot\beta$

18.



Let the two planes be at points A and B respectively

To find : AB

Let AB = x

$\therefore$ BC = AC – AB

In ΔBCD,

$\tan 45° = \dfrac{BC}{CD}$

$1 = \dfrac{4000 - x}{CD}$

$\therefore$ CD $= 4000 - x$ (i)

In ΔACD,

$\tan 60° = \dfrac{AC}{CD}$

$\sqrt{3} = \dfrac{4000}{4000 - x}$ (From (i))

$4000\sqrt{3} - \sqrt{3}x = 4000$

$\sqrt{3}x = 4000\sqrt{3} - 4000$

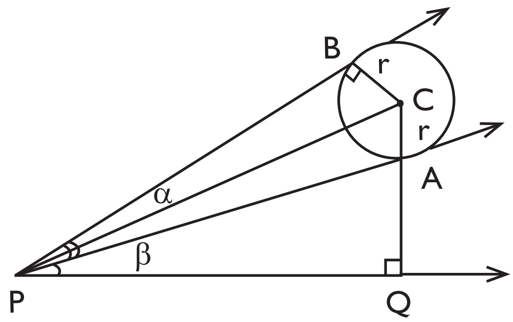
$x = \dfrac{4000}{\sqrt{3}}(\sqrt{3} - 1)$

$= \dfrac{4000}{3}\sqrt{3}(\sqrt{3} - 1)$

$\therefore$ AB $= x = \dfrac{4000}{3}\sqrt{3}(\sqrt{3} - 1)$ metres

SECTION-C

19.



Let P be the eye of observer.

As PB is tangent to circle at point B

$\therefore \quad \angle CBP = 90°$

Let $CA = CB = r$

Let h be the height of the center C.

Also, let $\angle APB = \alpha$

$\Rightarrow \quad \angle APC = \angle BPC = \dfrac{\alpha}{2}$

In ΔCBP,

$$\sin\frac{\alpha}{2} = \frac{BC}{PC} = \frac{r}{CP}$$

$\Rightarrow \quad CP = r \operatorname{cosec}\dfrac{\alpha}{2} \qquad$ (i)

In ΔCQP,

$$\sin \beta = \frac{CQ}{CP}$$

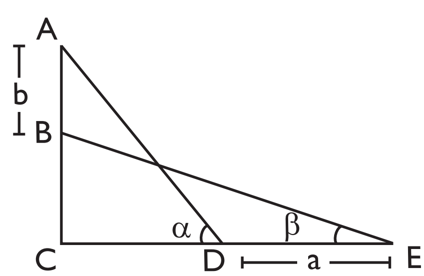
$\Rightarrow \quad CQ = CP \sin \beta$

$= r \operatorname{cosec}\dfrac{\alpha}{2} \sin \beta \quad$ (From (i))

$\therefore$ height of the centre

$= r \operatorname{cosec}\dfrac{\alpha}{2} \sin \beta$

20.



Let AD be the ladder such that when it's foot is pulled away from the wall through a distance 'a' such that it slides a distance 'b' down the wall making an angle 'β' with the horizontal, the ladder comes to position BE.

$\therefore \quad AD = BE$

To prove : $\dfrac{a}{b} = \dfrac{\cos\alpha - \cos\beta}{\sin\beta - \sin\alpha}$

Proof : In ΔACD,

$$\cos\alpha = \frac{CD}{AD}$$

$$\sin\alpha = \frac{AC}{AD}$$

$$= \frac{b + BC}{AD}$$

In ΔBCE,

$$\sin\beta = \frac{BC}{BE}$$

$$\cos\beta = \frac{CE}{BE}$$

$$= \frac{CD + a}{BE}$$

Consider

$$\frac{\cos\alpha - \cos\beta}{\sin\beta - \sin\alpha}$$

$$= \frac{\dfrac{CD}{AD} - \dfrac{CD + a}{BE}}{\dfrac{BC}{BE} - \dfrac{b + BC}{AD}}$$

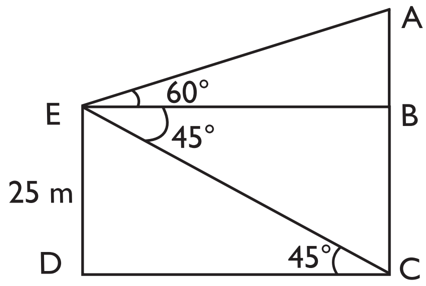
$$= \frac{CD - CD - a}{BC - b - BC}$$

[As AD = BE]

$$= \frac{-a}{-b}$$

$$= \frac{a}{b}$$

21.



Let DE denotes the ship and AC denotes the lighthouse.

Man is standing at point E.

In ΔEDC,

$$\tan 45° = \frac{DE}{CD}$$

$$1 = \frac{25}{CD}$$

$\therefore$ CD = 25 m

$\Rightarrow$ BE = CD = 25 m

In ΔABE,

$$\tan 60° = \frac{AB}{BE}$$

$$\sqrt{3} = \frac{AB}{25}$$

$\Rightarrow$ AB $= 25\sqrt{3}$ m

$\therefore$ Height of the lighthouse

= AC

= AB + BC

$= 25\sqrt{3} + 25$

[BC = DE = 25 m]

$= 25(\sqrt{3} + 1)$ m

In AB denotes the hill such that the two stones are at points C and D

Let BC = x km

$\therefore$ BD = x + 1 km

In ΔABC,

$$\tan 45° = \frac{AB}{BC}$$

$$1 = \frac{AB}{x}$$

$\therefore$ AB = x ...(i)

In ΔABD,

$$\tan 30° = \frac{AB}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{x}{x+1} \quad \text{(From (i))}$$

$\Rightarrow$ $x + 1 = \sqrt{3}x$

$\Rightarrow$ $1 = x(\sqrt{3} - 1)$

$\Rightarrow$ $x = \frac{1}{\sqrt{3}-1}$ km

So, AB $= x = \frac{1}{\sqrt{3}-1}$ km

$= \frac{\sqrt{3}+1}{2}$ km

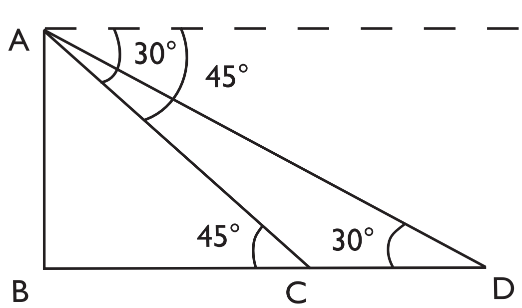
$= \frac{1.732+1}{2}$ km

$= \frac{2.732}{2}$ km

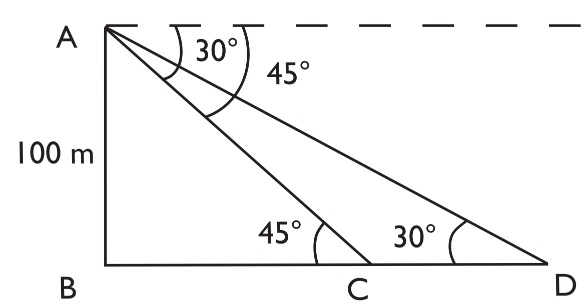
= 1.366 km

SECTION-D

22.



23.



Let the light house be at point A. CD denotes the distance travelled by the ship during the period of observation.

In ΔABC,

$$\tan 45^\circ = \frac{AB}{BC}$$

$$1 = \frac{100}{BC}$$

$$\therefore \quad BC = 100 \text{ m}$$

In ΔABD,

$$\tan 30^\circ = \frac{AB}{BD}$$

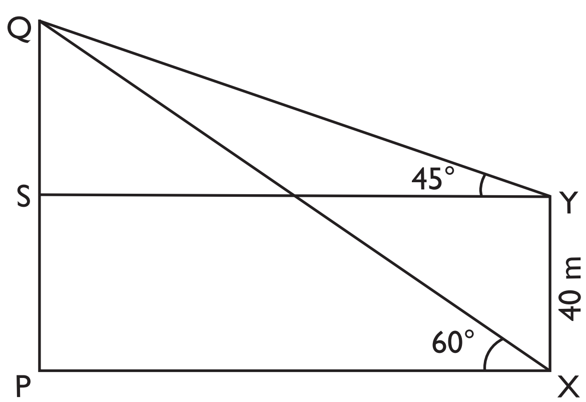
$$\frac{1}{\sqrt{3}} = \frac{100}{BC + CD}$$

$$BC + CD = 100\sqrt{3}$$

$$100 + CD = 100\sqrt{3}$$

$$CD = 100(\sqrt{3} - 1) \text{ m}$$

24.



PQ denotes the tower

PS = XY = 40 m

In ΔQSY,

$$\tan 45^\circ = \frac{QS}{SY}$$

$$1 = \frac{QS}{SY}$$

$$QS = SY \qquad \text{(i)}$$

In ΔQPX,

$$\tan 60^\circ = \frac{PQ}{PX}$$

$$\sqrt{3} = \frac{PS + SQ}{PX}$$

$$\therefore \quad \sqrt{3} = \frac{40 + QS}{SY} \quad [\text{As } PX = SY]$$

$$\sqrt{3} = \frac{40 + QS}{QS} \qquad \text{(From (i))}$$

$$\sqrt{3}\,QS = 40 + QS$$

$$QS(\sqrt{3} - 1) = 40$$

$$QS = \frac{40}{\sqrt{3}} - 1$$

$$= 20(\sqrt{3} + 1) \text{ m}$$

$$\text{So, } PQ = PS + SQ$$

$$= 40 + 20(\sqrt{3} + 1)$$

$$= 20(\sqrt{3} + 3) \text{ m}$$

In ΔQPX,

$$\sin 60^\circ = \frac{PQ}{XQ}$$

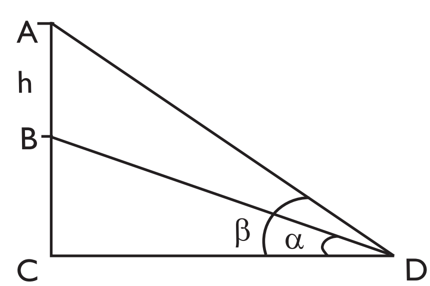
$$\frac{\sqrt{3}}{2} = \frac{20\left(\sqrt{3}+3\right)}{XQ}$$

$$\Rightarrow \quad XQ = \frac{2}{\sqrt{3}}(20)\left(\sqrt{3}+3\right)$$

$$= \frac{40}{\sqrt{3}}\sqrt{3}\left(\sqrt{3}+1\right)$$

$$= 40\left(\sqrt{3}+1\right) \text{ m}$$

25.



Let BC denotes the tower and AB denotes the flag.

To prove : $BC = \dfrac{h\tan\alpha}{\tan\beta - \tan\alpha}$

In ΔBCD,

$\tan\alpha = \dfrac{BC}{CD}$

$\therefore \; BC = CD \tan\alpha$ (i)

In ΔACD,

$\tan\beta = \dfrac{AC}{CD}$

$\tan\beta = \dfrac{AB + BC}{CD}$

$= \dfrac{h + BC}{CD}$

$\tan\beta = \dfrac{h + CD\tan\alpha}{CD}$ [From (i)]

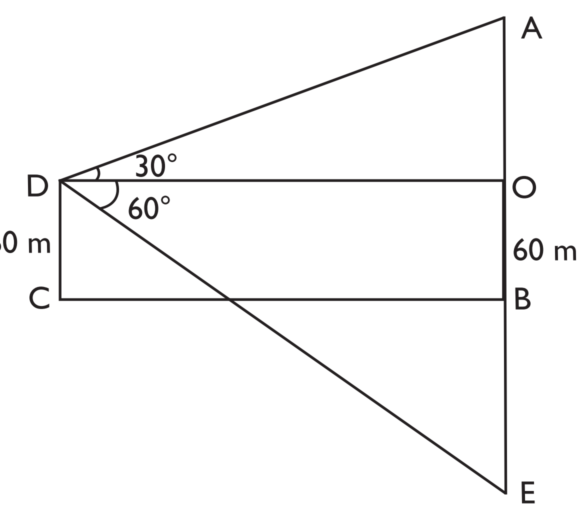
$CD \tan\beta = h + CD \tan\alpha$

$\therefore \; CD = \dfrac{h}{\tan\beta - \tan\alpha}$

From (i), $BC = CD \tan\alpha$

$= \dfrac{h\tan\alpha}{\tan\beta - \tan\alpha}$

26.



In ΔAOD,

$\tan 30° = \dfrac{AO}{OD}$

$\dfrac{1}{\sqrt{3}} = \dfrac{AO}{OD}$

$\Rightarrow \; OD = \sqrt{3}\, AO$ ...(i)

In ΔDOE,

$\tan 60° = \dfrac{OE}{OD}$

$\sqrt{3} = \dfrac{60 + BE}{\sqrt{3}AO}$ [From (i)]

$\Rightarrow \; \sqrt{3}\, AO = 60 + BE$

$\therefore \; 3\,AO = 60 + AO + 60$

[$\because$ BE = AB = AO + 60]

$2\,AO = 120$

$AO = 60$ m

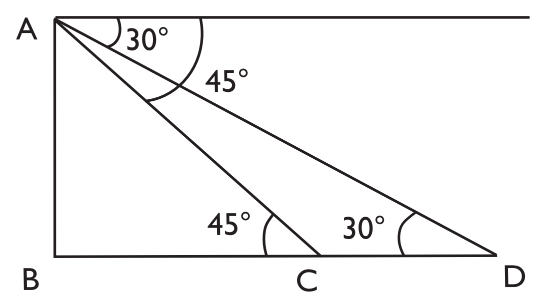
So, height of cloud = AB

= AO + OB

= 60 + 60

= 120 m

27.



Let speed of car be x m/s

So, $CD = 12 \times 60\, x$

$= 720\, x$ metre

In ΔABC,

$\tan 45° = \dfrac{AB}{BC}$

$1 = \dfrac{AB}{BC}$

$\therefore \; AB = BC$ (i)

In ΔABD,

$\tan 30° = \dfrac{AB}{BD}$

$\dfrac{1}{\sqrt{3}} = \dfrac{AB}{BC + CD}$

$\frac{1}{\sqrt{3}} = \frac{BC}{BC + 720x}$ [From (i)]

$\Rightarrow \sqrt{3}\ BC = BC + 720\ x$

$\Rightarrow BC = \frac{720x}{\sqrt{3}-1}$

$= 360\ (\sqrt{3} + 1)\ x$ metre

New, time taken to travel distance BC

$= \frac{720x}{(\sqrt{3}-1)x}$

$= \frac{720}{\sqrt{3} - 1}$

$= 360\ (\sqrt{3} + 1)$ seconds

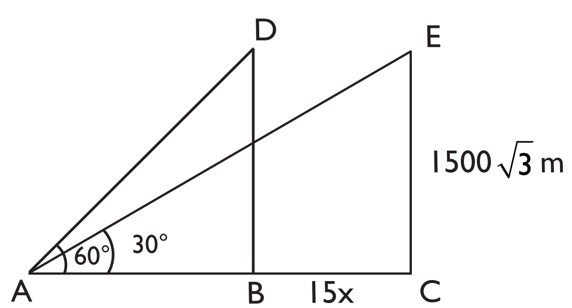
$= 360\ (1.732 + 1)$

$= 360\ (2.732)$

$= 983.52$ seconds

$= 984$ seconds

28.



Let the speed of the jet plane be x m/s

Distance BC = 15 x metre

In ΔABD,

$\tan 60° = \frac{BD}{AB}$

$\sqrt{3} = \frac{1500\sqrt{3}}{AB}$

$\therefore$ AB = 1500 m

In ΔACE,

$\tan 30° = \frac{CE}{AC}$

$\frac{1}{\sqrt{3}} = \frac{1500\sqrt{3}}{AB + 15x}$

$\frac{1}{\sqrt{3}} = \frac{1500\sqrt{3}}{1500 + 15x}$

$1500 + 15x = 1500 \times 3$

$15x = 4500 - 1500]$

$15x = 3000$

$x = 200$ m/s

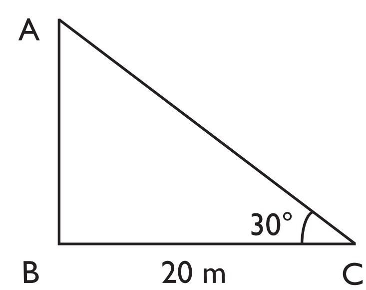
$= \frac{200}{1000} \times 3600$

$= 720$ km/hr

WORKSHEET - 2

SECTION-A

1.



Let AB denotes the tower

In ΔABC,

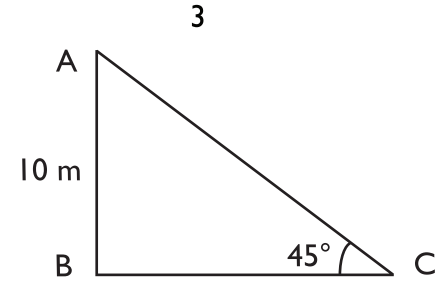
$\tan 30° = \frac{AD}{BC}$

$\frac{1}{\sqrt{3}} = \frac{AB}{20}$

$AB = \frac{20}{\sqrt{3}}$

$= \frac{20\sqrt{3}}{3}$

2.



Let AB and AC denote the vertical pole and wire respectively.

In ΔABC,

$$\sin 45^\circ = \frac{AB}{AC}$$

$$\frac{1}{\sqrt{2}} = \frac{10}{AC}$$

$$\therefore \quad AC = 10\sqrt{2} \text{ m}$$

3. $$BD = AB - AD$$
$$= 6 - 2.54$$
$$= 3.46 \text{ m}$$

In ΔCBD,

$$\sin 60^\circ = \frac{BD}{CD}$$

$$\frac{\sqrt{3}}{2} = \frac{3.46}{CD}$$

$$\therefore \quad CD = \frac{3.46 \times 2}{\sqrt{3}}$$
$$= \frac{3.46 \times 2}{1.73}$$
$$= 4 \text{ m}$$

4. AB denotes the pole and BC denotes shadow of the pole.

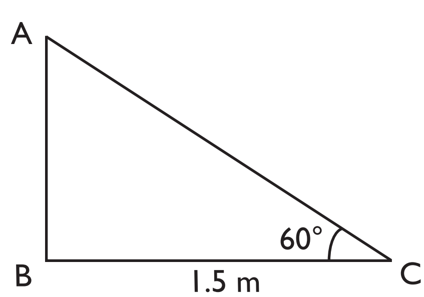
In ΔABC,

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{AB}{20\sqrt{3}}$$

$$\therefore \quad AB = 60 \text{ m}$$

5.



Let AB denotes the wall and AC denotes the ladder.

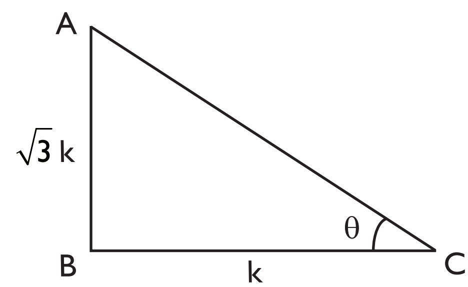
In ΔABC,

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{AB}{1.5}$$

$$\therefore \quad AB = 1.5\sqrt{3} \text{ m}$$

6.



Let AB denotes the tower and BC denotes the shadow of tower.

$$AB : BC = \sqrt{3} : 1$$

$$\text{Let } AB = \sqrt{3}k$$

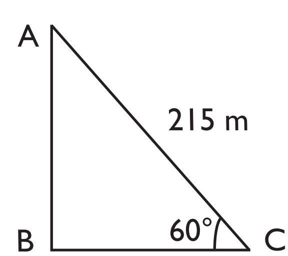
$$BC = k$$

In ΔABC,

$$\tan\theta = \frac{AB}{BC}$$
$$= \frac{\sqrt{3}k}{k}$$
$$= \sqrt{3}$$

$$\therefore \quad \theta = 60^\circ$$

7.



Let the ball be at point A and AC denotes the cable.

To find : AB

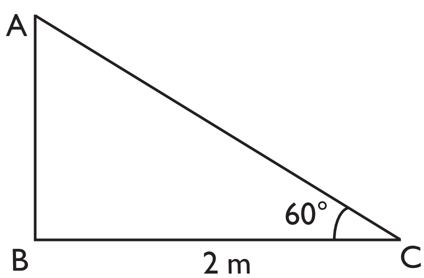
In ΔABC,

$$\sin 60° = \frac{AB}{AC}$$

$$\frac{\sqrt{3}}{2} = \frac{AB}{215}$$

$$AB = \frac{215}{2}\sqrt{3} \text{ m}$$

8.



Let AB denotes the wall and AC denotes the ladder.

To find : AC

In ΔABC,

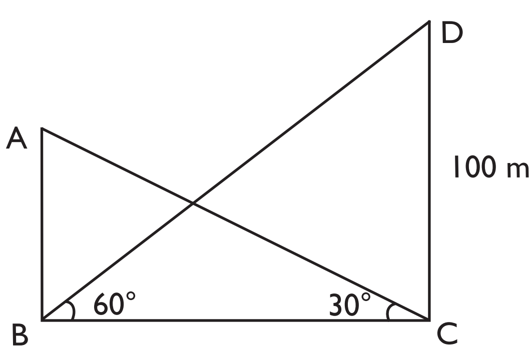
$$\cos 60° = \frac{BC}{AC}$$

$$\frac{1}{2} = \frac{2}{AC}$$

$$\therefore \quad AC = 4 \text{ m}$$

SECTION-B

9.



Let AB and CD denote building and tower respectively.

In ΔBCD,

$$\tan 60° = \frac{CD}{BC}$$

$$\sqrt{3} = \frac{100}{BC}$$

$$BC = \frac{100}{\sqrt{3}}$$

$$= \frac{100}{3}\sqrt{3} \text{ m}$$

In ΔCBA,

$$\tan 30° = \frac{AB}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{\frac{100\sqrt{3}}{3}}$$

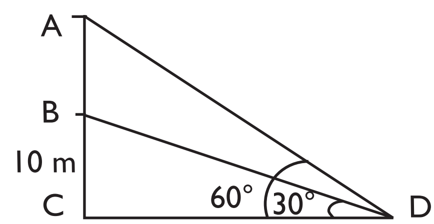
$$\frac{1}{\sqrt{3}} = \frac{3AB}{100\sqrt{3}}$$

$$3\,AB = 100$$

$$AB = \frac{100}{3} \text{ m}$$

So, height of building = $\frac{100}{3}$ m

10.



Let BC and AB denote the building and tower respectively.

In ΔBCD

$$\tan 30° = \frac{BC}{CD}$$

$$\frac{1}{\sqrt{3}} = \frac{10}{CD}$$

$$\therefore \quad CD = 10\sqrt{3} \text{ m}$$

In ΔACD,

$$\tan 60° = \frac{AC}{CD}$$

$$\sqrt{3} = \frac{10 + AB}{10\sqrt{3}}$$

$30 = AB + 10$

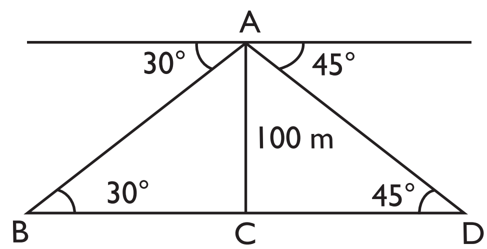
$AB = 30 - 10$

$= 20$ m

So, height of tower = AB

$= 20$ m

11.



Let AC denotes the tower and the two buses be at points B and D respectively.

To find : BD

In ΔABC,

$\tan 30° = \frac{AC}{BC}$

$\frac{1}{\sqrt{3}} = \frac{100}{BC}$

$\therefore \quad BC = 100\sqrt{3}$

In ΔACD,

$\tan 45° = \frac{AC}{CD}$

$1 = \frac{100}{CD}$

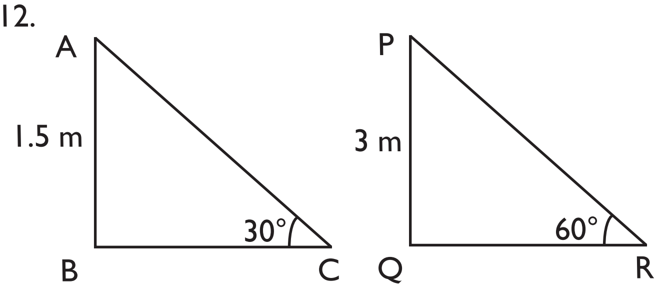
$CD = 100$ m

So, $BD = BC + CD$

$= 100\sqrt{3} + 100$

$= 100(\sqrt{3} + 1)$ m

12.



In ΔABC,

$\sin 30° = \frac{AB}{AC}$

$\frac{1}{2} = \frac{1.5}{AC}$

$\therefore \quad AC = 3$ m

In ΔPQR,

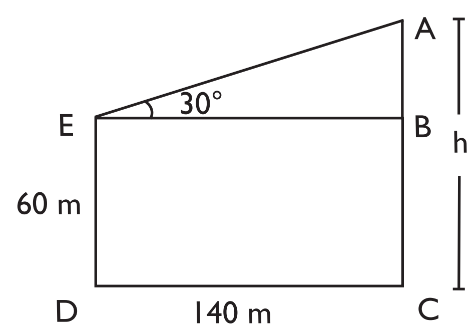
$\sin 60° = \frac{PQ}{PR}$

$\frac{\sqrt{3}}{2} = \frac{3}{PR}$

$\therefore \quad PR = \frac{6}{\sqrt{3}}$

$= 2\sqrt{3}$ m

13.



Let AC and DE denote two towers.

In ΔABE,

$\tan 30° = \frac{AB}{BE}$

$\frac{1}{\sqrt{3}} = \frac{AB}{140}$

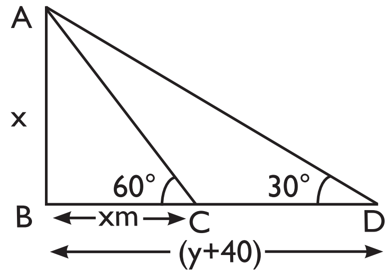
$[\because BE = CD = 140 \text{ m}]$

$\therefore \quad AB = \frac{140}{\sqrt{3}}$ m

So, $AC = BC + AB$

$= 60 + \frac{140}{\sqrt{3}}$

$\therefore \quad DE = BC = 60 + \frac{140}{\sqrt{3}}$ m

14. 

Let AB is the height of tower = x

In ΔABC

$$\tan 60° = \frac{AB}{BC} = \frac{x}{y}$$

$$\boxed{x = y\sqrt{3}} \qquad ...(i)$$

In ΔABD

$$\tan 30° = \frac{AB}{BD} = \frac{x}{y+40}$$

$$\frac{1}{\sqrt{3}} = \frac{x}{y+40}$$

$$(y + 40) = x\sqrt{3}$$

$$y + 40 = \left(y\sqrt{3}\right)\sqrt{3}$$

$$y + 40 = 3y$$

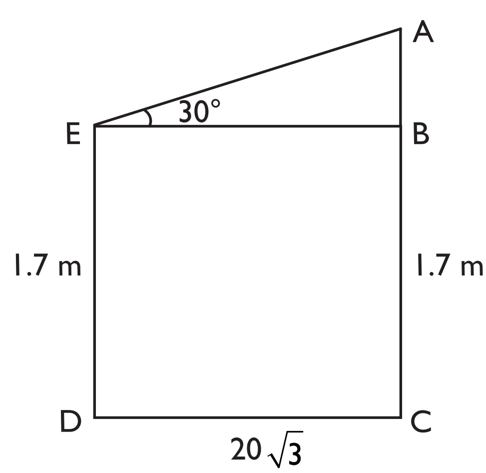
$$2y = 40$$

$$y = 20$$

$$\therefore \quad x = 20\sqrt{3} \text{ m}$$

$$= 20 \times 1.73 \text{ m}$$

$$= 24.6 \text{ m}$$

15. 

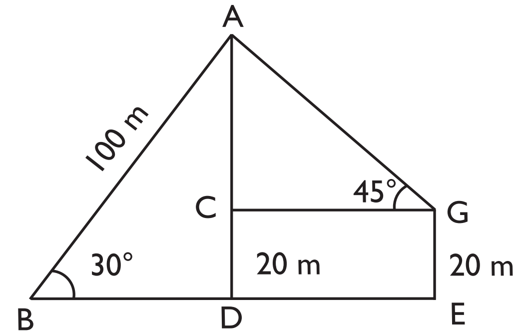
Let AB = x

In ΔABE

$$\tan 30° = \frac{AB}{BE}$$

$$\frac{1}{\sqrt{3}} = \frac{x}{20\sqrt{3}} \Rightarrow \boxed{x = 20 \text{ m}}$$

The height of tower AC is AB + BC i.e. 20 m + 1.7 m = 21.7 m

16. 

Let the girl is standing at point G and the boy is standing at point B.

In ΔADB

$$\frac{AD}{BA} = \sin 30$$

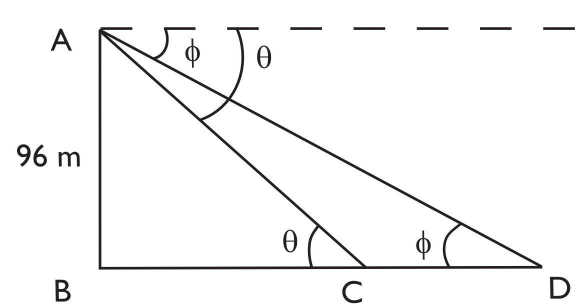
$$\frac{AC \quad CD}{100} = \frac{1}{2} \Rightarrow \frac{AC+20}{50} = 1$$

$$\Rightarrow \boxed{AC = 30 \text{ m}}$$

In ΔACG

$$\sin 45 = \frac{AC}{AG} = \frac{30}{AG} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \boxed{AG = 30\sqrt{2} \text{ m}}$$

17. 

Let CB and CD are the distance of cars from the foot of tower respectively.

$\tan\theta = \frac{3}{4}$ and $\tan\phi = \frac{1}{3}$

In ΔABC

$\tan\theta = \frac{AB}{BC}$

$\frac{3}{4} = \frac{96}{BC}$ and

$BC = \frac{96 \times 4}{3}$

In ΔABD

$\tan\phi = \frac{AB}{BD} = \frac{AB}{BC + CD}$

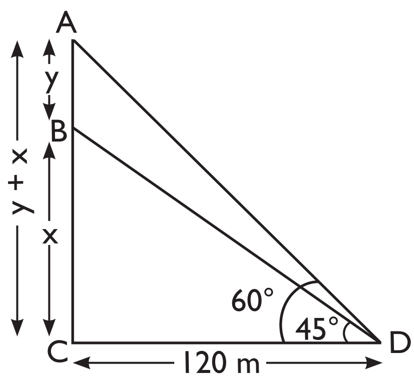
$\frac{1}{3} = \frac{96}{128 + CD}$

$\Rightarrow 128 + CD = 288$

$\boxed{CD = 160 \text{ m}}$

The distance between cars is CD = 160 m

18.



In ΔBCD

$\tan 45 = \frac{x}{120} \Rightarrow x = 120$ m

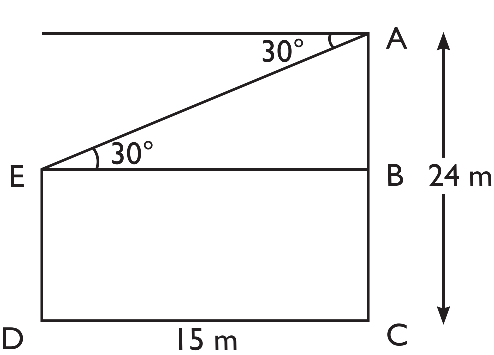
In ΔACD

$\tan 60 = \frac{x + y}{120}$

$\frac{x + y}{120} = \sqrt{3} \Rightarrow 120 + y = 120\sqrt{3}$

$y = 120(\sqrt{3} - 1)$ m

19.



In ΔABE

$$\frac{AB}{BE} = \tan 30$$

$$\frac{AB}{15} = \frac{1}{\sqrt{3}} = \boxed{AB = \frac{15}{\sqrt{3}} m} = \frac{15 \times \sqrt{3}}{3}$$

$$= 5\sqrt{3} \text{ m}$$

$$AB + BC = AC = 24 \text{ m}$$

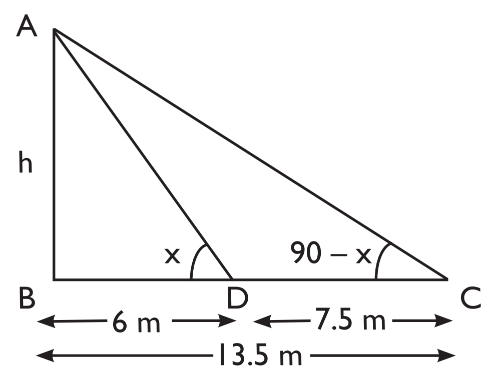
$$5\sqrt{3} + BC = 24$$

$$BC = 24 - 5\sqrt{3}$$

Let ED be the height of first pole.

From the figure, BC = ED

20.



Let AB be the tower

In ΔABD

$\frac{h}{6} = \tan x \Rightarrow h = 6 \tan x$ ...(i)

In ΔABC

$$\frac{h}{13.5} = \tan(90 - x)$$

$$\frac{h}{13.5} = \cot x$$

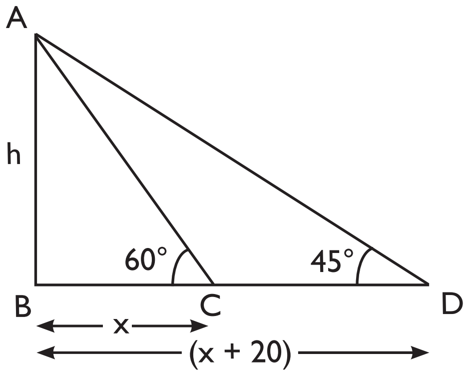
$h = 13.5 \cot x$ ...(ii)

Multiply equation (i) and (ii)

$$h^2 = 81$$

$$\boxed{h = 9 \text{ m}}$$

21.



Let AB = h be the height of tower.

In ΔABC

$$\frac{AB}{BC} = \tan 60$$

$$\frac{h}{x} = \sqrt{3}$$

$$\boxed{h = x\sqrt{3}\text{ m}} \qquad ...(i)$$

In ΔABD

$$\frac{AB}{BD} = \tan 45$$

$$AB = BD$$

$$h = x + 20 \qquad ...(ii)$$

from equation (i) and (ii)

$$x\sqrt{3} = x + 20$$

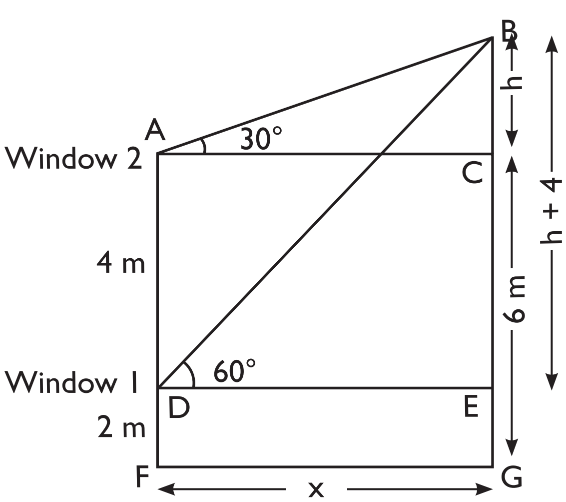
$$x(\sqrt{3} - 1) = 20$$

$$x = \frac{20}{\sqrt{3}-1}\,\frac{\sqrt{3}+1}{\sqrt{3}+1} = 10\left(\sqrt{3}+1\right)m$$

$$h = 10\sqrt{3}\left(\sqrt{3}+1\right)m$$

$$\boxed{h = 30 + 10\sqrt{3}\text{ m}}$$

22.



In ΔABC

$$\tan 30 = \frac{BC}{AC} = \frac{h}{x}$$

$$\frac{h}{x} = \frac{1}{\sqrt{3}} \quad \boxed{h\sqrt{3} = x} \qquad ...(i)$$

In ΔBDE

$$\tan 60 = \frac{BE}{DE} = \frac{h+4}{x}$$

$$\sqrt{3} = \frac{h+4}{x} \qquad ...(ii)$$

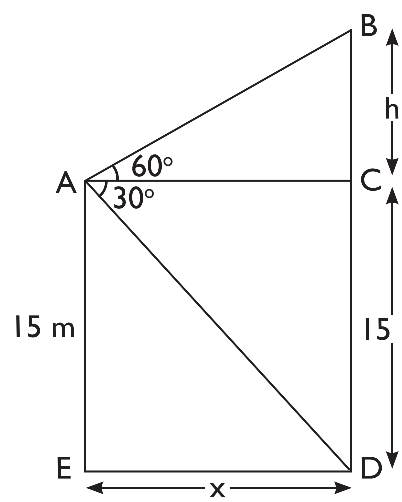
From equation (i) and (ii)

$$\sqrt{3} = \frac{h+4}{h\sqrt{3}} \Rightarrow 3h = h + 4 \Rightarrow 2h = 4$$

$$\boxed{h = 2\text{ m}}$$

$\therefore$ The height of balloon above the ground = h + 6 = 2 + 6 = 8 m.

23.



Let BD represents the cable tower.

In ΔABC

$$\frac{BC}{AC} = \tan 60$$

$$\Rightarrow \frac{h}{x} = \sqrt{3} \Rightarrow \boxed{h = x\sqrt{3}} \qquad ...(i)$$

In ΔACD

$$\tan 30 = \frac{15}{x} \Rightarrow \frac{1}{\sqrt{3}} = \frac{15}{x}$$

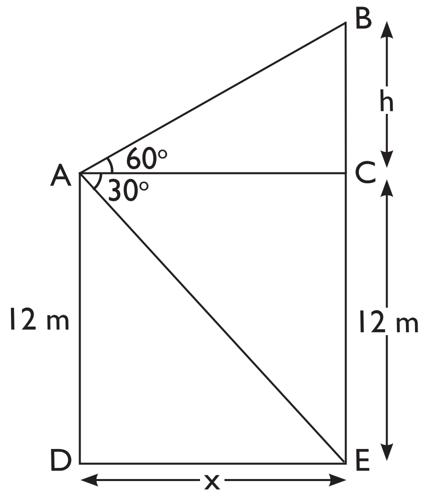
$$\boxed{x = 15\sqrt{3}} \qquad ...(ii)$$

From equation (i) and (ii)

$$h = (15\sqrt{3})\sqrt{3} = 45 \text{ m}$$

$\therefore$ The height of tower from the ground is $45 + 15 = 60$ m.

24.



Let BE represents the height of cliff and DE represents the distance between cliff and ship.

In ΔABC

$$\frac{BC}{AC} = \tan 60$$

$$\frac{h}{x} = \sqrt{3} \Rightarrow \boxed{h = x\sqrt{3}}$$

In ΔACE

$$\frac{CE}{AC} = \tan 30$$

$$\frac{12}{x} = \frac{1}{\sqrt{3}} \Rightarrow \boxed{x = 12\sqrt{3} \text{ m}}$$

$$\therefore \quad h = x\sqrt{3}$$

$$h = (12\sqrt{3})\sqrt{3} \Rightarrow 36$$

Therefore, the height of cliff is $36 + 12 = 48$m and distance between ship and cliff is $12\sqrt{3}$ m.

CASE STUDY-1

(i) (a) In ΔABC

$$\frac{AB}{AC} = \sin 30^\circ$$

$$\frac{1.5}{AC} = \frac{1}{2}$$

$$AC = 3 \text{ m}$$

$$AC = l_1 = 3\text{m}$$

(ii) (c) In ΔPQR

$$\frac{PQ}{PR} = \sin 60^\circ$$

$$\frac{3}{l_2} = \frac{\sqrt{3}}{2}$$

$$l_2 = \frac{b}{\sqrt{3}}$$

$$= 2\sqrt{3} \text{ m}$$

(iii) (b) In ΔABC

$$\frac{AB}{BC} = \tan 30$$

$$\frac{1.5}{BC} = \frac{1}{\sqrt{3}}$$

$$BC = 1.5\sqrt{3} \text{ m}$$

(iv) (c) In ΔPQR

$$\frac{PQ}{QR} = \tan 60$$

$$\frac{3}{QR} = \sqrt{3}$$

$$QR = \sqrt{3} \text{ m}$$

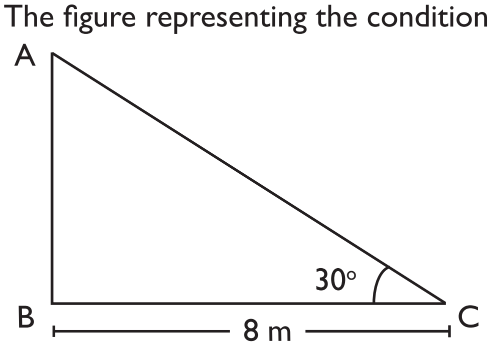
(v) (b) $l_1 = 3\text{m}$

$$l_2 = \sqrt[2]{3}$$

$$l_1 + l_2 = 3 + \sqrt[2]{3} \text{ m}$$

CASE STUDY-2

(i) (b) The figure representing the condition is



Height of broken tree = AC

$$\frac{BC}{AC} = \cos 30$$

$$\frac{8}{AC} = \frac{\sqrt{3}}{2}$$

$$AC = \frac{16}{\sqrt{3}} \text{ m}$$

(ii) (d) Height of remaining tree = AB

$$\frac{AB}{BC} = \tan 30$$

$$AB = BC \tan 30$$

$$= 8\left(\frac{1}{\sqrt{3}}\right) \text{m}$$

$$= \frac{8}{\sqrt{3}} \text{m}$$

(iii) (a) Total height of tree is AB + AC

$$AB + AC = \frac{8}{\sqrt{3}} + \frac{16}{\sqrt{3}}$$

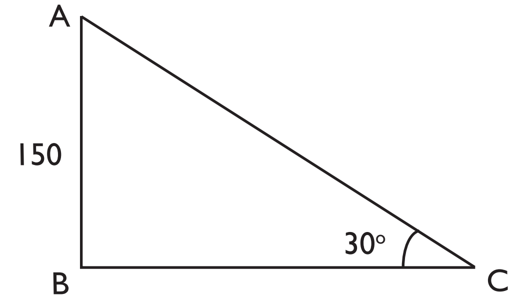
$$= \frac{24}{\sqrt{3}} \text{m}$$

$$= \frac{24\sqrt{3}}{\sqrt{3} \times \sqrt{3}} \text{m}$$

$$= \frac{24\sqrt{3}}{3} \text{m}$$

$$= 8\sqrt{3} \text{ m}$$

(iv) (c) Let the height of tree is AB and length of shadow is BC.

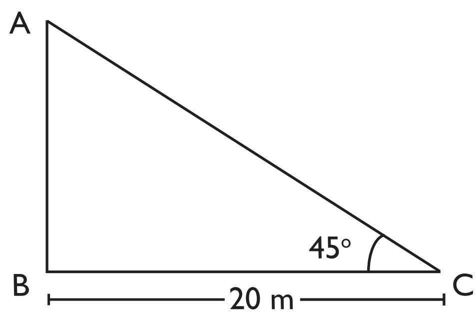


$$\frac{AB}{AC} = \tan 30$$

$$\frac{150}{AC} = \frac{1}{\sqrt{3}}$$

$$AC = 150\sqrt{3}m$$

(v) (b)



$$\frac{AB}{BC} = \tan 45$$

$$\frac{AB}{BC} = 1$$

$$AB = BC = 20 \text{ m}$$

Chapter 10 Circle

Multiple Choice Questions

1. (b) From

$$37 + r = \frac{44r}{7}$$
$$37 = \frac{44r}{7} - r$$
$$37 = \frac{37r}{7}$$
$$r = 7 \text{ cm}$$
$$c = 2\pi r$$
$$= 2 \times \frac{22}{7} \times 7$$
$$= 44 \text{ cm}$$

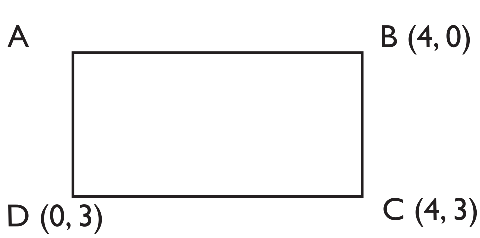
2. (b)

$$\pi r_1^2 + \pi r_2^2 = \pi r^2$$
$$r_1^2 + r_2^2 = r^2$$
$$5^2 + (12)^2 = r^2$$
$$25 + 144 = r^2$$
$$r^2 = 169$$
$$r = 13 \text{ cm}$$
$$\therefore \quad \text{Diameter} = 2r = 26 \text{ cm}$$

3. (b) Distance covered in one revolution

$$= 2\pi r$$
$$= 2 \times \frac{22}{7} \times \frac{35}{2}$$
$$= 110 \text{ cm}$$

4. (a)



$$\text{Diagonal} = BD = \sqrt{(4-0)^2 + (0-3)^2}$$
$$= \sqrt{16+9}$$
$$= \sqrt{25}$$
$$= 5$$

5. (c)

$$\text{Radius} = \frac{18}{2} = 9 \text{ cm}$$
$$\therefore \quad \text{Perimeter} = 2\pi r$$
$$= 2\pi\,(9)$$
$$= 18\pi \text{ cm}$$

WORKSHEET - 1

SECTION-A

1.

$$\text{Arc length} = \frac{\theta}{360} 2\pi r$$
$$3\pi = \frac{\theta}{360} 2\pi \times 6$$
$$\Rightarrow \quad 3\pi = \frac{\theta\pi}{30}$$
$$\Rightarrow \quad \theta = \frac{3\pi \times 30}{\pi}$$
$$= 90^\circ$$

2.

$$\text{Diameter} = 14 \text{ cm}$$
$$\Rightarrow \quad \text{radius} = \frac{14}{2} = 7 \text{ cm}$$

Perimeter of semi – circle protractor

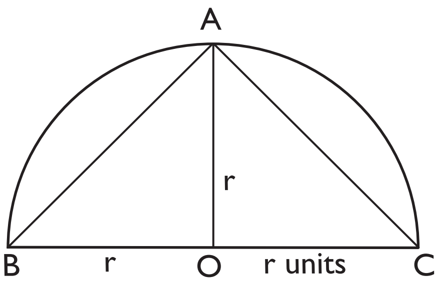
$$= 2r + \frac{1}{2}(2\pi r)$$
$$= 2r + \pi r$$

$$= 2(7)+\frac{22}{7}\times 7$$

$$= 14 + 22$$

$$= 36 \text{ cm}$$

3.



Area of $\Delta BAC = \frac{1}{2} \times BC \times AO$

$$= \frac{1}{2} \times 2r \times r$$

$$= r^2 \text{ sq. units}$$

4. Perimeter of sector

$$= 2r + \frac{\theta}{360} 2\pi r$$

$$= 2r\ \ 1+\frac{\pi\theta}{360}$$

$$= 2(10.5)\left(1+\frac{22}{7}\times\frac{60}{360}\right)$$

$$= 21\ \ 1+\frac{11}{21}$$

$$= \frac{21\times 32}{21}$$

$$= 32 \text{ cm}$$

5. $r = 10$ cm

$\theta = 108°$

area of sector $= \frac{\theta}{360°}\pi r^2$

$$= \frac{108}{360}\pi(100)$$

$$= 3\pi 10$$

$$= 30\pi \text{ cm}^2$$

6. Distance covered in one revolution

$$= 2\pi r$$

$$= 2\times\frac{22}{7}\pi$$

Number of revolutions in covering a distance of x metres.

$$= \frac{x}{2\times\frac{22}{7}\times r}$$

$$= \frac{7x}{44r}$$

7. Let the diameter and a side be x units.

So, radius of circle $= \frac{x}{2}$ units

$\therefore$ Area of circle $= \pi\left(\frac{x}{2}\right)^2$

$$= \frac{\pi x^2}{4}$$

Area of equilateral triangle

$$= \frac{\sqrt{3}}{4}\ (\text{side})^2$$

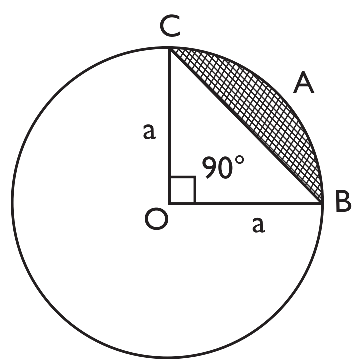
$$= \frac{\sqrt{3}}{4}\ x^2$$

$$\therefore \frac{\text{Area of circle}}{\text{Area of equilateral triangle}}$$

$$= \frac{\frac{\pi x^2}{4}}{\frac{\sqrt{3}}{4}x^2}$$

$$= \frac{\pi}{\sqrt{3}}$$

8.



Perimeter of segment ABC

= BC + length of arc $\overset{\frown}{BAC}$

In ΔBOC

$$BC^2 = OC^2 + OB^2$$

(By Pythagoras theorem)

$$BC^2 = a^2 + a^2$$
$$= 2a^2$$
$$BC = \sqrt{2}a$$

Also length of arc $\overset{\frown}{BAC}$

$$= \frac{90}{360} \times 2 \times \frac{22}{7} \times a$$

$$= \frac{11a}{7}$$

So, Perimeter of segment ABC

$$= \sqrt{2}a + \frac{11a}{7}$$

SECTION-B

9. We know that AD = AF

BD = BE

CE = CF

Let AD = AF = x

BD = BE = y

CE = CF = z

Then x + y = 12

y + z = 8

x + z = 10

On solving above equation we get

x = 7, y = 5, z = 3

So AD = 7, BE = 5, CF = 3

10. BP = AP = 5 cm

(The lengths of tangents drawn from an external point to a circle are equal.)

$\therefore \angle PAB = \angle PBA$

(Angle opposide to equal sides are equal.)

In PAB,

$\angle P + \angle PAB + \angle PBA = 180°$

(Angle sum property)

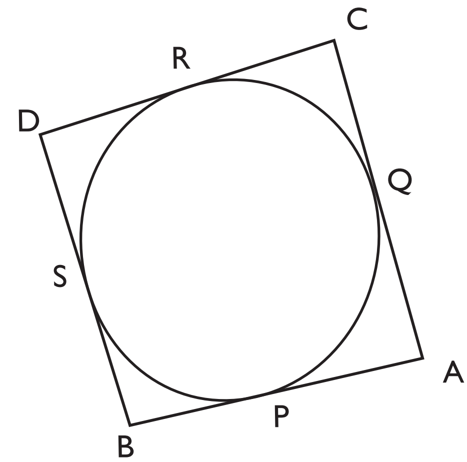
$$60° + 2\angle PAB = 180°$$
$$2\angle PAB = 120°$$
$$\angle PAB = 60°$$

As all the angles of triangle ABC are 60°, hence triangle ABC is an equilateral triangle and thus all the sides are equal.

$\therefore \angle PAB = \angle PBA = 60°$

$\therefore$ AB = PA = PB = 5 cm

11.



As we know that the length of tangents drawn from an external point to a circle are equal.

AP = AS ...(i)

BP = BQ ...(ii)

CR = CQ ...(ii)

DR = DS ...(iv)

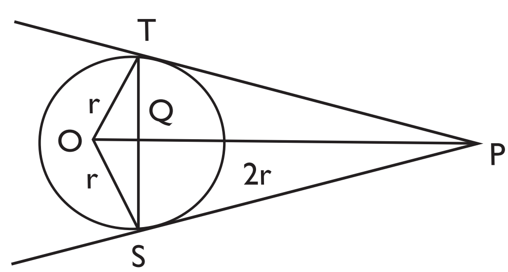
On adding both sides of (i), (ii), (iii) and (iv), we get

AP + BP + CR + DR = AS + BQ + CQ + DS

$\therefore$ (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

AB + CD = BC + AD

12.



$\angle TOP = \theta$

As we know that radius is perpendicular to the tangent at the point of contact.

$$\angle OTP = 90°$$

So in ΔOTP

$$\cos\theta = \frac{OT}{OP} = \frac{r}{2r} = \frac{1}{2}$$

$\therefore$ $\cos\theta = \frac{1}{2}$ cos 60°

$\Rightarrow$ $\theta = 60°$

$\therefore$ $\angle TOS = 60° + 60° = 120°$

As $OT = OS$

$\Rightarrow$ $\angle OTS = \angle OST$

(Angles opposite to equal sides are equal)

In $\angle OTS$,

$\angle TOS + \angle OTS + \angle OST = 180°$

$120° + 2\angle OTS = 180°$

$2\angle OTS = 60°$

$\angle OTS = 30°$

$\therefore$ $\angle OTS = \angle OST = 30°$

13. In ΔOTP and ΔOSP

OT = OS (Radii of same circle)

OP = PO (Common)

PT = PS

(The lengths of tangents drawn from an external point to a circle are equal)

$\therefore$ $\Delta OTP \cong \Delta OSP$ (SSS)

$\therefore$ $\angle OPS = \angle OPT$

$= \frac{1}{2}\angle TPS$ (CPCT)

$= \frac{1}{2}(120°)$

$= 60°$

In ΔOSP,

$OS \perp PS$

(Radius is perpendicular to the tangent at the point of contact)

$$\cos(\angle OPS) = \frac{PS}{OP}$$

$\Rightarrow$ $\cos 60° = \frac{PS}{OP}$

$\Rightarrow$ $\frac{1}{2} = \frac{PS}{OP}$

$\Rightarrow$ $OP = 2\,PS$

14. In ΔOAP

OA = 6 cm

AP = 8 cm

$\therefore$ $OP^2 = OA^2 + AP^2$

(By Pythagoras theorem)

$= 6^2 + 8^2$

$= 36 + 64$

$= 100$

$\Rightarrow$ OP = 10 cm

Now, In ΔOBP,

$OP^2 = OB^2 + BP^2$

$10^2 = 4^2 + BP^2$

(By Pythagoras theorem)

$100 = 16 + BP^2$

$BP^2 = 100 - 16$

$= 84$

$\therefore$ $BP = 2\sqrt{21}$ cm

15. $\angle OAC = 90°$ (as radius $\perp$ tangent)

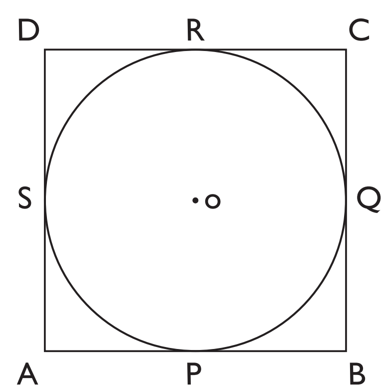
$\angle BOC = \angle OAC + \angle ACO$

(Exterior angle property)

$130° = 90° + \angle ACO$

$\angle ACO = 130° - 90° = 40°$

16.



A rhombus is a parallelogram with all equal sides.

In ll gm ABCD

AB = CD and AD = BC

Hence AP = AS; BP = BQ; CR = CQ; DR = DS

Adding we get AP + BP + CR + DR = AS + BQ + CQ + DS

(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

$$AB + CD = AD + BC$$

$$AB + AB = AD + AD$$

$$2AB = 2AD$$

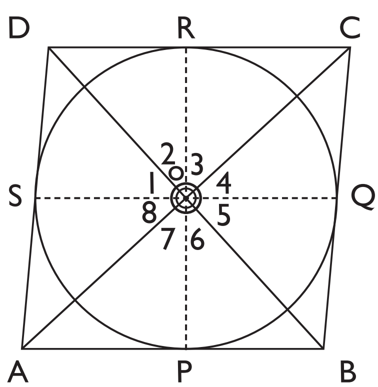
So AB = AD and AB = CD and AD = BC

So AB = CD = AD = CD

So ABCD is || gm with equal sides.

∴ ABCD is a rhombus.

∴ Proved.

17. 

Const : join OP, OQ, OR and OS.

Proof : Since, the two tangents drawn from an external point to a circle subtend equal angles at the centre.

∴ $\angle 1 = \angle 2, \angle 3 = \angle 4, \angle 5 = \angle 6, \angle 7 = \angle 8$

Since sum of all the angles subtended at a point is 360°.

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8$

$= 360°$

$\Rightarrow 2\angle 2 + 2\angle 3 + 2\angle 6 + 2\angle 7 = 360°$

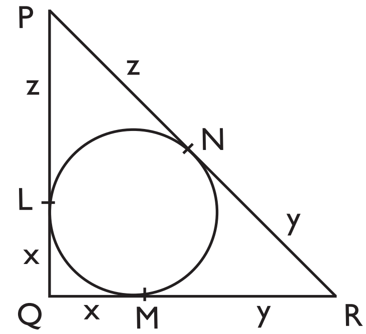
$\Rightarrow 2(\angle 2 + \angle 3 + \angle 6 + \angle 7) = 360°$

$\Rightarrow \angle 2 + \angle 3 + \angle 6 + \angle 7 = 180°$

$\Rightarrow (\angle 6 + \angle 7) + (\angle 2 + \angle 3) = 180°$

$\Rightarrow \angle AOB + \angle COD = 180°$

Similarly, we can prove $\angle AOD + \angle BOC = 180°$

18. 

QL = QM

RM = RN

PL = PN

We know that the tangents drawn to a circle from an external point are equal in length.

Let QL = QM = x

Let RM = RN = y

Let PL = PN = z

Consider PQ + QR + PR = 60

$\Rightarrow x + z + x + y + z + y = 60$

$\Rightarrow 2x + 2y + 2z = 60$

$\Rightarrow x + y + z = 30$

PQ = 20

x + z = 20

∴ RN = 10 cm

Also, QR = 16

x + y = 16

∴ z = 30 – (x + y)

= 30 – 16

= 14 cm

∴ PL = 14 cm

Again, PR = 24

y + z = 24

∴ x = 30 – (y + z)

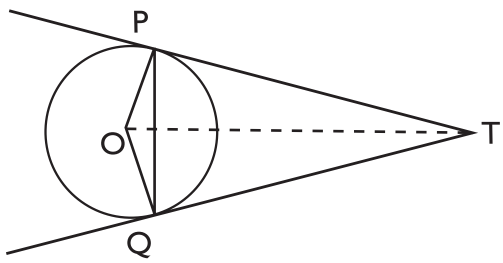
= 30 – 24

= 6

∴ QM = 6 cm

SECTION-C

19.



In ΔOPT and ΔOTQ

$OP = OQ$ (Both are radius of same circle)

$OT = OT$ (Common)

$PT = QT$ (Tangents drawn from same points are eqaual)

$\therefore \Delta OPT \cong \Delta OTQ$ (SSS)

$\angle OPT = 90°$ (Tangent is perpendicular to radius through point of contact).

From the figure

$\angle OPT = \angle OPQ + \angle TPQ = 90°$

$\Rightarrow \angle TPQ = 90 - \angle OPQ$

In ΔPTQ

$\angle PTQ + \angle TPQ + \angle TQP = 180°$ (angle sun property)

$\angle PTQ + \angle TPQ + \angle TQP = 180$

$\angle PTQ + \angle TPQ + \angle TPQ = 180$

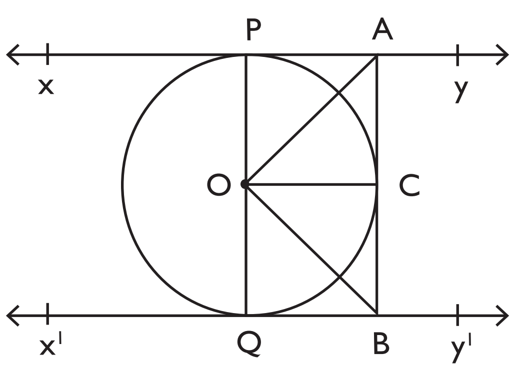
[$\angle TPQ = \angle TQP$ as TP and QT are equal]

$\angle PTQ + 2\angle TPQ = 180$

$\angle PTQ + 2\,(90 - \angle OPQ) = 180$

$\angle PTQ = 2\,\angle OPQ$

20.



Given: A circle with center O to which XY and XY' are tangents.

To prove: $\angle AOB = 90°$

Construction: Join point O to Point C.

Proof: $\angle OCB = \angle OCA = 90°$ (Radius is perpendicular to the tangents at the point of contact).

Consider ΔOCB and OQB

$OC = OQ$ (radius of same circle)

$\angle OQB = \angle OCB = 90°$ (Radius is perpendicular to the tangent at the point of contact)

$BQ = BC$ (Tangents from the same point are equal)

$\therefore \Delta OCB \cong \Delta OQB$ by SAS rule.

Consider ΔAPO and ΔAOC

$OP = OC$ [Radius of same circle]

$OA = OA$ [Common]

$AP = AC$ [Tangent from the same points are equal]

$\therefore \Delta APO \cong \Delta AOC$ by SSS rule

$\angle POA + \angle COA + \angle QOB + \angle BOC = 180°$

As ΔOCB is congurent to ΔOQB

$\therefore \angle QOB = \angle BOC$

Similarly $\angle POA = \angle COA$

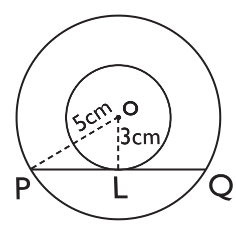
$\therefore \quad 2\angle COA + 2\angle COB = 180$

$\angle COA + \angle COB = 90°$

From the figure it is clear that $\angle COA + \angle COB = \angle AOB$.

$\therefore \angle AOB = 90°$

21.



Let O be the common centre of the two concentric circle.

Let PQ be a chord of the larger circle which touches the smaller circle at L.

Join OL and OP.

Since, the tangent at any point of a circle is perpendicular to the radius through the point of contact.

Therefore,

$\angle OLP = 90°$

Now,

In ΔOLP, we have

$OP^2 = OL^2 + PL^2$

[Using Pythagoras theorem]

$\Rightarrow \quad (5)^2 = (3)^2 + PL^2$

$\Rightarrow \quad 25 = 9 + PL^2$

$\Rightarrow \quad PL^2 = 16$

$\Rightarrow \quad PL = 4$ cm

Since, the perpendicular from the centre of a circle to a chord bisects the chord.

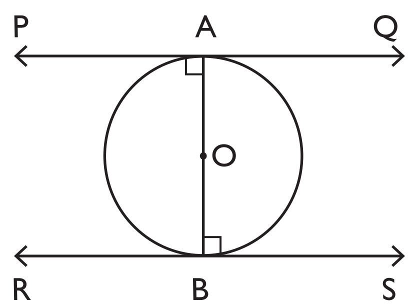
Therefore,

$PL = LQ = 4$ cm

$\therefore \quad PQ = 2\,PL = 2 \times 4 = 8$ cm

Hence, the required length = 8 cm.

22.



Let AB be the diameter of a circle, with centre O. The tangents PQ and RS are drawn at point A and B, respectively.

We know that a tangent at any point of a circle is perpendicular to the radius through the point of contact.

$\therefore$ OA $\perp$ PQ and OB $\perp$ RS

$\Rightarrow \angle OBR = 90°$

$\angle OBS = 90°$

$\angle OAP = 90°$

$\angle OAQ = 90°$

We can observe the following:

$\angle OBR = \angle OAQ$ and $\angle OBS = \angle OAP$

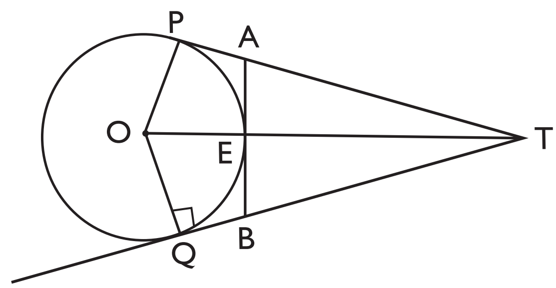
Also, these are the pair of alternate interior angles.

Since alternate angles are equal, the lines PQ and RS are parallel to each other.

Hence, proved.

SECTION-D

23.



$OP = OQ = 5$ cm

$OT = 13$ cm

$\angle OPT = \angle OQT = 90°$

[$\because$ radius is perpendicular to tangent at the point of contact]

In ΔOPT

$OT^2 = OP^2 + PT^2 \Rightarrow PT^2 = OT^2 - OP^2$

$= 169 - 25 = 144$

$\boxed{PT = 12 \text{ cm}}$

AP = AE [tangents drawn from same point are equal in length]

Similarly, BE = BQ

$\angle AEO = \angle AET = 90°$ [Radius is perpendicular to the tangent at the point of contact]

$\therefore$ ΔAET is a right angled triangle.

$AE^2 + ET^2 + AT^2 \Rightarrow AE^2 = AT^2 - ET^2$

$AE^2 = (PT - PA)^2 - (OT - OE)^2$

$AE^2 = (12 - AE)^2 - (B - 5)^2$

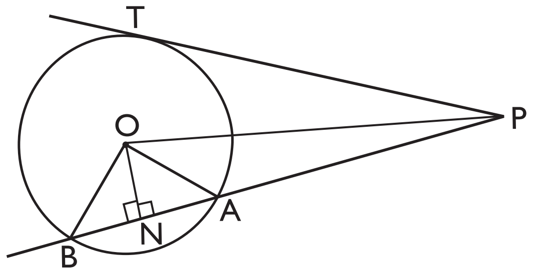
$AE^2 = 144 + AE^2 - 24AE - 64$

$= 24\,AE$

$AE = \frac{80}{24} = \frac{10}{3}$ cm

Similarly, $\quad BE = \frac{10}{3}$ cm

$\therefore$ $AB = AE + BE = \frac{20}{3}$ cm

24. 

To prove: PA.PB = $PN^2 - AN^2$

Construction: Join OP

Proof: Consider ΔAON and ΔBON

OA + OB (radius of same circle)

ON = ON (Common)

∠ONB = ∠ONA = 90°

[As ON is perpendicular to chord]

∴ ΔAON ≅ ΔBON

AN = BN (CPCT)

PA = PN – AN ⇒ PN – AN

PB = PN + BN = PN + AN

∴ PA.PB = (PN – AN) (PN + AN)

= $PN^2 - AN^2$

$PN^2 - AN^2 = OP^2 - OT^2$

Consider ΔOPN

$OP^2 = PN^2 + ON^2$

∴ $PN^2 = OP^2 - ON^2$

$OP^2 - ON^2 - AN^2 = OP^2 - OT^2$

⇒ $ON^2 + AN^2 = OT^2$

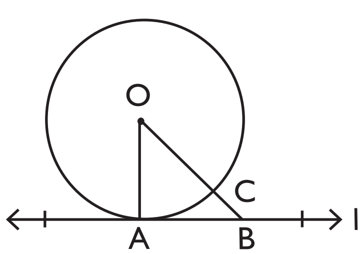
Also OA = OT [Radius]

In ΔOAN

$OA^2 = AN^2 + ON^2$

∴ $ON^2 + AN^2 = OT^2$

Hence proved.

25. 

Given: A circle C (0, r) and a tangent l at point A.

To prove: OA ⊥ l

Construction: Take a point B, other than A. On the tangent l join OB. Suppose OB meets the circle at C.

Proof: We know that, among all line segment joining the point O to a point on l, the perpendicular is shortest to l.

OA = OC (Radius of the same circle)

Now, OB = OC + BC

∴ OB > OC

⇒ OB > OA

⇒ OA > OB

B is an arbitrary point on the tangent l. Thus, OA is shorter than any other line segment joining O to any point on l.

Here, OA ⊥ l

26. We know that ∠ADO = 90° (Since O'D is perpendicular to AC)

∠ACO = 90° (OC (radius) perpendicular to AC (tangent))

In triangles ADO' and ACO,

∠ADO = ∠ACO (each 90°)

∠DAO = ∠CAO (common)

by AA criterion, triangles ADO' and ACO are similar to each other.

$$\frac{AO'}{AO} = \frac{DO'}{CO}$$

(corresponding sides of similar triangles)

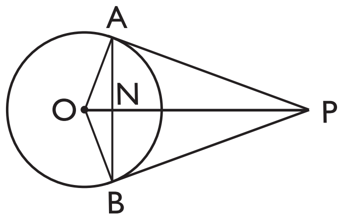
AO = AO' + O'X + OX

= 3AO' (since AO' = O'X = OX because radii of the two circles are equal)

$$\frac{AO'}{AO} = \frac{1}{3}$$

$$\frac{DO'}{CO} = \frac{AO'}{AO} = \frac{1}{3}$$

$$\frac{DO'}{CO} = \frac{1}{3}$$

27. 

$$OA = 10 \text{ cm}$$

$ON \perp AB$

$$AN = NB = \frac{16}{2} = 8 \text{ cm}$$

Pythagoras Theorem

In ΔONA,

$$ON^2 + NA^2 = OA^2$$
$$ON^2 = OA^2 - NA^2$$
$$ON^2 = 10^2 - 8^2$$
$$ON^2 = 36 \text{ cm}$$
$$ON = 6 \text{ cm}$$
$$\tan\angle AON = \frac{AN}{ON} = \frac{8}{6} = \frac{4}{3}$$

ΔOAP

$$\tan\angle AON = \frac{PA}{OA}$$
$$\frac{4}{3} = \frac{PA}{10}$$
$$\Rightarrow \quad PA = \frac{40}{3} \text{ cm}$$

28. Given $\angle RPQ = 30°$ and PR and PQ are tangents drawn from P to the same circle.

Hence PR = PQ [Since tangents drawn from an external point to a circle are equal in length]

$\therefore$ $\angle PRQ = \angle PQR$ [Angles opposite to equal sides are equal in a Δ]

In ΔPQR

$$\angle RQP + \angle QRP + \angle RPQ = 180°$$
[Angle sum property of a Δ]
$$2\angle RQP + 30° = 180°$$
$$2\angle RQP = 150°$$
$$\angle RQP = 75°$$

So $\angle RQP = \angle QRP = 75°$

$\angle RQP = \angle RSQ = 75°$

[By Alternate Segment Theorem]

Given, RS || PQ

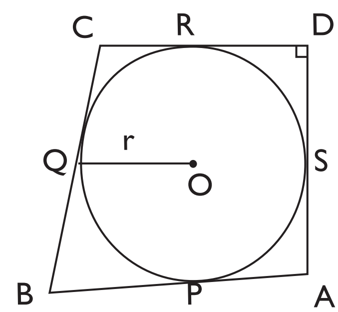
$\therefore$ $\angle RQP = \angle SRQ = 75°$ [Alternate angles]

$\angle RSQ = \angle SRQ = 75°$

$\therefore$ QRS is also an isosceles triangle.

[Since sides opposite to equal angles of a triangle are equal.]

$$\angle RSQ + \angle SRQ + \angle RQS = 180°$$
[Angle sum property of a triangle]
$$75° + 75° + \angle RQS = 180°$$
$$150° + \angle RQS = 180°$$
$$\therefore \quad \angle RQS = 30°$$

29. 

Given: ABCD is a quadrilateral such that $\angle D = 90°$.

BC = 38 cm, CD = 25 cm and BP = 27 cm

BP = BQ = 27 cm

[Tangents from an external point]

BC = 38

$\Rightarrow$ BQ + QC = 38

$\Rightarrow$ 27 + QC = 38

$\Rightarrow$ QC = 38 – 27

$\Rightarrow$ QC = 11 cm

$\therefore$ QC = 11 cm = CR

[Tangents from an external point]

CD = 25 cm

CR + RD = 25

$\Rightarrow$ 11 + RD = 25

$\Rightarrow$ RD = 25 – 11

$\Rightarrow$ RD = 14 cm

Also,

RD = DS = 14 cm

$\therefore$ OR and OS are radii of the circle.

From tangents R and S, $\angle ORD = \angle OSD = 90°$

Now, ORDS is a square.

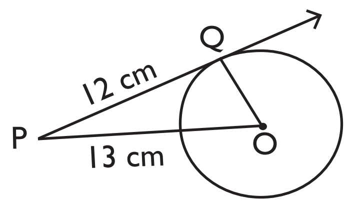
$\therefore \quad OR = DS = 14$ cm

Thus, radius , $r = 14$ cm

WORKSHEET - 2

SECTION-A

1.



As we know that tangent to a circle is perpendicular to the radius through the point of contact.

$\therefore \quad OQ \perp PQ$

$\therefore \quad \angle OQP = 90°$

So, In ΔOQP,

$OP^2 = OQ^2 + PQ^2$

$13^2 = OQ^2 + 12^2$

$169 = OQ^2 + 144$

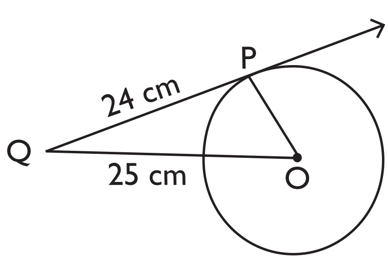
$OQ^2 = 169 - 144$

$= 25$

$\therefore \quad OQ = 5$ cm

So, radius of circle = 5 cm

2.



As we know that tangent to a circle is perpendicular to the radius through the point of contact.

$\therefore \quad OP \perp PQ$

i.e. $\angle OPQ = 90°$

In ΔOPQ,

$OQ^2 = OP^2 + PQ^2$

$25^2 = OP^2 + 24^2$

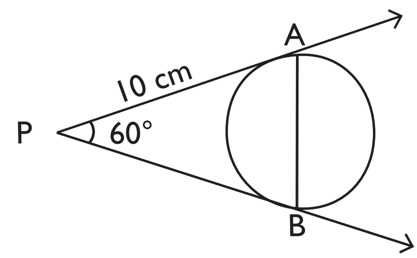
$625 = OP^2 + 576$

$OP^2 = 625 - 576$

$= 49$

$\therefore \quad OP = 7$ cm

3.



PB = PB

= 10 cm

(Length of tangents drawn from an external point to a circle are equal.)

$\Rightarrow \quad \angle PAB = \angle PBA$

(Angles opposite to equal sides are equal)

In ΔPBA,

$\angle P + \angle PAB + \angle PBA = 180°$

(Angle sum property)

$60° + \angle PAB + \angle PBA = 180°$

$\angle PAB + \angle PBA = 180° - 60°$

$= 120°$

$\Rightarrow \quad \angle PAB = \angle PBA = 60°$

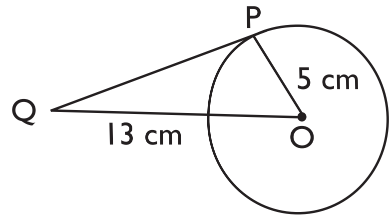
So, $\angle PAB = \angle PBA = \angle P = 60°$

$\Rightarrow \quad \Delta APB$ is equilateral

$\Rightarrow \quad AB = AP = 10$ cm

(Sides of equilateral triangle are equal.)

4.



As we know that tangent to a circle is perpendicular to the radius through the point of contact.

$\therefore \quad OP \perp PQ$

i.e. $\angle OPQ = 90°$

So, In ΔOPQ,

$OQ^2 = OP^2 + PQ^2$

$13^2 = 5^2 + PQ^2$

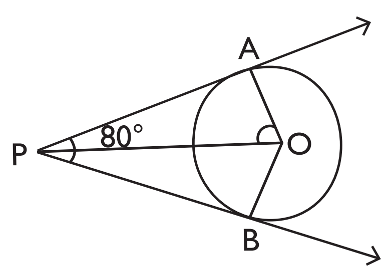
$169 = 25 + PQ^2$

$\therefore \quad PQ^2 = 169 - 25$

$= 144$

$\Rightarrow \quad PQ = 12$ cm

5.



In ΔPOA and ΔPOB,

PA = PB

(Length of tangents drawn from an external point to a circle are equal.)

OP = PO (Common)

OA = OB (Radii of same circle)

$\therefore \quad \Delta POA \cong \Delta POB$

(SSS congruence criteria)

$\therefore \quad \angle APO = \angle BPO$ (CPCT)

$= \frac{1}{2} \angle APB$

$= \frac{1}{2} (80°)$

$= 40°$

Also, $OA \perp AP$

i.e. $\angle OAP = 90°$

(As tangent is perpendicular to radius through point of contact.)

In ΔOAP,

$\angle OAP + \angle APO + \angle AOP = 180°$

(Angle sum property)

$\therefore \quad 90° + 40° + \angle AOP = 180°$

$\Rightarrow \quad 130° + \angle AOP = 180°$

$\Rightarrow \quad \angle AOP = 180° - 130°$

$= 50°$

6. PA = PB (Tangents drawn from same point are equal)

PA = PC + CA

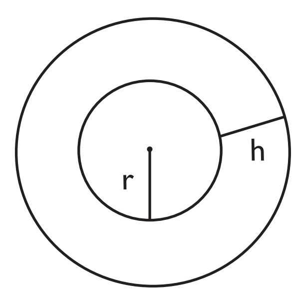
12 = PC + CA

CA = CQ (Tangents from same point are equal)

12 = PC + CQ

PC = 12 – CQ = 12 – 3 = 9 cm

7.



Radius of inner circle = r

Area of inner circle = πr^2

Radius of outer circle = r + h

Area of outer circle = $\pi (r + h)^2$

So, area of circular path

= area of outer circle

– area of inner circle

$= \pi (r + h)^2 - \pi r^2$

$= \pi (r^2 + h^2 + 2rh - r^2)$
$= \pi (h^2 + 2rh)$
$= \pi h (h + 2r)$
$= \pi h (2r + h)$

8. Area of sector $= \frac{\theta}{360} \pi r^2$

$$20\pi = \frac{\theta}{360} \pi r^2$$

$$20 = \frac{\theta}{360} r^2 \quad ...(i)$$

Length of arc is $\frac{\theta}{360} 2\pi r$

$$\frac{\theta}{360} 2\pi r = 5\pi$$

$$2\frac{\theta r}{360} = 5$$

$$\Rightarrow \quad \frac{r\theta}{360} = \frac{5}{2} \quad ...(ii)$$

From (i) and (ii)

$$20 = \frac{r\theta}{360} \quad r$$

$$20 = \frac{5}{2} r \Rightarrow r = 8 \text{ cm}$$

9. A circle can have infinitely many tangents.

10. Remark: If AB and CD are two common tangents to the two circles of unequal radii then they will always intersect each other.

Given: Two circles with centre's O_1 and O_2. AB and CD are common tangents to the circles which intersect in P.

To prove: AB = CD

Proof:

AP = PC ...(i)

(Length of tangents drawn from an external point to the circle are equal)

PB = PD ...(ii)

(Length of tangents drawn from an external point to the circle are equal)

Adding (i) and (ii), we get

AP + PB = PC + PD

$\Rightarrow$ AB = CD

SECTION-B

11. $\angle ABQ = \frac{1}{2} \angle AOQ = \frac{1}{2} (58°) = 29°$

($\because$ Angle subtended by an arc at the centre is double the angle subtended by it on the remaining part of the circle.)

$\angle OAT = 90°$ [Radius is perpendicular to tangent at point of contact]

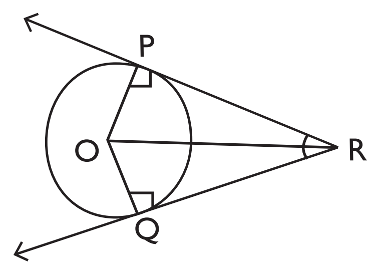
In ΔABT

$\angle BAT + \angle ABT + \angle BTA = 180$

$90° + 29° + \angle BTA = 180$

$\angle BTA = 61°$

$\angle BTA = \angle ATQ = 61°$

12. 

Construction: Join OP and OQ

PR and RQ are tangents to circle at points P and Q respectively.

$\Rightarrow$ $OP \perp PR$ and $OQ \perp QR$

(As tangent is perpendicular to radius through point of contact.)

In ΔOPR and ΔOQR

OP = OQ

(Radii of same circle)

OR = OR (Common)

$\angle OPR = \angle OQR = 90°$ (Proved above)

$\therefore$ $\Delta OPR \cong \Delta OQR$

(RHS congruence criteria)

$\Rightarrow$ $\angle ORP = \angle ORQ = \frac{1}{2} \angle PRQ$

$= \frac{1}{2} (120°)$

$= 60°$

In ΔPRO,

$\cos 60° = \frac{PR}{OR}$

$\frac{1}{2} = \frac{PR}{OR}$

$\Rightarrow \quad PR = \frac{1}{2} OR$ ...(i)

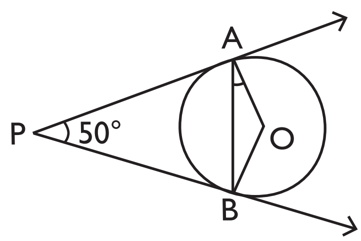
In ΔQRO

$RQ = \frac{1}{2} OR$ ...(ii)

On adding (i) and (ii), we get

$PR + RQ = \frac{1}{2} OR + \frac{1}{2} OR = RO$

13.



$\Rightarrow \quad PA = PB$

(Length of tangents drawn from an external point to a circle are equal.)

$\Rightarrow \quad \angle PAB = \angle PBA$ ...(i)

(Angles opposite to equal sides are equal.)

In ΔAPB,

$\angle P + \angle PAB + \angle PBA = 180°$

(Angle sum property)

$50° + \angle PAB + \angle PAB = 180°$ (From (i))

$2 \angle PAB = 130°$

$\therefore \quad \angle PAB = \angle PBA = 65°$

$OA \perp AP$

(As tangent is perpendicular to radius through point of contact.)

$\Rightarrow \quad \angle OAP = 90°$

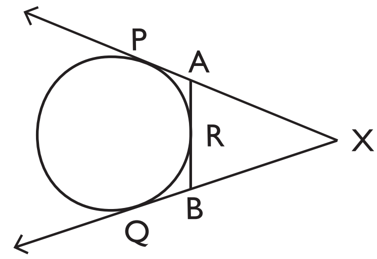
$\Rightarrow \quad \angle OAB + \angle PAB = 90°$

$\Rightarrow \quad \angle OAB + 65° = 90°$

$\Rightarrow \quad \angle OAB = 90° - 65°$

$= 25°$

14.



As we know that lengths of tangents drawn from an exterior point to a circle are equal.

$\therefore \quad XP = XQ,\ AP = AR$ and $BQ = BR$

$\Rightarrow \quad XA + AP = XB + BQ$

$\Rightarrow \quad XP + AR = XB + BR$

15. As we know that lengths of tangents drawn from an exterior point to a circle are equal.

$CE = CD = 9$ cm

$BF = BD = 6$ cm

$AE = AF = x$ cm

Also, $OE \perp AC$, $OD \perp BC$ and $OF \perp AB$

(As tangent is perpendicular to radius through point of contact.)

Area of $\Delta BOC = \frac{1}{2} \times BC \times OD$

$= \frac{1}{2} \times (9 + 6) \times 3$

$= \frac{1}{2} \times 15 \times 3$

$= \frac{45}{2}$ cm²

Area of $\Delta AOC = \frac{1}{2} \times AC \times OE$

$= \frac{1}{2} \times (9 + x) \times 3$

$$= \frac{3}{2}\ (9 + x)\ \text{cm}^2$$

$$\text{Area of } \Delta AOB = \frac{1}{2} \times AB \times OF$$

$$= \frac{1}{2} \times (x + 6) \times 3$$

$$= \frac{3}{2}\ (x + 6)\ \text{cm}^2$$

Area of ΔABC = area of ΔBOC

+ area of ΔAOC

+ area of ΔAOB

$$54 = \frac{45}{2} + \frac{1}{2}(9 + x) \times \frac{3}{2}(x + 6)$$

$$54 = \frac{45}{2} + \frac{27}{2} + \frac{18}{2} + \frac{3}{2}x + \frac{3}{2}x$$

$$54 = 45 + 3x$$

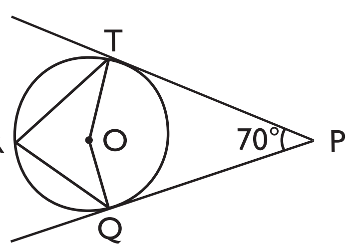
$$9 = 3x$$

$$\therefore \quad = 3$$

So, $AB = x + 6 = 3 + 6 = 9$ cm

$AC = x + 9 = 3 + 9 = 12$ cm

16.



Construction: Join O to T and O to Q.

As we know that tangent is perpendicular to the radius through the point of contact.

$\therefore$ $OT \perp PT$ and $OQ \perp PQ$

i.e. $\angle OTP = \angle OQP = 90°$

In quadrilateral TOQP

$\angle TOQ + \angle OQP + \angle QPT + \angle PTO = 360°$

(Angle sum property of quadrilateral.)

$\Rightarrow \angle TOQ + 90° + 70° + 90° = 360°$

$\Rightarrow \angle TOQ + 250° = 360°$

$\Rightarrow \angle TOQ = 360° - 250°$

$= 110°$

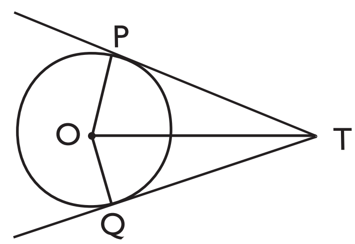
$$\angle TRQ = \frac{1}{2}\ \angle TOQ$$

(Angle subtended an arc at the centre is double the angle subtended by it on the remaining part of the circle.)

$$= \frac{1}{2}\ (110)$$

$$= 55°$$

17.



Construction: Join OT

$OP \perp PT$

(As tangent is perpendicular to the radius through point of contact)

i.e. $\angle OPT = 90°$

In ΔOPT,

$OT^2 = OP^2 + PT^2$

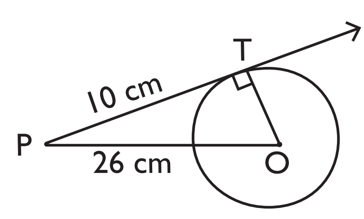
$= 5^2 + 8^2$

$= 25 + 64$

$= 89$

$\therefore$ $OT = \sqrt{89}$ cm

18.



$OT \perp PT$ i.e. $\angle OTP = 90°$

(Tangent is perpendicular to radius through point of contact.)

In ΔOTP,

$OP^2 = OT^2 + PT^2$

(By Pythagoras theorem)

$26^2 = OT^2 + 10^2$

$676 = OT^2 + 100$

$OT^2 = 576$

$OT = 24$ cm

$\therefore$ Radius of the circle = 24 cm

19. AE and CE are tangents to the circle with center O,

$\therefore$ AE = CE (i)

($\because$ Lengths of tangents drawn from an exterior point to a circle are equal.)

Also, DE and BE are tangents to the circle with centre O_2.

$\therefore$ BE = DE (ii)

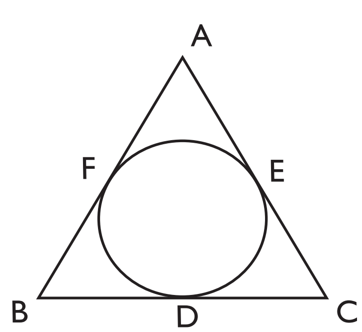
($\because$ Lengths of tangents drawn from an exterior point to a circle are equal.)

On adding (i) and (ii), we get

AE + BE = CE + DE

$\therefore$ AB = CD

20.



AF = AE ...(i)

(Lenghts of tangents drawn from an exterior point to a circle are equal.)

Also, AB = AC ...(ii)

(Given)

On subtracting (i) from (ii), we get

AB – AF = AC – AE

BF = CE ...(iii)

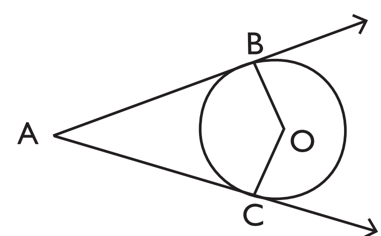
But BF = BD and CE = CD

(Lengths of tangents drawn from an exterior point to a circle are equal.)

$\therefore$ BD = CD

SECTION-C

21.



AB and AC are tangents to a circle.

$OB \perp AB$ and $OC \perp AC$

(Tangent is perpendicular to the radius through the point of contact.)

i.e. $\angle OBA = \angle OCA = 90°$ ...(i)

In quadrilateral ABOC,

$\angle A + \angle B + \angle O + \angle C = 360°$

(Angle sum property of quadrilateral)

$\Rightarrow \angle A + \angle O + \angle B + \angle C = 360°$

$\Rightarrow \angle A + \angle O + 90° + 90° = 360°$

From (i)

$\Rightarrow \angle A + \angle O + 180° = 360°$

$\Rightarrow \angle A + \angle O = 360° - 180°$

$= 180°$

22. BP and BQ are tangents to the circle

$\therefore$ BP = BQ ...(i)

(Lenghts of tangents drawn from an exterior point to a circle are equal.)

Also, CP = CR ...(ii)

(Lenghts of tangents drawn from an exterior point to a circle are equal.)

Consider

Perimeter of ΔABC = AB + BC + AC

= AB + (BP + CP) + AC

= AB + (BQ + CR) + AC

From (i) and (ii),

= AQ + AR

= AQ + AQ

= 2AQ

$\because$ AQ = AR as lengths of tangents drawn from an exterior point to a circle are equal.

$\therefore$ AQ $= \frac{1}{2}$ (Perimeter of ΔABC)

23. Consider ΔOAP and ΔOBP,

AP = BP

(Lenghts of tangents drawn from an exterior point to a circle are equal.)

OP = OP (Common)

AO = BO (Radii of same circle)

$\therefore$ $\Delta OAP \cong \Delta OBP$

(SSS congruence criteria)

$\Rightarrow$ $\angle$APO = $\angle$BPO (CPCT)

Now, Consider ΔACP and ΔBCP,

AP = BP

PC = CP (Common)

$\angle$APC = $\angle$BPC (Proved above)

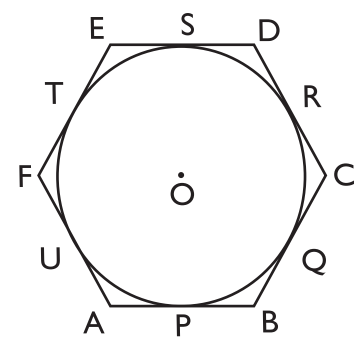
$\therefore$ $\Delta APC \cong \Delta BCP$ (SSS congruence criteria)

$\Rightarrow$ AC = BC and $\angle$ACP = $\angle$BCP = 90°

(CPCT)

So, OP is the perpendicular bisector of AB.

24.



(As we know that lengths of tangents drawn from an external point to a circle are equal.)

$\therefore$ AP = AU

BP = BQ

CQ = CR

DS = DR

ES = ET

FU = FT

Consider

AB + CD + EF

= (AP + BP) + (CR + DR) + (ET + TF)

= (AU + BQ) + (CQ + DS) + (ES + UF)

= (BQ + QC) + (DS + ES) + (AU + FU)

= BC + DE + AF

25. PR and CR are tangents to circle with centre A

$\therefore$ PR = CR ...(i)

(Lenghts of tangents drawn from an exterior point to a circle are equal.)

QR and CR are tangent to circle with center B.

$\therefore$ QR = CR (ii)

(Lenghts of tangents drawn from an exterior point to a circle are equal.)

From (i) and (ii), we get

PR = QR

$\therefore$ RC bisects PQ

Now, In ΔOCA and ΔOCB

$\angle OCA = \angle OCB = 90°$

OA = OB (Radii of the larger circle)

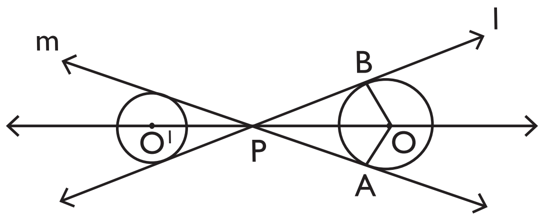
OC = OC (Common)

By RHS congruency

$\Delta OCA \cong \Delta OCB$

$\therefore$ CA = CB (By CPCT)

26.



In ΔOPA and ΔOBP,

OA = OB (Radii of circle)

PA = PB

(Lengths of tangents from an external point to a circle are equal.)

OP = PO (Common)

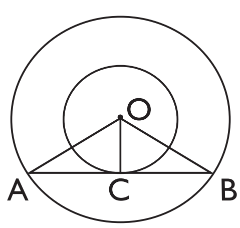
$\therefore$ $\Delta AOP \cong \Delta OBP$ (SSS congruence criteria)

$\Rightarrow$ $\angle APO \cong \angle BPO$ (CPCT)

$\Rightarrow$ OP is the bisector of $\angle APB$

$\therefore$ O lies on the bisector of the angle between l and m.

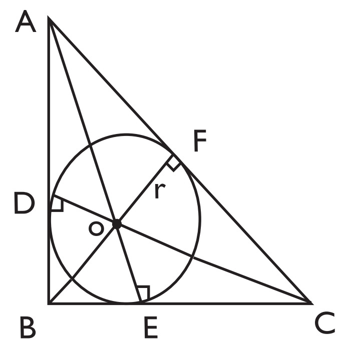
27.



We know that the radius and tangent are perpendicular at their point of contact.

$\therefore \angle OCA = \angle OCB = 90°$

28.



Construction: Join AO. Extend OC to OD, OP to BF.

In ΔABC, right angles at B

$AC^2 = AB^2 + BC^2$

$= 24^2 + 10^2$

$= 576 + 100$

$= 676$

$\therefore$ AC = 26 cm

Area of $\Delta ABC = \frac{1}{2} \times BC \times AB$

$= \frac{1}{2} \times 10 \times 24$

$= 120$ cm²

Also, $OF \perp AC$, $OE \perp BC$ and $OD \perp AB$

($\because$ Tangent is perpendicular to the radius through point of contact.)

$$\text{Area of } \Delta BOC = \frac{1}{2} \times BC \times OE$$

$$= \frac{1}{2} \times 10 \times r$$

$$= 5r$$

$$\text{Area of } \Delta AOC = \frac{1}{2} \times 26 \times r$$

$$= 13r$$

$$\text{Area of } \Delta AOB = \frac{1}{2} \times AB \times OD$$

$$= \frac{1}{2} \times 24 \times r$$

$$= 12r$$

Area of ΔABC = area of ΔBOC

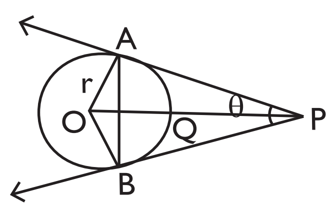
+ area of ΔAOC

+ area of ΔAOB

$\therefore \quad 120 = 5r + 13r + 12\, r$

$30r = 120$

$r = 4$ cm

29. 

AP is tangent to the circle

$\therefore \quad OA \perp AP$

i.e. $\angle OAP = 90°$

(Tangent is perpendicular to radius through point of contact.)

In ΔOAP,

$$\sin\theta = \frac{OA}{OP} = \frac{r}{2r}$$

$$= \frac{1}{2}$$

(OP = Diameter = 2r)

$\therefore \quad \theta = 30°$

$\Rightarrow \quad \angle OPA = 30°$

Similarly, $\angle OPB = 30°$

$\therefore \quad \angle APB = 30° + 30°$

$= 60°$

Also, AP = BP

(Lengths of tangent drawn from an external point to a circle are equal.)

So, In ΔAPB,

$\angle PAB = \angle PBA$ ...(i)

(Angles opposite to equal sides are equal.)

In ΔAPB,

$\Rightarrow \quad \angle PAB + \angle PBA + \angle APB = 180°$

(Angle sum property)

$\Rightarrow \quad \angle PAB + \angle PAB + 60° = 180°$

$\Rightarrow \quad 2\,\angle PAB = 180° - 60°$

$= 120°$

$\Rightarrow \quad \angle PAB = \frac{120}{2}$

$= 60°$

So, $\angle PAB = \angle PBA = \angle APB = 60°$

$\Rightarrow \quad \Delta APB$ is equilateral.

30. As we know that lengths of tangents drawn from an external point to a circle are equal,

$\therefore \quad PD = PF, \quad RF = RE, \quad QD = QE$

Consider

Perimeter of ΔPQR

= PQ +QR + PR

= (PD + DQ) + (QE + ER) + (PF + FR)

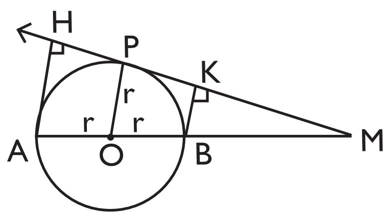
= (PD + PF) + (RF + RE) + (QD + QE)

= (PF +PF) + (RE + RE) + (QD + QD)

= 2 PF + 2 RE + 2 QD

= 2 (PF + ER + QD)

SECTION-D

31. 

PM is the tangent to circle.

$\therefore \quad \angle MPO = 90°$

(Tangent is perpendicular to radius through point of contact)

Let AH = x, BK = y, BM = z

Let r be the radius of circle

In ΔMKB and ΔMHA

$\angle M = \angle M$ (Common)

$\angle MKB = \angle MHA = 90°$

$\therefore \quad \Delta MKB \cong \Delta MHA$

(AA similarity criteria.)

$$\Rightarrow \quad = \frac{MK}{MH} = \frac{KB}{HA} = \frac{MB}{MA}$$

(Corresponding sides of similar triangles are proportional)

$$\Rightarrow \quad \frac{BK}{AH} = \frac{MB}{MA}$$

$$\Rightarrow \quad \frac{y}{x} = \frac{z}{2r + z}$$

$$\Rightarrow \quad 2r\,y + y\,z + = x\,z$$

$$\Rightarrow \quad z = \frac{2ry}{x - y} \qquad ...(i)$$

Now, In ΔMKB and ΔMPO,

$\angle M = \angle M$ (Common)

$\angle MKB = \angle MPO = 90°$

$\therefore \quad \Delta MKB$ and ΔMPO,

(AA similarity criteria.)

$$\Rightarrow \quad \frac{MK}{MP} = \frac{BK}{OP} = \frac{MB}{MO}$$

(Corresponding sides of similar triangles are proportional.)

$$\Rightarrow \quad \frac{BK}{PO} = \frac{BM}{OM}$$

$$\Rightarrow \quad \frac{y}{r} = \frac{z}{r + z}$$

$$\Rightarrow \quad yr + yz = rz$$

$$\Rightarrow \quad z = \frac{ry}{r - y} \qquad ...(ii)$$

From (i) and (ii), we get

$$\frac{2ry}{x - y} = \frac{ry}{r - y}$$

$$\Rightarrow \quad \frac{2}{x - y} = \frac{1}{r - y}$$

$$\Rightarrow \quad 2r - 2y = x - y$$

$$\Rightarrow \quad x + y = 2r$$

$$\Rightarrow \quad AH + BK = AB \qquad (\because AB = 2r)$$

32. Consider ΔOEA and ΔOEP

OA = OP (Radii of same circle)

OE = OE (Common)

AE = PE (OE bisects AP)

$\therefore \quad \Delta OEA \cong \Delta OEP$

(SSS congruence criteria)

$\Rightarrow \quad \angle OEA = \angle OEP$ (CPCT)

$\therefore \quad \angle OEA = \Delta OEP = 90° \qquad ...(i)$

($\angle OEA$ and $\angle OEP$ are linear pair)

Also, $AB \perp BC$ as BC is a tangent to the circle

(Tangent is perpendiculer to the radius through the point of contact.)

$\Rightarrow \quad \angle ABC = 90° \qquad ...(ii)$

Now, In ΔAEO and ΔABC

$\angle EAO = \angle BAC$ (Common)

$\angle AEO = \angle ABC$

$= 90°$ (From (i) and (ii))

$\Rightarrow \Delta AEO \sim \Delta ABC$

(By SS Similarity criteria)

33. Given: d_1, d_2 ($d_2 > d_1$) be the diameters of two concentric circles and C be the length of a chord of a circle which is tangent to the circle.

To prove: $d_2 = d_1^2 + c^2$

Now,

$$OQ = \frac{d_2}{2}, OR = \frac{d_1}{2} \text{ and } PQ = c$$

Since PQ is tangent to the circle therefore OR is perpendicular to PQ

$$\Rightarrow QR = \frac{PQ}{2} = \frac{c}{2}$$

Using pythagorus theorem in triangle OQR

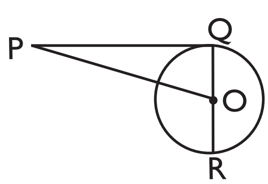
$OQ^2 = OR^2 + QR^2$

$$\Rightarrow \left(\frac{d_2}{2}\right)^2 = \left(\frac{d_1}{2}\right)^2 + \left(\frac{c}{2}\right)^2$$

$$\Rightarrow \frac{1}{4}(d_2)^2 = \frac{1}{4}(d_1)^2 + (c)^2$$

$$\Rightarrow d_2^2 = d_1^2 + c^2$$

Hence Proved.

34. 

$OQ : PQ = 3 : 4$

Let $OQ = 3k$, $PQ = 4k$

PQ is tangent to the circle

$\therefore OQ \perp PQ$

i.e. $\angle OQP = 90°$

(Tangent is perpendicular to radius through point of contact.)

In ΔOQP,

$OP^2 = OQ^2 + PQ^2$

(Pythagoras theorem)

$= (3k)^2 + (4k)^2$

$= 9k^2 + 16k^2$

$= 25k^2$

$\therefore OP = 5k$

Also, Perimeter of ΔPOQ = 60 cm

$\Rightarrow PO + OQ + PQ = 60$

$\Rightarrow 5k + 3k + 4k = 60$

$\Rightarrow 12k = 60$

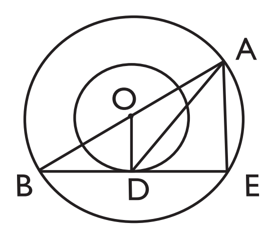
$$\Rightarrow k = \frac{60}{12} = 5$$

So, $PQ = 4k = 4 \times 5 = 20$ cm

$QR = 2(OQ) = 2(3k) = 6k$

$= 6 \times 5 = 30$ cm

$OP = 5k = 5 \times 5 = 25$ cm

35. 

BE is tangent to circle

$\therefore OD \perp BE$

i.e. $\angle ODB = 90°$

(Tangent is perpendicular to radius through point of contact)

$\Rightarrow BD = DE$

(As perpendicular from centre to the chord bisects the chord)

$\Rightarrow$ D is a midpoint of BE

Also, O being the centre is a midpoint of AB

So, By midpoint theorem,

$$OD \parallel AE \text{ and } OD = \frac{1}{2} AE$$

$$\therefore \quad AE = 2\,(OD)$$
$$= 2\,(8)$$
$$= 16 \text{ cm}$$

In ΔODB, $\angle ODB = 90°$

$$\therefore \quad OB^2 = OD^2 + BD^2$$

(By Pythagoras theorem)

$$\Rightarrow \quad 13^2 = 8^2 + BD^2$$
$$\Rightarrow \quad 169 = 64 + BD^2$$
$$\Rightarrow \quad BD^2 = 169 - 64$$
$$\Rightarrow \quad BD^2 = 105$$
$$\Rightarrow \quad BD = \sqrt{105} \text{ cm}$$
$$\Rightarrow \quad DE = \sqrt{105} \text{ cm}$$

($\because$ BD = DE)

In ΔAED, $\angle AED = 90°$

$$\therefore \quad AD^2 = AE^2 + DE^2$$
$$= (16)^2 + (\sqrt{105})^2$$
$$= 256 + 105$$
$$= 361$$
$$\therefore \quad AD = 19 \text{ cm}$$

36. BD is tangent to the circle

$\therefore$ $OC \perp BD$

i.e. $\angle OCD = 90°$

(Tangent is perpendicular to radius through point of contact.)

$$\Rightarrow \quad \angle OCA + \angle ACD = 90° \quad \text{...(i)}$$

Now, OA = OC

(Being radii of same circle.)

$\therefore$ In ΔAOC,

$$\angle OCA = \angle OAC$$

(Angles opposite to equal sides are equal.)

$$\Rightarrow \quad \angle OCA = \angle BAC \quad \text{...(ii)}$$

From (i) and (ii), we get

$$\angle BAC + \angle ACD = 90°$$

CASE STUDY-1

(i) (c) $\angle BAC + \angle COB + \angle ABU + \angle ACU = 360°$

[sum of all angles of quadrilateral is 360°]

$$40° + \angle BOC + 90° + 90° = 360°$$
$$\angle BOC = 140°$$

(ii) (b) $\angle BAO + \angle BOA + \angle ABO = 180°$

[sum of all angles of triangle is 180°]

$$40° + \angle BOA + 90° = 180°$$
$$\angle BOA = 50°$$

In ΔABO and ΔAOC

AC = AB [Tangents drawn from same point to a circle are equal in length]

OC = OB [radius]

AO = AO [common]

$\therefore$ $\Delta ABO \cong \Delta AOC$ by SSS rule.

Hence $\angle BOA = \angle COA$ [CPCT]

$$\therefore \quad \angle COA = 50°$$

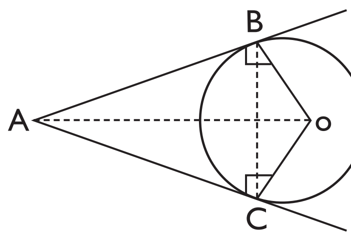
$$\angle BOC = \angle BOA + \angle COA$$
$$= 2\,\angle BOA$$
$$= 100°$$

(iii) (d) $\angle BAC + \angle COB + \angle ABO + \angle OCA = 360°$

[sum of all angles of quadrilateral is 360°]

$50° + \angle COB + 90° + 90° = 360$

$\angle COB = 130°$



As $OB = OC$ (radius)

$\therefore \angle OBC = \angle CBO$

[Angles opposite to equal sides are equal]

$2\angle OBC + \angle COB = 180°$

[Angle sum property of triangle]

$2\angle OBC + 130° = 180°$

$2\angle OBC = 50°$

$\angle OBC = 25°$

(iv) (a) In ΔAOB, $\angle B = 90°$

[Radius is $\perp$ to tangent at the point of contact]

$\therefore \Delta AOB$ is right triangle.

$AO^2 = AB^2 + OB^2$

$25^2 = 24^2 + OB^2$

$OB^2 = 49$

$OB = 7$ m

(v) (b) $\cos 30° = \frac{AB}{OA}$

$\frac{\sqrt{3}}{2} = \frac{AB}{4m}$

$AB = 2\sqrt{3}m$

CASE STUDY-2

(i) (c) $\angle PAO + \angle AOB + \angle OBP + \angle BPA = 360°$

$\angle PAO = \angle OBP = 90°$

$\therefore \angle AOB = 360° - (90° + 90° + 60°)$

$= 120°$

In OAB

$OA = OB$ [radius of circle]

$\therefore \angle OAB = \angle OBA$

[Angles opposite to equal sides are equal]

$\angle OAB + \angle OBA + \angle AOB = 180°$

$2\angle OAB = 180° - \angle AOB$

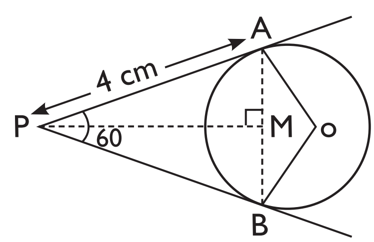
$= 180° - 120°$

$\angle OAB = 30°$

As $\angle OAB = \angle OBA$

$\therefore \angle OBA = 30°$

Draw a perpendicular PM on AB



$\angle MAP + \angle OAM = \angle PAO = 90°$ [tangent is $\perp^r$ to radius at point of contact]

$\angle OAM = \angle OAB = 30°$

$\therefore \angle MAP = 90° - 30° = 60°$

$\angle MAP = \angle BAP = 60°$

$\angle BAP + \angle APB + \angle PBA = 180°$

[sum of all angles of triangle is 180°]

$\angle PBA + 60° + 60° = 180°$

$\angle PBA = 60°$

Hence, ΔPAB is an equilateral triangle and thus $AB = BP = PA = 4$ cm

(ii) (a) As $\angle PAB = \angle BPA = \angle ABP = 60°$, hence ΔPAB is an equilateral triangle.

(iii) (b) Perimeter of ΔPAB sum of all sides

$= 4 \text{ cm} + 4 \text{ cm} + 4 \text{ cm}$

$= 12 \text{ cm}$

(iv) (a) Area of equilateral triangle is $\frac{\sqrt{3}}{4}a^2$, where a is side of triangle.

Area $= \frac{\sqrt{3}}{4}(4)^2$

$= 4\sqrt{3}$

(v) (c) As $\angle PAO = 90°$ (tangent is $\perp^r$ to radius)

and

$\angle PAB = 60°$ (ΔPAB is equilateral triangle)

Also

$\angle PAO = \angle PAB + \angle OAB$

$\angle OAB = \angle PAO - \angle PAB$

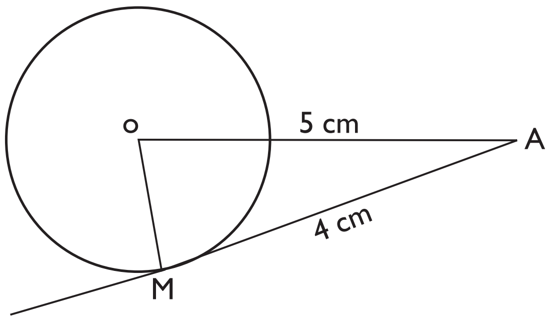
$= 90° - 60°$

$= 30°$

Chapter 11 Constructions

Multiple Choice Questions

1. (c)

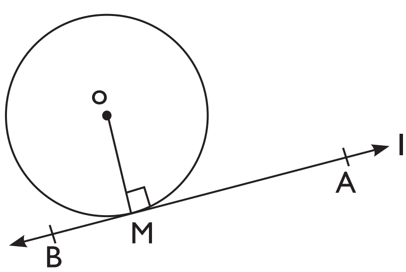


$\therefore \Delta OMA \angle M = 90°$

By Pythagoras Theorem,

$$OM^2 = OA^2 - AM^2$$
$$= 5^2 - 4^2$$
$$\therefore \quad OM^2 = 9 = 3^2$$
$$\therefore \quad OM = r = 3 \text{ cm}$$

2. (c)

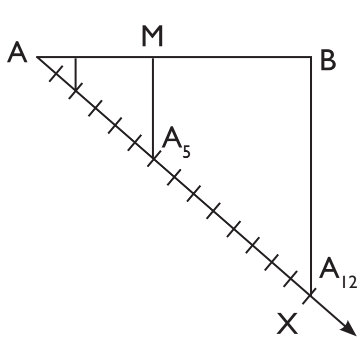


Line AB is a tangent of circle with centre O.

$\therefore OM \perp AR$

$\therefore \angle OMA = 90°$

3. (d)



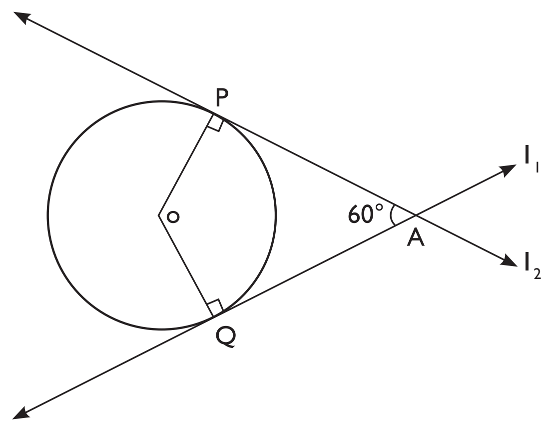
For 5 : 7

Given, $\angle BAX \angle 90°$

$\therefore A_5M \parallel BA_{12}$

$\therefore$ Total equal arc lengths are (5 + 7) = 12

4. (d)

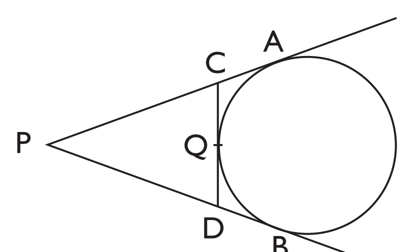


Given, $\angle PAQ = 60°$

Where AP and AQ are two tangents of the circle with centre O.

$$\therefore \quad \angle POQ = 360° - (90° + 90° + 60°)$$
$$= 360° - (240°)$$
$$\angle POQ = 120°$$

5. (a)



Given, PB = 12 cm, CQ = 3 cm

PA and PB are two tangents of the circle with centre O and CD is a third tangent.

We know that the two tangents draw to a circle from an external point are equal.

$\therefore$ PA = PB = 12 cm

Same, CQ = CA = 3 cm

$\therefore$ P – C – A,

$\therefore$ PA = PC + CA

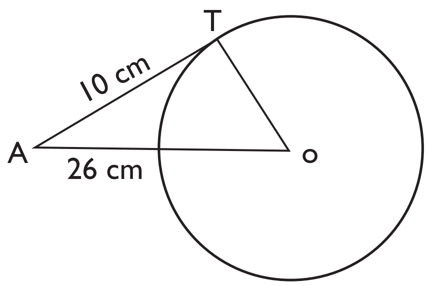
$\therefore$ 12 = PC + 3

$\therefore$ 12 – 3 = PC = 9 cm

WORKSHEET - 1

SECTION-A

1.



Since the tangent to a circle is perpendicular to the radius through the point of contact.

$\therefore \angle OTA = 90°$

In right ΔOTA, we have

$OA^2 = OT^2 + AT^2$

$\Rightarrow OT^2 = 676 - 100$

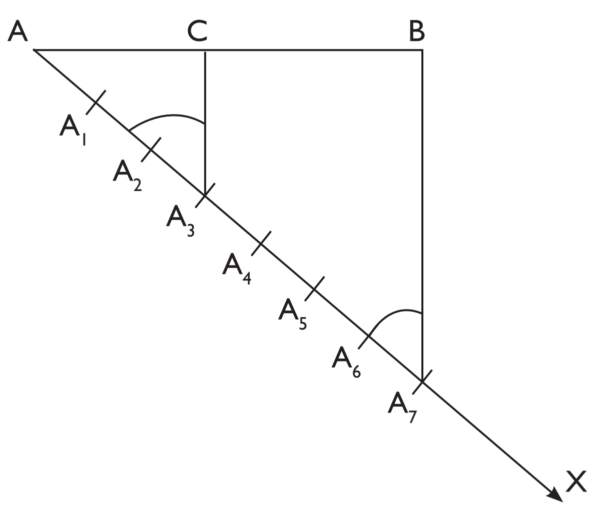
$\Rightarrow 576$

$\Rightarrow (24)2$

$\Rightarrow OT = 24$

Hence the radius of the circle is 24 cm.

2.



(i) Draw AB = 6 cm.

(ii) Draw a ray AX making an acute $\angle BAX$.

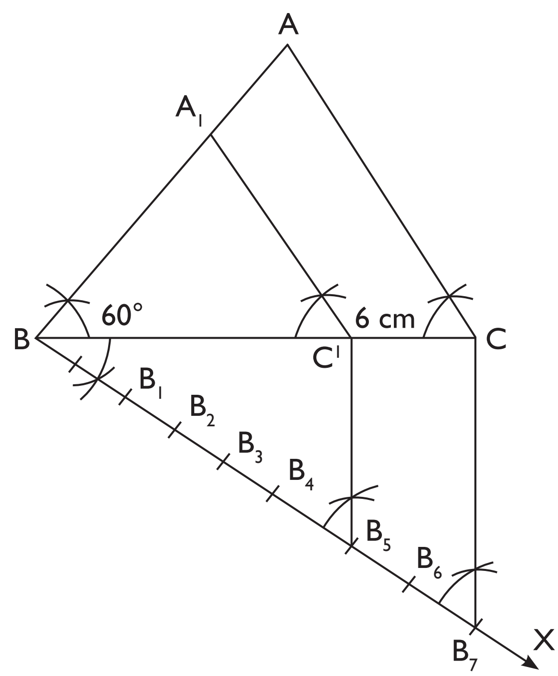
(iii) Along AX, mark point $A_1, A_2, A_3 ..., A_7$. Such that $AA_1 = A_1A_2 ... = A_6A_7$.

(iv) Join A_7B.

(v) Through A_3 draw a line $A_3C \parallel A_7B$ intersecting AB at C.

Thus, points C so obtained is the required point which divides internally in the ratio 3:4.

3.



Given, BC = 6 cm, AB = 5 cm, $\angle ABC = 60°$ and $\frac{5}{7}$ of the corresponding sides of the triangle ABC.

Steps of construction:

(i) BC = 6 cm is drawn

(ii) At point B, AB = 5 cm is drawn angle $\angle ABC = 60°$ with BC.

(iii) AC is joined to form ΔABC.

(iv) A ray BX is drawn making an acute angle with BC opposite to vertex A.

(v) 7 Points $B_1, B_2, B_3 ..., B_7$ at equal distance are marked on BX.

(vi) B_5 joined with C^1 to form B_5C^1.

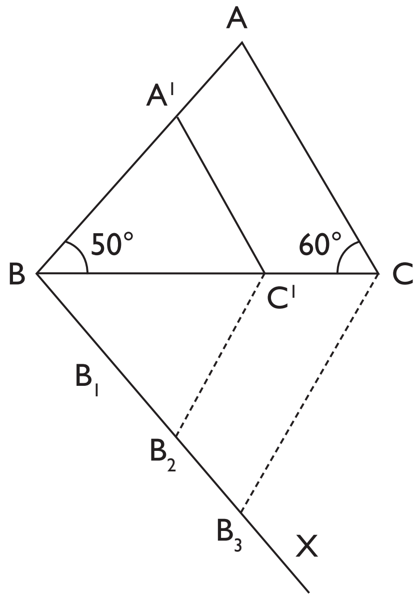
(vii) C^IA^I is drawn parallel to CA.

Thus A^IBC^I is the required triangle.

4. Given that

Construct triangle of given data, BC = 6 cm, ∠B = 50° and ∠C = 60° and then a triangle similar to it whose sides are (2/3)rd of the corresponding sides of ΔABC.

We follow the following steps to construct the given



Steps of construction

(i) First of all we draw a line segment BC = 60°.

(ii) With B as centre draw an angle ∠B = 50°.

(iii) With C as centre draw an angle ∠C = 60° which intersecting the line drawn in step ii at A.

(iv) Join AB and AC to obtain ΔABC.

(v) Below BC, makes an acute angle ∠CBX = 60°.

(vi) Along BX, mark off three points B_1, B_2 and B_3 such that $BB_1 = B_1B_2 = B_2B_3$.

(vii) Join B_3C.

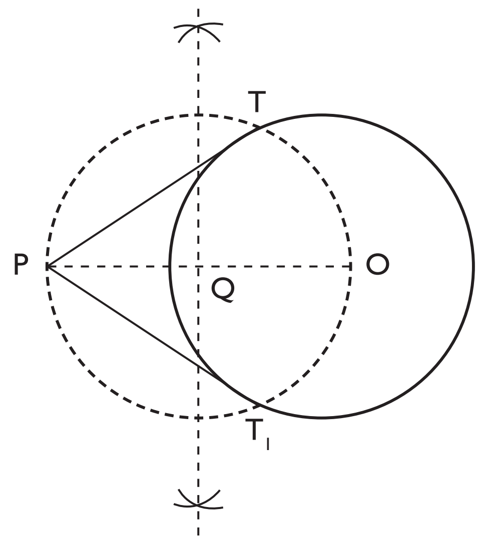
(viii) Since we have to construct a triangle each of whose sides is two-third of the corresponding sides of ΔABC.

So, we take two parts out of three equal parts on BX from point B_2 draw $B_2C \parallel B_3C$ and meeting BC at C.

(ix) From C^I draw $C^IA^I \parallel AC$ and meeting AB at A^I.

Thus, ΔABC is the required triangle, each of whose side is two third of the corresponding side of ΔABC.

5.



(i) Take a point O in the plane of the paper and draw a circle of radius 3 cm.

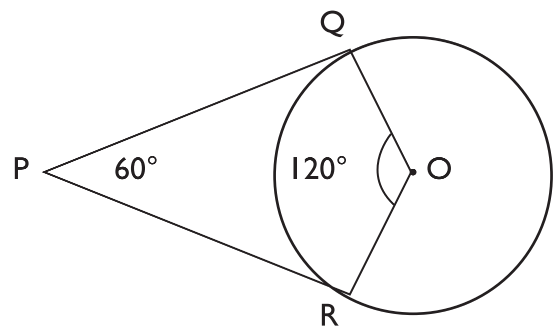
(ii) Mark a point P at a distance of 5.5 cm from the centre O and join OP.

(iii) Draw the right bisector of OP, intersection OP at Q.

(iv) Taking Q as centre and OQ = PQ as radius, draw a circle to intersect the given circle at T and T^I.

(v) Join PT and PT^I to get the required tangents.

6.



Given angle between tangents is 60°
i.e. ∠QPR = 60°

Since Angle at centre is double the angle between tangents

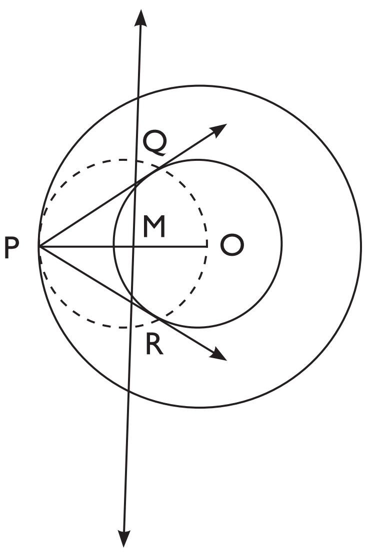
$\therefore \angle QPR = 2 \times 60° = 120°$

So, we need to draw $\angle QPR = 120°$

∴ We draw a radius, then second radius at 120° from first.

SECTION-B

7.



Steps of construction

(i) Draw a concentric circle two with circles radii are 3 cm and 5 cm.

(ii) Let P is a point on the circumference of center circle. Joined O to P, then OP formed.

(iii) Perpendicular bisector of OP. M is a mid point of the OP.

(iv) Draw a circle M in a centre and OM is a radius. This circle is intersected with three circles in Q and P points.

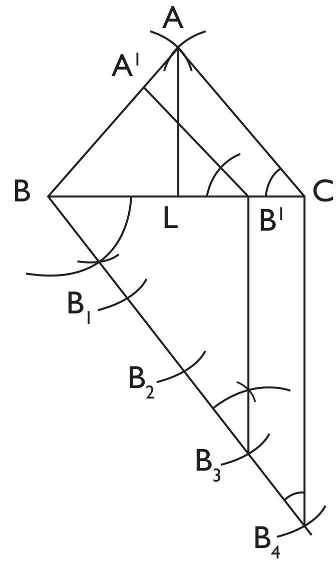
(v) Joined P to Q and P to R.

Thus, PQ and PR are required two tangents. OQ = 3 cm and OP = 5 cm,

In ΔOPQ,

$$PQ^2 = OP^2 - OQ^2$$
$$= (5)^2 - (3)^2$$
$$= 25 - 9$$
$$PQ^2 = 16 - 4$$
$$\therefore PQ = 4 \quad ...(i)$$

8.



(i) Make an line BC 5.5 cm

(ii) Take its bisector and now cut it 3 cm.

(iii) Join BA and CA.

(iv) Now from B make an circle ($\angle 60°$) angle.

(v) Extend that line.

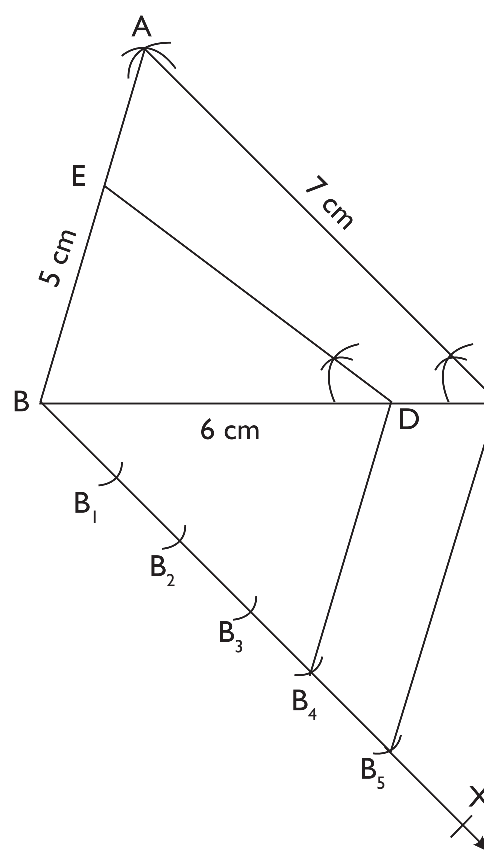
(vi) Cut 4 arcs.

(vii) From B_3 make a parallel line to CB_4.

(viii) Do same formation inside the triangle.

(ix) Your triangle is ready.

9.



Steps of construction

(i) Draw a line segment BC = 6 cm

(ii) With B as centre and radius equal to 5 cm, draw an arc.

(iii) With C as centre and radius equal to 7 cm, draw an arc.

(iv) Mark the point where the two arcs intersect as A.

Join AB and AC.

Thus, ΔABC is obtained.

(v) Below BC, make an acute $\angle CBX$

(vi) Along BX, mark off five points B_1, B_2, B_3, B_4, B_5 such that $B B_1 = B_1 B_2 = B_2 B_3 = B_3 B_4 = B_4 B_5$.

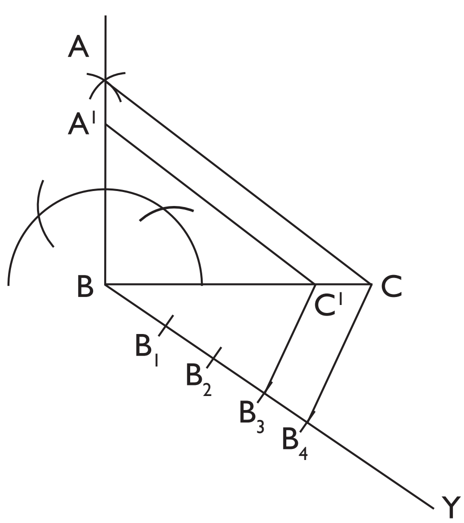
(vii) Join B_5C.

(viii) From B_4, draw $B_4D \parallel B_5C$, meeting BC at D.

(ix) From D, draw DE || CA, meeting AB at E.

Then ΔEBD is the required triangle each of whose sides is $\frac{4}{5}$ of the corresponding side of ΔABC.

10.



Steps of construction

(i) Draw a line segment BC = 8 cm

(ii) Draw line segment BX making an angle of 90° at the point B of BC.

(iii) From B mark an arc on BX at a distance of 6 cm. Let it is A.

(iv) Join A to C.

(v) Making an acute angle draw a line segment BY from B.

(vi) Mark B_1, B_2, B_3, B_4 on BX such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.

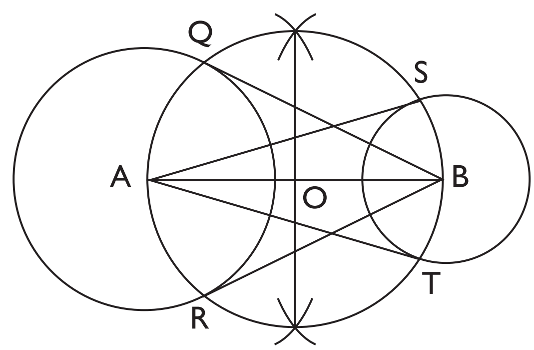
(vii) Join B_4 to C.

(viii) Draw a line segment $B_3C' \parallel$ to B_4C to meet BC at C'.

(ix) Draw line segment $CA' \parallel$ to CA to meet AB at A'.

$A'BC'$ is the required triangle.

11.



Steps of construction

(i) Take AB = 7 cm

(ii) With A as centre and 3 cm as radius, draw a circle.

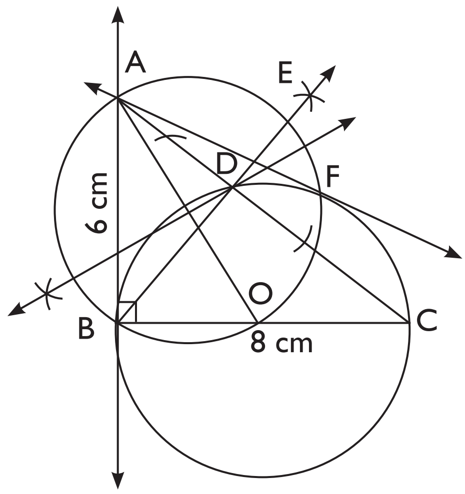
(iii) Similarly, with B as centre and 2 cm as radius, draw a circle.

(iv) Now, draw the perpendicular bisector of AB and mark the point of intersection O.

(v) With O as centre and OA as radius, draw a circle. Mark the 2 points where the circles with centre O and A meet as Q and R. Similarly, mark the points where the circles with centres O and B meet as S and T respectively.

(vi) Join BR and BQ as well as AS and AT. Now, BR, BQ, AS and AT are the required tangents.

12.



Steps of construction:

- Draw BC = 8 cm
- Draw the perpendicular at B and cut BA = 6 cm on it. Join AC and right ΔABC is obtained.
- Draw BD perpendicular to AC.
- Since $\angle BDC = 90°$ and the circle has to pass through B, C and D, BC must be the diameter of this circle. So, take O as the midpoint of BC and with O as centre and OB as radius draw a circle that will pass through B, C and D.
- To draw tangents from A to the circle with centre O.
  - Join OA, and draw its perpendicular bisector to intersect OA at point E.
  - With E as centre and EA as radius draw a circle that intersects the previous circle at B and F.
  - Join AF.
  - Thus, AF and AB are the required tangents to the circle with centre O.

Proof:

$\angle ABO = \angle AFO = 90°$ (Angle in a semi-circle)

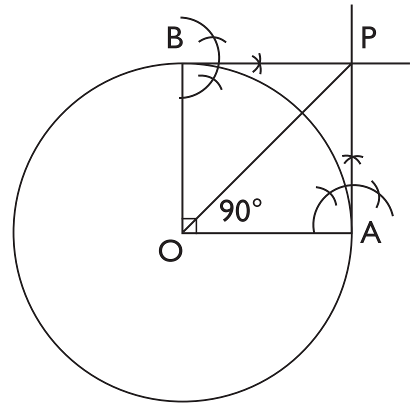
$\therefore$ AB $\perp$ OB and AF $\perp$ OF (We know that the line joining the centre of a circle to the tangent is always perpendicular)

Hence AB and AF are the tangents from A to the circle with centre O.

WORKSHEET - 2

SECTION-A

1.



Steps of construction

(i) Draw a circle of 3.5 cm radius with O as centre.

(ii) Take a point A on the circumference of the circle and join OA. Draw a perpendicular to OA at point A.

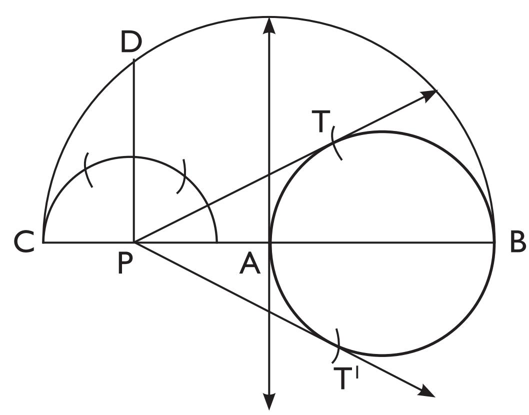
(iii) Draw a radius OB, making an angle of 90° with OA.

(iv) Draw a perpendicular to OB at point B. Let both the perpendicular intersect at point P.

(v) Join OP.

PA and PB are the required tangents, which make an angle of 45° with OP.

2.



Steps of construction

(i) Draw a circle of radius 4 cm.

(ii) Take a point P outside the circle and draw a secant PAB, intersecting the circle at A and B.

(iii) Produce AP to C such that AP = CP.

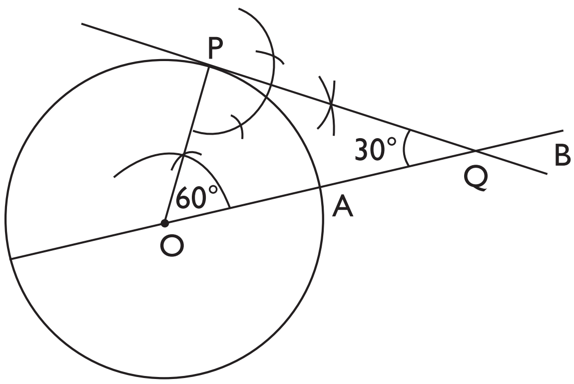
(iv) Draw a semi-circle with CB as diameter.

(v) Draw PD and CB intersecting the semicircle at D.

(vi) With P as centre and PD as radius draw arcs to intersect the given circle at T and T^{1}.

(vii) Join PT and PT^{1}. Then PT and PT^{1} are the required tangents.

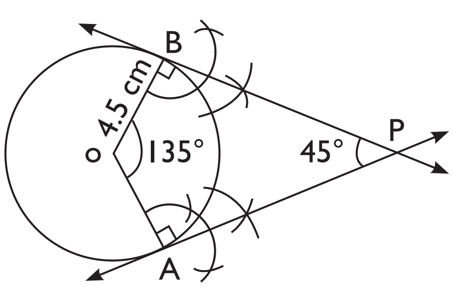
3.



Steps of construction:

1. Draw a circle with centre O and radius 6 cm.
2. Draw a radius OA of this circle and produce it to B.
3. Construct an angle ∠AOP equal to the complement of 30° i.e. equal to 60° [∴ ∠OPQ = 90°]
4. Draw perpendicular to OP at P which intersects OA produced at Q.
5. PQ is the required tangent such that ∠OQP = 30°.

4.



(i) Draw circle of radius 4.5 cm with centre O.

(ii) Take any points A on the circle. Join OA. Mark another point B on the circle such that ∠AOB = 135°, supplementary to the angle between the tangents.

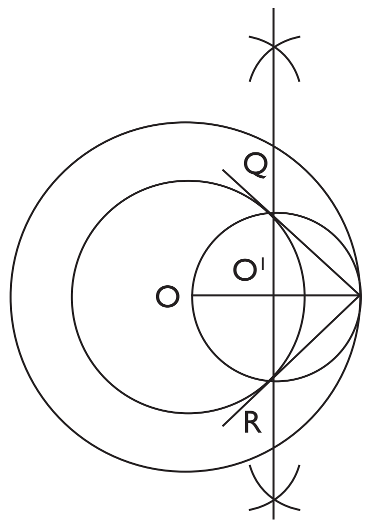
Since the angle between the tangents to be constructed is 45°.

∴ ∠AOB = 180° – 45° = 135°

(iii) Construct angles of 90° at A and B extend the lines so as to intersect at point P.

(iv) Thus AP and BP are the required tangents to the circle.

5.



Steps of construction

(i) Taking point O as a centre draw a circle of radius 4 cm.

(ii) Now taking O as centre draw a concentric circle of radius 6 cm.

(iii) Taking any point P on the outer circle join OP.

(iv) Draw a perpendicular bisector of OP.

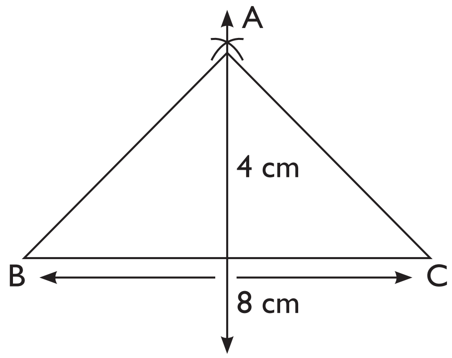
(v) Name the intersection of bisector and OP as O^{1}.

(vi) Now, draw a circle taking O^{1} as centre and O1P as radius.

(vii) Name the intersection point of two circles as R and Q.

(viii) Join PR and PQ. These are the required tangents.

6.



Steps of construction

(i) BC = 8 cm is drawn.

(ii) Perpendicular bisector of BC is drawn and it intersects BC at M.

(iii) At a distance of 4 cm a point A is marked on perpendicular bisector of BC.

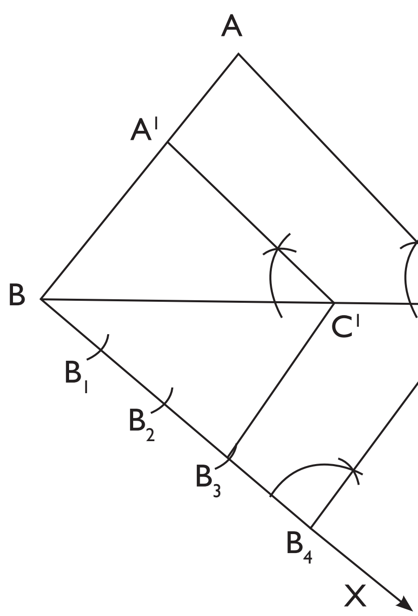
(iv) AB and AC are joined to form ΔABC.

(v) Ray BX is drawn making an acute angle with BC apposite to vertex A.

(vi) 4 points B_1, B_2, B_3 and B_4 are marked on BX.

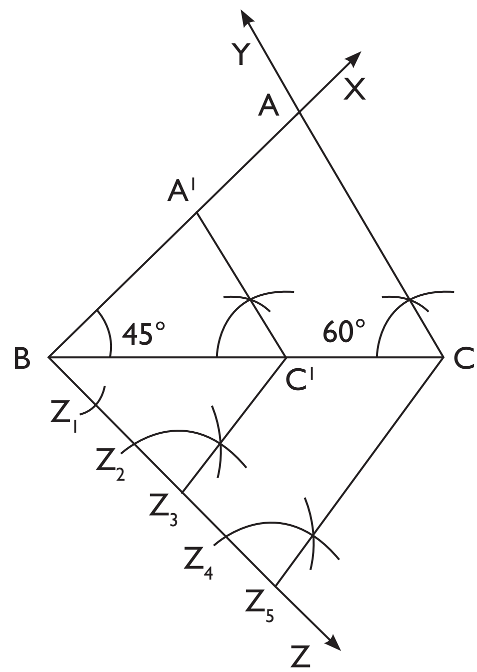
(vii) B_4 is joined with C to form B_4C.

(viii) B_4C^I is drawn parallel to B_4C and C^IA^I is drawn parallel to CA. Thus A^IBC^I is the required triangle formed.



SECTION-B

7.



Steps of construction

(i) Draw a line segment BC = 8 cm.

(ii) At B, draw ∠XBC = 45°.

(iii) At C, draw ∠YCB = 60°. Suppose BX and CY intersect at A.

Thus, ΔABC is the required triangle.

(iv) Below BC, draw an acute angle ∠ZBC.

(v) Along BZ, mark five points Z_1, Z_2, Z_3, Z_4 and Z_5 such that $BZ_1 = Z_1Z_2 = Z_2Z_3 = Z_3Z_4 = Z_4Z_5$

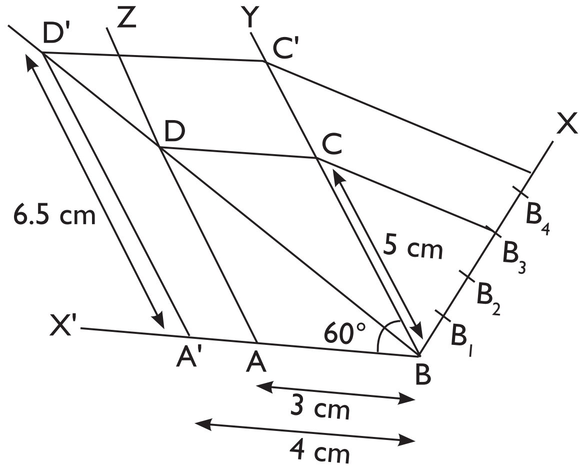
(vi) Join CZ_5.

(vii) From Z_3, draw $Z_3C^I \parallel CZ_5$ meeting BC at C^I.

(viii) From C^I, draw $A^IC^I \parallel AC$ meeting AB in A^I.

Here, ΔA^IBC^I is the required triangle whose sides are $\frac{3}{5}$ of the corresponding sides of ΔABC.

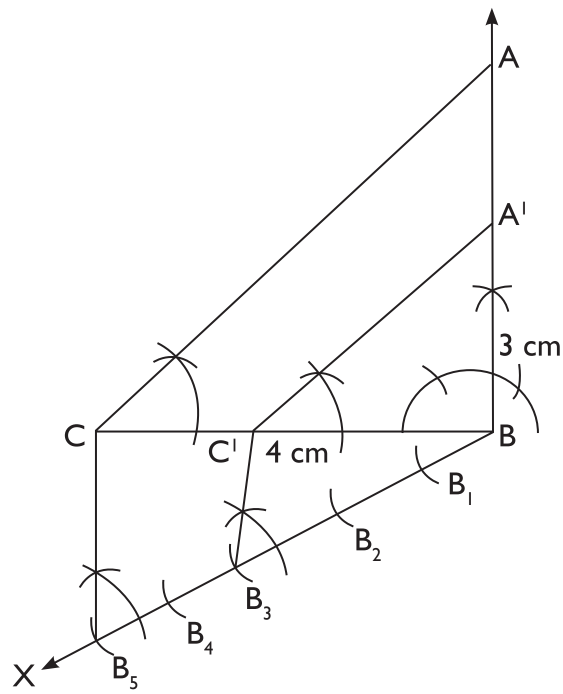
8.



Steps of Construction

1. Construct a line AB = 3 cm
2. Construct a ray BY which makes an acute angle $\angle ABY = 60^\circ$
3. With B as centre and 5 cm as radius, construct an arc which cuts the point C on BY
4. Construct a ray AZ which makes $\angle ZAX'$ = 60° as BY || AZ and $\angle YBX' = \angle ZAX'$ = 60°
5. With A as centre and 5 cm as radius, construct an arc which cuts the point D on AZ
6. Join CD
7. So we get a parallelogram ABCD
8. Join BD which is the diagonal of parallelogram ABCD
9. Construct a ray BX downwards which makes an acute angle $\angle CBX$
10. Now locate 4 points B_1, B_2, B_3, B_4 on BX where $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$
11. Join B_4C and from B_3C construct a line $B_4C' \parallel B_3C$ which intersects the extended line segment BC at C'
12. Construct C'D'||CD which intersects the extended line segment BD at D'. $\Delta D'BC'$ is the required triangle whose sides are 4/3 of the corresponding sides of ΔDBC
13. Construct a line segment D'A'||DA where A' lies on the extended side BA
14. We see that A'BC'D' is a parallelogram where A'D' = 6.5 cm, A'B = 4 cm and $\angle A'BD' = 60^\circ$
15. Divide it into triangles A'BD' and BC'D' by the diagonal BD'.

9.



Steps of construction

(i) BC = 4 cm is drawn.

(ii) At B, a ray making an angle of 90° with BC is drawn.

(iii) With B is centre and radius equal to 4 cm, an arc is made on provision ray intersecting it at point A.

(iv) AC is joined to form ABC.

(v) Ray BX is drawn making acute angle with BC opposite to vertex A.

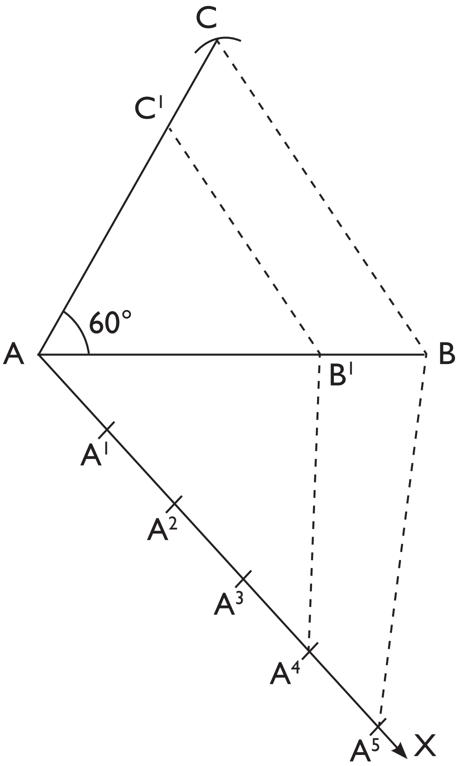
(vi) 5 points $B_1, B_2, \ldots, B_5$ at equal distance are marked on BX.

(vii) B_5C is joined and B_3C^I is made parallel to B_5C.

(viii) A^IC^I is joined together.

Thus, A^IBC^I is the required triangle.

10.



Step of construction

(i) First of all we draw a line segment AB = 4.6 cm

(ii) With A as centre draw an angle $\angle A = 60°$.

(iii) With B as centre and radius = BC = 5.1 cm, draw an arc, intersecting the arc drawn in step I at C.

(iv) Joins BC to obtain ΔABC.

(v) Below AB, makes acute angle $\angle BAX = 60°$.

(vi) Along AX, mark off five points A_1, A_2, A_3, A_4 and A_5 such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5$.

(vii) Join A_5B.

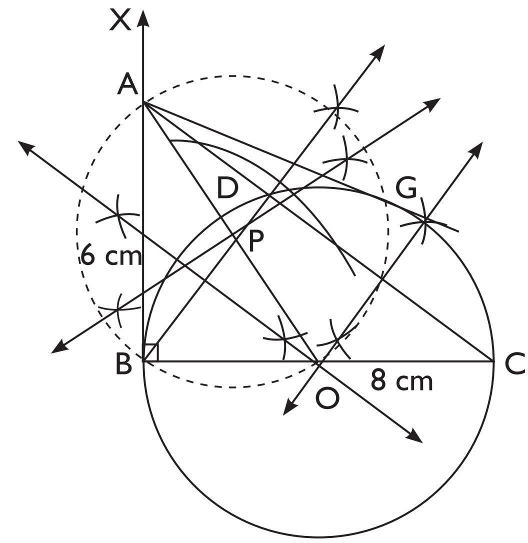
(viii) Since we have to construct a triangle each of whose sides is $(4/5)^{th}$ of the corresponding sides of ΔABC.

So, we take four parts out of five equal parts on AX from point A_4 draw $A_4B' \parallel A_5B$, and meeting AB at B'.

Step IX– From B' draw $B'C' \parallel BC$ and meeting AC at C'.

Thus, $\Delta AB'C'$ is the required triangle, each of whose sides is $(4/5)^{th}$ of the corresponding sides of ΔABC.

11.



Steps of construction

(i) Draw a line BC of 8 cm length.

(ii) Draw BX perpendicular to BC.

(iii) Mark an arc at the distance of 6 cm on BX. Mark it as A.

(iv) Join A and C. Thus ΔABC is the required triangle.

(v) With B as the centre, draw an arc on AC.

(vi) Draw the bisector of this arc and join it with B. Thus, BD is perpendicular to AC.

(vii) Now, draw the perpendicular bisector of BD and CD. Take the point of intersection as O.

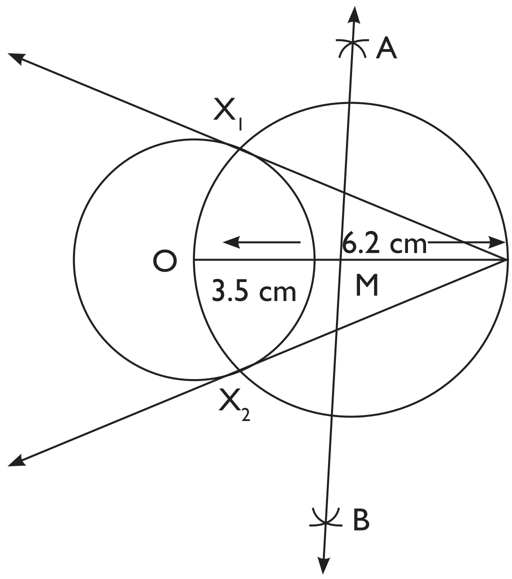
(viii) With O as the centre and OB as the radius, draw a circle passing through points B, C and D.

(ix) Join A and O and bisect it. Let P be the mid point of AO.

(x) Taking P as the centre and PO as its radius. Draw a circle which will intersect the circle at point B and G. Join A and G.

Here, AB and AG are the required tangents to the circle from A.

12.



Steps of construction

(i) Draw the circle with centre O and radius 3.5 cm.

(ii) Joint P from centre to outside the circle. OP = 6.2 cm.

(iii) Construct mid point of OP, M is the mid point of OP.

(iv) Draw a circle with centre M and radius OM intersects the given circle at X_1 and X_2.

(v) Join PX_1 and PX_2.

Thus, PX_1 and PX_2 are required two tangents from point P.

CASE STUDY-1

(i) (b) $\angle CBX$ is an acute angle as it is less than 90°.

(ii) (a) The points are marked at equal distance.

(iii) (d) ΔABC is $\frac{5}{7}$ of $\Delta A'BC'$ is the point B_5 should be joined to C so that $\Delta ABC \sim \Delta A'BC'$.

(iv) (d) B_7C' should be parallel to B_5C.

(v) (a) From the figure, it is obvious that A'C' is parallel to AC.

CASE STUDY-2

(i) (c) $\angle APB + \angle PAO + \angle PBO + \angle BOA = 360°$ [sum of all angles of quadrilateral is 360°]

$\angle APB = 45°$

$\angle PAO = 90°$

$\angle PBO = 90°$

$\therefore \angle BOA = 360° - (45° + 90° + 90°) = 135°$

(ii) (c) Only one tangent can be drawn from a point which lie on circle.

(iii) (d) Two tangents can be drawn from a point that lies outside the circle.

(iv) (c) No tangent can be drawn from a point inside the circle.

(v) (c) PA = PB [Length of tangents from external point are equal]

Chapter 12 Areas Related to Circles

Multiple Choice Questions

1. (d) $2(\pi)(r) - r = 37$

$$r\left(2 \times \frac{22}{7} - 1\right) = 37$$

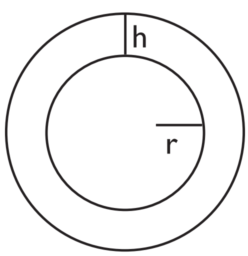
$$r = \frac{37 \times 7}{44 - 7}$$

$$= 7 \text{ cm}$$

$$\therefore \quad \text{Area} = \frac{22}{7} \times 7 \times 7$$

$$= 154 \text{ cm}^2$$

2. (b)



Area of circular path

$$= \pi (r + h)^2 - \pi r^2$$

$$= \pi (r^2 + h^2 + 2rh) - \pi r^2$$

$$= \pi (h^2 + 2rh)$$

$$= \pi h (h + 2rh)$$

3. (a) $2\pi r = 22$

$$2 \times \frac{22}{7} \times r = 22$$

$$r = \frac{22 x 7}{2 x 22}$$

$$= 3.5 \text{ cm}$$

$$\text{Area} = \frac{22}{7} \times 3.5 \times 3.5$$

$$= 38.5 \text{ cm}^2$$

4. (c) Perimeter of circle = 2 (perimeter of square)

$$\Rightarrow \quad 2\pi r = 2 (4x)$$

$$\Rightarrow \quad \pi r = 4x$$

$$\Rightarrow \quad r = \frac{4x}{\pi}$$

$$\text{Ratio of areas} = \frac{\pi r^2}{x^2}$$

$$= \pi \frac{16x^2}{\pi^2} \times \frac{1}{x^2}$$

$$= 16 : \pi$$

5. (d) $\dfrac{Area\ of\ Section\ 5_1}{Area\ of\ Section\ 5_2}$

$$= \frac{\frac{120}{360}\pi r^2}{\frac{150}{360}\pi r^2}$$

$$= 4 : 5$$

WORKSHEET - 1

SECTION-A

1. arc length = 3.5 cm

$$\frac{\theta}{360} 2\pi r = 3.5$$

$$\Rightarrow \quad \frac{\theta \pi r}{360} = \frac{3.5}{2} \qquad ...(i)$$

$$\text{Area of sector} = \frac{\theta}{360} \pi r^2$$

$$= \frac{3.5}{2} \pi r \ r$$

$$= \frac{3.5}{2} r \qquad \text{(From (i))}$$

$$= \frac{3.5}{2} \times 5$$

$$= 8.75 \text{ cm}^2$$

2. Length of arc $= \frac{\theta}{360} 2\pi r$

$= \frac{45}{360} \times 2 \times \pi \times 5$

$= \frac{45\pi}{36}$

$= \frac{5\pi}{4}$ cm

3. Area of section $= \frac{\theta}{360} \pi r^2$

$= \frac{120}{360} \times \frac{22}{7} \times 21 \times 21$

$= 462 \text{ cm}^2$

4. Area of circle = Circumference of circle

$\Rightarrow \quad \pi r^2 = 2\pi r$

$\Rightarrow \quad r = 2$

5. Circumference $= 2\pi r$ metres

Distance covered = 5 m

So, no. of revolutions $= \frac{\text{Distance Covered}}{\text{Circumference}}$

$= \frac{5}{2\pi r}$

6. Let r_1 = 19 cm and r_2 = 9 cm.

Circumference of circle

= Sum of circumferences of the two circles

$\Rightarrow \quad 2\pi r = 2\pi r_1 + 2\pi r_2$

$\Rightarrow \quad r = r_1 + r_2$

$= 19 + 9$

$= 28$ cm

7. Let r_1 = 12 cm, r_2 = 5 cm

Area of circle = Sum of areas of the two circles

$\Rightarrow \quad \pi r^2 = \pi r_1^2 + \pi r_2^2$

$\Rightarrow \quad r^2 = r_1 + r_2$

$= 12^2 + 5^2$

$= 144 + 25$

$= 169$

r = 13 cm = 13 cm

Diameter = 2r

= 26 cm

8. Circumference = 582 cm

$\Rightarrow \quad 2\pi r = 582$

$\Rightarrow \quad 2 \times \frac{22}{7} r = 582$

$\Rightarrow \quad r = \frac{291 \times 7}{2 \times 11}$

$= \frac{2037}{22} \text{ cm}^2$

$\therefore$ area of circle $= \pi r^2$

$= \frac{22}{7} \times \frac{2037}{22} \times \frac{2037}{22}$

$= 36943.95 \text{ cm}^2$

SECTION-B

9. Let r_1 = 8 cm, r_2 = 6 cm

Area of circle = Sum of area of 2 circles

$\pi r^2 = \pi r_1^2 + \pi r_2^2$

$r^2 = r_1^2 + r_2^2$

$= 8^2 + 6^2$

$= 64 + 36$

$= 100$

$\therefore \quad r = 10$ cm

$\therefore$ Circumference of circle

$= 2\pi r$

$= 2 \times \frac{22}{7} \times 10$

$= \frac{440}{7}$ cm

10. Time $= 10$ minutes

$= 10 \times 60$

$= 600$ seconds

Speed $= 66$ km/hr

$= \frac{\cancel{66}^{11} \times \cancel{1000}^{5}}{\cancel{3600}_{\cancel{6}\ 3}}$

$= \frac{55}{3}$ m/s

$\therefore$ Total distance covered = speed x time

$= \frac{55}{3} \times 600$

$= 11000$ m

Distance covered in one revolution

$= 2\pi r$

$= 2 \times \frac{22}{7} \times \frac{40}{100} = \frac{88}{35}$ m

$\therefore$ Number of revolutions

$= \frac{11000 \times 3}{88}$

$= 4375$

11. Circumference $= 22$

$\Rightarrow \quad 2\pi r = 22$

$\Rightarrow \quad 2 \times \frac{22}{7} r = 22$

$\Rightarrow \quad r = \frac{22 \times 7}{2 \times 22}$

$= \frac{7}{2}$

$= 3.5$ cm

area of quadrant $= \frac{1}{4} \pi r^2$

$= \frac{1}{4} \times \frac{22}{7} \times 3.5 \times 3.5$

$= 9.625\ cm^2$

12. Angle subtended in 60 minutes $= 360^\circ$

$\therefore$ Angle subtended in 10 minutes $= \frac{360 \times 10}{60}$

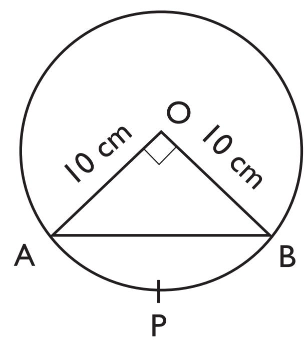
$= 60^\circ$

Area $= \frac{\theta}{360} \pi r^2$

$= \frac{60}{360} \times \frac{22}{7} \times 16 \times 16$

$= 134.095\ cm^2$

13.



(a) area of sector OAPB $= \frac{90}{360} \times \frac{22}{7} \times (10)^2$

$= \frac{550}{7}\ cm^2$

area of sector $\Delta AOB = \frac{1}{2} \times 10 \times 10$

$= 50\ cm^2$

$\therefore$ area of minor segment

$= \frac{550}{7} - 50$

$= \frac{550 - 350}{7}$

$= \frac{200}{7}\ cm^2$

$= 28.6\ cm^2$

(b) Area of major segment

= area of circle – area of minor segment

$$= \pi (10)^2 - \frac{200}{7}$$
$$= \frac{22}{7} \times 100 - \frac{200}{7}$$
$$= \frac{2000}{7} - \frac{200}{7}$$
$$= \frac{2000}{7} \text{ cm}^2$$
$$= 285.7 \text{ cm}^2$$

14. Area cleaned at each sweep of the blades

$$= \frac{\theta}{360} \pi r^2$$
$$= \frac{115}{360} \times \frac{22}{7} \times 2.5 \times 2.5$$
$$= 6.27 \text{ cm}^2$$

∴ Total area cleaned $= 2 \times 6.27$

$$= 12.54 \text{ cm}^2$$

15. Area of each semi circle

$$= \frac{1}{2} \times \frac{22}{7} \times \left(\frac{14}{2}\right)^2$$
$$= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$$
$$= 11 \times 7$$
$$= 77 \text{ cm}^2$$

Area of square $= (14)^2$

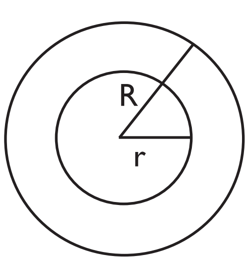
$$= 196 \text{ cm}^2$$

∴ area of shaded region

= area of square – 2x area of semi circle

$$= 196 - 2(77)$$
$$= 196 - 154$$
$$= 42 \text{ cm}^2$$

16.



$r = \frac{17.5}{2}$ 8.75 cm

width of path = 3.5 cm

⇒ R = 8.75 + 3.5

= 12.25 cm

area of path $= \pi (R^2 - r^2)$

$$= \frac{22}{7} [(12.25)^2 - (8.75)^2]$$
$$= \frac{22}{7} [150.0625 - 76.5625]$$
$$= 231 \text{ cm}^2$$

SECTION-C

17. Cost of fencing at the rate of ₹ 24 per metre

= ₹ 5280

⇒ Perimeter of circular field × 24 = 5280

⇒ Perimeter of circular field $= \frac{5280}{24}$

= 220 m

⇒ $2\pi r = 220$

⇒ $2 \times \frac{22}{7} \times r = 220$

⇒ $r = \frac{220 \times 7}{2 \times 22}$

= 35 m

∴ Area of field $= \pi r^2$

$$= \frac{22}{7} \times 35 \times 35$$
$$= 3850 \text{ m}^2$$

Cost of ploughing the field = 0.5 × 3850

= ₹ 1925

18. ∠QPR = 90° (Angle in a semi-circle is right angle.)

In ΔQPR,

$Qr^2 = PR^2 + PQ^2$ (Pythagoras theorem)

$= 7^2 + (24)^2$

$= 49 + 576$

$= 625$

$\therefore$ QR $= 25$ cm

$\therefore$ radius (r) $= \frac{QR}{2} = \frac{25}{2}$ cm

So, area of semi-circle

$= \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \frac{22}{7} \times \frac{25}{2} \times \frac{25}{2}$

$= 245.536$ cm²

area of $\Delta QPR = \frac{1}{2} \times PR \times QP$

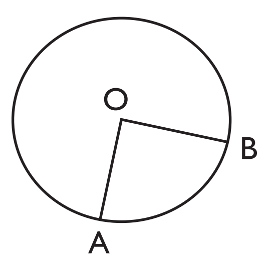
$= \frac{1}{2} \times 7 \times 24$

$= 84$ cm²

So, area of shaded region $= 245.536 - 84$

$= 161.536$ cm²

19.



r = 5.7 cm

Perimeter of sector of circle = 27.2 cm

OA + OB + length of arc $\widehat{AB}$ = 27.2

5.7 + 5.7 + length of $\widehat{AB}$ = 27.2

length of $\widehat{AB}$ = 27.2 – 5.7 – 5.7 = 15.8 cm

$\Rightarrow \frac{\theta}{360} 2\pi r = 15.8$

$\Rightarrow \frac{\theta}{360} \pi r = \frac{15.8}{2}$

$= 7.9$ cm

Area of sector OAB $= \frac{\theta}{360}\pi r^2$

$= \frac{\theta}{360}\pi r \; r$

$= 7.9 \; r$

$= 7.9 \; (5.7)$

$= 45.03$ cm²

20. area of sector OBD $= \frac{40}{360}\pi \; (7)^2$

area of sector OAC $= \frac{40}{360}\pi \; (14)^2$

$\therefore$ area of shaded region

$= \frac{40}{360}\pi \; [(14)^2 - 7^2]$

$= \frac{\pi}{9} \; (196 - 49)$

$= \frac{1}{9} \times \frac{22}{7} \; 147$

$= 51.33$ cm²

21. Area of each quadrant

$= \frac{1}{4}\pi (1)^2$

$= \frac{\pi}{4}$

area of circle of diameter 2 cm

$= \pi \; (1)^2$

$= \pi$ cm²

So, area of shaded region

= Area of square

$-4\left(\frac{\pi}{4}\right) - \pi$

$= 4^2 - \pi - \pi$

$= 16 - 2\pi$

$= 16 - \frac{22}{7} \times 2$

$= 16 - \frac{44}{7}$

$= \frac{68}{7}\ cm^2$

$= 9.71\ cm^2$

22. Area of quadrant OAB $= \frac{1}{4}\pi(21)^2$

$= \frac{441}{4}\pi\ cm^2$

Area of quadrant ODC $= \frac{1}{4}\pi(14)^2$

$= \frac{196}{4}\pi\ cm^2$

∴ Area of shaded region

$= \frac{441}{4}\pi - \frac{196}{4}\pi$

$= \frac{245}{4}\pi$

$= \frac{245}{4} \times \frac{22}{7}$

$= 192.5\ cm^2$

23. Angle subtended by minute hand

in 60 minutes $= 360°$

in 1 minute $= \frac{360°}{60}$

$= 6°$

in 5 minutes $= 5 \times 6$

$= 30°$

So, area swept by minutes hand

in 5 minutes $= \frac{30}{360}\pi(14)^2$

$= \frac{\pi}{12} \times 196$

$= \frac{196}{12} \times \frac{22}{7}$

$= 51.3\ cm^2$

24. (a) length of arc $= \frac{\theta}{360}\ 2\pi r$

$= \frac{60}{360} \times 2 \times \frac{22}{7} \times 21$

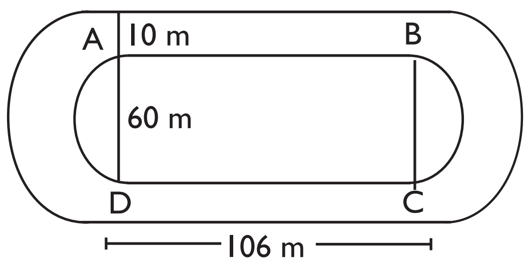
$= 22$ cm

(b) area of sector $= \frac{60}{360} \times \frac{22}{7} \times (21)^2$

$= 231\ cm^2$

SECTION-D

25.



(a) Distance around the track along its inner edge = AB + CD + 2 (semi-perimeter of inner circles ends)

$= 106 + 106 + 2\left(\frac{22}{7} \times 30\right)$

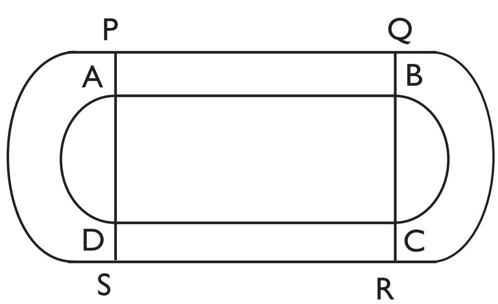
$= 212 + \frac{1320}{7}$

$= \frac{1484 + 1320}{7}$

$= \frac{2804}{7}$

$= 400.57$ cm

(b)



Area of track

= area of rectangle PQRS

– area of rectangle ABCD

+ 2 [Area of semi-circle with radius 40 cm]

[area of semicircle with radius 30 cm]

= (106 x 80) – (106 x 60)

$+ 2\left[\frac{\pi}{2}(40)^2 - \frac{\pi}{2}(30)^2\right]$

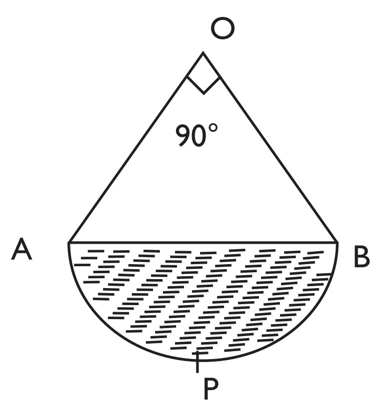
= 8480 – 6360 + π (1600 – 900)

$= 8480 - 6360 + \frac{22}{7} \times 700$

= 8480 – 6360 + 2200

= 4320 m^2

26.



ABCD is a square

∴ AC and BD bisect each other and are equal

∴ AO = OC = DO = BO

In ΔAOB,

$AB^2 = OA^2 + OB^2$

$(56)^2 = OA^2 + OA^2$ [∴ OA = OB]

$3136 = 20A^2$

$1568 = OA^2$

$OA = \sqrt{1568}$

$= 28\sqrt{2}$ m

So, area of sector OAPB

$= \frac{90}{360} \times \frac{22}{7} \times (28\sqrt{2})^2$

$= \frac{1}{4} \times \frac{22}{7} \times 784 \times 2$

= 1232 m^2

Also, area of ΔOAB

$= \frac{1}{2} \times OA \times OB$

$= \frac{1}{2} \times 28\sqrt{2} \times 28\sqrt{2}$

= 784 m^2

So, area of shaded part

= 2 [1232 – 784]

= 896 m^2

Also, area of square lawn + area of flower bets

= 896 + 3136

= 4032 m^2

27. Area of square = 8^2

= 64 cm^2

area of 1 quadrant

$= \frac{1}{4}\pi(1.4)^2$

$= \frac{1}{4} \times \frac{22}{7} \times \frac{14}{10} \times \frac{14}{10}$

= 1.54 cm

∴ Area of the shaded portion of the square

= Area of square – area of circle – 2 (area of a quadrant)

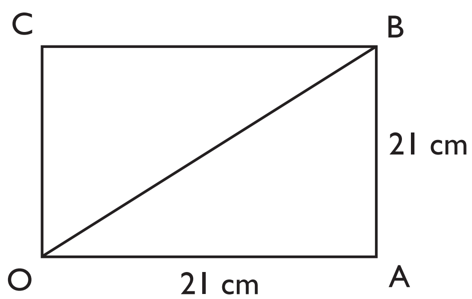
= 64 – 55.44 – 2 (1.54)

= 8.56 – 3.08

= 5.48 cm^2

28. Area of square OABC $= (21)^2$

$= 411 \text{ cm}^2$



In ΔOAB, right angled at A

$OB^2 = OA^2 + AB^2$

$= (21)^2 + (21)^2$

$= 441 + 441 = 882 \text{ cm}^2$

$\therefore \; OB = 21\sqrt{2}$

So, area of quadrant OPBQ with OB as radius

$= \frac{90}{360} \times \frac{22}{7} \times (21\sqrt{2})^2$

$= \frac{1}{4} \times \frac{22}{7} \times 441 \times 2$

$= 693 \text{ cm}^2$

$\therefore$ area of shaded part

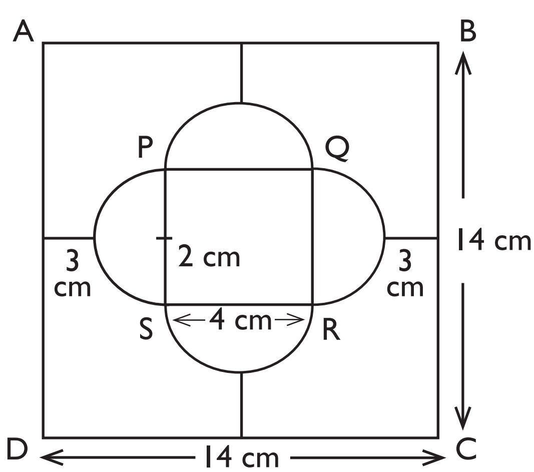
= area of quadrant OPBQ

– area of square OABC

$= 693 - 441$

$= 252 \text{ cm}^2$

29.



Area of square ABCD $= (14)^2$

$= 196 \text{ cm}^2$

Area of square PQRS $= 4^2$

$= 16 \text{ cm}^2$

Area of 4 semi-circles with radius 2 cm

$= 4\left(\frac{1}{2}\right) \frac{22}{7} \times 2 \times 2$

$= \frac{176}{7} \text{ cm}^2$

So, area of shaded part

$= 196 - \left(16 + \frac{176}{7}\right)$

$= 196 - 16 - \frac{176}{7}$

$= 180 - \frac{176}{7}$

$= \frac{1260 + 176}{7}$

$= \frac{1084}{7}$

$= 154.85 \text{ cm}^2$

30. Area of circle $= \pi r^2$

$= \pi (7)^2$

$= 49\pi \text{ cm}^2$

$= \frac{49 x 22}{7} \text{ cm}^2$

$= 154 \text{ cm}^2$

Area of sector

$= \frac{60}{360} \times \frac{22}{7} \times 7 \times 7$

$= \frac{154}{6} \text{ cm}^2$

$= \frac{77}{3} \text{ cm}^2$

$= 25.67 \text{ cm}^2$

Area of $\Delta ABC = \frac{\sqrt{3}}{4}(14)^2$

$= \frac{\sqrt{3}}{4} \times 196$

$= 84.87 \text{ cm}^2$

So, area of shaded part

= Area of circle + Area of triangle

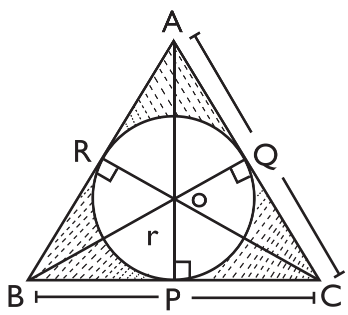
– 2 Area of sector

$= 154 + 84.87 - 2(25.67)$

$= 154 + 84.87 - 51.34$

$= 187.53 \text{ cm}^2$

31.



$AP \perp BC$, $BQ \perp AC$ and $CR \perp AB$

[As tangent is perpendicular to radius through point of contact.]

Area of $\Delta ABC = \frac{\sqrt{3}}{4}(\text{side})^2$

$= \frac{\sqrt{3}}{4}(12)^2$

$= \sqrt{3}(36)$

$= 36\sqrt{3} \text{ cm}^2$

Also, area of ΔABC

= area of ΔAOC + area of ΔBOC + area of ΔAOB

$36\sqrt{3} = \frac{1}{2}(12)r + \frac{1}{2}(12)r \frac{1}{2}(12)r$

$36\sqrt{3} = 6r + 6r + 6r$

$\frac{36\sqrt{3}}{18} = r$

$r = 2\sqrt{3} \text{ cm}$

$\therefore$ area of circle $= \pi r^2$

$= \frac{22}{7}(2\sqrt{3})^2$

$= \frac{22}{7} \times 12$

$= 37.71 \text{ cm}^2$

So, area of shaded part

= area of ΔABC

– area of circle

$= 36\sqrt{3} - 37.71$

$= 62.352 - 37.71$

$= 24.642 \text{ cm}^2$

32. Area of square OABC

$= 7^2$

$= 49 \text{ cm}^2$

Area of sector OAPC

$= \frac{90}{360} \times \frac{22}{7} \times 7 \times 7$

$= \frac{154}{4} \text{ cm}^2$

$= 38.5 \text{ cm}^2$

So, area of shaded region

$= 49 - 38.5$

$= 10.5 \text{ cm}^2$

WORKSHEET - 2

SECTION-A

1. Area of section $= \frac{p}{360}\pi r^2$

2. Perimeter of circle = Perimeter of square

(radius = r) (side = x)

$\Rightarrow 2\pi r = 4x$

$\Rightarrow \pi r = 2x$ ...(i)

So, $\frac{\text{area of circle}}{\text{area of square}} = \frac{\pi r^2}{x^2}$

$= \frac{\pi\left(\frac{2x}{\pi}\right)^2}{x^2}$

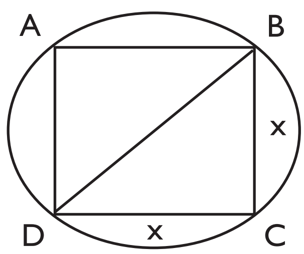
$= \frac{\pi\, 4x^2}{\pi^2} \times \frac{1}{x^2}$

$= \frac{4}{\pi}$

$= \frac{4\times 7}{22} = 14 : 11$

3. Area of sector $= \frac{60}{360} \times \frac{22}{7} \times 10 \times 10$

$= 52.38 \text{ cm}^2$

4. 

Circumference of circle = 100

$2\pi r = 100$

$2 \times \frac{22}{7} \times r = 100$

$r = \frac{100\times 7}{2\times 22}$

$= \frac{175}{11}$ cm

$= 2r$

$\Rightarrow$ Diameter $= \frac{350}{11}$ cm

In ΔBCD,

$BD^2 = BC^2 + CD^2$

$\left(\frac{350}{11}\right)^2 = x^2 + x^2$

$2x^2 = \left(\frac{350}{11}\right)^2$

$x^2 = \frac{1}{2}\left(\frac{350}{11}\right)^2$

$\Rightarrow \quad x = \frac{350}{11\sqrt{2}}$

$= \frac{350}{15.554}$

$= 22.50 \text{ cm}^2$

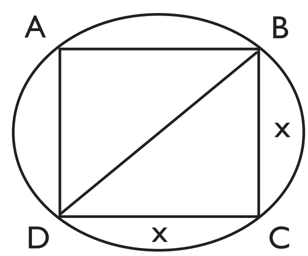
5. Area of circle = 220

$\Rightarrow \quad \pi r^2 = 220$

$\Rightarrow \quad r = \sqrt{\frac{220}{\pi}}$

$\Rightarrow$ Diameter $= 2r$

$= 2\sqrt{\frac{220}{\pi}}$ cm



In ΔABC,

$BD^2 = BC^2 + CD^2$

$\left(2\sqrt{\frac{220}{\pi}}\right)^2 = 2x^2$

$4\left(\frac{220}{\pi}\right) = 2x^2$

$x^2 = \frac{880}{\pi\,(2)}$

$= \frac{440}{\pi}$

So, area of square $= x^2$

$= \frac{440}{\pi}$

$= \frac{440}{\pi} \times 7$

$= 140 \text{ cm}^2$

6. $r = 0.25$ m

Distance covered in one revolution

$= 2\pi r$

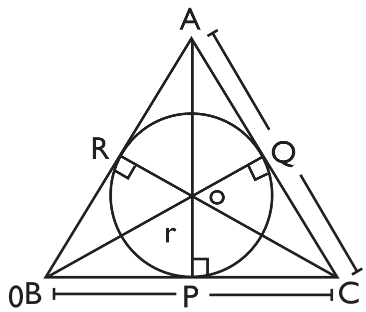
$= 2 \times \frac{22}{7} \times 0.25$

So, number of revolutions

$= \frac{11 \times 1000 \times 7}{2 \times 22 \times 0.25}$

$= 7000$

7.



Area of $\Delta ABC = \frac{\sqrt{3}}{4}$ (side)^2

$= \frac{\sqrt{3}}{4} \times 42 \times 42$

$= 441\sqrt{3}$ cm^2

Also, $AP \perp BC$, $BQ \perp AC$ and $CR \perp AB$

[As tangent is perpendicular to radius through point of contact.]

So,

area of ΔABC = area of ΔBOC

+ area of ΔAOC

+ area of ΔAOB

$\Rightarrow 441\sqrt{3} = \frac{1}{2} \times 42 \times r + \frac{1}{2} \times 42 \times r + \frac{1}{2} \times 42 \times r$

$\Rightarrow 441\sqrt{3} = 63 \quad r$ cm^2

$\Rightarrow r = \frac{441\sqrt{3}}{63}$

$= 7\sqrt{3}$ cm

So, area of circle $= \pi r^2$

$= \frac{22}{7} \times 7\sqrt{3} \times 7\sqrt{3}$

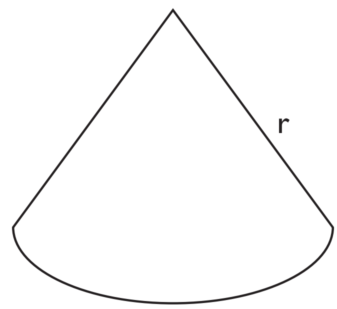
$= 462$ cm^2

8. Area of sector

$= \frac{90}{360} \times \frac{22}{7} \times 2 \times 2$

$= \frac{22}{7}$ cm^2

9.



arc length $= 5\pi$ cm

$\frac{\theta}{360} 2\pi r = 5$

$\frac{2\pi}{360} = 5$

$r\theta = 900$ (i)

Also, area of sector $= 20\pi$

$\frac{\theta}{360} \pi r^2 = 20\pi$

$\frac{\theta r^2}{360} = 20$

$r\theta^2 = 7200$ (ii)

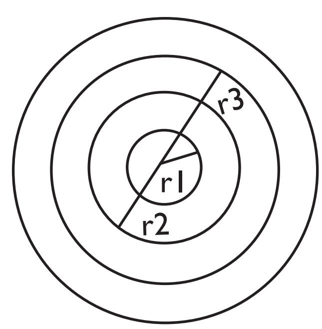
From (i) and (ii), we get

$r^2 \frac{900}{r} = 7200$

$900r = 7200$

$r = 8$ cm

10.



According to question,

$$\pi r^2 - \pi r_3^2 = \frac{1}{4}\pi r^2$$

$$\Rightarrow \quad \pi r^2 - \frac{1}{4}\pi r^2 = \pi r_3^2$$

$$\Rightarrow \quad \frac{3}{4}\pi r^2 = \pi r_3^2$$

$$\Rightarrow \quad \frac{3}{4} r^2 = r_3^2$$

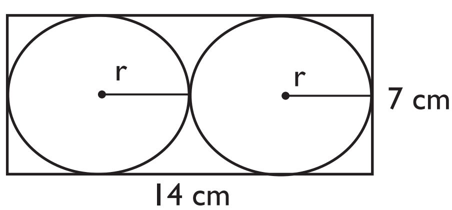
$$\Rightarrow \quad r^3 = \frac{\sqrt{3}}{2} r$$

$$= \frac{\sqrt{3}}{2}(20)$$

$$= 10\sqrt{3}$$

SECTION-B

11.



Area of rectangle $= 14 \times 7$

$= 98 \text{ m}^2$

Area of circle $= \pi\left(\frac{7}{2}\right)^2$

$$= \frac{22}{7} \times \frac{49}{7}$$

$$= \frac{77}{2} \text{ m}^2$$

So, area of remaining portion

= Area of rectangle

– 2 (area of circle)

$$= 98 - 2\left(\frac{7}{2}\right)$$

$= 98 - 77$

$= 21 \text{ cm}^2$

12. $R - r = 7$ ...(i)

$\pi R^2 - \pi r^2 = 286$

$$(R^2 - r^2)\frac{22}{7} = \frac{286 \times 7}{22}$$

$= 91$

$\Rightarrow \quad (R - r)(R + r) = 91$

$\Rightarrow \quad 7\,(R + r) = 91$

$\Rightarrow \quad R + r = 13$ ...(ii)

On solving (i) and (ii), we get

$R - r = 7$

$R + r = 13$

$2r = 20$

$r = 10 \text{ cm}$

$\therefore \quad R = 13 - r$

$= 13 - 10$

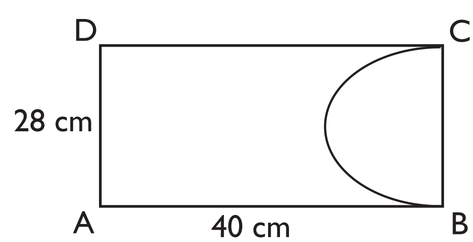
$= 3$ cm

So, sum of radii $= R + r$

$= 10 + 3$

$= 13$ cm

13.



Area of semi-circle $= \frac{1}{2}\pi r^2$

$$= \frac{1}{2}\pi(14)^2$$

$$= \frac{1}{2} \times \frac{22}{7} \times 196$$

$= 308 \text{ cm}^2$

Area of rectangle ABCD

$= AB \times BC$

$= 40 \times 28$

$= 1120 \text{ cm}^2$

So, area of remaining paper

$= 1120 - 308$

$= 812 \text{ cm}^2$

14. Perimeter of the top of the table

$= OA + OB + \frac{270}{360}(2\pi) 42$

$= 42 + 42 + 63\pi$

$= 84 + 63\pi$

$= 84 + 198$

$= 84 + 198$

$= 282$ cm

15. Circumference of circle $= 2\pi r$

$44 = 2 \times \frac{22}{7} \times r$

$r = \frac{44 \times 7}{2 \times 22}$

$= 7$ cm

So, area of quadrant $= \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \frac{22}{7} \times 7^2$

$= 38.5 \text{ cm}^2$

16. Perimeter of semi-circle with OA as radius

$= \pi (14)$

$= \frac{22}{7} \times 14$

$= 44$ cm

Perimeter of semi-circle with OA as diameter

$= \pi (7)$

$= \frac{22}{7} \times (7)$

$= 22$ cm

So, Total perimeter

$= 44 + 2(22) + AB$

$= 44 + 44 + 28$

$= 116$ cm

17. Area of shaded region = Area of trapezium – Area of quadrant

Area of trapezium $= \frac{1}{2}(AD + BC) \times AB =$ 24.5

$(AD + BC) AB = 49 \Rightarrow 14 (AB)$

$= 49 \Rightarrow AB = 3.5$ cm

Area of quadrant $= \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \frac{22}{7} \times (3.5)(3.5)$

$= \frac{11 \times 3.5}{4} = 9.625 \text{ cm}^2$

$\therefore$ Area of shaded region $= 24.5 - 9.625$

$= 14.875 \text{ cm}^2$

18. Area of quadrant OACB

$= \frac{1}{4} \times \frac{22}{7} \times 7 \times 7$

$= 38.5 \text{ cm}^2$

area of $\Delta AOD = \frac{1}{2} \times 7 \times 4$

$= 14 \text{ cm}^2$

$\therefore$ area of shaded part $= 38.5 - 14$

$= 24.5 \text{ cm}^2$

19. Length of arc $= \frac{\theta}{360} 2\pi r$

$8.5 = \frac{\theta}{360} \times 2 \times \frac{22}{7} \times r$

$\therefore \quad r = \frac{8.5 \times 360 \times 7}{30 \times 2 \times 22}$

$= 16.23$ cm

20. Cost of fencing 1 metre

Circular field $= ₹\ 12$

Total cost of fencing a circular field $= ₹\ 2640$

$\therefore$ Circumference of circular field

$$= \frac{2640}{12}$$

$= 220$ m

$\Rightarrow$ $2\pi r = 220$

$$2 \times \frac{22}{7} \times r = 220$$

$$r = \frac{220 \times 7}{2 \times 22}$$

$= 35$ m

Area of circular field $= \frac{22}{7}(35)^2$

$= 3850\ m^2$

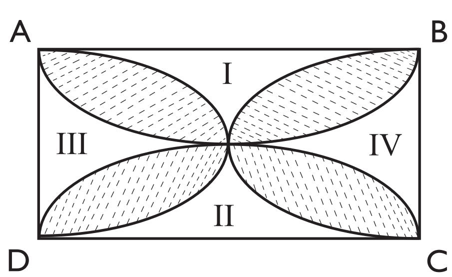
$\therefore$ cost of ploughing the field

$= 3850 \times 2$

$= ₹\ 7700$

SECTION-C

21.



Area of square ABCD $=(10)^2$

$= 100\ cm^2$

Area of semi-circle with AD as diameter

$$= \frac{1}{2} \times \frac{22}{7} \times 5 \times 5$$

$$= \frac{275}{7}\ cm^2$$

So, sum of area of part I and II

= area of square ABCD

– 2 (area of semi-circle)

$$= 100 - 2\left(\frac{275}{7}\right)$$

$$= 100 - \frac{550}{7}$$

$$= \frac{150}{7}\ cm^2$$

Also, sum of areas of part III and IV $= \frac{150}{7}\ cm^2$

So, area of shaded part

= area of square ABCD

– Sum of areas of part I and II

– Sum of areas of part III and IV

$$= 100 - \frac{150}{7} - \frac{150}{7}$$

$$= 100 - \frac{300}{7}$$

$$= \frac{400}{7}$$

$= 57.14\ cm^2$

22. In ΔQPR,

$\angle QPR = 90°$ (Angle in a semi circle is a right angle.)

$\therefore$ $QR^2 = PQ^2 + PR^2$ (Pythagoras theorem)

$= (12)^2 + 5^2$

$= 144 + 25$

$= 169$

$\Rightarrow$ QR = 13 cm

area of semi-circle with QR as diameter

$$= \frac{1}{2} \times \frac{22}{7} \times \left(\frac{13}{2}\right)^2$$

$$= \frac{\cancel{22}^{11}}{14} \times \frac{169}{\cancel{4}_2}$$

$= \frac{1859}{28}\ cm^2$

area of $\Delta QPR = \frac{1}{2} \times PR \times PQ$

$= \frac{1}{2} \times 5 \times 12$

$= 30\ cm^2$

$\therefore$ area of shaded part

$= \frac{1859 - 840}{28}$

$= \frac{1019}{28}\ cm^2$

$= 36.4\ cm^2$

23. Area of semi-circle PBQ

$= \frac{1}{2} \times \frac{22}{7} \times r^2$

$= \frac{11}{7} \times 5^2$

$= \frac{275}{7}\ cm^2$

$= 39.29\ cm^2$

As $OP = OQ = PQ = 10$ cm

$\Rightarrow$ ΔPOQ is an equilateral triangle

$\Rightarrow$ $\angle POQ = 60°$

Also, area of $\Delta POQ = \frac{\sqrt{3}}{4}(\text{side})^2$

$= \frac{\sqrt{3}}{4}(10)^2$

$= 25\sqrt{3}\ cm^2$

$= 43.3\ cm^2$

area of sector POQA

$= \frac{60}{360} \times \frac{22}{7} \times 10 \times 10$

$= 52.38\ cm^2$

So, area of part PMQA

= area of sector POQA

– area of ΔPOQ

$= 52.38 - 43.3$

$= 9.08\ cm^2$

area of shaded part

= area of semi-circle

– area of part PMQA

$= 39.29 - 9.08$

$= 30.21\ cm^2$

24. Area of sector (with radius 14 cm)

$= \frac{60}{360} \times \frac{22}{7} \times 14 \times 14$

area of sector (with radius 28 cm)

$= \frac{60}{360} \times \frac{22}{7} \times 28 \times 28$

$= \frac{11}{21}(784 - 196)$

$= \frac{11}{21}(588)$

$= 308\ cm^2$

25. Let $AO = OB = x$

$\Rightarrow$ $AB = 2x$

Perimeter of semi-circle (with AO as diameter)

$= \frac{1}{2} \times 2 \times \frac{22}{7} \times \frac{x}{2}$

$= \frac{11x}{7}$ cm

Perimeter of semi-circle (with AB as diameter)

$= \frac{1}{2} \times 2 \times \frac{22}{7}x$

$= \frac{22x}{7}$ cm

Given : Perimeter of figure $= 40$ cm

$\Rightarrow \quad \frac{11x}{7} + \frac{22x}{7} + OB = 40$ cm

$\Rightarrow \quad \frac{33x}{7} + x = 40$

$\Rightarrow \quad \frac{40x}{7} = 40$

$\Rightarrow \quad x = 7$ cm

Area of semi-circle (with AO as diameter)

$= \frac{1}{2} \times \frac{22}{7} \left(\frac{7}{2}\right)^2$

$= \frac{11}{7} \times \frac{49}{4}$

$= \frac{77}{4}$ cm²

Area of semi-circle (with AB as diameter)

$= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$

$= 77$ cm²

So, area of shaded region

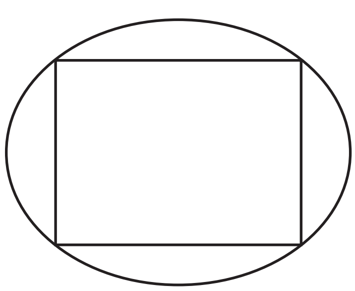
$= \frac{77}{4} + 77$

$= \frac{77 + 308}{4}$

$= \frac{308}{4}$ cm²

$= 96.25$ cm²

26.



As all the vertices of a rhombus lie on a circle

$\therefore$ it must be a square

$\Rightarrow$ both the diagonals must be equal

area of circle $= 1256$ cm²

$3.14 r^2 = 1256$

$\Rightarrow \quad r^2 = \frac{1256}{3.14}$

$= 400$

$\Rightarrow \quad r = 20$ cm

$\Rightarrow$ Diameter of circle $= 2r$

$= 40$ cm

Diameter must be same as diagonal of the square

$\Rightarrow$ Diagonal of square $= 40$ cm

So, area of rhombus

$= \frac{1}{2} \times 40 \times 40$

$= 800$ cm²

27. Radius of long hand $= 60$ cm

Distance travelled by long hand in 1 round

$= 2\pi (6)$

$= 12\pi$

Number of rounds made by long hand

In 24 hours $= 24$

So, Total distance travelled by long hand in 24 hours $= 24 \times 12\pi$

$= 288\pi$

Radius of short hand $= 4$ cm

Distance travelled by short hand in 1 round

$= 2\pi (4)$

$= 8\pi$

Number of rounds made by short hand

In 24 hours $= 2$

So, Total distance travelled by short hand in 24 hours $= 8\pi \times 2$

$= 16\pi$

So, Sum of distances $= 228\pi + 16\pi$

$= 304\pi$

$= 304 \times 3.14$

$= 954.56$ cm

28. Area of trapozium

$= \frac{1}{2}$ (Sum of parallel sides) x Distance between the parallel sides

$= \frac{1}{2}(AB + DC) \times 14$

$= \frac{1}{2}(18 + 32) \times 14$

$= 350 \text{ cm}^2$

area of a quadrant $= \frac{1}{4}\pi(7)^2$

$= \frac{1}{4} \times \frac{22}{7} \times 49$

$= \frac{154}{4} \text{ cm}^2$

So, area of shaded region

= area of trapezium

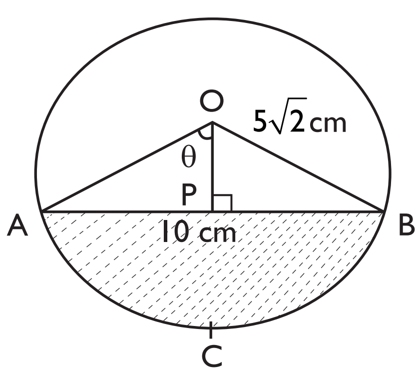
– 4 (area of a quadrant)

$= 350 - 4\left(\frac{154}{4}\right)$

$= 350 - 154$

$= 196 \text{ cm}^2$

29.



Draw $OP \perp AB$

In ΔOPA and ΔOPB

$OA = OB$ (radii of same circle)

$OP = OP$ (common)

$\angle OPA = \angle OPB = 90°$ (By construction)

$\therefore \Delta OPA \cong \Delta OPB$ (RHS)

$\Rightarrow AP = BP$ (CPCT)

$\Rightarrow AB = 2AP$

$\Rightarrow AP = \frac{10}{2} = 5$ cm

In ΔOPA,

$OA^2 = OP^2 + AP^2$

$(5\sqrt{2})^2 = OP^2 + 5^2$

$50 - 25 = OP^2$

$25 = OP^2$

$\therefore OA^2 = 5$ cm

So, area of $\Delta OAB = \frac{1}{2} AB \times OP$

$= \frac{1}{2} 10 \times 5$

$= 25 \text{ cm}^2$

In ΔAOP,

$\tan\theta = \frac{AP}{OP}$

$= \frac{5}{5}$

$= 1$

$\Rightarrow \theta = 45°$

So, $\angle AOB = 2(45°)$

$= 90°$

$\therefore$ area of sector AOBC

$= \frac{90}{360} \times \frac{22}{7} \times 25 \times 2$

$= \frac{275}{7} \text{ cm}^2$

So, area of shaded part

= area of sector AOBC

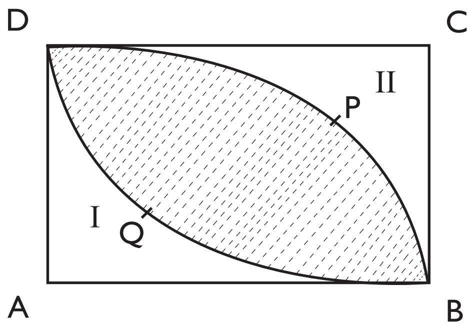
– area of ΔAOB

$= \frac{275}{7} - 25$

$= \frac{275 - 175}{7}$

$= \frac{100}{7}$ cm²

30.



Area of square ABCD $= 7^2$

$= 49$ cm²

Area of quadrant ABPD

$= \frac{90}{360} \pi (7)^2$

$= \frac{\pi}{4} (49)$

$= \frac{49}{4} \times \frac{22}{7}$

$= \frac{77}{2}$ cm²

$= 38.5$ cm²

So, area of part II

$= 49 - 38.5$

$= 10.5$ cm²

Similarly, area of part I $= 10.5$ cm²

∴ Area of the shaded region

= area of square ABCD

– area of I – area of I I

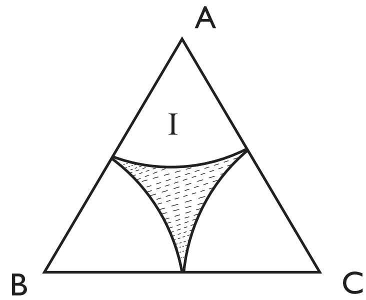
$= 49 - 10.5 - 10.5$

$= 49 - 21$

$= 28$ cm²

SECTION-D

31.



Area of $\Delta ABC = \frac{\sqrt{3}}{4} (\text{side})^2$

$= \frac{1.732}{4} (8)^2$

$= 27.712$ cm²

area of sector I

$= \frac{60}{360} \times 3.142 \times 4^2$

$= \frac{60}{360} \times 3.142 \times 16$

$= 8.38$ cm²

So, area of shaded part

= Area of ΔABC

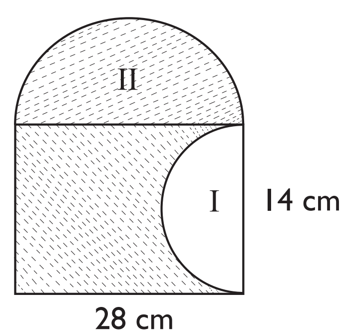
– 3 (area of sector I)

$= 27.712 - 3 (8.38)$

$= 27.712 - 25.14$

$= 2.572$ cm²

32.



Area of ractangle piece $= 28 \times 14$

$= 392$ cm²

area of part I $= \frac{1}{2} \times \frac{22}{7} \times 7^2$

$= 77$ cm²

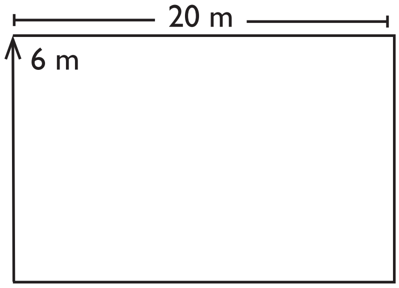
area of part II $= \frac{1}{2} \times \frac{22}{7} \times (14)^2$

$308\ cm^2$

So, area of shaded region

= area of rectangular piece – area of part I + area of part II

$= 392 - 77 + 308 = 623\ cm^2$

33. 

Area of the grassy lawn in which the calf can graze initially

$= \frac{90}{360} \times \frac{22}{7} \times 6 \times 6$
$= 28.286\ m^2$

Area of grassy lawn in which the calf can graze if the length of rope is increased by 5.5 m

$= \frac{90}{360} \times \frac{22}{7} \times 11.5 \times 11.5$
$= 103.911\ m^2$

∴ Increase in the area of the grassy lawn in which the calf can graze

$= 109.911 - 28.286$

$= 75.625\ cm^2$

34. Area of square $= (28)^2$

$= 784\ cm^2$

Area of part of circle inside square

= Area of sector

$= \frac{90}{360} \times \frac{22}{7} \times 14 \times 14$
$= 154\ cm^2$

Area of circle with center O'

$= \frac{22}{7} \times 14 \times 14$
$= 616\ cm^2$

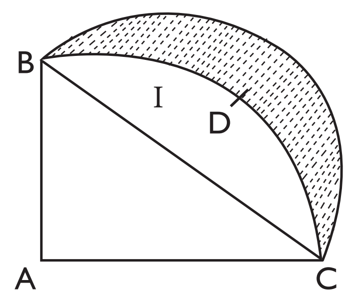
So, area of shaded part

= Area of square – 2 (area of sector) + (area of circle)

$= 784 - 2\ (154) + 2\ (616)$

$= 784 - 308 + 1232$

$= 1708\ cm^2$

35. 

In ΔBAC

$(BC)^2 = (AB)^2 + (AC)^2$

$= (14)^2 + (14)^2 = 392$

$BC = 14\sqrt{2}$

Area of semicircle with BC as diameter

$= \frac{1}{2}\pi\ \frac{14}{2}\sqrt{2}^{\ 2}\ \ \frac{14}{2}\sqrt{2}^{\ 2} = 154\ cm^2$

Area of ΔBAC =

Area of quadrant ACDB

$= 154\ cm^2$

Area of region I = area of quadrant area of

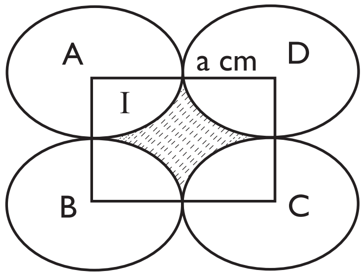
D

$= 154 - 98 = 56\ cm^2$

Area of shaded region = Area of semicircle – Area of region I

$= 154 - 56 = 98\ cm^2$

36.



To find : area of shaded region

area of part I

$$= \frac{90}{360} \times \frac{22}{7} \times a^2$$

$$= \frac{11}{14} a^2 \text{ cm}^2$$

area of square ABCD

(with side a + a = 2a cm)

$$= (2a)^2$$

$$= 4a^2 \text{ cm}^2$$

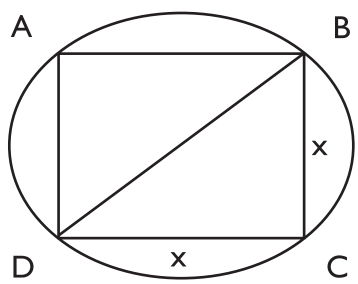
So, area of shaded region

= area of square

$$- 4a^2 - 4\left(\frac{11}{14}a^2\right)$$

$$= 4a^2 - \frac{22}{7}a^2 = \frac{6}{7}a^2$$

37.



Circumference of circle = 650 m

$$\Rightarrow \quad 2\pi r = 650$$

$$\Rightarrow \quad 2 \times \frac{22}{7} r = 650$$

$$\Rightarrow \quad r = \frac{\cancel{650}^{325} \times 7}{\cancel{2} \times 22}$$

$$= \frac{2275}{22} \text{ m}$$

$\therefore$ Diameter = 2 (radius)

$$= \frac{2275}{11} \text{ m}$$

In ΔBCD,

$$BC^2 + CD^2 = BD^2$$

$$x^2 + x^2 = \left(\frac{2275}{11}\right)^2$$

$$2x^2 = \left(\frac{2275}{11}\right)^2$$

$$\Rightarrow \quad x^2 = \frac{1}{2}\left(\frac{2275}{11}\right)^2$$

Area of square ABCD $= x^2$

$$= \frac{1}{2}\left(\frac{2275}{11}\right)^2 \text{ m}^2$$

$$= 21386.88 \text{ m}^2$$

38. (a) Area of square ABCD $= (22)^2$

$$= 484 \text{ cm}^2$$

$\therefore$ area of central part $= \frac{484}{5}$ cm²

$$\Rightarrow \quad \pi r^2 = \frac{484}{5}$$

$$\Rightarrow \quad \frac{22}{7} r^2 = \frac{484}{5}$$

$$\Rightarrow \quad r^2 = \frac{\cancel{484}^{22}}{5} \times \frac{7}{\cancel{22}}$$

$$= \frac{154}{5}$$

$$\Rightarrow \quad r = 5.55 \text{ cm}$$

Circumference of the central part

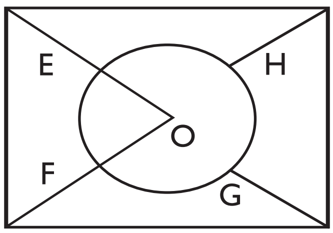
$$= 2\pi r^2 = 2 \times \frac{22}{7} \times 5.55$$

$$= 34.88 \text{ cm}$$

(b) $EF = \frac{1}{4}$ (Circumference of circle)

$= \frac{1}{4}(34.88)$

$= 8.72$ cm



BF = AE

= AO – OE

= AO – 5.55

$= 11\sqrt{2} - 5.55$

= 15.554 – 5.55 = 10.004 cm

$$\left[\begin{array}{l} In\ \Delta AOB,\quad \angle AOB = 90^\circ \\ \therefore AB^2 = AO^2 + OB^2 \\ (22)^2 = AO^2 + BO^2 \\ \Rightarrow AO = BO = 11\sqrt{2}\ cm \end{array}\right]$$

So, Perimeter of part ABEF

= 22 + 10.004 + 10.004 + 8.72

= 50.728 cm²

39. Area of rectangle ABCD = 20 x AD

In ΔADE, $\angle AED = 90°$,

$AE^2 + DE^2 = AD^2$

$9^2 + 12^2 = AD^2$

$81 + 144 = AD^2$

225 = AD

∴ AD = 15 cm

So, area of n $\Delta ADE = \frac{1}{2} \times 9 \times 12$

= 54 cm²

Also, area of rectangle = 20 x 15

= 300 cm²

So, area of semicircle $= \frac{1}{2} \times \frac{22}{7} \times \left(\frac{15}{2}\right)^2$

(∵ BC = AD = 15 cm)

$= \frac{1}{2} \times \frac{\cancel{22}^{11}}{7} \times \frac{225}{4}$

$= \frac{2475}{28}$ cm²

∴ Area of shaded region

$= 246 + \frac{2475}{28}$

$= \frac{9363}{28}$

= 334.393 cm²

40. Area of semicircle (with diameter CD)

$= \frac{1}{2} \times \frac{22}{7} \times 7^2$

= 77 cm²

area of rectangle ABCD

= AB x BC

= 14 x 7

= 98 cm²

area of semi-circle (with diameter BC)

$= \frac{1}{2} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2$

$= \frac{1}{2} \times \frac{22}{7} \times \frac{49}{4}$

= 19.25 cm²

Similarly, area of semicircle (with diameterAD)

= Area of rectangle ABCD

– area of semicircle (with diameter CD)

+ area of semicircle (with diameter BC)

+ area of semicircle (with diameter AD)

= 98 – 77 + 19.25 + 19.25

= 59.5 cm^2

CASE STUDY-1

(i) (a) Radius of semicircle a $= \frac{3}{2}$ cm

Radius of semicircle b and c $= \frac{3}{2}$ cm

Area of semicircle A $= \frac{\pi \left(\frac{3}{2}\right)^2}{2}$

$= \frac{\pi \times \frac{9}{4}}{2}$

$= \frac{9\pi}{8}$ cm^2

Area of semicircle B and C $= \frac{9\pi}{8}$ cm^2

Total area $= \frac{9\pi}{8} + \frac{9\pi}{8} + \frac{9\pi}{8} = \frac{27\pi}{8}$ cm^2

(ii) (b) Radius of semicircle E = Daimeter of circle D

Daimeter of semicircle E = Daimeter of semicircle A + Daimeter of semicircle B + Daimeter of semicircle C

= 3 cm + 3 cm + 3 cm

= 9 cm

Radius of semicircle E $= \frac{9}{2}$ cm

Daimeter of circle D $= \frac{9}{2}$ cm

Radius of circle D $= \frac{9}{4}$ cm

Area of circle D $= \pi h^2$

$= \pi \left(\frac{9}{4}\right)^2$

$= \frac{81}{16}\pi$ cm^2

(iii) (b) Radius of semicircle E $= \frac{9}{2}$ cm

Area of semicircle E $= \pi \frac{r^2}{2}$

$= \frac{\pi \left(\frac{9}{2}\right)^2}{2}$ cm^2

$= \frac{\pi \frac{81}{4}}{2}$ cm^2

$= \frac{81}{8}\pi$ cm^2

(iv) (d) Area of shaded region = Area of semicircle E – (Area of circle D + Area of semicircles A and C)

Area of semicircle A = Area of semicircle C $= \frac{9\pi}{8}$ cm^2

Area of shaded region

$= \frac{81}{8}\pi - \left(\frac{81}{16}\pi + \frac{18\pi}{8}\right)$ cm^2

$= \frac{162}{16}\pi - \frac{117}{16}\pi$

$= \frac{90}{32}\pi$ cm^2

(v) (b) Area of circle D $= \frac{81}{16}\pi\ cm^2$

Area of semicircle E $= \frac{81}{8}\pi\ cm^2$

Hence area of circle D is 2 times the area of semicircle E.

CASE STUDY-2

(i) (b) Area of sector $= \frac{\theta}{360}\pi r^2$

For sector OCD, $\theta = 60°$, r = 6 cm

Area $= \frac{60}{360}\pi(6)^2$

$= 6\pi\ cm^2$

(ii) (c) Area of blue region $= \frac{300°}{360°}\left[\pi(16)^2\right]$

$= \frac{30}{36}(36\pi)$

$= 30\pi\ cm^2$

(iii) (a) As AOB is an equilateral triangle $\therefore$ area of ΔAOB is $\frac{\sqrt{3}}{4}a^2$

Where 'a' is the side of triangle AOB

Area of $\Delta AOB = \frac{\sqrt{3}}{4}\times(10)^2$

$= 25\sqrt{3}\ cm^2$

Area of sector OCD $= 6\pi\ cm^2$

Area of red region = Area of ΔAOB – Area of sector OCD

$= \left(25\sqrt{3} - 6\pi\right) cm^2$

(iv) (a) Area of minor sector OCD (Yellow region) = $6\pi\ cm^2$

Area of major sector OCD (Blue region) = $30\pi\ cm^2$

$\therefore$ Area of major sector OCD is 5 times the area of minor sector.

(v) (a) Total area of red + yellow region is the area of ΔAOB i.e. $25\sqrt{3}\ cm^2$.

Chapter 13 Surface Areas and Volumes

Multiple Choice Questions

1. (a) Volume of piece of iron = $(49 \times 33 \times 24)$ cm²

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

A T Q;

$$\text{Volume of iron} = \text{Volume of sphere}$$

$$49 \times 33 \times 24 = \frac{4}{3}\pi r^3$$

$$49 \times 33 \times 24 = \frac{4}{3} \times \frac{22}{7} \times \pi r^3$$

$$r^3 = \frac{49 \times \cancel{33}^{3} \times \cancel{24}^{\cancel{6}\,3} \times 3 \times 7}{\cancel{4} \times \cancel{22}_{\cancel{2}}}$$

$$r^3 = 9261$$

$$r = \sqrt[3]{9261}$$

$$r = 21 \text{ cm}$$

2. (a) A.T.Q;

Volume of cone = Volume of cylinder

$$\frac{1}{3}\cancel{\pi}\cancel{r^2}h_1 = \cancel{\pi}\cancel{r^2}h_2$$

$$\frac{1}{3}h_1 = h_2$$

$$\frac{1}{3}h_1 = 5$$

$$h_1 = 15 \text{ cm}$$

3. (a) A T Q,

Volume of cylinder = Volume of cone

$$\cancel{\pi}r_1^2 h_1 = \frac{1}{3}\cancel{\pi}r_2^2 h_2$$

$$(18)^2\ (32) = \frac{1}{3} \times (r_2)^2 \times 24$$

$$(r_2)^2 = \frac{324 \times \cancel{32}^{4} \times \cancel{3}}{\cancel{24}_{\cancel{3}}}$$

$$(r_2)^2 = (18 \times 2)^2$$

$$r_2 = 36 \text{ cm}$$

4. (a) C. S. A. of cylinder = 264 m²

Volume of cylinder = 924 m³

$$\frac{\textit{C.S.A of cylinder}}{\textit{Volume of cylinder}} = \frac{264}{924}$$

$$\frac{2\cancel{\pi}\cancel{r}\cancel{h}}{\pi r^{\cancel{2}}\cancel{h}} = \frac{264}{924}$$

$$\frac{2}{r} = \frac{264}{924}$$

$$r = \frac{2 \times 924}{264}$$

$$r = 7 \text{ cm} \qquad \text{(i)}$$

We know;

C. S. A of cylinder = 264 cm²

$2\pi rh$ = 264 cm²

$$2 \times \frac{22}{7} \times 7 \times h = 264$$

$$h = \frac{\cancel{264}^{24} \times \cancel{7}}{2 \times \cancel{22}_{2} \times \cancel{7}} = 6 \text{ m}$$

$$\text{Ratio of Diameter to height} = \frac{2r}{h} = \frac{2 \times 7}{6}$$

$$= \frac{7}{3}$$

5. (a) In the right circle cone, the cross section made by a plane parallel to its base is a circle.

WORKSHEET - 1

SECTION-A

1. A.T. Q;

Radius of cylinder = Radius of sphere

Diameter of sphere = $2\pi r$

= $2r$

2. Surface area of cube = $6a^2$

Surface area of sphere = $4\pi r^2$

A T Q;

Surface area of cube = Surface area of sphere

$$6a^2 = 4\pi r^2$$

$$3a^2 = 2\pi r^2$$

$$\left(\frac{r}{a}\right)^2 = \frac{3}{2\pi}$$

$$\frac{r}{a} = \frac{\sqrt{3}}{\sqrt{2}\sqrt{\pi}}$$

$$\Rightarrow \frac{\textit{Volume of sphere}}{\textit{Volume of cube}} = \frac{\frac{4}{3}\pi r^3}{a^3}$$

$$= \frac{4}{3}\pi\left(\frac{r^3}{a^3}\right)$$

$$= \frac{4}{3}\pi\left(\frac{\sqrt{3}}{\sqrt{2}\sqrt{\pi}}\right)^2$$

$$= \frac{\cancel{4}^2}{\cancel{3}}\pi\left(\frac{\cancel{3}\sqrt{3}}{\cancel{2}\sqrt{2}\,\pi\sqrt{\pi}}\right)$$

$$\frac{\textit{Volume of sphere}}{\textit{Volume of cube}} = \frac{\sqrt{6}}{\sqrt{\pi}}$$

3. Volume of sphere = n × Volume of cones

$$\frac{4}{3}\cancel{\pi}r^3 = n\times\frac{1}{3}\cancel{\pi}r^2h$$

$$4\times(10.5)^3 = n\times(3.5)^2\,(3)$$

$$n = \frac{4\times(10.5)^3}{(3.5)^2\times 3} = 126 \text{ cones}$$

4. Volume of sphere = Volume of cone

$$\frac{4}{\cancel{3}}\cancel{\pi}r^3 = \frac{1}{\cancel{3}}\cancel{\pi}r_1^2\ (R)$$

$$4r^3 = r_1^{\,2}\ (R)$$

$$\frac{4r^3}{R} = r_1^{\,2}$$

$$r_1 = \frac{2r\sqrt{r}}{\sqrt{R}}$$

5. Volume of frustum = $\frac{\pi h}{3}\left[R_1^2+R_2^2+R_1R_2\right]$

$$R_1 = \frac{10}{2} = 5 \text{ m}$$

$$R_2 = \frac{4}{2} = 2 \text{ m}$$

$$h = 6 \text{ m}$$

Volume of frustum = $\frac{\pi\times 6}{3}\left[5^2+2^2+(5)(2)\right]$

$$= 2\pi\,[39]$$

$$= 245 \text{ m}^3$$

6. Total surface area of canvas

= Sum of curved surface area and curved surface area of cylinder

C. S.A of cone = πrl

= $(\pi)\,(105)\,(40)$

= 4200π m²

C. S.A of cylinder = $2\pi rh$

= $(2)\,(\pi)\,(105)\,(4)$

= $840\,\pi$ m²

Total surface area of canvas $= 4200\pi + 840\pi$

$$= 5040\pi$$

$$= 5040 \times \frac{22}{7}$$

$$= 720 \times 22$$

$$= 15840 \text{ m}^2$$

7. Surface area of hemisphere = Surface area of cone

$$3\pi r^2 = \pi rl + \pi r^2$$

$$2\not{\pi} r^{\not{2}} = \not{\pi}\not{r}l$$

$$2r = \sqrt{r^2+h^2} \quad [\,l = \sqrt{r^2+h^2}\,]$$

Squaring b/s :-

$$4r^2 = r^2 + h^2$$

$$3r^2 = h^2$$

$$\frac{r}{h} = \frac{1}{\sqrt{3}}$$

8. Volume of hemisphere

= Surface area of hemisphere

$$\frac{2}{3}\not{\pi} r^{\not{3}} = 3\not{\pi} r^{\not{2}}$$

$$\frac{2}{3}r = 3$$

$$r = \frac{9}{2}$$

$$2r = 9 \text{ cm}$$

Diameter of hemisphere = 9 cm.

SECTION-B

9. Width of canal = 30 m

Depth of canal = 12 m

Flow velocity = 10 km/hr = 10, 000 m/hr

Standing water required = 8 cm = 0.8 m

Time = 30 minutes $= \frac{1}{2}$ hr = 0.5 hr.

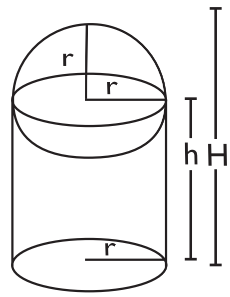
Area irrigated by 0.08 m

$$= 12 \times 30 \times 10,000 \times 0.5$$

$$A \times 0.08 = 1,800,000$$

$$A = \frac{1,800,000}{0.08}$$

$$A = 22,500,000 \text{ m}^2$$

10. 

Given,

Volume of air $= 41\frac{19}{21}$ m^3

$2r = H$ ...(i)

Total height of the building

= Height of cylinder

Total height of the building

= Height of hemisphere

$H = h + r$ ...(ii)

From (i) and (ii) :

$$2r = h + r$$

$$h = r$$

Volume of building = Volume of cylinder

+ Volume of hemisphere

$$\frac{880}{21} = \pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \pi r^2 \left[h + \frac{2}{3} r \right]$$

$$= \pi r^2 \left[\frac{5}{3} r \right]$$

$$\frac{880}{21} = \pi r^2 \left[\frac{5}{3} \right]$$

$$r^3 = \frac{880 \times \cancel{3} \times \cancel{7}}{\cancel{321} \times 5 \times 22} = 8$$

$$h = 2 \text{ m}$$

$$H = h + r$$

$$H = 2 + 2$$

$$H = 4 \text{ m}$$

11. Let radius cone Y = r

So, radius of cone X = 3r

Let volume of cone Y = V

So, volume of cone X = 2V

Let height of cone X be h_1 and

height of cone Y be h_2

$$\frac{\text{Volume of cone } X}{\text{Volume of cone } Y} = \frac{\frac{1}{3}\pi (3r)^2 (h_1)}{\frac{1}{3}\pi (r)^2 (h_2)}$$

$$\frac{2\cancel{V}}{\cancel{V}} = \frac{9h_1}{h_2}$$

$$\frac{h_1}{h_2} = \frac{2}{9}$$

12.
$$l \times b = x$$

$$b \times h = y$$

$$h \times l = z$$

$$(l \times b) \times (b \times h) \times (h \times l) = x\,y\,z$$

$$(l\,b\,h)^2 = x\,y\,z$$

$$l\,b\,h = \sqrt{xyz}$$

$$\text{Volume of cuboid} = \sqrt{xyz}$$

13. Total area without dimples = $\pi r^2 \times n$

$$= \frac{22}{7} (0.2)^2 \times 150$$

$$= 18.857 \text{ cm}^2$$

Total area of without dimples = $4\pi r^2$

$$= 4 \times \frac{22}{7} \times \left(\frac{4.1}{2}\right)^2$$

$$= 52.83 \text{ cm}^2$$

Area where there are no dimples

$$= (52.83 - 18.857) \text{ cm}^2$$

$$= 33.714 \text{ cm}^2$$

Surface area exposed to surroundings

$$= (33.973 + 37.714) \text{ cm}^2$$

$$= 71.687 \text{ cm}^2$$

14. Volume of resulting spheres

= Volume of three spheres

$$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 + \frac{4}{3}\pi r_3^3$$

$$\frac{4}{\cancel{3}}\cancel{\pi} r^3 = \frac{4}{\cancel{3}}\cancel{\pi} (r_1^3 + r_2^3 + r_3^3)$$

$$r^3 = (6^3 + 8^3 + 10^3)$$

$$r^3 = (216 + 512 + 1000)$$

$$r^3 = 1728$$

$$r = 12 \text{ cm}$$

15. Let the height of platform be 'h' metres.

Volume of mud dug out from the well

= Volume of platform

$$770 = 22 \times 14 \times h$$

$$h = \frac{770}{22 \times 14}$$

$$h = 2.5 \text{ m}$$

16.
$$2\pi R = 18 \text{ cm}$$

$$2 \times \frac{22}{7} \times R = 18 \text{ cm}$$

$$R = \frac{18 \times 7}{2 \times 22}$$

$$R = \frac{9}{\pi} \text{ cm}$$

$$2\pi r = 6$$

$$r = \frac{6}{2\pi}$$

$$r = \frac{3}{\pi}$$

Given; $l = 4$ cm

Curved surface area of frustum

$$= \pi (R + r) l$$

$$= \pi \left(\frac{9}{\pi} + \frac{3}{\pi}\right) (4)$$

$$= \not{\pi} \frac{12}{\not{\pi}} (4)$$

$$= 48 \text{ cm}^2$$

17. Given,

Area of valley $= 97280$ km²

Rainfall $= 10$ cm $= 0.00010$ km

Volume of rainwater

$= \text{Area of valley} \times \text{Rainfall}$

$= 97280 \times 0.00010 \text{ km}^3$

$= 0.97280 \text{ km}^3$

Volume in 1 day $= \frac{0.97280}{14} = 0.7 \text{ km}^3$

Volume of a river $= l \times b \times h$

$= (1072 \times \frac{75}{1000} \times \frac{3}{1000}) \text{ km}^3$

$= 0.2412 \text{ km}^3$

Volume of 3 rivers $= 3 \times 0.2412$

$= 0.7236 \text{ km}^3$

18. h of cone $= 12$ cm (Given)

r of cone $= 4.5$ cm

slant height (l) $= \sqrt{h^2 + r^2}$

$= \sqrt{(12)^2 + (4.5)^2}$

$= \sqrt{144 + 20.25}$

$= \sqrt{164.25}$

$l = 12.81$ cm

SECTION-C

19. Capacity of drinking glass

$$= \frac{1}{3}\pi h (r_1^2 + r_2^2 + r_1 r_2)$$

$$= \frac{1}{3}\pi (14) (2^2 + 1^2 + 2 \times 1)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 14 (7)$$

$$= \frac{308}{3} = 102.6 \text{ cm}^3$$

20. Radius of sphere $= \frac{6\,cm}{2} = 3$ cm

Radius of wire $= \frac{2\,cm}{2} = 1$ cm

Volume of sphere = Volume of cylinder (wire)

$$\frac{4}{3}\not{\pi} r^3 = \not{\pi} r^2 h$$

$$\frac{4}{3}(3)^3 = (1)^2 h$$

$$4 \times 9 = h$$

$$h = 36 \text{ cm}$$

$\therefore$ length of wire $= 36$ cm

21. Height of cone $= 9$ cm

Radius of cone $= \frac{24}{2} = 12$ cm

Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi (12)^2 (9)$

$= 432\pi \text{ cm}^3$

Height of cylinder = 110 cm

Radius of cylinder = 12 cm

Volume of cylinder = $\pi r^2 h$

$= \pi (12)^2 (110)$

$= 15840\ \pi\ cm^3$

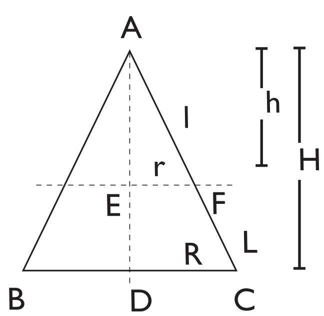
Volume of irom pole = $432\ \pi + 15840\ \pi$

$= 16272\ \pi$

$= 51120\ cm^3$

Mass of pole = 408960 g

22.



Let R, H and L be the radius, height and slant height of the larger cone. Let r, h and l be the radius, height and slant height of smaller cone.

Consider ΔADC and ΔAEF

$$\frac{r}{R} = \frac{h}{H} = \frac{l}{L} \qquad ...(i)$$

C. S. A. of smaller cone = πrl

C. S. A. of larger cone = πRL

$$RL - \pi rl = \frac{8}{9}$$

$$\Rightarrow \quad \frac{1}{9}\pi RL = \pi rl$$

$$\Rightarrow \quad \frac{L}{r} \times \frac{R}{r} = 9$$

$$\frac{H}{h} \times \frac{H}{h} = 9 \qquad \text{[From (i)]}$$

$$\frac{H}{h} = 3$$

$$\Rightarrow \quad \frac{H - h}{h} = \frac{3h - h}{h} = 2$$

Required ratio = h : (H – h)

= 1 : 2

23. Volume of sphere = Volume of cylinder

$$\frac{4}{3}\cancel{\pi} r_s^3 = \cancel{\pi} r_c^2 h \qquad (r_s = 4.2\text{ cm}) \quad (r_c = 6\text{ cm})$$

$$\frac{4}{3}(4.2)^3 = (6)^2 h$$

$$\frac{4}{3} \times \frac{(4.2)^3}{(6)^2} = h$$

$$2.744\text{ cm} = h$$

24. Radius of cylinder = 6 cm (Given)

Height of cylinder = 15 cm

Radius of cone and hemisphere = 3 cm (Given)

Height of cone = 12 cm

A.T.Q.;

Volume of cylinder = Volume of cone + volume of hemisphere

$$\cancel{\pi} r_{cy}^2 h = \left(\frac{1}{3}\cancel{\pi} r_c^2 h + \frac{2}{3}\cancel{\pi} r_h^3\right) \times n$$

$$(6)^2 \times 15 = \left(\frac{1}{3} \times (3)^2 \times 12 + \frac{2}{3}(3)^3\right) \times n$$

$$540 = (36 + 18) \times n$$

$$\frac{540}{54} = n$$

n = 10 cones.

25. Diameter of copper wire = 3 mm or 0.3 cm

Number of rounds of copper wire around cylinder

$$= \frac{\textit{Height of cylinder}}{\textit{Diameter of wire}} = \frac{12}{0.3} = 40 \text{ rounds}$$

Wire required in round

$= 2\pi r$ (Circumference of base cylinder)

$= 2 \times \pi \times 5$

$= 10\pi$ cm

Length required in 40 rounds

$= 40 \times 10\pi = 400\pi$

$= 400 \times \frac{22}{7}$

$= 1257.14$ cm

Radius of wire $= \frac{0.3}{2} = 0.15$ cm

Volume of wire = Area of wire × Length of wire

$= \pi r^2 \times 1257.14$

$= \frac{22}{7} \times (0.15)^2 \times 1257.14$

$= 88.898$ cm^3

Mass of wire = Density x Volume

$= 8.88 \times 88.898$

Mass of wire = 789.41 g.

26. Radius of hemisphere $= \frac{14}{2} = 7$ cm

Curved surface area of hemisphere

$= 2\pi 7^2$

$= 2 \times \frac{22}{7} \times (7)^2$

$= 308$ cm^2

Height of cylinder = Total height − Height of hemisphere

$= 13 - 7 = 6$ cm

Curved surface area of cylinder

$= 2\pi rh$

$= 2 \times \frac{22}{7} \times 7 \times 6$

$= 264$ cm^2

Inner surface area of the vessel

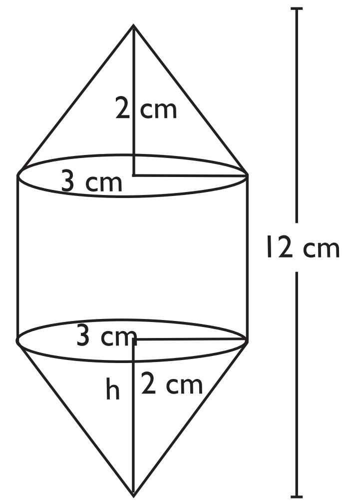
= C S A of cylinder + C S A of hemisphere

$= 264 + 308$

$= 572$ cm^2

SECTION-D

27.



Volume of model = Volume of cylinder + Volume of 2 cones

Volume of cylinder $= \pi r^2 h$

r = Radius of cylinder

$= \frac{3}{2}$ cm

h = Height of cylinder

$h = 12 - (2 + 2) = 12 - 4$

$= 8$ cm

Volume of cylinder $= \frac{22}{7} \times \left(\frac{3}{2}\right)^2 \times 8$

$= 56.57$ cm^3

Volume of cylinder $= \frac{1}{3} \times \frac{22}{7} \times \left(\frac{3}{2}\right)^2 \times 2$

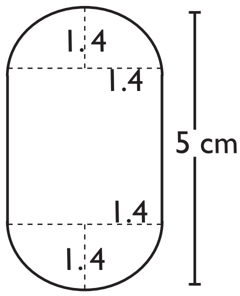
$= \frac{33}{7} = 4.7$ cm^3

Volume of model containing air

$= 56.57 + 2 \times 4.71$

$= 65.99$ cm^3

28.



Total Volume of gulab-jamun

= Volume of cylinder + Volume of 2 hemispheres

Volume of cylinder = $\pi r_c^2 h_c$

r_c = Radius of cylinder

$= \frac{2.8}{2} = 1.4$ cm

h_c = Height of cylinder

$= 5 - 2 \times (1.4)$

$= 5 - 2.8 = 2.2$ cm

Volume of cylinder $= \frac{22}{7} \times (1.4)^2 \times (2.2)$

$= 13.55$ cm³

Volume of 2 hemispheres

$= 2 \times \left(\frac{2}{3}\pi r_h^3\right)$

$= 2 \times \frac{2}{3} \times \pi \times (1.4)^3$

$= \frac{2}{3} \times 2 \times \frac{22}{7} \times (1.4)^3$

$= 11.50$ cm³

Volume of gulab-jamun = (11.50 + 13.55) cm³

$= 25.05$ cm³

Volume of sugar syrup in 1 gulabjamun

$= \frac{30}{100} \times 25.05$

$= 7.51$ cm³

Volume of sugar syrup for 45 gulabjamuns

$= 45 \times 7.51 = 337.95$

$= 338$ (approx)

29. Radius of cylindrical tank $= \frac{10}{2} = 5$ m

$= 500$ cm

Height of cylindrical tank = 200 cm = (2 m)

Volume of cylindrical tank $= \pi r^2 h$

$= \pi (500)^2 (200)$

Time taken $= \frac{\textit{Volume of cylindrical tank}}{\textit{Volume of water flowing in 1 hr}}$

Time taken $= \frac{\cancel{\pi}(500)^2(200)}{10 \times 300000 \times \cancel{\pi} \times 10}$

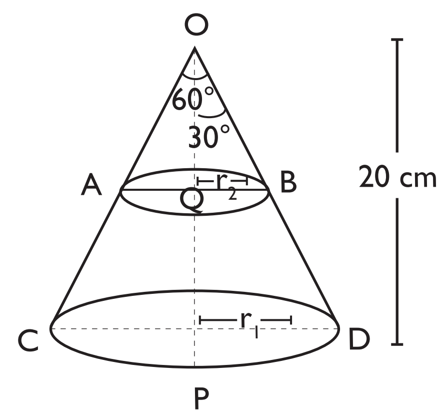
$= \frac{5\cancel{0}\cancel{0} \times 5\cancel{0}\cancel{0} \times 2\cancel{0}\cancel{0}}{1\cancel{0} \times 3\cancel{0}\cancel{0}\cancel{0}\cancel{0}\cancel{0} \times 10}$

$= \frac{50}{30} = \frac{5}{3}$ hr

$= \frac{5}{3} \times 60$

Time taken = 100 minutes

30.



Given,

OP = 20 cm

QO = QP = 10 cm

∠QOB = 30°

In ΔDOP,

$\tan\theta = \frac{PD}{OP}$

$\tan 30° = \frac{r_1}{20}$

$$r_1 = \frac{20}{\sqrt{3}}$$

In ΔBOQ

$$\tan\theta = \frac{BQ}{OQ}$$

$$\tan 30° = \frac{r_2}{10}$$

$$r_2 = \frac{10}{\sqrt{3}} \text{ cm}$$

Volume of frustum ADBC

$$= \frac{\pi h}{3}\left(r_1^2 + r_2^2 + r_1 r_2\right)$$

$$= \frac{\pi \times 10}{3}\left(\left(\frac{20}{\sqrt{3}}\right)^2 + \left(\frac{10}{\sqrt{3}}\right)^2 + \frac{20}{\sqrt{3}} + \frac{10}{\sqrt{3}}\right)$$

$$= \frac{10\pi}{3}\left(\frac{400}{3} + \frac{100}{3} + \frac{200}{3}\right)$$

$$= \frac{10\pi}{3} \times \frac{700}{3}$$

$$= \frac{7000\pi}{9} \text{ cm}^3$$

Volume of wire $= \pi r_w^2 h$

$$= \pi\left(\frac{1}{32}\right)^2 \times h \quad \left[r_w = \frac{\frac{1}{16}}{2} = \frac{1}{32} cm\right]$$

$$= \frac{\pi h}{32 \times 32}$$

Volume of frustum = Volume of wire

$$\frac{7000\not\pi}{9} = \frac{\not\pi h}{32 \times 32}$$

$$h = \frac{7000 \times 32 \times 32}{9} \text{ cm}$$

$$h = 796444.\overline{44} \text{ cm}$$

31. Radius of cylindrical pipe $= \frac{5}{2}$ mm

$$= \frac{5}{2} \times \frac{1}{10} \text{ cm}$$

$$= \frac{1}{4}$$

Speed of water = 10 m/ min

= 1000 cm/ min

Volume of water flowing in 1 minute

$$= \pi r^2 h$$

$$= \frac{22}{7} \times \left(\frac{1}{4}\right)^2 \times 1000$$

$$= \frac{1375}{7} \text{ cm}^3$$

Radius of cone $= \frac{40}{2} = 20$ cm

Depth of cone = 24 cm

Capacity of conical vessel $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times (20)^2 \times 24$$

$$= \frac{70400}{7} \text{ cm}^3$$

Time required

$$= \frac{\textit{Capacity of vessel}}{\textit{Volume of water flowing in 1 minute}}$$

$$= \frac{\frac{70400}{7}}{\frac{1375}{7}} = \frac{256}{5}$$

= 51 min 12 sec

32. Volume of water left = Volume of cylinder – (Volume of cone + Volume of hemisphere)

Volume of cylinder $= \pi r_c^2 h_c$

$r_c = 5$ cm

$h_c = 10.5$ cm

Volume of cylinder $= \pi (5)^2 (10.5)$

$= 262.5\pi$ cm³

Volume of cone $= \frac{1}{3}\pi (3.5)^2 (4)$

$= 16.33\pi$ cm³

Volume of hemisphere = $\frac{2}{3}\ \pi r_h^3$

r_h^3 = 3.5 cm

Volume of hemisphere = $\frac{2}{3}\ \pi\ (3.5)^3$

= 28.58 π cm^3

Volume of water left = 262.5 π

– (16.33 π + 28.58 π)

= π (217.59) cm^3

= $\frac{22}{7} \times 217.59\ cm^3$

= 683.579 cm^3

33. Radius of cylinder = $\frac{4.3}{2}$ = 2.15 m

Height of cylinder = 3.8 m

C.S.A. of cylinder = 2 πrh

= $2 \times \frac{22}{7} \times 2.15 \times 3.8$

= $\frac{359.48}{7}\ m^2$

= 51.3543 m^2

As vertical angle of cone is a right angle.

Let ABC be a triangle

l = AB = AC

$l^2 + l^2 = BC^2$

$2l^2 = 4.3^2$

$l^2 = 9.245$

l = 3.04 m

We know,

$l^2 = r^2 + h^2$

$(3.04) = (2.15)^2 + h^2$

$9.2416 = 4.6225 + h^2$

$h^2 = 9.2416 - 4.6225$

$h = \sqrt{4.6191}$

h = 2.149 m

C.S.A. of cone = πrl

= $\frac{22}{7} \times 2.15 \times 3.04$

= 20.5417 m^2

T.S.A. of building = C.S.A. of cylinder + C.S.A. of cone

= 51.3543 + 20.5417

= 71.896 m^2

Volume of building = Volume of cone + Volume of cylinder

Volume of cone = $\frac{1}{3}\ \pi r^2 h$

= $\frac{1}{3} \times \frac{22}{7} \times (2.15)^2 \times 2.15$

= 10.4116 m^3

Volume of cylinder = $\pi r^2 h$

= $\frac{22}{7} \times (2.15)^2 \times 3.8$

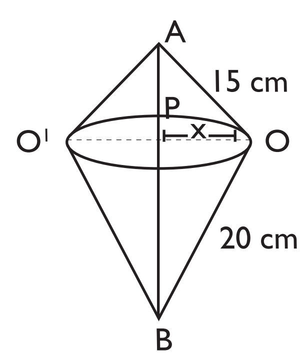
= 55.2059 m^3

Total Volume of building

= 10.4116 + 55.2059

= 65.6175 m^3

34.



Let ΔAOB is the right triangle with AB as hypotenuse.

Let OA = 15 cm

OB = 20 cm

In ΔAOB

$$AB = \sqrt{OA^2 + OB^2}$$

$$= \sqrt{15^2 + 20^2}$$

$= 25$ cm

Let $OP = x$ and $AP = y$

Area of ΔAOB = Area of ΔAPO + Area of ΔBPO

$$\frac{1}{2} \times 15 \times 20 = \frac{1}{2}(y)(x) + \frac{1}{2}(x)(25 - y)$$

$$300 = xy + 25x - xy$$

$$x = 12$$

In ΔAPO

$$AO^2 = AP^2 + PO^2$$

$$225 = y^2 + 144$$

$\boxed{y = 9 \text{ cm}}$

$AP = 9$ cm

$BP = 25 - 9$

$= 16$ cm

Volume of cone AOO' $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\pi (PO)^2 (AP)$$

$$= \frac{1}{3}\pi (144)(9)$$

Volume of cone BOO' = 1357.17 cm^3

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi (PO)^2 (BP)$$

$$= \frac{1}{3}\pi (144)(16)$$

$= 2412.74$ cm^3

Net volume of cone = Volume of BOO' + Volume of AOO'

$= 2412.74 + 1357.17 = 3770$ cm^3

Surface area of cone AOO' $= \pi rl$

$= \pi$ (PO) (AO)

$= \pi (12)(15) = 1$

$= 180\pi$

Surface area of cone BOO' $= \pi rl$

$= \pi (12)(20) = 240\pi$

Net surface area $= 100\pi + 240\pi = 320\pi$ cm^2

WORKSHEET - 2

SECTION-A

1. Volume of cone are in the ratio 1 : 4 diameters are in the ratio 4 : 5

So, radius are also in the ratio 4 : 5.

$$\frac{\text{Volume of cone 1}}{\text{Volume of cone 2}} = \frac{1}{4}$$

$$\frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \frac{1}{4}$$

$$\left(\frac{r_1}{r_2}\right)^2 \times \frac{h_1}{h_2} = \frac{1}{4}$$

$$\left(\frac{4}{5}\right)^2 \times \frac{h_1}{h_2} = \frac{1}{4}$$

$$\frac{16}{25} \times \frac{h_1}{h_2} = \frac{1}{4}$$

$$\frac{h_1}{h_2} = \frac{25}{64}$$

2.
$$4\pi r^2 = 616$$

$$4 \times \frac{22}{7} \times r^2 = 616$$

$$r^2 = \frac{616 \times 7}{22 \times 4}$$

$$r^2 = \frac{4312}{88}$$

$$r^2 = 49$$

$$r = 7 \text{ cm}$$

3. Radius of hemisphere = Radius of cone = r

Also, Height of cone = Height of Hemisphere = r

l (Slant height) = $\sqrt{h^2 + r^2} = \sqrt{r^2 + r^2} = r\sqrt{2}$

$$\frac{\text{Curved surface area of hemisphere}}{\text{Curved surface area of cone}} = \frac{2\pi r^2}{\pi r l}$$

$$= \frac{2\pi r^2}{\pi r\sqrt{2}r}$$

$$= 2 : \sqrt{2}$$

$$= \sqrt{2} : 1$$

4. The total thickness of the plates become the height of cylinder.

Net thickness = $\frac{1}{2}$ cm × 50 = 25 cm

radius = 7 cm

Total surface area of cylinder = $2\pi r (r + h)$

= 2π (7 cm) (7 cm + 25 cm)

= 1408 cm^2

Volume of cylinder = $\pi r^2 h$

= π (7 cm)^2 (25 cm)

= 3850 cm^3

5. Volume of water = Volume of conical flask

$$= \frac{1}{3}\pi r^2 h$$

As the water is poured into the cylindrical flask,

So, Volume of cylinder = Volume of water

$$\cancel{\pi}(mr)^2 H = \frac{1}{3}\cancel{\pi} r^2 h$$

$$H = \frac{h}{3m^2}$$

6. Height of cylinder (h) = 12 cm

Radius of cylinder (r) = $\frac{12}{2}$ = 6 cm

A.T.Q.,

Surface area of sphere = C. S.A. of cylinder

$$4\cancel{\pi}r_s^2 = 2\cancel{\pi}rh$$

$$2r_s^2 = (12)(6)$$

$$r_s^2 = (6)(6)$$

$$r_s = 6 \text{ cm}$$

7. Radius of greatest sphere inside the log

= radius of the log

So; Volume of sphere = $\frac{4}{3}\pi r^3$

$$= \frac{4}{3}\pi (1)^3$$

$$= \frac{4}{3} \times \frac{22}{7}$$

$$= \frac{88}{21} = 4.19 \text{ cm}^3$$

8. In the hemisphere,

Height of hemisphere = Radius of hemisphere

h = r

For the volume of cone,

Radius of cone = Radius of hemisphere

R = r

Volume of cone = $\frac{1}{3}\pi R^2 h$

$$= \frac{1}{3}\pi r^2 h \quad [R = r]$$

$$= \frac{1}{3}\pi r^3 \quad [h = r]$$

9. After r = 7 cm

Volume of sphere = $\frac{4}{3}\pi r^3$

$$= \frac{4}{3}\pi (7)^3$$

$$= 1436.75 \text{ cm}^3$$

10. Slant height (l) = 5 cm

$R = r_1 - r_2 = 4$ cm

$l^2 = R^2 + h^2$

$5^2 = R^2 + h^2$

$5^2 = 4^2 + h^2$

$h^2 = 5^2 - 4^2$

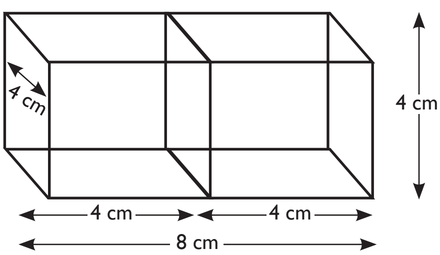
$h^2 = 25 - 16$

$h^2 = 9$

h = 3 cm

SECTION-B

11.



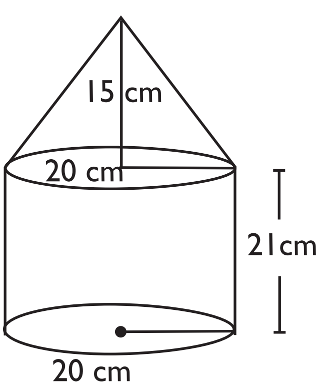
The length of the cuboid becomes 8 cm. Breadth and height will remain 4 cm. The surface area of cuboid = 2(lb + bh + hl)

= 2 [(8 × 4) + (4 × 4) + (4 × 8)]

= 2 [80 cm^2]

= 160 cm^2

12.



Slant height of cone = $\sqrt{r^2 + h^2}$

$l = \sqrt{(20)^2 + (15)^2}$

$l = \sqrt{400 + 225}$

$l = \sqrt{625}$

= 25 cm

Total surface area of toy

= Curved surface area of cone
+ Curved surface area of cylinder
+ Area of bottom part of cylinder

$= \pi rl + 2\pi r_c h_c + \pi r_c^2$

$= \pi [rl + 2\pi r_c h_c + r_c^2]$

$= \pi [(20)(25) + (2)(20)(21) + (20)^2]$

$= \pi [500 + 840 + 400]$

$= \frac{22}{7} \times 1740$

$= 22 \times 248.57$

$= 5468.54$ cm^2

13. Height of cylinder (h) = 10 cm

Radius of cylinder (r) = 3.5 cm

Radius of hemisphere (R) = Radius of cylinder

= 3.5 cm

Total surface area of article
= Curved surface area of cylinder
+ Curved surface area of 2 hemispheres

Total surface area of article = $2\pi rh + 2(2\pi r^2)$

$= 2\pi [rh + 2R^2]$

$= 2 \times \frac{22}{7} [3.5 \times 10 + 2(3.5)^2]$

$= 2 \times \frac{22}{7} [35 + 24.5]$

$= 2 \times \frac{22}{7} \times 59.5$

= 374 cm^2

14. Height = 18

Radius = R = 18 cm and r = 12 cm

$l = \sqrt{(R - r)^2 + h^2}$

$l = \sqrt{(18-12)^2+8^2}$

$l = \sqrt{6^2+8^2}$

$l = \sqrt{100}$

$l = 10$ cm

Total surface area = π (R + r) l + π (R^2 + r^2)

$= \pi [(R + r) l + (R^2 + r^2)]$

$= \pi [(18 + 12) (10) + (18^2 + 12^2)]$

$= \pi [300 + 468]$

$= \pi [768] = \frac{22}{7} \times 768 = 2413.71 \text{ cm}^2$

15. Total surface area of remaining solid

= C. S.A. of cylinder + Area of upper part

+ Curved surface area of cone

$= 2 \pi rh + \pi r^2 + (\pi r_c h)$

Radius of cylinder (r) = 6 cm

Height of cylinder (h) = 20 cm

Radius of cone (r_c) = 6 cm

Slant height of cone (l) $= \sqrt{r_c^2 + h}$

$= \sqrt{6^2+8^2}$

$= \sqrt{100}$

l = 10 cm

Total surface area of remaining solid

$= 2 \times \frac{22}{7} \times 6 \times 20 + \frac{22}{7} \times 6^2 + \left(\frac{22}{7} \times 6 \times 10\right)$

$= \frac{22}{7} [240 + 36 + 60]$

$= \frac{22}{7} \times 336$

$= 1056 \text{ cm}^2$

16. Water is flowing at 7 m/s

Radius of pipe (r) = 1 cm = 0.01 m

Radius of tank (R) = 40 cm = 0.4 m

Time = $\frac{1}{2}$ hr = 30 min = 1800 seconds

We know,

Volume of cylindrical tank

= Area of cross section

× speed of flowing water

× time

$\cancel{\pi} r^2 h = \cancel{\pi} R^2 \times$ Rate of flowing water × time

$(0.4)^2 h = (0.01)^2 \times 7 \times 1800$

$0.16h = 1.26$

$h = \frac{1.26}{0.16}$

$h = 7.875$ m

17. Let the radius be r and height be h.

ATQ

h = 6r

Total surface area is $2\pi r (r + h)$

$2\pi r (r + h) = 2\pi r (r + 6r)$

$= 14\pi r^2$

Cost of painting $= 14\pi r^2 \times \frac{60}{100}$

$14\pi r^2 \times \frac{60}{100} = 237.60$

r = 3 dm

Volume of cylinder $= \pi r^2 h$

$= \pi (3^2) (6 \times 3)$

$= 509 \text{ dm}^3$

18. Radius of well (r) = 5 m

Depth of well = 14 m

Volume of Earth taken out

$= \frac{22}{7} \times 5^2 \times 14$

$= 1100 \text{ m}^3$

As the Earth is spread around the embankment;

Inner radius $(r_1) = 5$ m

Outer radius $(r_2) = (5 + 5)$ m = 10 m

Height = h

Volume of Earth taken out = $\pi (r_2^2 - r_1^2) h$

$= 1100$

$= \frac{22}{7} (10^2 - 5^2) h$

$\frac{1100 \times 7}{22 \times (75)} = h$

$h = 4.67$ m

19. Radius of sphere (r) = $\frac{6}{2}$ = 3 cm

Radius of cylindrical vessel (R) = $\frac{12}{2}$ = 6 cm

Let water be raised by height 'h'

A.T.Q.,

Volume of water raised = Volume of sphere

$\cancel{\pi} R^2 h = \frac{4}{3} \cancel{\pi} r^3$

$R^2 h = \frac{4}{3} r^3$

$36 \times h = \frac{4}{3} \times (3)^3$

$h = \frac{4 \times 27}{36 \times 3} = \frac{108}{108}$

$= 1$ cm

20. Radius of hemisphere = $\frac{36}{2}$ = 18 cm

Radius of cylindrical bottle (R) = 3 cm

Height of cylindrical bottle (h) = 6 cm

A.T.Q.,

Volume of hemisphere = n × Volume of cylinder

$\frac{2}{3} \cancel{\pi} r^3 = n \times \cancel{\pi} R^2 h$

$\frac{2}{3} \times 18^3 = n \times 3^2 \times 6$

$\frac{2}{3} \times \frac{18^3}{3^2 \times 6} = n$

$\frac{11664}{162} = n$

n = 72 bottles.

SECTION-C

21. Let the radius of base of cylinder be 'r'

A.T.Q.,

Volume of cylinder = Volume of two cones

$\pi r^2 h = \frac{1}{3} \pi r^2 h + \frac{1}{3} \pi r_2^2 h$

$r^2 = \frac{1}{3} r_1^2 + \frac{1}{3} r_2^2$

$r^2 = \frac{r_1^2 + r_2^2}{3}$

$r = \sqrt{\frac{r_1^2 + r_2^2}{3}}$

22. Volume of bucket = $\frac{1}{3} \pi h (R^2 + r^2 + Rr)$

$5390 = \frac{1}{3} \pi (15) (196 + r^2 + 14r)$

$5390 \times 3 = \frac{22}{7} \times 15 (196 + r^2 + 14r)$

$\frac{5390 \times 3 \times 7}{22 \times 15} = 196 + r^2 + 14r$

$343 = 196 + r^2 + 14r$

$r^2 + 14r + 196 - 343 = 0$

$r^2 + 14r - 147 = 0$

$r^2 - 7r + 21r - 147 = 0$

$r (r - 7) + 21 (r - 7) = 0$

$r = -21$ or $r = 7$

As Radius can't be negative, So r = 7 cm

23. $r_1 = 18$ cm

$r_2 = 12$ cm

$h = 8$ cm

Total surface area of frustum = $\pi[(r_1 + r_2)\,l + r_1^2 + r_2^2]$

$= \pi\,[(18 + 12)\,l + 18^2 + 12^2]$

$l = \sqrt{h^2 + (r_1 - r_2)^2}$

$= \sqrt{8^2 + (18-12)^2}$

$= \sqrt{64+36}$

$= 10$ cm

Total surface area = $\pi\,[(30)\,(10) + 468]$

$= 2413.71$ cm^2

24. Volume of water in tank = $l \times b \times h$

$= 50 \times 44 \times \frac{21}{100}$

$= 462$ m^3

For cylindrical pipe,

$r = \frac{14}{2} = 7$ cm $= 0.07$ cm

Water is flowing at the rate of 15 km/hour.

Volume of cylindrical pipe = $\pi r^2 h$

$462 = \frac{22}{7} \times (0.07)^2 \times h$

$\frac{462 \times 7}{22 \times (0.07)^2} = h$

$h = 30{,}000$

Time $= \frac{30000}{15000} = 2$ hours

25. Surface area of sphere = $4\,\pi r^2$

$1386 = 4\,\pi r^2$

$\frac{1386 \times 7}{4 \times 22} = r^2$

$r^2 = 110.25$

$r = 10.5$ cm

Volume of sphere = $\frac{4}{3}\,\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times (10.5)^3$

$= \frac{4}{3} \times \frac{22}{7} \times 10.5 \times 10.5 \times 10.5$

$= 4851$ cm^3

A.T.Q.,

Volume of sphere = Volume of wire (cylinder)

$4851 = \pi r^2 h$

$4851 = \frac{22}{7} \times r^2 \times 31.5 \times 100$

[length of wire = 31.5 m]

$\frac{4851 \times 7}{22 \times 31.5 \times 100} = r^2$

$r^2 = 0.49$

$r = 0.7$ cm

$\therefore$ Diameter of wire = $0.7 \times 2 = 1.4$ cm

26. Sum of radius and height of cylinder = $(r + h)$ = 37 cm

Total surface area of cylinder = $2\,\pi r\,(r + h)$

$1628 = 2 \times \frac{22}{7} \times r\,(37)$

$\frac{1628 \times 7}{2 \times 22 \times 37} = r$

We know,

$r = 7$

$r + h = 37$

$7 + h = 37$

$h = 30$ cm

Volume of cylinder = $\pi r^2 h$

$= \frac{22}{7} \times 7^2 \times 30$

$= \frac{22}{7} \times 49 \times 30 = 4620$ cm^3

27. For cylindrical tub,

$$r = \frac{12}{2} = 6 \text{ cm}$$

$$h = 15 \text{ cm}$$

Volume of cylindrical tube = $\pi r^2 h$

$$= \pi \times 6^2 \times 15$$

$$= \pi \times 36 \times 15$$

$$= 540\pi$$

For ice-cream,

We know,

h = 2 × diameter = 2 × 2r = 4r

Radius of cone = Radius of hemisphere = R

Volume of ice- cream cone

= Volume of cone + Volume of hemisphere

$$= \frac{1}{3}\pi R^2 h + \frac{2}{3}\pi R^3$$

$$= \frac{1}{3}\pi R^2 (h + 2R)$$

$$= \frac{1}{3}\pi R^2 (4R + 2R)$$

$$= \frac{1}{3}\pi\, 6R^3$$

A.T.Q.,

Volume of cylindrical tub

= n × Volume of ice - cream cones

$$540 \not{\pi} = \frac{1}{3}\not{\pi}\, 6R^3 \times 10$$

$$540 \times 3 = 6R^3 \times 10$$

$$54 \times 3 = 6R^3$$

$$27 = R^3$$

$$R = 3 \text{ cm}$$

∴ Diameter of ice - cream cones

= 3 × 2 = 6 cm

28. Radius of hemisphere = Radius of cone

= r = 3.5 cm

Volume of total wood used

$$= 166\frac{5}{6} = \frac{1001}{6} \text{ cm}^3$$

Volume of wood used in toy

= Volume of hemisphere + Volume of cone

$$\frac{1001}{6} = \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$$

$$\frac{1001}{6} = \frac{1}{3}\pi r^2 (2r + h)$$

$$\frac{1001}{6} = \frac{22}{7} \times 3.5 \times 3.5\,(2 \times 3.5 + h)$$

$$\frac{1001 \times 7}{2 \times 22 \times 3.5 \times 3.5} = 7 + h$$

$$13 = 7 + h$$

$$h = 13 - 7$$

Height of cone = h = 6 cm

Height of toy = Height of cone + Height of hemispher

= 6 + 3.5

= 9.5 cm

Curved surface area of hemisphere = $2\pi r^2$

$$= 2 \times \frac{22}{7} \times (3.5)^2$$

$$= 77 \text{ cm}^2$$

∴ Cost of painting the hemispherical part

= 77 x 10

= ₹ 770.

29. Radius of cone (r) = $\frac{3.5}{2}$ cm

Height of cone (h) = 3 cm

A.T.Q.,

Volume of 504 cones = Volume of sphere

$$504 \times \frac{1}{3} \times \pi \times r^2 \times h = \frac{4}{3}\pi R^3$$

$$\frac{504}{3} \times \left(\frac{3.5}{2}\right)^2 \times 3 = \frac{4}{3} R^3$$

$$\frac{504 \times 3.5 \times 3.5 \times 3 \times 3}{3 \times 2 \times 4 \times 2} = R^3$$

$$1157.625 = R^3$$

$$R = 10.5 \text{ cm}$$

Total surface area of sphere = $4\pi R^2$

$$= 4 \times \frac{22}{7} \times 10.5 \times 10.5$$

$$= 1386 \text{ cm}^2$$

30. Height of cone (h) = 60 cm

Radius of cone (r) = 30 cm

Height of cylinder (H) = 180 cm

Radius of cylinder (R) = 60 cm

Volume water left = Volume of cylinder – Volume of cone

Volume of water left = $\pi R^2 H - \frac{1}{3}\pi r^2 h$

$$= \pi \left[(60)^2 (180) - \frac{1}{3}(30)^2 (60) \right]$$

$$= \pi [648000 - 18000]$$

$$= \frac{22}{7} \times 630000$$

$$= 1980000 \text{ cm}^3$$

$$= 1.98 \text{ m}^3$$

SECTION-D

31. Radius of well = $\frac{4}{2}$ = 2 m

Depth = 14 m

Volume of well = $\pi r^2 h$

$$= \frac{22}{7} \times 2^2 \times 14$$

$$= 176 \text{ m}^3$$

A.T.Q.,

Volume of well = Volume of embankment

$$176 = \pi (r_1^2 - r_2^2) h$$

[r^2 = inner radius = 2 m]

$$176 = \frac{22}{7}(r_1^2 - 4) \times \frac{40}{100}$$

$$140 = r_1^2 - 4$$

$$144 = r_1^2$$

$$r_1 = 12 \text{ m}$$

Width of embankment = (12 – 2) m

= 10 m

32. Radius of the water tank = 40 cm

Increase in water level = 3.15 m = 315 cm

Volume of water flowing in the tank in half on hour = $\pi r^2 h$

$$= \frac{22}{7} \times 40 \times 40 \times 315$$

$$= 1584000 \text{ cm}^3$$

Rate of water flow = 2.52 km/hr

Water flow in half an hour = $2.52 \times \frac{1}{2}$

= 1.26 km

= 126000 cm

Let internal diameter be 'd'

Water that flows in half an hour through pipe

$$= \left(\frac{d}{2}\right)^2 (126000)$$

We know,

Water flowing through pipe in half an hour

= Volume of water flowing in half an hour

$$\frac{22}{7} \times \frac{d^2}{4} \times 126000 = 1584000$$

$$d^2 = 16$$

$$d = \sqrt{16}$$

$$d = 4 \text{ cm}$$

33. Radius of hemisphere bowl $= \frac{36}{2} = 18$ cm

Volume of liquid in the bowl $= \frac{2}{3} \pi r^3$

$$= \frac{2}{3} \pi r^3$$

$$= \frac{2}{3} (18)^3 \pi$$

$$= 3888 \pi \text{ cm}^3$$

Diameter of the bottle = 6 cm

Radius of the bottle $(r_1) = \frac{6}{2} = 3$ cm

Volume of each bottle $= \pi r_1^2 h$

$$= \pi (3)^2 (h)$$

$$= 9 \pi h$$

A.T.Q.,

90 % of volume of liquid in bowl

= 72 × Volume of liquid in each bottle

$$\frac{90}{100} \times 3888 \not{\pi} = 72 \times 9 \not{\pi} h$$

$$3499.2 = 648 h$$

$$h = \frac{3499.2}{648}$$

$$h = 5.4 \text{ cm}$$

34. Volume of solid metal cylinder before scooping out $= \pi r^2 h$

$r = 4.2$ cm, $h = 10$ cm

$$\pi r^2 h = \pi \times (4.2)^2 \times 10$$

$$= 176.4 \pi \text{ cm}^3$$

Volume of scooped part $= \frac{4}{3} \pi r^3$

$$= \frac{4}{3} \pi (4.2)^3$$

$$= 98.8 \pi \text{ cm}^3$$

Volume of scooping metal cylinder

$$= 176.4 \pi - 98.8 \pi$$

$$= 77.6 \pi \text{ cm}^3$$

For wire,

Radius of wire $= \frac{1.4}{2} = 0.7$ cm

A.T.Q.,

Volume of wire = Volume of scooping metal cylinder

$$\not{\pi} r^2 h = 77.6 \not{\pi}$$

$$77.6 = r^2 h$$

$$77.6 = (0.7)^2 h$$

$$h = \frac{77.6}{0.49}$$

$$h = 158.36 \text{ cm}$$

∴ Length of the wire is 158.36 cm

35. Volume of water = Volume of cone

$$= \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 5^2 \times 8$$

$$= \frac{1}{3} \times \frac{22}{7} \times 25 \times 8$$

$$= 209.5 \text{ cm}^3$$

A.T.Q.,

Volume of 100 spherical balls

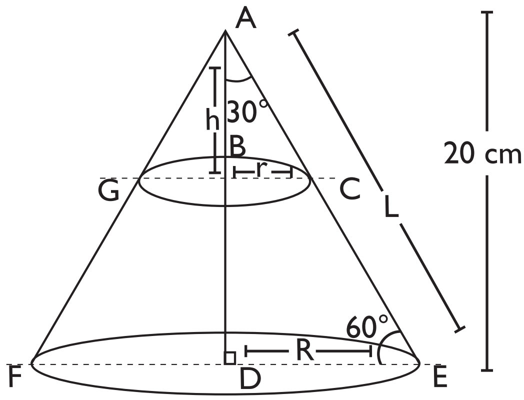
$$= \frac{1}{4} \text{ of volume of water}$$

$$100 \times \frac{4}{3} \pi R^3 = \frac{1}{4} \times 209.5$$

$$R^3 = \frac{209.5 \times 3 \times 7}{4 \times 22 \times 100 \times 4} = 0.125$$

$$R = \sqrt[3]{0.125} = 0.5 \text{ cm}$$

36.



Consider ΔABC & ΔADE;

$$\frac{h}{H} = \frac{r}{R} = \frac{l}{L}$$

$$\frac{h}{20} = \frac{r}{R}$$

In ΔADB

$$\angle ADE = 90° \ \& \ \angle DEA = 60°$$

$$\tan 60 = \frac{20}{DE}$$

$$\sqrt{3}\ DE = 20$$

$$DE = \frac{20}{\sqrt{3}}$$

$$R = \frac{20}{\sqrt{3}} \text{ cm}$$

In ΔABC

$$\tan 30 = \frac{r}{AB}$$

$$\frac{1}{\sqrt{3}} = \frac{r}{AB}$$

From (i)

$$r = \frac{10}{\sqrt{3}}$$

$$\text{Volume of frustum} = \frac{\pi h}{3}\ [R^2 + r^2 + Rr]$$

$$= \frac{\pi \times 10}{3}\left[\left(\frac{20}{\sqrt{3}}\right)^2 + \left(\frac{10}{\sqrt{3}}\right)^2 + \left(\frac{20}{\sqrt{3}}\right)^2\left(\frac{10}{\sqrt{3}}\right)^2\right]$$

$$= \frac{10\pi}{3}\left[\frac{400}{3} + \frac{100}{3} + \frac{200}{3}\right]$$

$$= \frac{10\pi}{3}\left[\frac{700}{3}\right]$$

$$= \frac{7000\pi}{9} \text{ cm}^3$$

Volume of cylinder = $\pi r^2 h$

$$\left[\ r = \frac{1}{\frac{12}{2}} = \frac{1}{24} \text{ cm}\right.$$

$$\text{Volume of cylinder} = \pi\left(\frac{1}{24}\right)^2 h$$

$$= \frac{\pi h}{24 \times 24}$$

$$= \frac{\pi h}{576} \text{ cm}^3$$

As Volume of frustum must be equal to volume of cylinder.

$$\frac{7000\pi}{9} = \frac{\pi h}{576}$$

$$h = \frac{7000 \times 576}{9}$$

$$h = 448000 \text{ cm}$$

or

(Length of wire) h = 4480 m.

37. Length of water flows in 40 minute

$$= \frac{20 \text{ km}}{60 \text{ mm}} \times 40 \text{ min}$$

$$= \frac{40}{3} \text{ km}$$

$$= \frac{40,000 \text{ m}}{3}$$

Volume of water flow = (length of water flow in 40 minutes) × (width of canal) × depth of canal)

$= \frac{40,000}{3} \times 5 \times 1 = \frac{200,000}{3}$ m^3

Let the area irrigated = xm^2

Height of standing water = 10 cm = $\frac{10}{100}$ m

$\therefore \frac{x \times 10}{100} = \frac{200,000}{3}$

$x = \frac{2000,000}{3}$ m^2

Since 10,000 m^2 = 1 hectare

$\therefore$ Area irrigated = $\frac{200}{3}$ hectare

38. h = 24 cm

Lower end = r = 8 cm

Upper end = R = 20 cm

Volume = $\frac{\pi h}{3}$ $(R^2 + r^2 + Rr)$

$= \frac{24\pi}{3}$ $[(20)^2 + (8)^2 + (20)(8)]$

$= 8\pi[400 + 64 + 160]$

$= 8\pi[624] = 8 \times \frac{22}{7} \times 624$

= 15689.142 cm^2

= 15.689142 l

Cost of milk per liter = ₹ 27

Total cost = 15.689142 × 27

= ₹ 423.606

39. ① Volume of bucket = Volume of frustum

$= \frac{1}{3}\pi h(R^2 + r^2 + Rr)$

$= \frac{1}{3} \times \frac{22}{7} \times 24(14^2 + 7^2 + 14 \times 7)$

$= \frac{1}{3} \times \frac{22}{7} \times 24(196 + 49 + 98)$

$= \frac{1}{3} \times \frac{22}{7} \times 24(343)$

= 8624 cm^3

② Slant height (l) = $\sqrt{h^2 + (R - r)^2}$

$= \sqrt{24^2 + (14 - 7)^2}$

$= \sqrt{576 + 49}$

$= \sqrt{625}$ = 25 cm

Area of metal sheet = C.S.A of frustum + Area of base

$= \pi l(R + r) + \pi r^2$

$= \pi[25(14 + 7) + 7^2]$

$= \frac{22}{7}[25(21) + 49]$

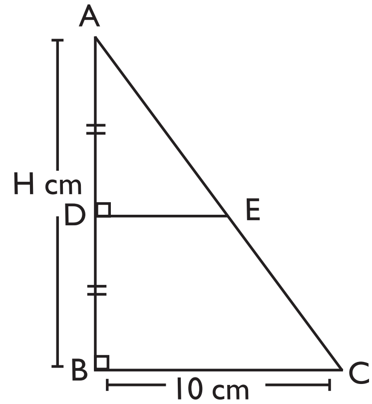
$= \frac{22}{7}[525 + 49]$

$= \frac{22}{7}[574]$

= 22 × 82

= 1804 cm^2

40.



Given that $AD = DB = \frac{h}{2}$ cm

Consider ΔADE and ΔABC

$\angle ADE = \angle ABC = 90°$

$\angle BAC = \angle DAE =$ (Common)

By Angle - Angle similarity [$\Delta ADE \sim \Delta ABC$]

So, $\frac{AD}{DB} = \frac{DE}{BC} = \frac{r}{10}$ (r is radius of cone)

$$\frac{\frac{H}{2}}{H} = \frac{r}{10}$$

$$= \frac{1}{2} = \frac{r}{10}$$

$r = 5$ cm

$\Rightarrow$ Volume of frustum = Volume of cone ABC – Volume of cone ADE

$$= \frac{1}{3}\pi R^2 H - \frac{1}{3}\pi\left(\frac{R}{4}\right)^2 \frac{H}{2}$$

Here, $\left[\frac{R}{2} = \frac{10}{2} = 5\,cm\right]$

Volume of frustum of cone

$$= \frac{1}{3}\pi(100) \times H - \frac{1}{3}\pi(25)\frac{H}{2} = \frac{175\pi H}{6}$$

Volume of cone

$$ADE = \frac{1}{3} \times \pi \times 5^2 \times \frac{H}{2} = \frac{25\pi H}{6}$$

$$\text{Ratio of Volume} = \frac{25\pi \cancel{H/6}}{\cancel{H/6}} = \frac{1}{7}$$

CASE STUDY-1

(i) (c) Area of hemisphere $= 2\pi r^2$

$= 2\pi\,(2.5\text{ cm})^2$

$= 12.5\pi\text{ cm}^2$

(ii) (b) Area of 1 hemispherical dome

$= 2\pi r^2$

$= 2\pi\,(1\text{ cm})^2$

$= 2\pi\text{ cm}^2$

Area of 7 hemispherical dome $= 14\pi\text{ cm}^2$

(iii) (a) Curved surface area of 1 cylinder $= 2\pi rh$

Curved surface area of 2 cylinder $= 4\pi rh$

$= 4\pi\,(1.4\text{ cm})\,(7\text{ cm})$

$= 123.15\text{ cm}^2$

$\approx 123.2\text{ cm}^2$

(iv) (d) Inner surface area of big dome $= 12.5\pi\text{ cm}^2$

Inner surface area of 1 small dome $= 2\pi\text{ cm}^2$

$$\frac{\text{Area of big dome}}{\text{Area of small dome}} = \frac{12.5\pi}{2\pi}$$

$$= \frac{25}{4}$$

(v) (d) Curved surface area of cylinder is $2\pi rh$.

CASE STUDY-2

(i) (a) Length of water in canal is 30 min = 5 km

= 5000 m

Volume of water flowing = (1.5 m) (6 m) (5000 m)

$= 45000\text{ m}^3$

(ii) (b) The volume of tank is given by $\pi r^2 h$, where "r" is the radius and "h" is height of cylinder.

Volume of water collected = Volume of tank

$= \pi\left(\frac{10}{2}\right)^2 (2)$

$= 50\pi$ m^3

(iii) (b) Let the area irrigated is 'x' m

Width of canal = 6 m

Depth of canal = 1.5 m

Length of water in canal in 30 minutes

= 5 km = 5000 m

Area irrigated by 8 cm = 6 × 1.5 × 5000

$6 \times 1.5 \times 5000 = \frac{8}{100}.x$

x = 562500 m^2

(iv) (d) The volume of water collected is equal to the volume of the frustum

Volume of frustum $= \frac{\pi}{3}\left(r_1^2 + r_2^2 + r_1 r_2\right)h$

$= \frac{\pi}{3}\left(14^2 + 7^2 + 19 \times 7\right)15$

= 5390 cm^3

(v) (a) $l = \sqrt{h^2 + \left(r_1 r_2\right)^2}$

$= \sqrt{h^2 + 4^2}$

25 = h^2 + 16

h^2 = 9

h = 3 cm

Chapter 14 Statistics

Multiple Choice Questions

1. (c) Mode is the most frequent value.

2. (c) $\overline{X} = \frac{\sum x_i}{n}$

$15 = \frac{1+2+3+\ldots+n}{n}$

$15n = \frac{n(n+1)}{2}$

$2 \times 15 = n + 1$

$n = 29$

3. (b) Median

4. (a) Median

5. (c) Mode = 3 median – 2 mean

WORKSHEET - 1

SECTION-A

1. Mode of the data is 8

if x = 8 because then 8 has highest frequency i.e 4.

2. Here, number of terms is 10 i.e. even

$$\text{So, Median} = \frac{\left(\frac{n}{2}\right)^{th} obs + \left(\frac{n}{2}+1\right)^{th} obs}{2}$$

$$= \frac{5^{th} obs + 6^{th} obs}{2}$$

$$= \frac{2x - 8 + 2x + 10}{2}$$

$$= \frac{4x+2}{2}$$

$$= 2x + 1$$

But median is 25

$\therefore$ 25 = 2x + 1

24 = 2x

x = 24

3. h is the class size

4. $\text{Mean} = \frac{\sum f_x}{\sum f}$

$7 = \frac{4p+63}{17}$

$\Rightarrow 119 = \frac{4p+63}{17}$ 4p + 63

$\Rightarrow 119 - 63 = 4p$

$\Rightarrow 56 = 4p$

$\Rightarrow 14 = p$

5. We know that

Mode = 3 median – 2 mean

15 = 3 median – 2 (30)

$\frac{75}{3}$ = median

25 = median

6. Mode is 2 because it has the highest frequency.

7. First five odd natural number 5 are 1, 3, 5, 7, 11

So, mean $= \frac{1+3+5+7+11}{5}$

= 5.4

8. Mean $= \frac{\Sigma f_x}{\Sigma f}$

$3 = \frac{3p+36}{15}$

45 = 3p + 36

45 – 36 = 3p

9 = 3p

p = 3

SECTION-B

9.

Class	Frequency
3 – 6	2
6 – 9	5
9 – 12	10 f_0
12 – 15	23 f_1
15 – 18	21 f_2
18 – 21	12
21 – 24	3

Here, Modal class is 12 – 15

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$$

l = 12

f_0 = 10

f_1 = 23

f_2 = 21

h = 3

$$\text{So, Mode} = 12 + \left(\frac{23-10}{46-10-21}\right)3$$

$$= 12 + \left(\frac{13}{15}\right)3$$

$$= 12 + \frac{13}{5}$$

= 12 + 2.6

= 14.6

10. We know that

mode = 3 median – 2 mean

21.4 = 3 (21.2) – 2 mean

2 mean = 63.6 – 21.4

= 42.2

∴ mean = 21.1

11.

Marks	Number of students	CF
0 – 10	5	5
10 – 30	15	20
30 – 60	30	50
60 – 80	8	58
80 – 100	2	60
	60	

$\frac{n}{2} = \frac{60}{2} = 30$

So, median class = 30 – 60

∴ l = 30

f = 30

cf = 20

h = 30

$$\text{Median} = l + \left(\frac{\frac{n}{2} - CF}{f}\right)h$$

$$= 30 + \left(\frac{30-20}{30}\right)$$

$$= 30 + \frac{10}{30}(30)$$

= 30 + 10

= 40

12.

Classes	Frequency (f_i)	x_i	$F_i x_i$
0 – 10	3	5	15
10 – 20	5	15	75
20 – 30	9	25	225
30 – 40	5	35	175
40 – 50	3	45	135
	25		**625**

Mean $= \frac{\sum f_i x_i}{\sum f_i}$

$= \frac{625}{25}$

$= 25$

13.

Class interval	Frequency	Cumulative frequency
0 –10	5	5
10 – 20	3	8
20 – 30	10	18
30 – 40	6	24
40 – 50	4	28
50 – 60	2	30

14.

x_i	f_i	$f_i x_i$
10	5	50
15	10	150
p	7	7p
25	8	200
30	2	60
	32	**460 + 7p**

Mean $= \frac{\sum f_i x_i}{\sum f_i}$

18.75 $= \frac{460 + 7p}{32}$

600 – 460 = 7p

140 = 7p

20 = 7p

15.

Number of branches (n)	Number of plants (f)	fn
2	49	98
3	43	129
4	57	228
5	38	190
6	13	78
	200	**723**

Average Number of branches per plant

$= \frac{\sum fn}{\sum f}$

$= \frac{723}{200}$

= 3.615

16.

Weekly wages	Number of workers (f)	x	$d_i = x_i - A$	$f_i d_i$
40 – 43	31	41.5	– 6	–186
43 – 46	58	44.5	– 3	–174
46 – 49	60	47.5	0	0
49 – 52	m	50.5	3	3m
52 – 55	27	53.5	6	162
	176 + m			–198 + 3m

Mean $= A + \frac{\sum f_i d_i}{\sum f_i}$

47.7 $= 47.5 + \frac{-198 + 3m}{176 + m}$

– 0.3 $= \frac{-198 + 3m}{176 + m}$

⇒ – 52.8 – 0.3 m = – 198 + 3m

⇒ 145.2 $= \frac{145.2}{3.3}$

= 44

SECTION-C

17.

Class intervals	x_i	f_i	$u_i = \frac{x_i - 50}{20}$	$f_i u_i$
0 – 20	10	17	– 2	– 34
20 – 40	30	f_i	– 1	– f_i
40 – 60	50	32	0	0
60 – 80	70	f_2	1	f_2
80 – 100	90	19	2	38
				4 f_i + f_2

$\sum f_i = 120$

$\Rightarrow 17 + f_i + 32 + f_2 + 19 = 120$

$\Rightarrow 68 + f_i + f_2 = 120$

$\Rightarrow f_i + f_2 = 52$ ...(i)

Also, mean = 50

$\Rightarrow a + \left(\frac{\sum f_i u_i}{\sum f_i}\right)h = 50$

$\Rightarrow 50 + 20\left(\frac{4 - f_i + f_2}{120}\right) = 50$

$\Rightarrow \frac{1}{6}(4 - f_1 + f_2) = 0$

$\Rightarrow f_1 - f_2 = 4$ ...(ii)

Solving eq. (i) and (ii), we get

$$f_i + f_2 = 52$$
$$f_i - f_2 = 4$$
$$2f_i = 56$$
$$f_i = 28$$

From (i), $f_2 = 52 - 28$

= 24

18.

Mark	Number of Students (f_I)	x_I	f_I	u_I	$f_I u_I$
1 – 10	20	5	– 20	– 2	– 40
10 – 20	24	15	– 10	– 1	– 24
20 – 30	40	25	0	0	0
30 – 40	36	35	10	1	36
40 – 50	20	45	20	2	40
	140				**12**

Mean $= a + \left(\frac{\sum f_i u_i}{\sum f_i}\right)h$

$= 25 + \left(\frac{12}{140}\right)^{10}$

$= 25 + \frac{12}{14}$

$= 25 + \frac{6}{7}$

$= \frac{181}{7}$

$= 25.86$

19.

Class Intervals	cf	f_i	x_i	$f_i x_i$
20 – 30	100	100	25	2500
30 – 40	220	120	35	4200
40 – 50	350	130	45	5850
50 – 60	750	400	55	22000
60 – 70	950	200	65	13000
70 – 80	1000	50	75	3750
		1000		51300

Mean $= \left(\frac{\sum f_i u_i}{\sum f_i}\right)$

$$= \frac{51300}{1000}$$

$$= 51.3$$

20.

Variable	Frequency	c f
10 – 20	12	12
20 – 30	30	42
30 – 40	f_i	$42 + f_i$
40 – 50	65	$107 + f_i$
50 – 60	52	$107 + f_i + f_2$
60 – 70	25	$132 + f_i + f_2$
70 – 80	18	$150 + f_i + f_2$
229		

Median is 46 which lies in interval 40 – 50

∴ Median class 40 – 50

$l = 40$

$f = 65$

$c f = 42 + f_i$

$h = 10$

Median $= 46$

$$l + \left(\frac{\frac{n}{2} - cf}{f}\right) h = 46$$

$$40 + \left(\frac{\frac{229}{2} - 42 - f_l}{65}\right) 10 = 46$$

$$\left(\frac{77.5 - f_l}{65}\right) 10 = 6$$

$$72.5 - f_i = 39$$

$$f_i = 33.5$$

Also, $\sum f_i = 229$

$\Rightarrow 12 + 30 + f_i + 65 + f_2 + 25 + 18 = 229$

$\Rightarrow 150 + f_i + f_2 = 229$

$\Rightarrow f_i + f_2 = 79$

$\Rightarrow 33.5 + f_2 = 79$

$\Rightarrow f_2 = 79 - 33.5$

$= 45.5$

21.

Class Interval	Frequency	cf
0 – 6	4	4
6 – 12	x	4 + x
12 – 18	5	9 + x
18 – 24	y	9 + x + y
24 – 30	1	10 + x + y
	10 + x + y	

As Median is 14.4 which lies in interval 12 – 18,
So, Median class is 12 – 18

$l = 12$

$f = 5$

$c f = 4 + x$

$h = 6$

Now, $\sum f_i = 20$

$\Rightarrow 10 + x + y = 20$

$\Rightarrow x + y = 10$ (i)

Also, $\sum f_i = 14..4$

$$l + \left(\frac{\frac{n}{2} - cf}{f}\right) h = 14.4$$

$$\Rightarrow 12 + \frac{10 - 4 - x}{5} \, 6 = 14.4$$

$$\Rightarrow \frac{6}{5}(6 - x) = 2.4$$

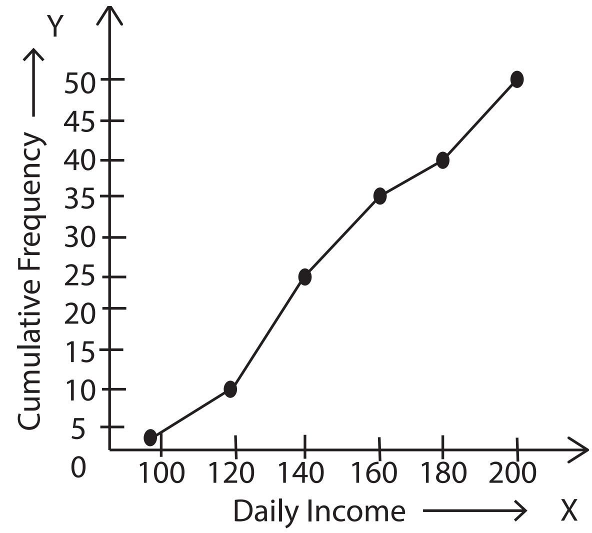
$\Rightarrow \quad 6 - x = \frac{2.4 \times 5}{6} = 2$

$\Rightarrow \quad x = 6 - 2 = 4$

Form (i), $y = 10 - x$

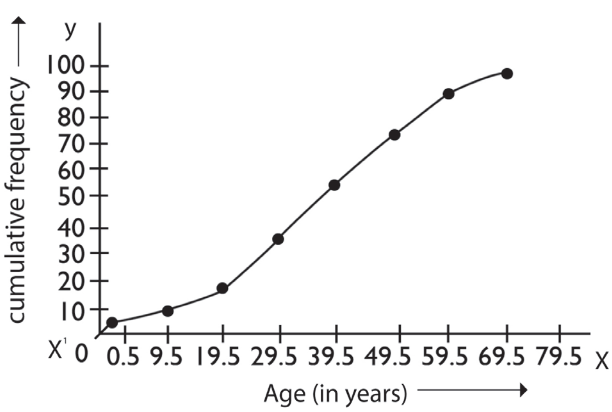
$= 10 - 4$

$= 6$



22.

Age (in year)	Number of persons	c f	
– 0.5 – 9.5	5	5	less than 9.5
9.5 – 19.5	15	20	less than 19.5
19.5 – 29.5	20	40	less than 29.5
29.5 – 39.5	23	63	less than 39.5
39.5 – 49.5	17	80	less than 49.5
49.5 – 59.5	11	91	less than 59.5
59.5 – 69.5	9	100	less than 69.5



23.

Daily Income	Number of workers	Cumulative frequency
less than 120	12	12
less than 140	14	26
less than 160	8	34
less than 180	6	40
less than 200	10	50

24.

Monthly Consumption of electricity	Number of Consumer	Cumulative Frequency (cf)
65 – 85	4	4
85 – 105	5	9
105 – 125	13	22
125 – 145	20	42
145 – 165	14	56
165 – 185	8	63
185 – 205	4	67

Median

Here $\frac{n}{2} = \frac{68}{2} = 34$

Median class is 125 – 145

$l = 125$

$cf = 22$

$f = 20$

$h = 20$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) h$$

$$= 125 + \left(\frac{34-22}{20}\right) 20$$

$$= 125 + 12$$

$$= 137$$

Mode

Modal class is 125 – 145 as it has highest frequency.

$l = 125$

$h = 20$

$f_0 = 13$

$f_1 = 20$

$f_2 = 14$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) h$$

$$= 125 + \left(\frac{20-13}{40-13-14}\right) 20$$

$$= 125 + \left(\frac{7}{13}\right) 20$$

$$= 125 + \frac{140}{13}$$

$$= 135.8$$

25.

Height (in cm)	Number of student (f_i)	x_i	$f_i x_i$
140 – 150	74	145	10730
150 – 160	163	155	25265
160 – 170	135	165	22275
170 – 180	28	175	4900
	400		**63170**

$$\text{Mean} = \frac{f_i x_i}{f_i}$$

$$= \frac{63170}{400} = 157.925$$

Modal class is 150 – 160 as this class has highest frequency

$l = 150$

$f_0 = 74$

$f_1 = 163$

$f_2 = 135$

$h = 10$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) h$$

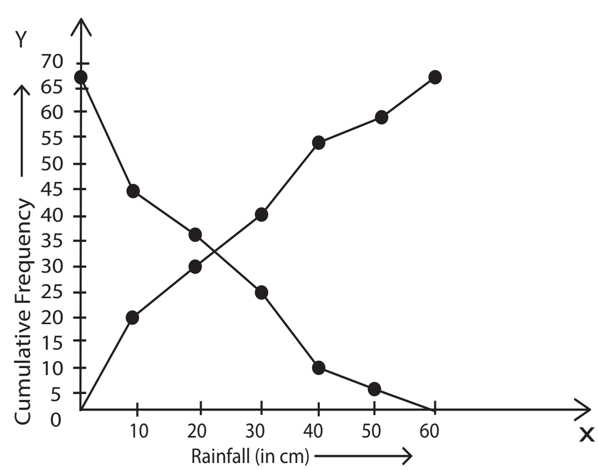
$$= 150 + \left(\frac{163-74}{326-74-135}\right)$$

$$= 150 + \left(\frac{89}{117}\right)$$

$$= \frac{17550+890}{117} = 157.61$$

26.

Rainfall (in cm) (less than)	cf	Rainfall (in cm) (more than)	cf
less than 10	22	more than 0	66
less than 20	32	more than 10	44
less than 30	40	more than 20	34
less than 40	55	more than 30	26
less than 50	60	more than 40	11
less than 60	66	more than 50	6



27.

Amount	Number of students (f_i)	Cumulative frequency
0 – 20	5	5
20 – 40	8	13
40 – 60	12	25
60 – 80	11	36
80 – 100	4	40
	40	

$\frac{n}{2} = \frac{40}{2} = 20$

Median Class is 40 – 60

$l = 40$

$cf = 13$

$f = 12$

$h = 20$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) h$$

$$= 40 + \left(\frac{20-13}{12}\right) 20$$

$$= 40 + \frac{7}{12} (20)$$

$$= 51.7$$

Modal Class is 40 – 60

$l = 40$

$f_0 = 8$

$f_1 = 12$

$f_2 = 11$

$h = 20$

$$\text{Mode} = \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) h$$

$$= 40 + \left(\frac{12-8}{24-8-11}\right)$$

$$= 40 + \left(\frac{4}{5}\right)$$

$$= 40 + 16$$

$$= 56$$

28.

Number of Mangoes	Number of boxes f_i	x_i	d_i	$f_i d_i$
50 – 52	15	51	– 6	– 90
53 – 55	110	54	– 3	– 330
56 – 58	135	57	0	0
59 – 61	115	60	3	345
62 – 64	25	63	6	150
	400			**75**

$$\text{Mean} = a + \frac{\Sigma f_i d_i}{\Sigma f_i}$$

$$= 57 + \frac{75}{400}$$

$$= 57.19$$

We use assumed mean method as values of x_i and f_i are large.

29.

Class Interval	Frequency (f_i)	x_i	f_i
0 – 20	17	10	170
20 – 40	f_1	30	30 f_1
40 – 60	$f_2 = 4x$	50	200x
60 – 80	$f_3 = 3x$	70	210x
80 – 100	19	90	1710
	120		**1880 + 30 f_1 + 410x**

$f_2 = f_3 = 4 : 3$

Let $f_2 = 4x$ and $f_3 = 3x$

$\sum f_1 = 120$

$\Rightarrow \quad 17 + f_1 + f_2 + f_3 + 19 = 120$

$\Rightarrow \quad 17 + f_1 + 4x + 3x + 19 = 120$

$\Rightarrow \quad f_1 + 7x = 84 \quad$ (i)

Also, mean $= 50$

$$\frac{\sum f_1 x_1}{\sum f_1} = 50$$

$$\frac{1880 + 30f_1 + 410x}{120} = 50$$

$1880 + 30f_1 + 410x = 6000$

$30f_1 + 410x = 4120$

$\Rightarrow \quad 15f_1 + 205x = 2060$ (ii)

Solving (i) and (ii), we get

$f_1 = 28$

$x = 8$

$\therefore \quad f_2 = 4x = 32$

$f_3 = 3x = 24$

30.

Class	Frequency (f_1)
0 – 20	6
20 – 40	8
40 – 60	f_1
60 – 80	12
80 – 100	6
100 – 120	5

Modal Class is 60 – 80 as this class has the highest frequency

$l = 60$

$f_0 = f_1$

$f_1 = 12$

$f_2 = 6$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) h$$

$$65 = 60 + \left(\frac{12 - f_1}{24 - f_1 - 6}\right) 20$$

$$65 = 60 + \left(\frac{12 - f_1}{18 - f_1}\right) 20$$

$$\frac{5}{20} = \frac{12 - f_1}{18 - f_1}$$

$$\frac{1}{4} = \frac{12 - f_1}{18 - f_1}$$

$18 - f_1 = 48 - 4f_1$

$3 f_1 = 30$

$f_1 = 10$

WORKSHEET - 2

SECTION-A

1. Mean $= 10.5$

Median $= 9.6$

We know that

3 Median = mode + 2 mean

3 (9.6) = mode + 2 (10.5)

28.8 = mode + 21

Mode = 28.8 – 21

= 7.8

2. $$\text{Mean} = a + \left(\frac{\sum f_1}{\sum f}\right) h$$

$$= 55 + \left(\frac{-3}{100}\right) 10$$

$$= 55 - \frac{-3}{10}$$

$$= 54.7$$

3.

Marks Obtained	Number of student (cf)	Marks Obtained (Class intervals)	Number of student (f)
Less than 20	8	10 – 20	8
Less than 30	13	20 – 30	5
Less than 40	19	30 – 40	6
Less than 50	24	40 – 50	5

4.

Classes	frequency (f)	C f
0 – 10	4	4
10 – 20	4	8
20 – 30	8	16
30 – 40	10	26
40 – 50	12	38
50 – 60	8	46
60 – 70	4	50
	50	

$$\frac{n}{2} = \frac{50}{2} = 25$$

Cf of class 30 – 40 is greatest than 25

So, Median class is 30 – 40

5. Modal class is 40 – 50 as this class has highest frequency.

So, lower limit is 40

6. Mean $= \frac{\sum x_I}{N}$

$18 = \frac{\sum x_I}{50}$

$\sum x_I = 900$

New $\sum x_I = 900 + 200$

$= 1100$

So, New mean $= \frac{1100}{50}$

$= 22$

7. Median is equal to the x – coordinate of point of intersection of less than ogive and more than ogive

Here, point of intersection is (18, 20)

So, Median = 18

8. We get less than ogive when upper limits are taken along x-axis and cumulative frequency along y-axis.

9. Mean $= \frac{\sum x_I}{n}$

$40 = \frac{\sum x_I}{5}$

$\sum x_I = 200$

If 88 is excluded,

New $\sum x_I = 200 - 88$

$= 112$

New n $= 5 - 1 = 4$

$\therefore$ New Mean $= \frac{112}{4} = 28$

10. Mean of 30 number $= \frac{10(12)+20(9)}{30}$

$= \frac{120+180}{30}$

$= \frac{300}{30}$

$= 10$

SECTION-B

11.

Class Interval	Frequency
100 – 110	6
110 – 120	35
120 – 130	72
130 – 140	48

Modal class is 120 – 130

Mode $= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) h$

$= 120 + \left(\frac{72-35}{144-35-48}\right) 10$

$= 120 + \left(\frac{37}{61}\right) 10$

$= 126.07$

12.

Marks	f_i	x_i	f_ix_i
0 – 10	20	5	100
10 – 20	24	15	360
20 – 30	40	25	1000
30 – 40	36	35	1260
40 – 50	20	45	900
	140		**3620**

mean $= \frac{\Sigma f_i x_i}{\Sigma f_i}$

$= \frac{3620}{140}$

$= 25.86$

13.

Age in years	Frequency (f_i)	x_i	f_ix_i
20 – 30	4	25	100
30 – 40	5	35	175
40 – 50	8	45	360
50 – 60	3	55	165
60 – 70	6	65	390
	26		**1190**

Mean $= \frac{\Sigma f_i x_i}{\Sigma f_i}$

$= \frac{1190}{26}$

$= 45.77$

14.

x_i	f	f x
5	6	30
15	4	60
25	3	75
35	k	35k
45	2	90
	15 + k	**255 + 35k**

Mean $= \frac{\Sigma fx}{\Sigma f}$

$= \frac{255+35k}{15+k}$

$322.5 + 21.5k = 225 + 35k$

$67.5 = 13.5k$

$k = \frac{67.5}{13.5} = 5$

15. Number of observations = 10 (even)

$$\text{So, Median} = \frac{\left(\frac{n}{2}\right)^{th} obs + \left(\frac{n}{2}+1\right)^{th} obs}{2}$$

$$= \frac{5^{th} obs + 6^{2} obs}{2}$$

$$= \frac{x+2 \; + \; x+4}{2}$$

= x + 3

Median = 24

∴ x + 3 = 24

x = 24 – 3

= 21

16. Number of observations = 10 (even)

$$\text{So, Median} = \frac{\left(\frac{n}{2}\right)^{th} obs + \left(\frac{n}{2}+1\right)^{th} obs}{2}$$

$$= \frac{5^{th} obs + 6^{th} obs}{2}$$

$$= \frac{48+35}{2}$$

$$= \frac{85}{2}$$

= 42.5

If 25 is replaced by 52 and 19 by 29, median remains same.

Median will get affected by 5^{th} and 6^{th} observation only.

17.

x_I	f	f x
15	2	30
17	3	51
19	4	76
20 + p	5p	5p (20 + p)
23	6	138
	15 + 5p	295 + 5p (20 + p)

Mean = 20

$$\frac{fx}{f} = 20$$

$$\frac{295+5p(20+p)}{15+5p} = 20$$

$$\Rightarrow \frac{59+p\,(20+p)}{3+p} = 20$$

$\Rightarrow$ $59 + 20p + p^2 = 60 + 20p$

$\Rightarrow$ $p^2 = 1$

$\Rightarrow$ p = 1

18. $x_1 = 5 + 7 = 12$

$x_1 + x_2 = 18$

$\Rightarrow$ $12 + x_2 = 18$

$x_2 = 18 - 12$

$= 6$

$18 + 5 = x_3$

$23 = x_3$

$x_3 + x_4 = 30$

$23 + x_4 = 30$

$x_4 = 30 - 23$

$= 7$

19.

Class	Frequency (f_I)	Cf
40 – 50	5	5
50 – 60	x	5 + x
60 – 70	15	20 + x
70 – 80	12	32 + x
80 – 90	7	39 + x

As mode is 67 which is in class 60 – 70,

So, Modal class is 60 – 70

l = 60

f_0 = x

$f_1 = 15$

$f_2 = 12$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) h$$

$$67 = 60 + \left(\frac{15 - x}{30 - x - 12}\right) 10$$

$$7 = 10 \left(\frac{15 - x}{18 - x}\right)$$

$$7\ (18 - x) = 150 - 10x$$

$$126 - 7x = 150 - 10x$$

$$3x = 24$$

$$x = 8$$

20.

Life time (in hours)	Frequency f_i	x_i	$f_i x_i$
0 – 20	15	10	150
20 – 40	10	30	300
40 – 60	35	50	1750
60 – 80	50	70	3500
80 – 100	40	90	3600
	150		**9300**

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$= \frac{9300}{150}$$

$$= 62$$

SECTION-C

21. Let number of boys be x and number of girls be y.

∴ Total number of students = x + y

Average score of boys = 71

⇒ Total score of boys = 71x

Average score of girls = 73

⇒ Total score of girls= 73y

Average score of school = 71.8

Total score of school = 71.8 (x + y)

So,

$$71x + 73y = 71.8\ (x + y)$$

$$71x + 73y = 71.8x + 71.8y$$

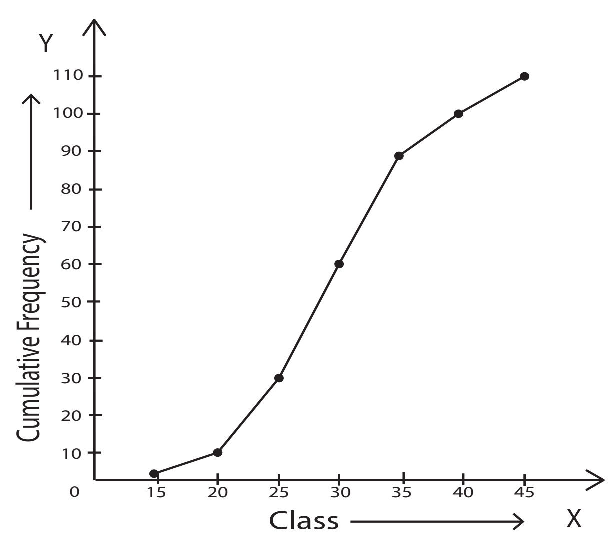
$$1.2y = 0.8x$$

$$3y = 2x$$

$$\therefore \quad \frac{x}{y} = \frac{3}{2}$$

22.

Class	Frequency f_i	Cumulative Frequency Cf
Less than 20	13	13
Less than 25	18	31
Less than 30	31	62
Less than 35	25	87
Less than 40	15	102
Less than 45	5	107



23.

Age in year	Number of patients (f_1)	Cf
0 – 8	10	10
8 – 16	12	22
16 – 24	8	30
24 – 32	25	55
32 – 40	15	70
40 – 48	11	81
48 – 56	21	102
56 – 64	30	132
64 – 72	22	154
	154	

$$\frac{n}{2} = \frac{154}{2} = 77$$

Median class is 40 – 48

l = 40

f = 11

cf = 70

h = 8

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) h$$

$$= 40 + \left(\frac{77 - 70}{11}\right) 8$$

$$= 40 + \frac{56}{11} = 45.1$$

24.

Class interval	Frequency f_1	Cf
0 – 10	2	2
10 – 20	3	5
20 – 30	x	5 + x
30 – 40	6	11 + x
40 – 50	5	16 + x
50 – 60	3	19 + x
60 – 70	2	21 + x
	21 + x	

Median is 35 which lies in class 30 – 40.

l = 30

f = 10

cf = 6

h = 5 + x

Median = 35

$$l + \left(\frac{\frac{n}{2} - cf}{f}\right) h = 35$$

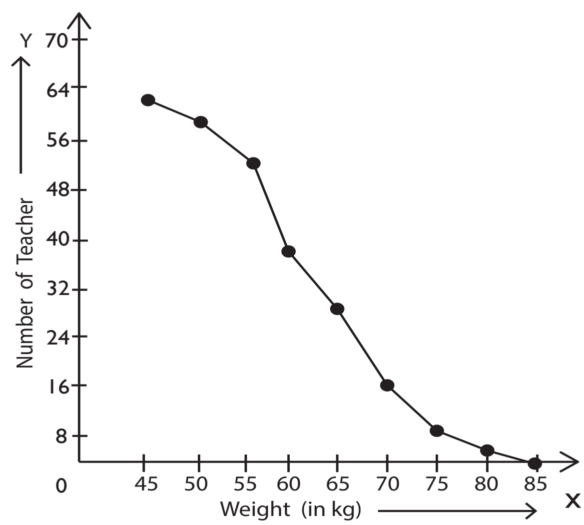
$$30 + \left(\frac{21 + x - 5 - x}{6}\right) 10 = 35$$

$$\frac{10}{6} + \left(\frac{21 + x - 10 - 2x}{6}\right) = 5$$

$$11 - x = 6$$

$$x = 5$$

25.



26.

Literacy rates	Number of cities (f_i)	x_i	f_ix_i
35 – 40	1	37.5	37.5
40 – 45	2	42.5	85
45 – 50	3	47.5	142.5
50 – 55	x	52.2	52.5x
55 – 60	y	57.5	57.5y
60 – 65	6	62.5	375
65 – 70	8	67.5	540
70 – 75	4	72.5	290
75 – 80	2	77.5	155
80 – 85	3	82.5	247.5
85 – 90	2	87.5	175
	40		**2047.5 + 52.5x + 57.5y**

$\sum f_i = 40$

$\therefore\ 31 + x + y = 40$

$x + y = 9$ (i)

Mean $= 63.5$

$$\frac{\sum f_ix_i}{\sum f_i} = 63.5$$

$$\frac{2047.5 + 52.5x + 57.5y}{40} = 63.5$$

$52.5x + 57.5y = 493.5$

$\Rightarrow\ 525x + 575y = 4925$

$\Rightarrow\ 105x + 115y = 985$

$\Rightarrow\ 21x + 23y = 197$ (ii)

Solving (i) and (ii), we get

$x = 5$

$y = 4$

27.

Class	Frequency f_i	x_i	f_ix_i
10 – 30	5	20	100
30 – 50	8	40	320
50 – 70	f_1	60	$60 f_1$
70 – 90	20	80	1600
90 – 110	f_2	100	$100 f_2$
110 – 130	2	120	240
	50		$2260 + 60 f_1 + 100 f_2$

$\sum f_i = 50$

$\Rightarrow\ 35 + f_1 + f_2 = 50$

$\Rightarrow\ f_1 + f_2 = 15$ ...(i)

Mean $= 65.6$

$$\Rightarrow\ \frac{\sum f_ix_i}{\sum f_i} = 65.5$$

$$\Rightarrow\ \frac{2260 + 60f_1 + 100f_2}{50}$$

$= 65.5$

$\Rightarrow\ 60 f_1 + 100 f_2 = 1020$

$\Rightarrow\ 15 f_1 + 25 f_2 = 255$

$\Rightarrow\ 3 f_1 + 5 f_2 = 51$ ...(ii)

On solving (i) and (ii), we get

$f_1 = 12$

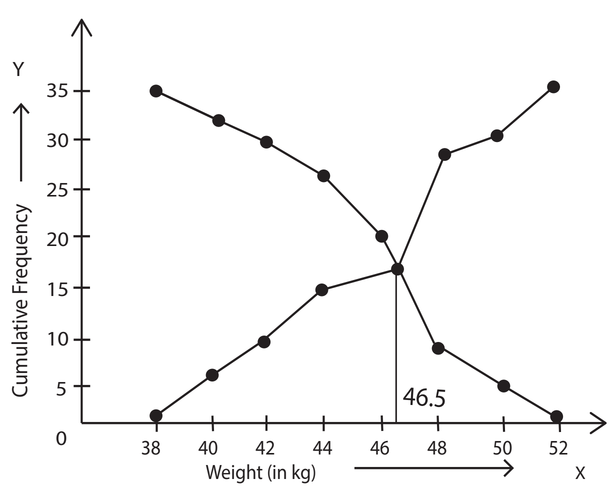
$f_2 = 3$

SECTION-D

28.

Weight (in kg)	Number of Students
38 – 40	3
40 – 42	2
42 – 44	4
44 – 46	5
46 – 48	14
48 – 50	4
50 – 52	3

Weight (in kg)	Cf	Weight (in kg)	Cf
lass than 40	3	more than 38	35
lass than 42	5	more than 40	32
lass than 44	9	more than 42	30
lass than 46	14	more than 44	26
lass than 48	28	more than 46	21
lass than 50	32	more than 48	7
lass than 52	35	more than 50	3



∴ Median = 46.5cs

29.

Marks obtained	Number of students	Cf
25 – 35	7	7
35 – 45	31	38
45 – 55	33	71
55 – 65	17	88
65 – 75	11	99
75 – 80	1	100
	100	

Mode

Modal class is 45 – 55 as this class has the highest frequency

$l = 45$

$f_0 = 31$

$f_1 = 33$

$f_2 = 17$

$h = 5$

$$\text{Mode} = \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) h$$

$$= 45 + \left(\frac{33-31}{66-31-17}\right) 5$$

$$= 45 + \left(\frac{2}{18}\right) 5$$

$$= 45 + \frac{5}{9}$$

$$= 45.6$$

Median

$$\frac{n}{2} = \frac{100}{2} = 50$$

Median class is 45 – 55

Median $= l + \left(\dfrac{\frac{n}{2} - cf}{f}\right) h$

$= 45 + \left(\dfrac{50 - 38}{33}\right) 5$

$= 45 + \left(\dfrac{12}{33}\right) 5$

$= 46.82$

30. Mean $= \overline{x}$

(a) $\Rightarrow \dfrac{x_1}{n} = \overline{x}$

$\Rightarrow \sum x_1 = n\overline{x}$

New $\sum x_1$

$= (x_1 + 1) + (x_2 + 1) + \_\_\_\_ + (x_n + n)$

$= \sum x_1 + (1 + 2 + 3 + ... + n)$

$= \sum x_1 + \dfrac{n(n+1)}{2}$

$= n\overline{x} + \dfrac{n(n+1)}{2}$

$\therefore$ New mean $= \dfrac{n\overline{x} + \frac{n(n+1)}{2}}{2}$

$= \overline{x} + \left(\dfrac{n+1}{2}\right)$

(b) Mode $= 7.88$

Mean $= 8.32$

We know that

3 Median = Mode + 2 Mean

$\Rightarrow$ 3 Median $= 7.88 + 2\,(8.32)$

$= 7.88 + 16.64$

$\Rightarrow$ Median $= 8.17$

31.

Height (in cm)	Number of girls (Cf)	Class Intervals	f
less than 140	4	135 – 140	4
less than 145	11	140 – 145	7
less than 150	29	145 – 150	18
less than 155	40	150 – 155	11
less than 160	46	155 – 160	6
less than 165	51	160 – 165	5

Here $\dfrac{n}{2} = \dfrac{51}{2} = 25.5$

Median class is 145 – 150

Median $= l + \left(\dfrac{\frac{n}{2} - cf}{f}\right) h$

$= 145 + \left(\dfrac{25.5 - 11}{18}\right) 5$

$= 145 + 4.03$

$= 149.03$

CASE STUDY-1

(i) (b) Maximum number of students have marks in the range 35–45. Hence the modal class is 35–45.

(ii) (b) Lower limit is 25.

(iii) (d) Maximum number of students is 24, hence it is the maximum frequency.

(iv) (c) Class mark $= \dfrac{45 + 55}{2}$

$= 50$

(v) (a) Class size of each class is 10.

CASE STUDY-2

(i) (a)

No. of apples	No. of boxes (frequency)	Cumulative frequency
0–20	6	6
20–40	8	14
40–60	10	24
60–80	12	36
80–100	6	42
100–120	5	47
120–140	3	50

N = 50

$\frac{N}{2} = 25$

Cumulative frequency more than or equal to 25 is 36 which belongs to class interval 60–80. Thus Median class is 60–80.

(ii) (b) Median = $l + \left[\frac{\left(\frac{N}{2} - cf\right)}{f}\right] \times h$

l = lower limit of median class

$\frac{N}{2}$ = half of sum of cumulative frequencies

Cf = Cumulative frequency of class preceding the Median class

f = frequency of Median class

h = class height

l = 60, N = 50, Cf = 24, f = 12, h = 20

$$\text{Median} = 60 + \left[\frac{\left(\frac{50}{2} - 24\right)}{12}\right] \times 20$$

$$= 60 + \frac{1}{\cancel{12}_3} \times \cancel{20}\,5$$

$$= \frac{185}{3} = 61.6$$

(iii) (b) The maximum frequency is 12, which lies in the class internal 60–80. Hence the modal class is 60–80.

(iv) (d) Mode = $l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$

Where l = lower limit of modal class

f_1 = frequency of modal class

f_2 = frequency of class following modal class

f_0 = frequency of class precedding modal class

h = width of modal class

$$\text{Mode} = 60 + \left[\frac{12 - 10}{2(12) - 10 - 6}\right] \times 20$$

$$= 60 + \left[\frac{2 \times 20}{8}\right]$$

$$= 65$$

(v) (a) Mean = $\frac{3\text{ Median - Mode}}{2}$

$$= \frac{3(61.6) - 65}{2}$$

$$= 59.9$$

Chapter 15

Probability

Multiple Choice Questions

1. (c) When two dices are thrown together, total number of outcomes are:

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

P (Getting the same number)

$$= \frac{\textit{Number of favourable outcomes}}{\textit{Total number of outcomes}}$$

$$= \frac{6}{36} = \frac{1}{6}$$

2. (d) Total Number of cards = 52

Cards that are not ace = 48

P (card not an arc)

$$= \frac{\textit{Number of favourable outcomes}}{\textit{Total number of outcomes}}$$

$$= \frac{48}{52} = \frac{12}{13}$$

3. (d) When two dices are rolled together, total number of outcomes are:

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

P (Getting even number on both dices)

$$= \frac{\textit{Number of favourable outcomes}}{\textit{Total number of outcomes}}$$

Number of favourable outcomes

= (2, 2), (2, 4), (2, 6), (4, 2)

(4, 6), (6, 2), (6, 4), (4, 6)

P (Getting even number on both dices)

$$= \frac{9}{36} = \frac{1}{4}$$

4. (c) Number from 1 to 15 that are multiple of 4 = 4, 8 , 12

P (Multiple of 4) $= \frac{3}{15} = \frac{1}{5}$

5. (c) Prime number from 1 to 30

= 2, 3, 5, 7, 11, 13, 17, 19, 23, 29

P (Prime number between 1 and 30)

$$= \frac{10}{30} = \frac{1}{3}$$

WORKSHEET - 1

SECTION-A

1. If an event cannot occur, then its probability is 0.

2. Total number of face cards = 12 cards

Total number of red face cards = 6 cards

P (red face cards) $= \frac{6}{52} = \frac{3}{26}$

3. Total number of outcomes when a die is thrown = 1, 2, 3, 4, 5, 6

Odd number less than 3 = 1

P (odd number less than 3) = $\frac{1}{6}$

4. If three coins are tossed simultaneously, total number of outcomes are (HHH), (HHT), (HTH), (THH), (HTT), (THT), (TTT), (TTH)

Outcomes for at least two heads

= (HHH), (HHT), (HTH), (THH)

P (at least two heads) = $\frac{4}{8} = \frac{1}{2}$

5. A non-leap year has 365 days

For 364 days, there are 52 weeks i.e. 52 Sundays

For the remaining 1 day, only one Sunday can exist.

So,

P (Getting 53 Sundays in non - leap year) = $\frac{1}{7}$

6. Number of aces in a deck of cards = 4

P (ace) = $\frac{4}{52} = \frac{1}{13}$

7. Given number = 3, 5, 5, 7, 7, 9, 9, 9, 9

Average of the given number

$= \frac{3+5+5+7+7+9+9+9+9}{9}$

$= \frac{63}{9} = 7$

So, 7 comes two times in these numbers

Thus, P (selecting their average) = $\frac{2}{9}$

8. In a single throw of dice, total number of outcomes are 6 namely 1, 2, 3, 4, 5, and 6

Perfect squares = 1, 4

P (Getting perfect square) = $\frac{2}{6} = \frac{1}{3}$

SECTION-B

9. When two coins are tossed together, total number of outcomes are (H, H), (H, T), (T, H) (T, T)

Outcomes for at least 1 head and 1 tail

= (H, T), (T, H)

P (at least 1 head and 1 tail) = $\frac{2}{4} = \frac{1}{2}$

10. Tickets are numbered from 1 to 20

Multiples of 2 between 1 and 20

= 2, 4, 6, 8, 10, 12, 14, 16, 18, 20

Multiples of 7 between 1 and 20 = 7, 14

Outcomes which are multiple of 2 or 7

= 2, 4, 6, 7, 8, 10, 12, 14, 16, 18, 20

P (multiples of 2 or 7) = $\frac{11}{20}$

11. Number of red marbles = 3

Number of blue marbles = 2

Total number of marbles = 5

P (blue marble) = $\frac{2}{5}$

12 a) When a die is thrown, total number of outcomes are 6 namely 1, 2, 3, 4, 5 and 6

Outcomes which are multiple of 3 = 3, 6

P (multiple of 3) = $\frac{2}{6} = \frac{1}{3}$

b) Outcomes which are even number or a multiple of 3 = 2, 3, 4, 6

P (even number or multiple of 3)

$= \frac{4}{6} = \frac{2}{3}$

13. Total number of children = 3

P (number girl) $= \frac{0}{3} = 0$

$P\text{ (one girl)} = \frac{1}{3}$

$P\text{ (two girls)} = \frac{2}{3}$

$P\text{ (three girls)} = \frac{3}{3} = 1$

So, the probability of each cannot be $\frac{1}{4}$

$\therefore$ the given statement is incorrect.

14. No, the given statement is false and we do want a higher chance of getting tail in the 4 th because every coin toss has an equal probability of getting head and tail which is $\frac{1}{2}$

$\therefore$ There are equal chances of getting head and tail in the 4th toss.

15. Prizes available in 1000 tickets = 5

$P\text{ (winning a prize)} = \frac{5}{1000} = \frac{1}{200}$

16. Given number = – 2, – 1, 0, 1, 2

Number when $x^2 < 2$ = – 2, – 1, 0, 1

$P(x^2 < 2) = \frac{4}{5}$

SECTION-C

17. Word 'Assassination' has 6 vowels and 7 consonants

6 Vowels = { A, A, I, A, I, O }

7 Consonants = { S, S, S, S, N, T, N }

i) $P\text{ (vowels)} = \frac{6}{13}$

ii) $P\text{ (consonants)} = \frac{7}{13}$

18. (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

Sum of number greater than 10 = (5, 6), (6, 5), (6, 6)

$P\text{ (sum greater than 10)} = \frac{3}{26} = \frac{1}{12}$

19. A leap year has 366 days in which there are 52 weeks and 2 days

These 2 days can be filled as :

{ Monday, Tuesday }

{ Tuesday, Wednesday }

{ Wednesday, Thursday }

{ Thursday, Friday }

{ Friday, Saturday }

{ Saturday, Sunday }

$P\text{ (53 Sundays and 53 Mondays)} = \frac{1}{7}$

20. Total number of marbles = 225

Let 'x' marbles be green

Probability of green marbles = $\frac{2}{3}$

$P\text{ (green)} = \frac{2}{3}$

$\frac{x}{225} = \frac{2}{3}$

$x = \frac{225 \times 2}{3}$

$x = 75 \times 2$

x = 150 green marbles

Number of blue marbles = 225 – 150

= 75 blue marbles

21. a) Total number of cards = (60 – 13) + 1

= 48 cards

Cards divisible by 5 = 15, 20, 25, 30, 35, 40, 45, 50, 55, 60

P (disible by 5) = $\frac{10}{48}$ = $\frac{5}{24}$

b) Cards which are perfect square = 16, 25, 36, 49

P (perfect square) = $\frac{4}{48}$ = $\frac{1}{12}$

22. a) When two dices are thrown, the total number of outcomes are :

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)

(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)

(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)

(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)

(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)

(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

Getting a number greater than 3 on each dice

= (4, 4) (4, 5), (4, 6)

(5, 4) (5, 5), (5, 6)

(6, 4) (6, 5), (6, 6)

P (greater than 3 on each dice) = $\frac{9}{36}$ = $\frac{1}{9}$

b) Getting a total of 6 or 7

= (1, 5), (1, 6), (2, 4), (2, 5),

(3, 3), (3, 4), (4, 2), (4, 3),

(5, 1), (5, 2), (6, 1)

P (total of 6 or 7) = $\frac{11}{36}$

23. As king, queen, jack of clubs are removed from the deck of cards, total number of cards becomes 49.

a) Heart,

Number of cards (Heart) = 13

P (heart) = $\frac{13}{49}$

b) Queen,

Number of cards (queen) = 3

P (queen) = $\frac{3}{49}$

c) Clubs

Number of cards (clubs) = 10

P (clubs) = $\frac{10}{49}$

24. Given card number are from 1 to 20

a) Number divisible by 2 or 3

= 2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20

P (divisivle by 2 or 3) = $\frac{13}{20}$

b) Prime number between 1 and 20

= 2, 3, 5, 7, 11, 13, 17, 19

P (prime numbers) = $\frac{8}{20}$ = $\frac{2}{5}$

SECTION-D

25. Total number of red face cards = 6

When red face cards are removed, the total number of cards now becomes 46.

a) A red card

Total number of remaining red cards = 20

P (red card) = $\frac{20}{46}$ = $\frac{10}{23}$

b) A face card

Total number of remaining face cards

= 12 – 6 = 6

P (face card) = $\frac{6}{46}$ = $\frac{3}{23}$

c) A red card

Total number of remaining clubs cards = 13

P (clubs) $= \frac{13}{46}$

26 When a dice is thrown two times, total number of outcomes are :

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)

(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)

(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)

(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)

(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)

(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

a) P (5 will not come either time) $= \frac{25}{36}$

b) P (5 will come exactly once) $= \frac{10}{36} = \frac{5}{18}$

27. Total number of cards = (45 – 5) + 1 = 41

a) Odd number cards = 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45

P (odd number) $= \frac{21}{41}$

b) Perfect square numbers = 9, 16, 25, 36

P (perfect square) $= \frac{4}{41}$

c) Multiples of 5 = 5, 10, 15, 20, 25, 30, 35, 40, 45

P (multiple of 5) $= \frac{9}{41}$

d) 0, as 2 is the only even prime number and cards are numbered from 5 to 45.

28. Cards are numbered as 3, 5, 7, _ _ _ _ 37.

Total number of cards = 19

Prime numbered cards = 3, 5, 7, 11, 13, 17, 17, 23, 29, 31, 37.

P (prime numbers) $= \frac{11}{19}$

29. Total number of red balls = 5

Total number of white balls = 3

Total number of black balls = 7

Total number of balls = 5 + 3 + 7 = 15

a) Red or white

Red number of red and white balls = 5 + 3 = 8

P (red or white) $= \frac{8}{15}$

b) Not black = Total number of red and white balls

P (red or white) $= \frac{8}{15}$

c) Neither white nor black = Red balls

P (Neither white nor black) = P (Red)

$= \frac{5}{15} = \frac{1}{3}$

30. a) P (queen) $= \frac{1}{5}$

b) i) P (Ace) $= \frac{\textit{Number of aces}}{\textit{Total number of cards}} = \frac{1}{4}$

ii) P (king)

$= \frac{\text{Number of Kings in second draw}}{\text{Total number of cards in second draw}}$

$= \frac{0}{4} = 0$

31. Number of red balls = 4

Number of black balls = 5

Number of white balls = 6

Total number of balls = 4 + 5 + 6 = 15

a) P (white) $= \frac{6}{15} = \frac{2}{5}$

b) P (red) $= \frac{4}{15}$

c) P (not black) = P (red and white)

$= \frac{4+6}{15} = \frac{10}{15} = \frac{2}{3}$

d) P (red or white) $= \frac{4+6}{15} = \frac{10}{15} = \frac{2}{3}$

32. a) Total number of cards = 52

Number of black kings = 2

P (black king) = $\frac{2}{52} = \frac{1}{26}$

b) Cards which are neither red nor queen = 24

P (neither red nor queen) = $\frac{24}{52} = \frac{6}{13}$

c) Cards which are neither king nor queen = 44

P (neither king nor queen) = $\frac{44}{52} = \frac{11}{13}$

d) Cards which are either black or a king = 28

P (either a black card or a king) = $\frac{28}{52} = \frac{7}{13}$

WORKSHEET - 2

SECTION-A

1. Total number of discs = 90

Prime number less than 23 = 2, 3, 4, 5, 7, 11, 13, 17, 19

P (prime number less than 23) = $\frac{8}{90} = \frac{4}{45}$

2. If two dice are thrown together, the total number of outcomes are :

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

P (even number on both dice) = $\frac{9}{36} = \frac{1}{4}$

3. Let B be Boy and G be Girl

Total number of outcomes = GGG, GGB, GBB, GBG, BBB, BBG, BGB, BGG

Favourable outcomes for at least 1 boy = 7

Total number of outcomes = 8

P (at least 1 boy) = $\frac{7}{8}$

4. Cards are numbered from 1 to 25

Total number of outcomes = 25

Cards divisible by both 2 and 3 = 6, 12, 18, 24

P (divisible by both 2 and 5) = $\frac{4}{25}$

5. Total numbers = – 3, – 2, – 1, 0, 1, 2, 3

Number of total outcomes = 7

Number less than 2 = – 3, – 2, – 1, 0, 1

P ($x < 2$) = $\frac{5}{6}$

6. Total number of cards = 52

Total number = 4

Total number of jack = 4

Total number of cards which are neither ace nor jack = 52 – (4 + 4)

= 52 – 8

= 44

P (neither ace nor jack) = $\frac{44}{52} = \frac{11}{13}$

7. Number of red balls = 5

Number of green balls = 8

Number of white balls = 7

Total of white balls = 5 + 8 + 7 = 20

P (getting a white balls or green balls) = $\frac{8+7}{20}$

$= \frac{15}{20} = \frac{3}{4}$

8. When a dice is thrown once, the total number of outcomes is 6.

P (number less than 3) = $\frac{2}{6} = \frac{1}{3}$

9. Total number of alphabets = 26

Number of consonants = 21

Number of vowels = 5

P (consonants) = $\frac{21}{26}$

10. Probability of two students not having the same birthday = P (B') = 0. 992

Probability of two students having the same birthday = P (B') = 1 – P (B')

= 1 – 0. 992

= 0. 008

SECTION-B

11. Total number = – 3, – 2, – 1, 0, 1, 2, 3

Number whose square is less than or equal to 1

i) $(-1)^2 = 1$

ii) $(0)^2 = 0$

iii) $(1)^2 = 1$

P (square is less than or equal to 1) = $\frac{3}{7}$

12. When two coins are tossed, total number of outcomes are { H, H }, { H,T }, { T, H }, { T, T }

Outcomes for at least one tail

= { H,T }, { T, H }, { T, T }

P (at least one tail) = $\frac{3}{4}$

13. When two dice are tossed together, total number of outcomes are :

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

a) Outcomes for number on each dice
= { 2, 2 }, { 2, 4 }, { 2, 6 },
{ 4, 2 }, { 4, 4 }, { 4, 6 },
{ 6, 2 }, { 6, 4 }, { 6, 6 }

P (both numbers are even) = $\frac{9}{36} = \frac{1}{4}$

b) Outcomes for sum on two dices is 5

P (sum on two dices is 5) = $\frac{4}{36} = \frac{1}{9}$

14. When two dices are rolled simultaneously, total number of outcomes are :

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

Outcomes for sum on the two dices is 10

= (4, 6), (5, 5), (6, 4)

P (sum on the two dice is 10) = $\frac{3}{36} = \frac{1}{12}$

15. Total number of jacks = 4

Total number of ace = 4

Number of cards is neither ace nor jack
= 52 – (4 + 4) = 44

P (neither jack nor ace) = $\frac{44}{52} = \frac{11}{13}$

16. When three coins are tossed simultaneously, the total outcomes are :

(H H H), (H H T), (H T H), (T H H),
(T T T), (T T H), (T H T), (H T T)

Outcomes for exactly 2 heads

= (H H T), (H T H), (T H H)

P (exactly 2 heads) = $\frac{3}{8}$

17. Total number of cards = 52

Total number of spades = 13

After losing 3 spades, number of spades left

= 13 – 3 = 10

Total number of black cards after losing 3 spades

= 26 – 3 = 23

P (black colour card) = $\frac{23}{52}$

18. Cards are numbered from 1 to 20

Number which are multiples of 3 or 7

= 3, 6, 7, 9, 12, 14, 15, 18

P (multiple of 3 or 7) = $\frac{8}{20} = \frac{2}{5}$

19. a) Total number of cards = 52

Total number of red king = 2

P (red king) = $\frac{2}{52} = \frac{1}{26}$

b) Total number of queen = 4

Total number of jack = 4

P (queen or jack) = $\frac{4+4}{52} = \frac{8}{52} = \frac{2}{13}$

20. Total number of red cards = 100

Total number of yellow cards = 200

Total number of blue cards = 50

Total number of cards = 100 + 200 + 50 = 350

a) P (blue card) = $\frac{50}{350} = \frac{1}{7}$

b) P (not a yellow card) = P (red and blue card)

$= \frac{100+50}{350} = \frac{150}{350} = \frac{3}{7}$

c) P (neither yellow nor blue card) = P (red card)

$= \frac{100}{350} = \frac{2}{7}$

SECTION-C

21. When two dices are thrown together, the total number of outcomes are :

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

a) Outcomes for prime number on each dice

= (3, 1), (3, 5) (5, 2)
(5, 2), (5, 5), (3, 3)
(2, 2), (2, 3), (2, 5)

P (prime number on each dice) = $\frac{9}{36} = \frac{1}{4}$

b) Outcomes for total of 9

= (3, 6) (4, 5), (5, 4), (6, 3)

Outcomes for total of 11 = (5, 6) (6, 5)

P (total of 9 or 11) = $\frac{4+2}{36} = \frac{6}{36} = \frac{1}{6}$

22. When two dices are thrown together, the total number of outcomes are :

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

a) Outcomes for a number greater than 3 on each dice = (4, 4), (4, 5), (4, 6)

(5, 4), (5, 5), (5, 6)

(6, 4), (6, 5), (6, 6)

P (number greater than 3 on each dice)

$= \frac{9}{36} = \frac{1}{4}$

b) Outcomes for getting a total of 6 on both dice

= (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)

P (getting a total of 6 or 7 on both dice)

$= \frac{5+6}{36} = \frac{11}{36}$

23. Total number of shirts = 100

Shirts which are good = 88

Shirts with minor defects = 8

Shirts with major defects = 4

a) P (Ramesh buys the selected shirt)

$= \frac{Number\ of\ good\ shirts}{Total\ number\ of\ shirts} = \frac{88}{100} = \frac{22}{25}$

b) P (Kewal buys the selected shirt)

$= \frac{Number\ of\ good\ shirts + shirts\ with\ minor\ defects}{Total\ number\ of\ shirts}$

$= \frac{88+8}{100}$

$= \frac{96}{100} = \frac{24}{25}$

24. When three coins are tossed together, the total number of outcomes are :

(H H H), (H H T), (H T H), (T H H),

(T T T), (T T H), (T H T), (H T T)

a) Outcomes for exactly two heads

= (H H T), (H T H), (T H H)

P (at least two heads) = $\frac{3}{8}$

b) Outcomes for at least two heads

= (H H H), (H H T), (H T H), (T H H)

P (at least to heads) = $\frac{4}{8} = \frac{1}{2}$

c) Outcomes for at least two tails

= (T T T), (T T H), (T H T), (H T T)

P (at least two tails) = $\frac{4}{8} = \frac{1}{2}$

25. Total number of cards = 52

Total number of jack, king = 2 + 2 + 2

And queen of red colour = 6

After removing these 6 cards, the total number of cards become 52 – 2 = 46

a) A black king

Total number of black kings = 2

P (black king) = $\frac{2}{46} = \frac{1}{23}$

b) A card of red colour

Number remaining red cards = 26 – 6 = 20

P (red cards) = $\frac{20}{46} = \frac{10}{23}$

c) A card of black colour

Number of black cards = 26

P (black king) = $\frac{26}{46} = \frac{13}{23}$

26. Cards are numbered from 1 to 100

Number divisible by 9 and is a perfect square

= 36, 81

a) P (divisible by 9 and a perfect square)

$= \frac{2}{100} = \frac{1}{50}$

b) Prime number greater than 80 = 83, 89, 97

P (prime number greater than 80) = $\frac{3}{100}$

27. When a coin tossed 3 times, the total number of outcomes are :

(H H H), (H H T), (H T H), (T H H),
(T T T), (T T H), (T H T), (H T T)

Ramesh wins if all the tosses show same result

= P (A) = (H H H), (T T T)

P (A) = $\frac{2}{8}$

P (Ramesh lossing the game) = 1 – P (A)

$= 1 - \frac{2}{8} = \frac{6}{8} = \frac{3}{4}$

28. Eight equal parts of the game are numbered as

= 1, 2, 3, 4, 5, 6, 7, 8,

a) An odd number

Outcomes for odd number = 1, 3, 5, 7

P (odd number) = $\frac{4}{8} = \frac{1}{2}$

b) A number greater than 3

Outcomes for number greater than 3

= 4, 5, 6, 7, 8

P (number greater than 3) = $\frac{5}{8}$

c) A number less than 9

Outcomes for number less than 9

= 1, 2, 3, 4, 5, 6, 7, 8,

P (number less than 9) = $\frac{8}{8} = 1$

29. A number ' x ' can be selected from 1, 2, 3, and 4

A number ' y ' can be selected from 1, 3, 9 and 16

Total number of Outcomes = 4 × 4 = 16

Cases for product ' x y ' to be less than 16 :

1) (1, 1) = 1 × 1 = 1
2) (1, 3) = 1 × 3 = 3
3) (1, 9) = 1 × 9 = 9
4) (2, 1) = 2 × 1 = 2
5) (2, 3) = 2 × 3 = 6
6) (3, 1) = 3 × 1 = 3
7) (3, 3) = 3 × 3 = 9
8) (4, 1) = 4 × 1 = 4
9) (4, 3) = 4 × 3 = 12

P (product of x and y less than 16) = $\frac{9}{16}$

30. When three coins are tossed together, the total number of outcomes are :

(H H H), (H H T), (H T H), (T H H),
(T T T), (T T H), (T H T), (H T T)

a) Outcomes for at least 2 heads
= (H H H), (H H T), (H T H), (T H H)

P (at least 2 heads) = $\frac{4}{8} = \frac{1}{2}$

b) Outcomes for at most 2 heads
= (H H T), (H T H), (T H H), (T T H),
(T H T), (H T T), (T T T)

P (at most 2 heads) = $\frac{7}{8}$

SECTION-D

31. Probability of selecting red balls P (R) = $\frac{1}{4}$

Probability of selecting blue balls P (B) = $\frac{1}{3}$

Probability of selecting orange balls P (O)

$$= 1 - \frac{1}{4}+\frac{1}{3}$$

$$= 1 - \frac{7}{12}$$

$$= 1 - \frac{12-7}{12}$$

$$= 1 - \frac{5}{12}$$

P (O) = $\frac{5}{12}$

Let there be ' n ' balls in a jar.
So,

P (O) = $\frac{5}{12}$

$\frac{5}{12} \times n = 10$

$n = \frac{10 \times 12}{5}$

n = 24 balls

P (B) = $\frac{1}{3}$

$\frac{1}{3} \times 24 = 8$ balls which are blue in colour

∴ 8 blue balls are present in the jar.

32. Total number of balls in a bag = 18 balls

Total number of red balls = x

Total number of balls which are not red
= 18 – x

a) P (ball is not red) = $\frac{18-x}{18}$

b) P (ball is red) = $\frac{x}{18}$

As 2 red balls are further added in the bag,
Total number of red balls = x + 2
Total number of balls in the bag = 18 + 2 = 20

P (red ball) = $\frac{x+2}{20}$

A.T.Q., $= \frac{9}{8} \times \frac{x}{18} = \frac{(x - 2)}{20}$

$\frac{x}{16} = \frac{x+2}{20}$

20 x = 16 x + 32

4 x = 32

x = 8 balls

∴ Initial number of red balls = 8

33. Cards are numbered from 1 to 25

a) Outcomes for number divisible by 3 or 5
= 3, 5, 6, 9, 10, 12, 15, 18, 20, 21, 24, 25

P (numbers divisible by 3 or 5) = $\frac{12}{25}$

b) Outcomes for a perfect square number
= 1, 4, 9, 16, 25

P (perfect square number) = $\frac{5}{25} = \frac{1}{25}$

34. a) Total number of cards = 52

Total number of spades = 13

Total number of aces excluding spades = 3

P (spade or ace) = $\frac{13+3}{52} = \frac{16}{52} = \frac{4}{13}$

b) P (black king) = $\frac{2}{52} = \frac{1}{26}$

c) Total number of jack = 4

Total number of king = 4

P (Either jack or king) = P (J) = $\frac{4+4}{52}$

$= \frac{8}{52} = \frac{2}{13}$

P (neither jack noe king) = 1 – P (J)

$= 1 - \frac{2}{13}$

$= \frac{13-2}{13}$

$= \frac{11}{13}$

d) Number of king = 4

Number of queen = 4

p (Either a king or a queen) = $\frac{4+4}{52}$

$= \frac{8}{52} = \frac{2}{13}$

35. Cards are numbered from 1 to 49

Total number of outcomes = 49

Outcomes for odd number cards = 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49

a) Total odd number cards = 25

P (odd number) = $\frac{25}{49}$

b) Outcomes for multiple of 5

= 5, 10, 15, 20, 25, 30, 35, 40, 45

P (multiple of 5) = $\frac{9}{49}$

c) Outcomes for perfect square number cards

= 1, 4, 9, 16, 25, 36, 49

P (perfect square) = $\frac{7}{49} = \frac{1}{7}$

d) Even prime numbered cards = 0

p (even prime number) = $\frac{0}{49}$

= 0

36. The sample space is

{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}

a) P (5 will not come either time) = $\frac{25}{36}$

b) P (5 will come up exactly one time) = $\frac{5}{36}$

37. Total number of persons = 12

Number of persons who are extremely patient = 3

Number of persons who are extremely honest = 6

Number of persons who are extremely kind

= 12 – (3 + 6)

= 12 – 9

= 3

a) P (person who is extremely patient) = $\frac{3}{12}$

$= \frac{1}{4}$

b) P (persons who are extremely kind or honest)

$$= \frac{3+6}{12}$$

$$= \frac{9}{12}$$

$$= \frac{3}{4}$$

CASE STUDY-1

(i) (b) Sample space when pair of dice is thrown

{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}

Probability that 4 will come either of them

$$= \frac{11}{36}$$

(ii) (b) P (5 will come at least once) = $\frac{11}{36}$

(iii) (a) (5, 5) is the only outcome in which 5 is coming on both dice $P\left[(5,5)\right] = \frac{1}{36}$

(iv) (c) Probabily both number are odd = $\frac{9}{36} = \frac{1}{4}$

(v) (d) Probability both numbers are prime

$$= \frac{9}{36} = \frac{1}{4}$$

CASE STUDY-2

(i) (c) Let A: getting a King

Let B: getting a club

P (either A or B) = P (A ∪ B)

P (A ∪ B) = P(A) + P(B) – P(A.B)

$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$$

$$= \frac{4}{13}$$

(ii) (a) Number of black Queens = 2

P (Black Queen) = $\frac{2}{52} = \frac{1}{26}$

(iii) (c) There are 8 cards which are either ace or jack

P (Ace or jack) = $\frac{8}{52} = \frac{2}{13}$

(iv) (b) Probability of red king = $\frac{2}{52} = \frac{1}{26}$

(v) (a) No. of King and Queen = 4 + 4 = 8

No. of card that don't have King and Queen = 44

P (neither King nor Queen) = $\frac{44}{52} = \frac{11}{13}$