ADDITIONAL™ PRACTICE

MATHEMATICS 10

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1. Real Numbers

MULTIPLE CHOICE QUESTIONS

1. (a) Here, a = Dividend, b = Divisor, q = Quotient and r = Remainder
Using Euclid's Divison Lemma,
$a = bq + r, 0 \le r < b$
$a = 3q + r$
Here $b = 3;$
So, possible values of r = 0, 1, 2.
$\therefore 0 \le r < 3.$

2. (c) LCM of 23 and 33 = 23 × 33

3. (b)

$$\begin{array}{r|l|l} n+7 & 2n+13 & 2 \\ & \underline{\overset{-}{}2n \overset{-}{+} 14} & \\ & -1 \;|\; n+7 & -n-7 \\ & \quad \underline{-n} & \\ & \quad 0+7 & \\ & \quad \underline{-+7} & \\ & \quad\quad 0 & \end{array}$$

$\therefore$ HCF = –1

4. (d) The two numbers 51 and 34
Their L. C. M is
102 = 17 x 3 x 2
51 = 17 x 3
34 = 17 x 2

5. (a) 70 – 5 = 65
(b) 125 – 8 = 117
Now, we will compute the H.C.F. of 65 and 117.

(a) 65

5	65,
13	13
	1

(b) 117

3	117
3	39
13	13
	1

13 is the only common factor of 65 and 117. Hence H.C.F. of 65 and 117 is 13.
Therefore, 13 is the largest number which divides 70 and 125 leaving the remainder of 5 and 8 respectively.
Let us check it.

(a) 70 ÷ 12
Quotient = 5
Remainder = 5

(b) 125 ÷ 12
Quotient = 9
Remainder = 8

WORKSHEET 1

Section A

1. Here, a is a dividend.
2. Number 13233343563715 is a composite number as it has more than two factors and a number which has more than two factors is a composite number and it is also divisible by 5 besides 1 and the number itself.
3. Dividend = Divisor × Quotient + Remainder
   = 53 × 34 + 21
   = 1802 + 21
   = 1823
4. y = 5 × 13 = 65
   x = 3 × 195 = 585
5. HCF (k, 2k, 3k, 4k, 5k) = k
6. Smallest composite no. = 4
   Smallest prime no. = 2
   ∴ HCF (2, 4) = 2
7. 6n = (2 × 3)n
   We know that a number ends with digit 0 only if it has both 2 and 5 as factors. As 6n does not have 5 as a prime factor, so, 6n does not end with digit 0.
8. $P = ab^2$ $Q = a^3b$
   FACTORS OF $P(ab^2)$ = a × b × b
   FACTORS OF $Q(a^3b)$ = a × a × a × b
   so, LCM OF PQ = a × a × a × b × b
   $= a^3b^2$
9. HCF of $a = x^3y^2$
   $b = xy^3$
   a = x x x y y
   b = x y y y
   The highest common factors of a and b are
   xy^2
10. $$\text{LCM }(a, b) = \frac{a \times b}{\text{HCF }(a, b)}$$
    $$= \frac{1800}{12} = 150$$

Section B

11. Let a be a given positive number.
    On dividing a by 4, let q be the quotient and r be the remainder.
    Then,by Euclid's algorithm,we have:
    a = 4q + r where 0 < = r < 4
    a = 4q + r where r = 0, 1, 2, 3
    a = 4q + 2 = 2 (2q + 1)
    It is clearly shown that 2q +1 is divisible by 2. Therefore, 4q + 2 is a positive integer.
12. Using Euclid's Algorithm,
    240 = 228 × 1 + 12

228 = 12 × 19 + 0
Here, remainder = 0, Divisor = 12
So, HCF (240, 228) = 12

13. If a, b are any two positive numbers
Their HCF (a, b) = h and LCM (a, b) = l then
a × b = h × l
Given
a = 253, b = 440
h = 11
l = 253 × R
Therefore

$$h \times l = a \times b$$
$$11 \times 253 \times R = 253 \times 440$$
$$R = \frac{(253 \times 440)}{(11 \times 253)}$$
$$R = 40$$

14. 3 × 12 × 101 + 4
= 4 × (3 × 3 × 101 + 1)
So, 4 is also a factor of 3 × 12 × 101 + 4 besides 1 and the no. itself.
So, 3 × 12 × 101 + 4 is a composite number.

15. Step by step explanation:
We have 1200,
We fist factorise the number "1200".
1200 = 2 × 2 × 2 × 2 × 3 × 5 × 5
We can see that 3 has no pair.
To make it a perfect square, we will multiply by 3 on both the sides.
1200 × 3 = 2 × 2 × 2 × 2 × 3 × 3 × 5 × 5

$$\sqrt{3600} = \sqrt{2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5}$$
$$\sqrt{3600} = 2 \times 2 \times 3 \times 5$$
$$\sqrt{3600} = 60$$

Hence, 3 is the smallest natural number by which 1200 should be multiplied so that to make it a perfect square.

16. Factors of 1 to 10 numbers
1 = 1
2 = 1 × 2
3 = 1 × 3
4 = 1 × 2 × 2
5 = 1 × 5
6 = 1 × 2 × 3
7 = 1 × 7
8 = 1 × 2 × 2 × 2
9 = 1 × 3 × 3
10 = 1 × 2 × 5
LCM of number 1 to 10 = LCM (1, 2, 3, 4, 5, 6, 7, 8, 9, 10)
= 1 × 2 × 2 × 2 × 3 × 3 × 5 × 7 = 2520

17. Let x and x + 1 be two consecutive positive integers.
If x is even, x + 1 is odd, so, x (x + 1) is even

If x is odd, x + 1 is even, so, x (x + 1) is even.
Therefore, the product of two consecutive positive integers is always divisible by 2.

18. $3 \times 5 \times 13 \times 46 + 23 = 23 \times (3 \times 5 \times 13 \times 2 + 1)$
So, 23 is a factor of $3 \times 15 \times 13 \times 46 + 23$ besides 1 and the no. itself.
Therefore, $3 \times 5 \times 13 \times 46 + 23$ is a composite number.

19. As least prime factor of a is 3, a is an odd no. (because if a is even then it's least prime factor must be 2). Also, as least prime factor of b is 5, b is an odd no.
Therefore, a + b is even such that it's least prime factor is 2.

20. No, two numbers can not have 15 as their HCF and 175 as their LCM because 15 is not a factor of 175.
(HCF of two numbers is always the factor of their LCM)

Section C

21. Using Euclid's Division lemma.
$a = 6q + r; o < r < 6$
$r = 0, a = 6q = 2(3q)$, even
$r = 1, a = 6q + 1 = 2(3q) + 1$, odd
$r = 2, a = 6q + 2 = 2(3q + 1)$, even
$r = 3, a = 6q + 3 = 2(3q + 1) + 1$, odd
$r = 4, a = 6q + 4 = 2(3q + 2)$, even
$r = 5, a = 6q + 5 = 2(3q + 2) + 1$, odd
So, any positive even integer can be written in the form of 6q, 6q + 2 or 6q + 4.

22. We know that any positive odd integer (say a) is of form 4q + 1 or 4q + 3
Case 1
$a = 4q + 1$
$a^2 = (4q + 1)^2 = 16q^2 + 1 + 8q = 8(2q^2 + q) + 1$
$= 8m + 1$ $(m = 2q^2 + q)$
Case 2
$a = 4q + 3$
$a^2 = (4q + 3)^2 = 16q^2 + 9 + 24q = 8(2q^2 + 3q + 1) + 1$
$= 8m + 1$ $(m = 2q^2 + 3q + 1)$
So, square of an odd positive integer is of form 8m + 1.

23. Consider 252 and 324.
Here, a = 324 and b = 252
by euclid's division lemma
$a = bq + r, \quad 0 \le \text{ or } = r < b$
$324 = 252 \times 1 + 72$
$252 = 72 \times 3 + 36$
$72 = 36 \times 2 + 0$
Therefore, HCF (252, 324) = 36
Now consider 36 and 180.
Here a = 180 and b = 36.
by euclid's division lemma- $a = bq + r, 0 < \text{ or } = r < b$
$180 = 36 \times 5 + 0$
Therefore, HCF (180, 36) = 36
Hence, HCF (180, 252, 324) = 36

24. Using Euclid's Division lemma,

$a = 5q + r; 0 < r < 5$

$r = 0, a = 5q, a^2 = 25q^2 = 5m \ (m = 5q^2)$

$r = 1, a = 5q + 1, a^2 = 25q^2 + 1 + 10q$

$= 5\ (5q^2 + 2q) + 1$

$= 5m + 1 \ (m = 5q^2 + 2q)$

$r = 2, a = 5q + 2, a^2 = 25q^2 + 4 + 20q$

$= 5\ (5q^2 + 4q) + 4$

$= 5m + 4 \ (m = 5q^2 + 4q)$

$r = 3, a = 5q + 3, a^2 = 25q^2 + 9 + 30q$

$= 5\ (5q^2 + 6q + 1) + 4$

$= 5m + 4 \ (m = 5q^2 + 6q + 1)$

$r = 4, a = 5q + 4, a^2 = 25q^2 + 16 + 40q$

$= 5\ (5q^2 + 8q + 3) + 1$

$= 5m + 1 \ (m = 5q^2 + 8q + 3)$

So, square of positive integer cannot be of form 5m + 2 or 5m + 3.

25. Minimum distance each should walk so that each can cover the same distance.

= LCM (40, 42, 45)

= 2520 cm

2	40,	42,	45
2	20,	21,	45
5	10,	21,	45
2	2,	21,	9
3	1,	21,	9
3	1,	7,	3
7	1,	7,	1
	1	1	1

26. $7 \times 19 \times 11 + 11 = 11\ (7 \times 19 \times 1 + 1)$

So, 11 is also a factor of $7 \times 19 \times 11 + 11$besides 1 and number itself.

So, $(7 \times 19 \times 11 + 11)$ is a composite number.

$7 \times 6 \times 4 \times 3 \times 2 \times 1 + 3 = 3\ (7 \times 6 \times 4 \times 2 \times 1 + 1)$

So, 3 is also a factor of $7 \times 6 \times 4 \times 3 \times 2 \times 1 + 3$ besides 1 and number itself.

So, $7 \times 6 \times 4 \times 3 \times 2 \times 1 + 3$ is a composite number.

27. Here, we have to find LCM (12, 15, 18) which indicates after how long they all again toll together.

LCM (12, 15, 18) = 180

So, three bells will toll together after 180 minutes i.e. 3 hours.

2	12,	15,	18
3	6,	15,	9
2	2,	5,	3
3	1,	5,	3
5	1,	5,	1
	1	1	1

28. Using Euclid's division algorithm,

$1170 = 650 \times 1 + 520$

$650 = 520 \times 1 + 130$

$520 = 130 \times 4 + 0$

So, HCF (650, 1170) = 130

Therefore, the largest number which divides 650 and 1170 exactly is 130.

29. Consider $\frac{1}{3 + 2\sqrt{2}} = \frac{1}{3 + 2\sqrt{2}} \times \frac{3 - 2\sqrt{2}}{3 - 2\sqrt{2}}$

$$= \frac{3 - 2\sqrt{2}}{3^2 - \left(2\sqrt{2}\right)^2} = \frac{3 - 2\sqrt{2}}{1} = 3 - 2\sqrt{2}$$

Let if possible $3 - 2\sqrt{2}$ is rational

$3 - 2\sqrt{2} = \frac{p}{q}$, p and q are integers and $q \neq 0$

$\frac{1}{2}\left(3 - \frac{p}{q}\right) = \sqrt{2}$

Here, $\frac{1}{2}\left(3 - \frac{p}{q}\right)$ is rational but $\sqrt{2}$ is irrational which is not possible.

So, we get a contradiction.

Therefore, $3 - 2\sqrt{2}$ is irrational.

i.e. $\frac{1}{3 + 2\sqrt{2}}$ is irrational.

30. Using Euclid's division lemma,

$117 = 65 \times 1 + 52$
$65 = 52 \times 1 + 13$
$52 = 13 \times 4 + 0$

Here, remainder = 0, divisor = 13

So, HCF (117, 65) = 13

To find : m, n

$13 = 65 - 52(1)$
$= 65 - (117 - 65(1))$
$= 65(2) + 117(-1)$
$= 65m + 117n$

So, m = 2, n = –1

Section D

31. Using Euclid's Division Algorithm, we get

$256 = 36 \times 7 + 4$
$36 = 4 \times 9 + 0$

Here, remainder = 0, divisor = 4

So, HCF (256, 36) = 4

LCM (256, 36) $= 2^8 \times 3^2$
$= 2304$

2	256,	36
2	128,	18
2	64,	9
2	32,	9
2	16,	9
2	8,	9
2	4,	9
2	2,	9
3	1,	9
3	1,	3
	1,	1

Now,

HCF × LCM $= 4 \times 2304$
$= 9216$

Product of numbers $= 256 \times 36$
$= 9216$

So, HCF × LCM = Product of numbers

32. We know that every positive even integer is of form 2q and every positive odd integer is of form 2q + 1.

Case 1 $n = 2q$

Consider $n^2 - n = 4q^2 - 2q = 2(2q^2 - q)$

$\therefore n^2 - n$ is divisible by 2

Case 2 $n = 2q + 1$

Consider $n^2 - n = (2q + 1)^2 - (2q + 1)$

$= 4q2 + 1 + 4q - 2q - 1$

$= 4q2 + 4q - 2q$

$= 2(2q2 + 2q - q)$

$= 2(2q2 + q)$

$\therefore n^2 - n$ is divisible by 2.

From Case 1, Case 2, we get $n^2 - n$ is divisible by 2 for every positive integer n.

33. According to Euclid's Division lemma,

$a = 3q + r, 0 \leq r < 3$

For $r = 0$

$a = 3q \Rightarrow a^3 = 27q^3 \Rightarrow a^3 = 9(3q^3)$

$= 9m \ (m = 3q^3)$

For $r = 1$

$a = 3q + 1 \Rightarrow a^3 = 27q^3 + 1 + 27q^2 + 9q$

$= 9(3q^3 + 3q^2 + q) + 1$

$= 9m + 1 \ (m = 3q^3 + 3q^2 + 2)$

For $r = 2$

$a = 3q + 2 \Rightarrow a^3 = 27q^3 + 8 + 54q^2 + 36q$

$= 9(3q^3 + 6q^2 + 4q) + 8$

$= 9m + 8 \ (m = 3q^3 + 6q^2 + 4q)$

Therefore, cube of any positive integer is of form 9m, 9m + 1 or 9m + 8 for some integer m.

34. (i) Greatest possible length of each plank

$= \text{HCF}(42, 49, 56)$

$= \text{HCF}(2 \times 3 \times 7, 7^2, 2^3 \times 7)$

$= 7$

So, greatest possible length of each plank is 7m.

(ii) HCF (182, 169)

$= \text{HCF}(2 \times 7 \times 13, 13^2)$

$= 13$

2	182
7	91
13	13
	1

13	169
13	13
	1

35. To find the no. of required baskets such that Each basket contains only one of the two fruits but equal in number.

We will find the H.C.F i.e. highest common factor

H.C.F :The largest common factor of two or more numbers is called the highest common factor

Thus $990 = 2 \times 3 \times 3 \times 4 \times 5$

$945 = 3 \times 3 \times 3 \times 5 \times 7$

Thus $\text{HCF} = 3 \times 3 \times 5 = 45$

Thus the no. of fruits to be put in each basket in order to have minimum no. of baskets = 45.

36. Let the three consecutive positive integers be n, n + 1 and n +2.

If number is divided by 3, remainder can be 0, 1 or 2. i.e. $n = 3q + r, 0 \le r < 3$

If $r = 0$, $n = 3q$ divisible by 3

If $r = 1$, $n + 2 = 3q + 1 + 2$

$= 3q + 3$

$= 3(q + 1)$ divisible by 3

If $r = 2$, $n + 1 = 3q + 2 + 1 = 3(q + 1)$ divisible by 3

So, one of numbers n, n + 1, and n + 2 must be divisible by 3 i.e. n (n + 1) (n + 2) is divisible by 3

Now, if a number is divided by 2, remainder is 0 or 1

i.e. $n = 2q + r$; $0 \le r < 2$

$r = 0$, $n = 2q$ divisible by 2

Also, $n + 2 = 2q + 2 = 2(q + 1)$ divisible by 2

So, one of n, n + 1 or n + 2 is divisible by 2 i.e.

n (n + 1) (n + 2) is divisible by 2.

Since, n (n + 1) (n + 2) is divisible by 2 and 3

implies n (n + 1) (n + 2) is divisible by 6.

37. (a)

2	420
2	210
5	105
3	21
7	7
	1

2	180
2	90
3	45
3	15
5	5
	1

2	378
3	189
3	63
3	21
7	7
	1

So, HCF of 378, 180 and 420 is $2 \times 3 = 6$

And LCM of 378, 180 and 420 is $2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 7 = 3780$

2	378,	180,	420
3	189,	90,	210
3	63,	30,	70
7	21,	10,	70
2	3,	10,	10
5	3,	5,	5
3	3,	1,	1
	1	1	1

So, let us check whether

LCM (378, 180 and 420) × HCF (378, 180 and 420)

is equal to the product of the three numbers

$3780 \times 6 = 378 \times 180 \times 420$

$22680 \neq 28576800$

Hence LCM × HCF is not equal to the product of the three numbers.

Now HCF × LCM

$= 6 \times 3780 = 22680$

Product of numbers $= 378 \times 180 \times 420$

$= 28576800$

So, HCF × LCM ≠ Product of numbers.

(b) Let if possible $2\sqrt{2}$ is rational.

$$2\sqrt{2} = \frac{p}{q}, \text{ p and q are integers, } q \neq 0$$

$$\Rightarrow \quad \sqrt{2} = \frac{p}{q}$$

Here, $\frac{p}{2q}$ is rational but $\sqrt{2}$ is irrational

So, we get a contradiction.

$\therefore 2\sqrt{2}$ is irrational.

38. (i) Let if possible $\frac{2\sqrt{3}}{5}$ is rational.

$$\frac{2\sqrt{3}}{5} = \frac{p}{q}; \text{ p and q are integers, } q \neq 0$$

$$\sqrt{3} = \frac{5p}{2q}$$

Here, $\frac{5p}{2q}$ is rational but $\sqrt{3}$ is irrational which is not possible, so we get a contradiction.

$\therefore \frac{2\sqrt{3}}{5}$ is irrational.

(ii) 3 rational numbers between 1.12 and 1.13 are 1.1210, 1.1211, 1.1213.
3 irrational numbers between 1.12 and 1.13 are 1.121121112111..., 1.1221222..., 1.123123312333...

WORKSHEET 2

Section A

1. Here, denominator = $2^2 \cdot 5^7 \cdot 7^2$. As denominator is not of the form $2^m \times 5^n$, so, the given rational number has a nonterminating repeating decimal expansion.

2. $$\frac{2\sqrt{45} + 2\sqrt{20}}{2\sqrt{5}} = \frac{6\sqrt{5} + 4\sqrt{5}}{2\sqrt{5}}$$

$$= \frac{10\sqrt{5}}{2\sqrt{5}}$$

= 5 which is rational.

3. HCF (a, b) × LCM (a, b) = a × b

15 × LCM = 45 × 105

$$\text{LCM} = \frac{45 \times 105}{15} = 315$$

4. Decimal expansion will terminate after 4 places of decimal.

5. HCF × LCM = 100 × 170 = 17000.

6. Here, denominator = 1500 = $2^2 \times 3 \times 5^3$
As denominator is not of the form $2^m \times 5^n$, so, it has non-terminating repeating decimal expansion.

7. HCF (a, b) × LCM (a, b) = a × b

$$9 \times 360 = a \times 45$$

$$\frac{9 \times 360}{45} = a$$

$$72 = a$$

8. $\frac{7}{625} = 0.0112$

9. $$\frac{95}{40} + \frac{15}{4} = \frac{95 + 150}{40}$$

$$= \frac{245}{40} = 6.125$$

10. Decimal expansion will terminate after 5 places of decimal.

Section B

11.

$$\begin{array}{r|l} 8 & 3 \\ & -0 \\ \hline & 30 \\ & -24 \\ \hline & 60 \\ & -56 \\ \hline & 40 \\ & -40 \\ \hline & 0 \end{array} \quad \text{Quotient} = 0.375$$

$\therefore \frac{3}{8} = 0.375$

12. Let if possible $5\sqrt{6}$ is rational.

$$5\sqrt{6} = \frac{p}{q}; \text{p, q are integers, } q \neq 0$$

$$\sqrt{6} = \frac{p}{5q}$$

Here, $\frac{p}{5q}$ is rational but $\sqrt{6}$ is irrational which is not possible. So, we get a contradiction i.e. $5\sqrt{6}$ is irrational.

13. Let $x = 1.\overline{41}...$ (1)

$x \times 100 = 1.\overline{41} \times 100$

$100x = 141.1\overline{41}$ (2)

On subtracting (1) from (2) , we get

$99x = 140$

$x = \frac{140}{99}$

14. Maximum capacity $= \text{HCF}(850, 680)$

$= \text{HCF}(2 \times 5^2 \times 17, 2^3 \times 5 \times 17)$

$= 2 \times 5 \times 17$

$= 170\ l.$

15. (i) $(-1) + (-1)^{2n} + (-1)^{2n} + 1 + (-1)^{4n+1}$

$= (-1) + (1) + (-1) + (-1)$

$= -2$

(ii) $\left(2^3\right)^{\frac{-5}{3}} = 2^{3 \times \frac{-5}{3}} = 2^{-5} = \frac{1}{2^5} = \frac{1}{32}$

16. The given rational number is $\frac{13}{64}$

Now $\frac{13}{64} = \frac{13}{2^6} = \frac{13}{2^6 \times 5^0}$

The denominator of the given rational number is of the form

$2^m \times 5^n$, i.e. $2^6 \times 5^0$

$\therefore$ The decimal expansion of $\frac{13}{64}$ is of the form of terminating.

The decimal expansion of $\frac{13}{64}$ terminates after 6 places of decimal.

$$\begin{array}{r|l} & 0.203125 \\ 64 & 13.000000 \\ & 128 \\ \hline & 200 \\ & 192 \\ \hline & 80 \\ & 64 \\ \hline & 160 \\ & 128 \\ \hline & 320 \\ & 320 \\ \hline & 0 \end{array}$$

17. Using Euclid's Algorithm.

$$\begin{aligned} 4052 &= 420 \times 9 + 272 \\ 420 &= 272 \times 1 + 148 \\ 272 &= 148 \times 1 + 124 \\ 148 &= 124 \times 1 + 24 \\ 124 &= 24 \times 5 + 4 \\ 24 &= 4 \times 6 + 0 \end{aligned}$$

Here remainder = 0, Divisor = 4

So, HCF (4052, 420) = 4

18. Let if possible $\frac{3}{\sqrt{5}}$ is rational

$\frac{3}{\sqrt{5}} = \frac{p}{q}$, p, q are integers, $q \neq 0$

$\sqrt{5} = \frac{3q}{p}$

Here, $\frac{3q}{p}$ is rational but 5 is irrational which is not possible, so we get a contradiction.

$\therefore \frac{3}{\sqrt{5}}$ is irrational.

19. Using Euclid's Division Algorithm,

$$\begin{aligned} 180 &= 144 \times 1 + 36 \\ 144 &= 36 \times 4 + 0 \end{aligned}$$

Here, remainder = 0, divisor = 36

So, HCF (144, 180) = 36

We can write

$$\begin{aligned} 36 &= 180 - 144 \ (1) \\ &= 36 \\ &= 39 - 3 \\ &= 13\ (3) - 3 \\ &= 13\,m - 3 \\ m &= 3 \end{aligned}$$

20. $9^n = (3 \times 3)^n$

Since, prime factorization does not contain 2 and 5, so, it cannot end with digit 0.

Section C

21. Let if possible $\sqrt{3} + \sqrt{5}$ is rational

$$\sqrt{3} + \sqrt{5} = \frac{p}{q} \text{ p and q are integers and } q \neq 0$$

$$\sqrt{3} = \frac{p}{q} - \sqrt{5}$$

$$\Rightarrow \quad \left(\sqrt{3}\right)^2 = \left(\frac{p}{q} - \sqrt{5}\right)^2$$

$$3 = \frac{p^2}{q^2} + 5 - \frac{2p}{q}\sqrt{5}$$

$$\frac{2p}{q}\sqrt{5} = \frac{p^2}{q^2} + 2$$

$$\sqrt{5} = \frac{q}{2p}\left(\frac{p^2}{q^2} + 2\right)$$

Here, $\frac{q}{2p}\left(\frac{p^2}{q^2} + 2\right)$ is rational but $\sqrt{5}$ is irrational, which is not possible.

Therefore, $\sqrt{3} + \sqrt{5}$ is irrational.

22. Let if possible $2\sqrt{3} + \sqrt{7}$ is rational

$$2\sqrt{3} + \sqrt{7} = \frac{p}{q}, \text{ q are integers, } q \neq 0$$

$$\sqrt{7} = \frac{p}{q} - 2\sqrt{3}$$

$$7 = \frac{p^2}{q^2} + 12 - \frac{4p}{q}\sqrt{3}$$

$$\frac{4p}{q}\sqrt{3} = \frac{p^2}{q^2} + 5$$

$$\sqrt{3} = \frac{q}{4p}\left(\frac{p^2}{q^2} + 5\right)$$

Here, $\frac{q}{4p}\left(\frac{p^2}{q^2} + 5\right)$ is rational but $\sqrt{3}$ is irrational which is not possible. So, we get a contradiction

$\therefore 2\sqrt{3} + \sqrt{7}$ is irrational.

$$\left(2\sqrt{3} + \sqrt{7}\right)\left(2\sqrt{3} - \sqrt{7}\right) = \left(2\sqrt{3}\right)^2 - \left(\sqrt{7}\right)^2$$

$$= 12 - 7 = 5 \text{ which is rational}$$

23. $5 \times 7 \times 13 \times 17 + 289 = 17 (5 \times 7 \times 13 \times 1 + 17)$
Here, 17 is also a factor of $5 \times 7 \times 13 \times 17 + 289$ besides 1 and number itself. So, it is a composite number.
Also, $7 \times 11 \times 13 \times 15 + 225 = (7 \times 11 \times 13 \times 1 + 15) 15$
Here, 15 is also a factor of $7 \times 11 \times 13 \times 15 + 225$ besides 1 and number itself. So, it is a composite number.

24. LCM (20, 30, 40) = 120
So, all the three bells will toll together after 120 minutes i.e. 2 hours.

2	20,	30,	40
2	10,	15,	20
5	5,	15,	10
2	1,	3,	2
3	1,	3,	1
	1	1	1

25. Using Euclid's Division algorithm.
$2058 = 378 \times 5 + 168$
$378 = 168 \times 2 + 42$
$168 = 42 \times 4 + 0$
Here, remainder = 0, divisor = 42
So, HCF (2058, 378) = 42

26. Let HCF = x

$\therefore$ LCM = 14x

LCM + HCF = 600

14x + x = 600

15x = 600 x = 40

We know that HCF (a, b) × LCM (a, b) = a × b

40 × 14 × 40 = a × 280

$$a = \frac{40 \times 14 \times 40}{280}$$

= 80

27. According to Euclid's division lemma,

a = bq + r and 0 ≤ r < b

let a = Some integer

b = 4

r = 0, 1, 2, 3

a = 4q, 4q + 1, 4q + 2, 4q + 3

Therefore, a is a positive integer if

a = 4q + 1, 4q + 3

28. Let if possible $7 - 2\sqrt{3}$ is rational.

$$7 - 2\sqrt{3} = \frac{p}{q}$$, p and q are integers and q ≠ 0

$$2\sqrt{3} = 7 - \frac{p}{q}$$

$$\sqrt{3} = \frac{1}{2}\left(7 - \frac{p}{q}\right)$$

Here, $\frac{1}{2}\left(7 - \frac{p}{q}\right)$ is rational but $\sqrt{3}$ is irrational which is not possible.

So, we get a contradiction

$\therefore$ $7 - 2\sqrt{3}$ is irrational.

Section D

29. (i) Let if possible $\frac{1}{\sqrt{2}}$ is rational

$$\frac{1}{\sqrt{2}} = \frac{p}{q}$$, p and q are integers and q ≠ 0

$$\sqrt{2} = \frac{q}{p}$$

Here, $\frac{q}{p}$ is rational but $\sqrt{2}$ is irrational which is not possible. So, we get a contraction.

$\therefore$ $\frac{1}{\sqrt{2}}$ is irrational

(ii) Let if possible $7\sqrt{5}$ is rational

$$7\sqrt{5} = \frac{p}{q}$$, p and q are integers and q ≠ 0

$$\sqrt{5} = \frac{p}{7q}$$

Here, $\frac{p}{7q}$ is rational but $\sqrt{5}$ is irrational which is not possible. So, we get a contraction.

$\therefore 7\sqrt{5}$ is irrational.

30. Using Euclid's division algorithm

$$237 = 81 \times 2 + 75$$
$$81 = 75 \times 1 + 6$$
$$75 = 6 \times 12 + 3$$
$$6 = 3 \times 2 + 0$$

So, HCF (237, 81) = 3

Consider $3 = 75 - 6(12)$
$= (81 - 6) - 6(12)$
$= 81 - 13(6)$
$= 81 - 13(81 - 75)$
$= 81 - 81(13) + 13(237 - 81(2))$
$= 81(1 - 13 - 26) + 237(13)$
$= 81(-38) + 237(13)$
$= 81x + 237y$

where $x = -38, y = 13$

31. HCF (96, 240, 336)
$= \text{HCF}(2^5 \times 3, 2^4 \times 3 \times 5, 2^4 \times 3 \times 7)$
$= 2^4 \times 3$
$= 48$

So, number of stacks of English books $= \frac{96}{48} = 2$

Number of stacks of Hindi books $= \frac{240}{48} = 5$

Number of stacks of Mathematics books $= \frac{336}{48} = 7$

32. (i) ⇨ AS we have to find 5 rational numbers between 1 and 2
⇨ We can consider = 1.1, 1.2, 1.3, 1.4, 1.5

$= \frac{11}{10}, \frac{12}{10}, \frac{13}{10}, \frac{14}{10}, \frac{15}{10}$

$= \frac{11}{10}, \frac{6}{5}, \frac{13}{10}, \frac{7}{5}, \frac{3}{2}$

(ii) HCF (70 – 5, 125 – 8)
= HCF (65, 117)
= HCF (5 × 13, 32 × 13)
= 13

33. Let if possible $\sqrt{3}$ is rational

$\sqrt{3} = \frac{p}{q}$, p and q are integers and $q \neq 0$

HCF (p, q) = 1

$q\sqrt{3} = p$

$3q^2 = p^2$

3 divides p^2 ⇨ 3 divides p

$p = 3c$

$p^2 = 9c^2 \Rightarrow 3q^2 = 9c^2$

$q^2 = 3c^2$

⇨ 3 divides q^2 ⇨ 3 divides q

So, p and q have atleast 3 in common which is a contradiction to the fact that HCF (p, q) = 1

So, our supposition was wrong,

$\sqrt{3}$ is irrational.

34. According to Euclid's division lemma, for any positive integer n, we have

$n = bq + r, \quad 0 \leq r < b$

Take b = 5

$n = 5q + r, 0 < r < 5$

For r = 0

$n = 5q$, divisible by 5

$n + 4 = 5q + 4$, not divisible by 5

$n + 8 = 5q + 8$, not divisible by 5

$n + 12 = 5q + 12$, not divisible by 5

$n + 16 = 5q + 16$, not divisible by 5

So, for r = 0, only n is divisible by 5

For r = 1

$n = 5q + 1$, not divisible by 5

$n + 4 = 5q + 1 + 4$

$= 5q + 5$

$= 5(q + 1)$, divisible by 5

$n + 8 = 5q + 1 + 8$

$= 5q + 9$, not divisible by 5

$n + 12 = 5q + 1 + 12$

$= 5q + 13$, not divisible by 5

$n + 16 = 5q + 1 + 16$

$= 5q + 17$, not divisible by 5.

So, for r = 1, only n + 4 is divisible by 5

For r = 2,

$n = 5q + 2$, not divisible by 5

$n + 4 = 5q + 6$, not divisible by 5

$n + 8 = 5q + 10$

$= 5(q + 2)$, divisible by 5

$n + 12 = 5q + 14$, not divisible by 5

$n + 16 = 5q + 18$, not divisible by 5

So, for r = 2, only n + 8 is divisible by 5

For r = 3

$n = 5q + 3$, not divisible by 5

$n + 4 = 5q + 7$, not divisible by 5

$n + 8 = 5q + 11$, not divisible by 5

$n + 12 = 5q + 15$

$= 5(q + 3)$, divisible by 5

$n + 16 = 5q + 19$, not divisible by 5.

So, for r = 3, only n + 12 is divisible by 5.

For r = 4

$n = 5q + 4$, not divisible by 5

$n + 4 = 5q + 8$, not divisible by 5

$n + 8 = 5q + 12$, not divisible by 5

$n + 12 = 5q + 16$, not divisible by 5

$n + 16 = 5q + 20$

$= 5(q + 4)$, divisible by 5

So, for $r = 4$, only $n + 16$ is divisible by 5.

35. Let if possible $n + \sqrt{m}$ is rational

$n + \sqrt{m} = \frac{p}{q}$, p and q are integers and $q \neq 0$

$\therefore \quad \sqrt{m} = \frac{p}{q} - n$

Here, $\frac{p}{q} - n$ is rational (as p, q are integers and n is rational) but $\sqrt{m}$ is irrational.

So, we get a contradiction.

Therefore, $n + \sqrt{m}$ is irrational.

36. To prove: $\sqrt{p} + \sqrt{q}$ is irrational.

Let if possible $\sqrt{p} + \sqrt{q}$ is rational

$\sqrt{p} + \sqrt{q} = \frac{a}{b}$, a and b are integers and $b \neq 0$

$\sqrt{p} = \frac{a}{b} - \sqrt{q}$

On squaring both sides, we get

$$p = \frac{a^2}{b^2} + 2 - \frac{2a}{b}\sqrt{q}$$

$$\frac{2a}{b}\sqrt{q} = \frac{a^2}{b^2} + q - p$$

$$\sqrt{q} = \frac{b}{2a}\left(\frac{a^2}{b^2} + q - p\right)$$

Here, $\frac{b}{2a}\left(\frac{a^2}{b^2} + q - p\right)$ is rational but $\sqrt{q}$ is irrational (as square root of a prime number is irrational) which is not possible.

So, we get a contradiction.

Therefore, $\sqrt{p} + \sqrt{q}$ is irrational.

37. = 72/24 km/h = 3 km/h

Time = Distance/Speed

Time for 1st cyclist = $\frac{360}{2}$ = 180 hrs

Time for 2nd cyclist = $360 \div \frac{60}{24}$ = 144 hrs

Time for 3rd cyclist = $\frac{360}{3}$ = 120 hrs

L.C.M of 180, 144, 120 = 720 hrs

Total Time taken in days = $\frac{720}{24}$ = 30 days

38. (i) In order to find the maximum number of columns in which they can march, we will find HCF (32, 616).

$32 = 2^5$

$616 = 2^3 \times 7 \times 11$

So, HCF (32, 616) = 2^3 = 8
Hence, maximum number of columns = 8

(ii) We know that for any two positive integers a and b,
LCM (a, b) × HCF (a, b) = a × b
LCM (306, 657) × HCF (306, 657)
= 306 × 657
LCM (306, 657) × 9 = 306 × 657

$$\text{LCM}(306, 657) = \frac{306 \times 657}{9} = 22338$$

39. (i) According to Euclid's Division lemma,
$a = bq + r; \quad 0 \le r < b$
Take b = 6
$a = 6q + r; 0 < r < 6$
For r = 0
a = 6q
= 2 (3q) which is even
For r = 1
a = 6q + 1
= 2 (3q) + 1 which is odd
For r = 2
a = 6q + 2
= 2 (3q + 1) which is even
For r = 3
a = 6q + 3
= 6q + 2 + 1
= 2(3q + 1) + 1 which is odd
For r = 4
a = 6q + 4
= 2 (3q + 2) which is even
For r = 5
a = 6q + 5
= 6q + 4 + 1
= 2 (3q + 2) + 1 which is odd
Therefore, every positive integer is of form 6q + 1 or 6q + 3 or 6q + 5.

(ii) LCM ($x^3 y^3$, $x^3 y^5$) = $x^3 y^5$

40. (i) 135 and 225
225 = 135 × 1 + 90
135 = 90 × 1 + 45
90 = 45 × 2 + 0
So, HCF (135, 225) = 45

(ii) 196 and 38220
38220 = 196 × 195 + 0
So, HCF (196, 38220) = 196

(iii) 867 and 255
867 = 255 × 3 + 102
255 = 102 × 2 + 51
102 = 51 × 2 + 0
So, HCF (867, 255) = 51

Chapter 02

Polynomials

MULTIPLE CHOICE QUESTIONS

1. (a) Let α, β be the zeroes of f(x)

$$\therefore \quad \alpha\beta = 3$$

$$\Rightarrow \quad \frac{K}{1} = 3$$

$$\Rightarrow \quad K = 3,$$

2. (c) $\alpha + \beta = \frac{-3}{4}, \quad \alpha\beta = \frac{7}{4}$

So, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{\frac{-3}{4}}{\frac{7}{4}} = \frac{-3}{7}$

3. (d) both a and c.

4. (b) Let $p(x) = 2x^2 + 2ax + 5x + 10$

As (x + a) is a factor of p(x),

$\therefore$ p(–a) = 0

$$2a^2 - 2a^2 - 5a + 10 = 0$$

$$5a = 10$$

$$a = 2$$

5. (b) If c = 0,

Discriminant (D) = $b^2 - 4a(0)$

< 0 (as f(x) has no real zeroes)

$\Rightarrow \quad b^2 < 0$ not possible

So, $\quad c \neq 0$

If $\quad c > 0$

In the discriminant, b^2 is positive. Discriminant (D) will be negative only if a > 0

Consider $\quad a + b + c < 0$

$\Rightarrow$ b < – a – c

$\Rightarrow$ – b > a + c

$\Rightarrow b^2 > (a + c)^2 = a^2 + c^2 + 2ac$

$\Rightarrow b^2 - 4ac > a^2 + c^2 - 2ac$

$\Rightarrow b^2 - 4ac > (a - c)^2 \geq 0$

Discriminant cannot be negative and positive simultaneously.

$\therefore$ a cannot be greater than 0.

$\Rightarrow$ c cannot be greater than 0.

So, only possibility is c < 0

WORKSHEET 1

Section A

1. $b^2 - 4ac = 0$

f(x) has two equal zeroes

2. A quadratic polynomial is of form

k {x^2 – (sum of zeroes) x + product of zeroes

$= k\{x^2 - \left(\frac{-1}{2}\right)x + (-3)\}$

$= \frac{k}{2}\{2x^2 + x - 6\}$

3. Let $p(x) = x^4 + x^3 - 2x^2 + x + 1$

Remainder is p(1) = 1 + 1 – 2 + 1 + 1 = 2

4. A binomial of degree 6 is $x^6 + 4x^2$

5. $3x^3 - x^2 - 3x + 1$

$= x^2(3x - 1) - 1(3x - 1)$

$= (x^2 - 1)(3x - 1)$

$= (x + 1)(x - 1)(3x - 1)$

6. a + b = 11, ab = 30

$a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$= (a + b) [a + b)^2 - 3ab]$

$= 11 (121 - 90)$

$= 11 (31)$

$= 341$

7. $f(x) = 6x^2 - 3 - 7x$

$= 6x^2 - 7x - 3$

$= 6x^2 - 9x + 2x - 3$

$= 3x (2x - 3) + 1 (2x - 3)$

$= (2x - 3) (3x + 1)$

Now, $f(x) = 0 \Rightarrow x = \frac{3}{2}, \frac{-1}{3}.$

So, zeroes are $x = \frac{3}{2}, \frac{-1}{3}$

8. $p(x) = 4x^2 - 5x - 1$

$\alpha + \beta = \frac{5}{4}, \quad \alpha\beta = \frac{-1}{4}$

$\alpha^2\beta + \alpha\beta^2 = \alpha\beta (\alpha + \beta) = \frac{-1}{4}\left(\frac{5}{4}\right) = \frac{-5}{16}$

9. $f(x) = 6x^3 + 3x^2 - 5x + 1$

$\alpha + \beta + \gamma = \frac{-3}{6} = \frac{-1}{2}$

$\alpha \beta \gamma = \frac{-1}{6}$

So, $\alpha^{-1} \beta^{-1} \gamma^{-1} = \frac{1}{\alpha\beta\gamma} = -6$

$\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2$

$= \alpha\beta\gamma (\alpha + \beta + \gamma)$

$= \frac{-1}{6}\left(\frac{-1}{2}\right) = \frac{1}{12}$

Section B

10. Dividend = Divisor × Quotient + Remainder

$x^3 + 2x^2 + 4x + b = (x + 1) (x^2 + ax + 3) + (2b - 3)$

$= (x^3 + ax^2 + 3x + x^2 + ax + 3 + 2b - 3)$

On comparing coefficients of x^2 and constant terms we get,

$a + 1 = 2 \Rightarrow a = 1$

$b = 3 + 2b - 3 \Rightarrow b = 0$

11. $p(x) = 3x^2 - 6x + 4$

$\alpha + \beta = 2, \quad \alpha\beta = \frac{4}{3}$

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{2} + \frac{1}{\beta}\right) + 3\alpha\beta$

$= \frac{\alpha^2 + \beta^2}{\alpha\beta} + 2\left(\frac{\alpha + \beta}{\alpha\beta}\right) + 3\alpha\beta$

$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} + 2\left(\frac{\alpha + \beta}{\alpha\beta}\right) + 3\alpha\beta$

$= \frac{4 - \frac{8}{3}}{\frac{4}{3}} + 2\left(\frac{2}{\frac{4}{3}}\right) + 3\left(\frac{4}{3}\right)$

$= 1 + 3 + 4$

$= 8$

12. Let the zeroes be $\alpha, \frac{1}{\alpha}$

$\alpha + \frac{1}{\alpha} = \frac{-13}{a^2 + 9}, \quad \alpha\frac{1}{\alpha} = \frac{6a}{a^2 + 9}$

So, $1 = \frac{6a}{a^2 + 9} \Rightarrow a^2 - 6a + 9 = 0$

$a^2 - 3a - 3a + 9 = 0$

$a (a - 3) - 3 (a - 3) = 0$

$(a - 3) (a - 3) = 0$

$a = 3$

13. Let the two zeroes of the $f(t) = kt^2 + 2t + 3k$ α and β.

Sum of zeroes $(\alpha + \beta)$

Product of the zeroes $\alpha\beta$

$\frac{-2}{k} = \frac{3k}{k}$

$-2k = 3k^2$

$2k + 3k^2 = 0$

$k(3k + 2) = 0$

$k = 0$

$k = \frac{-2}{3}$

14. Long division:

$$\begin{array}{r|l} & 2x^2 + 2x - 1 \\ \hline 4x^2 + 3x - 2 & 8x^4 + 14x^3 - 2x^2 + 8x - 12 \\ & 8x^4 + 6x^3 - 4x^2 \\ & - \quad - \quad + \\ \hline & 8x^3 + 2x^2 + 8x - 12 \\ & 8x^3 + 6x^2 - 4x \\ & - \quad - \quad + \\ \hline & -4x^2 + 12x - 12 \\ & -4x^2 - 3x + 2 \\ & + \quad + \quad - \\ \hline & 15x - 14 \end{array}$$

15. Cubic polynomial is of form

$\{x^3 - (\alpha + \beta + \gamma) x^2 + (\alpha\beta + \beta\gamma + \alpha\gamma) x - \alpha\beta\gamma\}$

$= k \{x^3 - (5 + 6 - 1) x^2 + (30 - 6 - 5) x - 30\}$

$= \quad k \{x^3 - 10x^2 + 19x + 30\}$

16. Let the zeros of the polynomial be:

$a - d$, a and $a + d$, so that the roots are in AP.

$f(x) = x^3 + 3px^2 + 3qx + r.$

The standard form of a cubic equation is:

$x^3 (a + b + c)x^2 + (ab + bc + ca)x - abc - 0.$

Comparing this equation with the given polynomial:

We find:

$3p = -(a - d + a + a + d)$

$\Rightarrow 3p = -3a$

$\Rightarrow p = -a$

$3q - (a - d) a + a (a + d) + (a + d)(a - d)$

$\Rightarrow 3q = a^2 - ad + a^2 + ad + a^2 - d^2$

$\Rightarrow 3q = 3a^2 - d^2$

$\Rightarrow d^2 = 3a^2 - 3q$

or, $d^2 = 3p^2 - 3q$

And, $r = -(a - d)a(a + d)$

Or, $r = ad^2 - a^3$

Or, $r = (-p) (3p^2 - 3q) - (-p)^3$

Or, $r = 3p^3 + 3pq + p^3$

Thus, $r = 3pq - 2p^3$.

Section C

17. $\alpha^2 + \beta^2$ can be written as $(\alpha + \beta)^2 - 2\alpha\beta$

$p(x) = 2x^2 - 5x + 7$

$a = 2$, $b = - 5$, $c = 7$

α and β are the zeros of $p(x)$

We know that ,

Sum of zeros $= \alpha + \beta = \frac{-b}{a} = \frac{5}{2}$

Product of zeros $= \frac{c}{a} = \frac{7}{2}$

$2\alpha + 3\beta$ and $3\alpha + 2\beta$ are zeros of a polynomial.

Sum of zeros $= 2\alpha + 3\beta + 3\alpha + 2\beta$

$= 5\alpha + 5\beta$

$= 5 [\alpha + \beta]$

$= 5 \times \frac{5}{2}$

$= \frac{25}{2}$

Product of zeros $= (2\alpha + 3\beta)(3\alpha + 2\beta)$

$= 2\alpha [3\alpha + 2\beta] + 3\beta [3\alpha + 2\beta]$

$= 6\alpha^2 + 4\alpha\beta + 9\alpha\beta + 6\beta^2$

$= 6\alpha^2 + 13\alpha\beta + 6\beta^2$

$= 6 [\alpha^2 + \beta^2] + 13\alpha\beta$

$= 6 [(\alpha + \beta)^2 - 2\alpha\beta] + 13\alpha\beta$

$= 6 \left[\left(\frac{5}{2}\right)^2 - 2 \times \frac{7}{2}\right] + 13 \times \frac{7}{2}$

$= 6 \left[\frac{25}{4} - 7\right] + \frac{91}{2}$

$= 6 \left[\frac{25}{4} - \frac{28}{4}\right] + \frac{91}{2}$

$= 6 \left[\frac{-3}{4}\right] + \frac{91}{2}$

$= \frac{-18}{4} + \frac{91}{2}$

$= \frac{-9}{2} + \frac{91}{2}$

$= \frac{82}{2}$

$= 41$

$\frac{-18}{4} = \frac{-9}{2}$ [Simplest form]

a quadratic polynomial is given by :-

k { x² – (sum of zeros)x + (product of zeros)}

$k\{x^2 - \frac{5}{2x} + 41\}$

k = 2

$2\{x^2 - \frac{5}{2x} + 41]$

$2x^2 - 5x + 82$ is the required polynomial.

18. Dividend = Divisor × Quotient + Remainder

$x^4 + 2x^3 - 2x^2 + x - 1 = (x^2 + 2x - 3)$ Quotient + Remainder

Quotient: $x^2 + 1$

$x^2 + 2x - 3 \,\big)\, x^4 + 2x^3 - 2x^2 + x - 1$

$x^4 + 2x^3 - 3x^2$ (– – +)

$x^2 + x - 1$

$x^2 + 2x - 3$ (– – +)

$-x + 2$

So, $x^4 + 2x^3 - 2x^2 + x - 1 = (x^2 + 2x - 3)(x^2 + 1) + (-x + 2)$

So, $-(-x + 2) = x - 2$ must be added to the polynomial f(x).

19. Quotient: $2x^2 + 5$

$3x^2 + 4x + 1 \,\big)\, 6x^4 + 8x^3 + 17x^2 + 21x + 7$

$6x^4 + 8x^3 + 2x^2$ (– – –)

$15x^2 + 21x + 7$

$15x^2 + 20x + 5$ (– – –)

$x + 2$

On comparing x + 2 with ax + b, we get

a = 1, b = 2

20. Let the quotient be $q(x) = ax^2 + bx + c$ and remainder $r(x) = px + q$

Using division algorithm,

f(x) = g (x) q(x) + r(x)

$3x^4 + 5x^3 - 7x^2 + 2x + 2$

$= (x^2 + 3x + 1)(ax^2 + bx + c) + px + q$

$= ax^4 + bx^3 + cx^2 + 3ax^3 + 3bx^2 + 3cx + ax^2 + bx + c + px + q$

$a = 3$

$5 = b + 3a \Rightarrow b = 5 - 3a \Rightarrow b = -4$

$-7 = c + 3b + a$

$-7 = c - 12 + 3 \Rightarrow c = 2$

$2 = 3c + b + p$

$2 = 6 - 4 + p \Rightarrow p = 0$

$2 = q + c \Rightarrow q = 2 - 2 = 0$

So, Remainder = px + q = 0

As remainder is zero, g(x) is a factor of p(x).

21. Let $f(x) = x^3 + 2x^2 + kx + 3$

Remainder = f(3) = 21

$3^3 + 2(3)^2 + 3k + 3 = 21$

$27 + 18 + 3k + 3 = 21$

$3k = 21 - 48 = -27$

$k = -9$

Now, we will find the quotient.

Dividend $= x^3 + 2x^2 + kx + 3$

$= x^3 + 2x^2 - 9x + 3$

Divisor = x – 3

Quotient: $x^2 + 5x + 6$

$x - 3 \,\big)\, x^3 + 2x^2 - 9x + 3$

$x^3 - 3x^2$ (– +)

$5x^2 - 9x + 3$

$5x^2 - 15x$ (– +)

$6x + 3$

$6x - 18$ (– +)

21

So, quotient $= x^2 + 5x + 6$

22. Zeroes are $-\sqrt{3}$ and $\sqrt{3}$

So, factors are $(x + \sqrt{3})$, $(x - \sqrt{3})$

i.e. $(x + \sqrt{3})(x - \sqrt{3})$ is also a factor

i.e. $x^2 - 3$ is a factor of given polynomial.

$$\begin{array}{r|l} & 2x + 1 \\ \hline x^2 - 3 & 2x^3 + x^2 - 6x - 3 \\ & \underline{\overset{-}{2x^3} \quad \overset{+}{-6x}} \\ & x^2 - 3 \\ & \underline{\overset{-}{x^2} \overset{+}{- 3}} \\ & 0 \end{array}$$

For the remaining zero,

put $2x + 1 = 0$

$$x = \frac{-1}{2}$$

23. As $\sqrt{2}$ is a zero of given polynomial, $x - \sqrt{2}$ is a factor of the polynomial.

$$\begin{array}{r|l} & 6x^2 + 7\sqrt{2}x + 4 \\ \hline x - \sqrt{2} & 6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2} \\ & \underline{\overset{-}{6x^3} \overset{+}{- 6\sqrt{2}x^2}} \\ & 7\sqrt{2}x^2 - 10x \\ & \underline{\overset{-}{7\sqrt{2}x^2} \overset{+}{- 14x}} \\ & 4x - 4\sqrt{2} \\ & \underline{\overset{-}{4x} \overset{+}{- 4\sqrt{2}}} \\ & 0 \end{array}$$

For other zeroes,

$6x^2 + 7\sqrt{2}x + 4 = 0$

$6x^2 + 3\sqrt{2}x + 4\sqrt{2}x + 4 = 0$

$3x(2x + \sqrt{2}) + 4\sqrt{2}x + 4 = 0$

$3\sqrt{2}x(\sqrt{2}x + 1) + 4(\sqrt{2}x + 1) = 0$

$(3\sqrt{2}x + 4)(\sqrt{2}x + 1) = 0$

$$x = \frac{-4}{3\sqrt{2}} = \frac{-4\sqrt{2}}{6} = \frac{-2\sqrt{2}}{3}$$

and $x = \dfrac{-1}{\sqrt{2}} = \dfrac{-\sqrt{2}}{2}$

24. According to division algorithm,

Dividend = Divisor × Quotient + Remainder

$x^3 - 3x^2 + x + 2 = g(x)(x - 2) + (-2x + 4)$

$$g(x) = \frac{x^3 - 3x^2 + x + 2 + 2x - 4}{x - 2}$$

$$= \frac{x^3 - 3x^2 + 3x - 2}{x - 2}$$

$$\begin{array}{r|l} & x^2 - x + 1 \\ \hline x - 2 & x^3 - 3x^2 + 3x - 2 \\ & \underline{\overset{-}{x^3} \overset{+}{- 2x^2}} \\ & -x^2 + 3x - 2 \\ & \underline{\overset{+}{-x^2} \overset{-}{+ 2x}} \\ & x - 2 \\ & \underline{\overset{-}{x} \overset{+}{- 2}} \\ & 0 \end{array}$$

So, $g(x) = x^2 - x + 1$

Section D

25. $f(x) = x^2 - px + q$

$\alpha + \beta = p, \quad \alpha\beta = q$

Consider

LHS

$$\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2} = \frac{\alpha^4 + \beta^4}{\alpha^2\beta^2}$$

$$= \frac{(\alpha^2)^2 + (\beta^2)^2}{\alpha^2\beta^2}$$

$$= \frac{[\alpha^2 + \beta^2]^2 - 2\alpha^2\beta^2}{(\alpha\beta)^2}$$

$$= \frac{[(\alpha + \beta)^2 - 2\alpha\beta]^2 - 2(\alpha\beta)^2}{(\alpha\beta)^2}$$

$$= \frac{[p^2 - 2q]^2 - 2(q)^2}{(q)^2}$$

$$= \frac{p^4 + 4q^2 - 4p^2q - 2q^2}{q^2}$$

$$= \frac{p^4}{q^2} - \frac{4p^2}{q} + 2 \quad = \text{RHS}$$

26. Let $p(x) = x^3 - 2x^2 + qx - r$

$\alpha + \beta + \gamma = 2$

For $\alpha + \beta = 0 \Rightarrow 0 + r = 2$

$\Rightarrow r = 2$

Also,

$$\alpha\beta\gamma = r$$
$$2\alpha\beta = r$$
$$\alpha\beta + \beta\gamma + \alpha\gamma = q$$
$$\alpha\beta + \gamma(\alpha + \beta) = q$$
$$\alpha\beta + \gamma(0) = q \quad [\text{As } \alpha + \beta = 0]$$
$$\alpha\beta = q$$
$$\frac{r}{2} = q$$
$$2q = r$$

27.

$$\begin{array}{r|l}
 & 2x^2 - 3x + (-8 - 2k) \\
\hline
x^2 + 2x + k & 2x^4 + x^3 - 14x^2 + 5x + 6 \\
 & 2x^4 + 4x^3 + 2kx^2 \\
 & - \quad - \quad - \\
\hline
 & -3x^3 + x^2(-14 - 2k) + 5x + 6 \\
 & -3x^3 - 6x^2 - 3kx \\
 & + \quad + \quad + \\
\hline
 & x^2(-8 - 2k) + x(5 + 3k) + 6 \\
 & x^2(-8 - 2k) + x(-16 - 4k) + k(-8 - 2k) \\
 & - \quad - \quad - \\
\hline
 & x(5 + 3k + 16 + 4k) + 6 + 8k + 2k^2 \\
\hline
\end{array}$$

Remainder = $(21 + 7k)x + 6 + 8k + 2k^2$

As $x^2 + 2x + k$ is a factor of

$2x^4 + x^3 - 14x^2 + 5x + 6$,

So, Remainder should be zero

$(21 + 7k)x + 6 + 8k + 2k^2 = 0$

$= 0x + 0$

On comparing coefficient of x, we get
$21 + 7k = 0$

$k = -3$

Now, we will find zeroes of the two polynomials.

$2x^4 + x^3 - 14x^2 + 5x + 6$

$= (x^2 + 2x + k)\,[2x^2 - 3x + (-8 - 2k)]$

$= (x^2 + 2x - 3)\,(2x^2 - 3x - 2)$

$= (x^2 + 3x - x - 3)\,(2x^2 - 4x + x - 2)$

$= [x(x + 3) - 1(x + 3)]\,[2x(x - 2) + 1(x - 2)]$

$= (x + 3)(x - 1)(2x + 1)(x - 2)$

So, zeroes are $-3, 1, \frac{-1}{2}, 2$

Again consider $x^2 + 2x + k$

$= x^2 + 2x - 3$

$= x^2 + 3x - x - 3$

$= x(x + 3) - 1(x + 3)$

$= (x - 1)(x + 3)$

So, zeroes are 1, – 3.

28.

$$f(x) = x^2 - 2x + 3$$
$$\alpha + \beta = 2$$
$$\alpha\beta = 3$$

(a) Roots are $(\alpha + 2, \beta + 2)$

Polynomial is

k {x^2 – (sum of zeroes) x + product of zeroes}

$= k\{x^2 - (\alpha + 2 + \beta + 2)x + (\alpha + 2)(\beta + 2)\}$

$= k\{x^2 - (\alpha + \beta + 4)x + \alpha\beta + 2(\alpha + \beta) + 4\}$

$= k\{x^2 - (2 + 4)x + 3 + 2(2) + 4\}$

$= k\{x^2 - 6x + 11\}$

(b) Sum of zeroes

$$= \frac{\alpha - 1}{\alpha + 1} + \frac{\beta - 1}{\beta + 1}$$

$$= \frac{(\alpha - 1)(\beta + 1) + (\alpha + 1)(\beta - 1)}{(\alpha + 1)(\beta - 1)}$$

$$= \frac{\alpha\beta + \alpha - \beta - 1 + \alpha\beta - \alpha + \beta - 1}{\alpha\beta + \alpha + \beta + 1}$$

$$= \frac{2\alpha\beta - 2}{\alpha\beta + \alpha + \beta + 1}$$

$$= \frac{6 - 2}{3 + 2 + 1}$$

$$= \frac{4}{6} = \frac{2}{3}$$

As

$\alpha + \beta = 2$

$\alpha\beta = 3$

Product of zeroes $= \left(\frac{\alpha-1}{\alpha+1}\right)\left(\frac{\beta-1}{\beta+1}\right)$

$= \frac{(\alpha-1)(\beta-1)}{(\alpha+1)(\beta+1)}$

$= \frac{\alpha\beta-(\alpha+\beta)+1}{\alpha\beta+(\alpha+\beta)+1}$

$= \frac{3-2+1}{3+2+1}$

$= \frac{2}{6}$

$= \frac{1}{3}$

A quadratic polynomial is of form

k {x^2 – (sum of zeroes) x + product of zeroes}

$= \left\{x^2 \ \frac{-2}{3}x\frac{+1}{3}\right\}$

$= \frac{k}{3}\{3x^2-2x+1\}$

29.

$$
\begin{array}{r|l}
 & x^2 - 2\sqrt{5}x + 3 \\
x-\sqrt{5} & x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5} \\
 & x^3 - \sqrt{5}x^2 \\
 & - \quad + \\
 & \quad -2\sqrt{5}x^2 + 13x - 3\sqrt{5} \\
 & \quad -2\sqrt{5}x^2 + 10x \\
 & \quad + \quad - \\
 & \qquad 3x - 3\sqrt{5} \\
 & \qquad 3x - 3\sqrt{5} \\
 & \qquad - \quad + \\
 & \qquad\quad 0
\end{array}
$$

For other zeroes,

Consider $x^2 - 2\sqrt{5}x + 3 = 0$

$x = \frac{2\sqrt{5}\pm\sqrt{20-12}}{2}$

$= \frac{2\sqrt{5}\pm\sqrt{8}}{2}$

$= \frac{2\sqrt{5}\pm 2\sqrt{2}}{2}$

$= \sqrt{5}\pm\sqrt{2}$

30. $ax^3 + 3x^2 - bx - 6$

$x = -1, -2$

Put the values of x in Equation

We get,

$a(-1)^3 + 3(-1)^2 - b(-1) - 6 = 0$

$\Rightarrow -a + 3 + b - 6 = 0$

$\Rightarrow b - a = 3$ --------(1)

Now $x = -2$

$a(-2)^3 + 3(-2)^2 - b(-2) - 6 = 0$

$\Rightarrow -8a + 12 + 2b - 6 = 0$

$\Rightarrow 2b - 8a + 6 = 0$

$\Rightarrow b - 4a = -3$ --------(2)

from (1) and (2)

$$
\begin{array}{r}
b - a = 3 \\
b - 4a = -3 \\
- \quad + \qquad + \\
\hline
3a = 6
\end{array}
$$

$\Rightarrow a = 2$ put in --------(1)

$b - 3 = 2$

$\Rightarrow$ b = 5 now put this value in Equation

$ax^3 + 3x^2 - bx - 6 = 0$

$\Rightarrow 2x^3 + 3x^2 - 5x - 6 = 0$

Two zeroes are given (–1, –2)

$(x + 1)(x + 2) = x^2 + 2x + x + 2 = 0$

$\Rightarrow x^2 + 3x + 2 = 0$

$$
\begin{array}{r|l}
 & 2x - 3 \\
x^2+3x+2 & 2x^3 + 3x^2 - 5x - 6 \\
 & 2x^3 + 6x^2 + 4x \\
 & - \quad - \quad - \\
 & \quad -3x^2 - 9x - 6 \\
 & \quad -3x^2 - 9x - 6 \\
 & \quad + \quad + \quad + \\
 & \qquad 000
\end{array}
$$

Hence, another zeroes is

$2x - 3 = 0$

$\Rightarrow x = \frac{3}{2}$

31. As zeroes of q(x) are also the zeroes of p(x), so, remainder should be zero. (As q(x) is a

factor of p(x)

$$
\begin{array}{r|l}
 & x^2 - 3x + 2 \\
x^3 + 2x^2 + a & x^5 + x^4 - 4x^3 + 3x^2 + 3x + b \\
 & x^5 + 2x^4 + ax^2 \\
 & - \quad - \quad - \\
\hline
 & - 3x^4 - 4x^3 + (3 - a)x^2 + 3x + b \\
 & - 3x^4 - 6x^3 \qquad - 3ax \\
 & + \quad + \qquad + \\
\hline
 & 2x^3 + (3 - a)x^2 + (3 + 3a)x + b \\
 & 2x^3 + 4x^2 \qquad + 2a \\
 & - \quad - \qquad - \\
\hline
 & (-a - 1)x^2 + (3 + 3a)x + (b - 2a) \\
\hline
\end{array}
$$

Remainder = 0

$(-a - 1)x^2 + (3 + 3a)x + (b - 2a) = 0$

$\Rightarrow \quad -a - 1 = 0, \quad b - 2a = 0$

$\Rightarrow \quad a = -1, \qquad b + 2 = 0$

$\Rightarrow \quad a = -1, \qquad b = -2$

Now,

$$p(x) = (x^3 + 2x^2 + a)(x^2 - 3x + 2) + 0$$
$$= (x^3 + 2x^2 - 1)(x^2 - 3x + 2)$$

For other zeroes of p(x),

Put $\quad x^2 - 3x + 2 = 0$

$$x^2 - 2x - x + 2 = 0$$
$$x(x - 2) - 1(x - 2) = 0$$
$$(x - 1)(x - 2) = 0$$
$$x = 1, 2$$

So, x = 1, 2 are zeroes of p(x) but not of q(x)

32. (i) $p(x) = x^3 - 5x^2 - 16x + 80$

Let the two zeroes be $\alpha, -\alpha$ and the third zero be γ.

$$\alpha + (-\alpha) + \gamma = 5$$
$$\gamma = 5$$

Also $\quad \alpha(-\alpha)\gamma = -80$

$$-\alpha^2 (5) = -80$$
$$\alpha^2 = \frac{80}{5} = 16$$
$$\alpha = \pm 4$$

For $\alpha = -4, \qquad -\alpha = -(-4) = 4$

For $\alpha = 4, \qquad -\alpha = -4.$

So, zeroes are –4, 4, 5

(ii)
$$f(x) = x^2 - p(x + 1) - c$$
$$= x^2 - px - (p + c)$$
$$\alpha + \beta = p, \quad \alpha\beta = -(p + c)$$

Consider

$$(\alpha + 1)(\beta + 1) = \alpha\beta + (\alpha + \beta) + 1$$
$$= -(p + c) + p + 1$$
$$= 1 - c$$

WORKSHEET 2

Section A

1. f(x) has 2 real zeroes.
2. $x^2 + 7x + 12$
   $= x^2 + 3x + 4x + 12$
   $= x^2 + (x + 3) + 4(x + 3)$
   $= (x + 3)(x + 4)$
   For zeroes of polynomial,
   $x + 3 = 0, \quad x + 4 = 0$
   $x = -3, \qquad x = -4$
3. Let a, $\frac{1}{\alpha}$ be the zeroes of p(x)
   $$\alpha \frac{1}{\alpha} = \frac{-a}{5}$$
   $$1 = \frac{-a}{5}$$
   $$a = -5$$
4. f(x) has 2 distinct real zeroes
5. Let $p(x) = x^3 + ax^2 + bx + c$
   Let α, β, γ be zeroes of p(x)
   Such that $\quad \alpha = -1$
   $$\alpha\beta\gamma = -C$$
   $$(-1)\beta\gamma = -C$$
   $$\beta\gamma = C$$

So, product of other two zeroes = C

6. Quadratic polynomial is of form

k {x^2 – (Sum of zeroes) x + product of zeroes}

$$= k\left\{x^2 - \left(\frac{2}{3} - \frac{1}{4}\right)x + \frac{2}{3}\left(\frac{-1}{4}\right)\right\}$$

$$= k\left\{x^2 - \left(\frac{5}{12}\right)x - \frac{1}{6}\right\}$$

$$= k\left\{\frac{12x^2 - 5x - 2}{12}\right\}$$

$$= \frac{k}{12}(12x^2 - 5x - 2)$$

7. $2y^2 + 7y + 5 = 0$

Here a = 2 , b = 7 , c = 5

α, β are 2 zeroes

So sum of the zeroes = $\alpha + \beta = \frac{-b}{ca} = \frac{-7}{2}$

Product of zeroes = $\alpha\beta = \frac{c}{a} = \frac{5}{2}$

Now just put the values

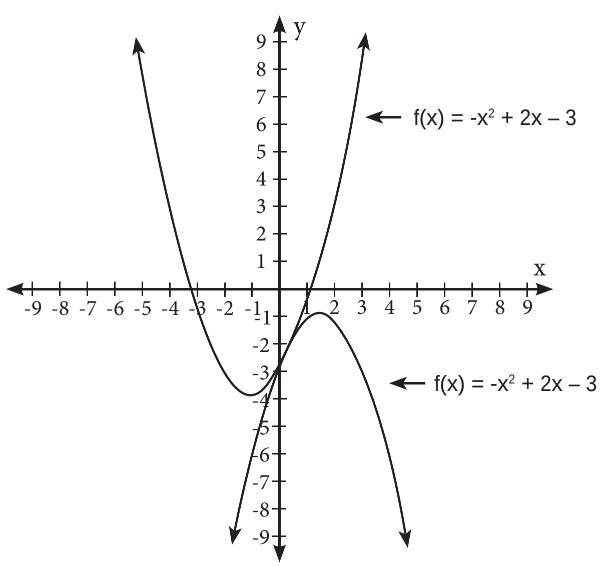
$$\alpha + \beta + \alpha\beta = (c + \beta) + \alpha\beta$$

$$= \frac{-7}{2} + \frac{5}{2}$$

$$= \frac{-2}{2} = -1$$

So required answer is –1

8.



In the above graph, for both the curves we can observe that the sign of c is negative only.

9. $f(x) = (k^2 + 4)x^2 + 13x + 4k$

Let the two zeroes be $\alpha, \frac{1}{\alpha}$

$$1 = \alpha\left(\frac{1}{\alpha}\right) = \frac{4k}{k^2 + 4}$$

$$k^2 - 4k + 4 = 0$$

$$(k - 2)^2 = 0$$

$$k = 2$$

10. $x^2 + 99x + 127$

$\alpha + \beta = -99, \quad \alpha\beta = 127$

α, β are either both positive or both negative

If α, β are both positive then $\alpha + \beta = -99$ is not possible

So, α and β must be negative.

Section B

11. $f(x) = x^2 - px + q$

$\alpha + \beta = p, \quad \alpha\beta = q$

(i) Consider $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
$= p^2 - 2q$

(ii) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{p}{q}$

12. We know that,

Sum of zeroes = $\frac{-b}{a}$

$\alpha + \beta = 5$ ------(1)

also,

Product of zeroes = $\frac{c}{a}$

$\alpha\beta = k$ ------(2)

Given:

$\alpha - \beta = 1$ ------(3)

From (1) and (3)

$\alpha + \beta = 5$

$\alpha - \beta = 1$ (on adding)

$2\alpha = 6$

$\alpha = 3$

Put this in (3)

$3 - \beta = 1$

$-\beta = -2$

$\beta = 2$

Now put this value in (3)

$\alpha\beta = k$

$2 \times 3 = k$

$6 = k$

13. Quadratic polynomial is of form

$k\{x^2 - (\text{sum of zeroes})x + \text{Product of zeroes}\}$

$$\text{Sum of zeroes} = \frac{4+\sqrt{2}}{2} + \frac{4-\sqrt{2}}{2} = 4$$

$$\text{Product of zeroes} = \left(\frac{4+\sqrt{2}}{2}\right)\left(\frac{4-\sqrt{2}}{2}\right) = \frac{16-2}{4} = \frac{14}{4} = \frac{7}{2}$$

So, quadratic polynomial is

$$k\left\{x^2 - 4x + \frac{7}{2}\right\} = \frac{k}{2}\{2x^2 - 8x + 7\}$$

14.

```
                  3x² – x
3x² + x – 1 ) 9x⁴ – 4x² + 4
              9x⁴ + 3x³ – 3x²
             –     –     +
             -----------------
                 – 3x³ – x² + 4
                 – 3x³ – x² + 4
                 +     +    –
                 ---------------
                       – x  + 4
```

Quotient $= 3x^2 - x$

Remainder $= -x + 4$

15. $f(x) = x^2 - 1 = x^2 + 0\,x - 1$

$\alpha + \beta = 0, \quad \alpha\beta = -1$

$$\text{Sum of zeroes} = \frac{2\alpha}{\beta} + \frac{2\beta}{\alpha} = 2\left(\frac{\alpha^2+\beta^2}{\alpha\beta}\right) = 2\left[\frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}\right]$$

$$= \frac{2}{-1}[0+2] = -4$$

$$\text{Product of zeroes} = \frac{2\alpha}{\beta}\;\frac{2\beta}{\alpha} = 4$$

A quadratic polynomial is of form $k\{x^2 - (\text{sum of zeroes})\,x + \text{product of zeroes}\}$

$k\{x^2 + 4x + 4\}$

16. $f(x) = ax^2 + bx + c$

$$\alpha + \beta = \frac{-b}{a}, \quad \alpha\beta = \frac{c}{a}$$

$$\text{Consider } \frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta}$$

$$= \frac{(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)}{\alpha\beta}$$

$$= \frac{(\alpha+\beta)\left[(\alpha+\beta)^2 - 3\alpha\beta\right]}{\alpha\beta}$$

$$= \frac{\frac{-b}{a}\left[\frac{b^2}{a^2} - \frac{3c}{a}\right]}{\frac{c}{a}}$$

$$= \frac{-b}{c}\left(\frac{b^2-3ac}{a^2}\right)$$

17. As 1 is a zero of f(x),

so, (x – 1) is a factor of f(x)

```
          –x² – x + 6
x – 1 ) –x³ + 7x + 6
        –x³ + x²
        +     –
        ----------------
           – x² + 7x – 6
           – x² + x
           +      –
           -------------
                6x – 6
                6x – 6
               –     +
               ---------
                     0
```

For other zeroes of f(x),

put $\quad -x^2 - x + 6 = 0$

$x^2 + x - 6 = 0$

$$x^2 + 3x - 2x - 6 = 0$$
$$x(x + 3) - 2(x + 3) = 0$$
$$(x - 2)(x + 3) = 0$$
$$x = 2, \quad -3$$

So, other zeroes are $x = 2, \quad -3$

18. $f(x) = x^2 - 13x + k$

Let α, β be two zeroes of f(x)

$$\alpha\beta = k = 40$$

So, $$f(x) = x^2 - 13x + 40$$
$$= x^2 - 5x - 8x + 40$$
$$= x(x - 5) - 8(x - 5)$$
$$= -(x - 5)(x - 8)$$

For zeroes of f(x), put f(x) = 0

i.e. $(x - 5) \quad (x - 8) = 0$

$$x = 5, 8$$

19.

$$\begin{array}{r|l}
 & 2x^2 - 2x - 1 \\
\hline
4x^2 + 3x - 2 & 8x^4 + 14x^3 - 2x^2 + 7x - 8 \\
 & 8x^4 + 6x^3 - 4x^2 \\
 & - \quad - \quad + \\
\hline
 & 8x^3 + 2x^2 + 7x - 8 \\
 & 8x^3 + 6x^2 - 4x \\
 & - \quad - \quad + \\
\hline
 & -4x^2 + 11x - 8 \\
 & -4x^2 + 4x + 2 \\
 & + \quad + \quad - \\
\hline
 & 15x - 10 \\
\hline
\end{array}$$

So, $15x - 10$ must be subtracted from $8x^4 + 14x^3 - 2x^2 + 7x - 8$. So, that the resultant polynomial is exactly divisible by $4x^2 + 3x - 2$.

20. $f(t) = t^2 - 4t + 3$

$\alpha + \beta = 4, \quad \alpha\beta = 3$

Consider

$$\alpha^4\beta^3 + \alpha^3\beta^4 = \alpha^3\beta^3(\alpha + \beta)$$
$$= (\alpha\beta)^3(\alpha + \beta)$$
$$= 27(4) = 108$$

And $$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{4}{3}$$

Section C

21. Let a – d, a and a + d be the zeroes of f(x)

$$a - d + a + a + d = 12$$
$$3a = 12$$
$$a = 4$$

Also, $$(a - d)\,a\,(a + d) = 28$$
$$(4 - d)\,4\,(4 + d) = 28$$
$$16 - d^2 = 7$$
$$d^2 = 9$$
$$d = \underline{+}3$$

Case 1

a = 4, d = 3

So, zeroes are

a – d, a, a + d = 1, 4, 7

Case 2

a = 4, d = –3

So, zeroes are 7, 4, 1

Therefore, zeroes of polynomial are 1, 4 and 7.

22.

$$\begin{array}{r|l}
 & 10x^2 + \frac{19}{3}x - \frac{8}{9} \\
\hline
3x^2 - x + 1 & 30x^4 + 9x^3 + x^2 + 2 \\
 & 30x^4 - 10x^3 + 10x^2 \\
 & - \quad + \quad - \\
\hline
 & 19x^3 - 9x^2 + 2 \\
 & 19x^3 - \frac{19}{3}x^2 + \frac{19}{3}x \\
 & - \quad + \quad - \\
\hline
 & -\frac{8}{3}x^2 - \frac{19}{3}x + 2 \\
 & -\frac{8}{3}x^2 + \frac{8}{9}x - \frac{8}{9} \\
 & + \quad - \quad + \\
\hline
 & -\frac{65}{9}x + \frac{26}{9} \\
\hline
\end{array}$$

$$\text{Dividend} = 30x^4 + 9x^3 + x^2 + 2$$
$$\text{Divisor} = 3x^2 - x + 1$$
$$\text{Quotient} = 10x^2 + \frac{19}{3}x - \frac{8}{9}$$

$$\text{Remainder} = -\frac{65}{9}x + \frac{26}{9}$$

According to divisor algorithm,

Dividend = Divisor × Quotient + Remainder

Consider

Divisor × Quotient + Remainder

$$= (3x^2 - x + 1)\left(10x^2 + \frac{19}{3}x - \frac{8}{9}\right) - \frac{8x}{9} - \frac{31}{9}$$

$$= 30x^4 + 19x^3 - \frac{8}{3}x^2 - 10x^3 - \frac{19}{3}x^2 + \frac{8}{9}x + 10x^2 + \frac{19}{3}x - \frac{8}{9} - \frac{65}{9}x + \frac{26}{9}$$

$$= 30x^4 + x^3(19 - 10) + x^2\left(-\frac{8}{3} - \frac{19}{3} + 10\right) + x\left(\frac{8}{9} + \frac{19}{3} - \frac{65}{9}\right) + \left(-\frac{8}{9} + \frac{26}{9}\right)$$

$$= 30x^4 + 9x^3 + x^2 + 0x + 2$$

$$= 30x^4 + 9x^3 + x^2 + 2$$

= Dividend Hence verified.

23. $f(x) = 4\sqrt{3}x^2 + 5x - 2\sqrt{3}$

Sum = 5 (8 – 3)

Product = – 24 (8 × –3)

$$f(x) = 4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3}$$

$$= 4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2)$$

$$= (4x - \sqrt{3})(\sqrt{3}x + 2)$$

For zeroes of f(x), put f(x) = 0

$$(4x - \sqrt{3})(\sqrt{3}x + 2) = 0$$

$$x = \frac{\sqrt{3}}{4}, \frac{-2}{\sqrt{3}}$$

$$\text{Sum of zeroes} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$= \frac{\sqrt{3}}{4} - \frac{2}{\sqrt{3}} \qquad = -\frac{5}{4\sqrt{3}}$$

$$= \frac{3-8}{4\sqrt{3}} \qquad = -\frac{5}{4\sqrt{3}}$$

$$\therefore \quad \text{Sum of zeroes} = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$\text{Product of zeroes} = \left(\frac{\sqrt{3}}{4}\right)\left(\frac{-2}{\sqrt{3}}\right)$$

$$= -\frac{1}{2}$$

$$\frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$= -\frac{2\sqrt{3}}{4\sqrt{3}}$$

$$= -\frac{1}{2}$$

$$\therefore \quad \text{Product of zeroes} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

Hence, the relationship between zeroes and its coefficient is verified.

24. $f(x) = x^2 - x - 2$

$\alpha + \beta = 1, \quad \alpha\beta = -2$

Sum of zeroes $= 2\alpha + 1 + 2\beta + 1$

$= 2(\alpha + \beta) + 2$

$= 2(1) + 2$

$= 4$

Product of zeroes $= (2\alpha + 1)(2\beta + 1)$

$= 4\alpha\beta + 2(\alpha + \beta) + 1$

$= -8 + 2 + 1$

$= -5$

Quadratic polynomial is of form

k {x^2 – (sum of zeroes) x + Product of zeroes}

= k {$x^2 - 4x - 5$}

Now, we need to find $\alpha^3 + \beta^3$

$= (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta)$

$= (\alpha + \beta)[(\alpha + \beta)^2 - 3\alpha\beta]$

$= 4(16 + 15)$

$= 4(31)$

$= 124$

25. $f(x) = 3x^2 - 4x + 1$

$\alpha + \beta = \frac{4}{3}, \quad \alpha\beta = \frac{1}{3}$

$$\text{Sum of zeroes} = \frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$$

$$= \frac{\alpha^3 + \beta^3}{\alpha\beta}$$

$$= \frac{(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)}{\alpha\beta}$$

$$= \frac{(\alpha+\beta)\left[(\alpha+\beta)^2 - 3\alpha\beta\right]}{\alpha\beta}$$

$$= \frac{\frac{4}{3}\left(\frac{16}{9} - 1\right)}{\frac{1}{3}}$$

$$= 4\left(\frac{16-9}{9}\right) = \frac{28}{9}$$

$$\text{Product of zeroes} = \frac{\alpha^2\beta^2}{\alpha\beta} = \alpha\beta = \frac{1}{3}$$

Quadratic polynomial is of the form k {x^2 – (sum of zeroes) x + product of zeroes}

$$= k\left\{x^2 - \frac{28}{9}x + \frac{1}{3}\right\}$$

$$= \frac{k}{9}\left\{9x^2 - 28x + 3\right\}$$

26.

$$\begin{array}{r|l}
 & x^2 + x + 7 \\
x^2 + 1 & x^4 + x^3 + 8x^2 + ax + b \\
 & \underline{\overset{-}{x^4} + \overset{-}{x^2}} \\
 & x^3 + 7x^2 + ax + b \\
 & \underline{\overset{-}{x^3} + \overset{-}{x}} \\
 & 7x^2 + (a-1)x + b \\
 & \underline{\overset{-}{7x^2} + \overset{-}{7}} \\
 & (a-1)x + (b-7)
\end{array}$$

As $x^4 + x^3 + 8x^2 + ax + b$ is exactly divisible by $x^2 + 1$

$\therefore$ Remainder = 0

$(a-1)x + (b-7) = 0$

$a = 1, \quad b = 7$

27. Let α, β be the zeroes of f(x)

$$\alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a}$$

Zeroes of the required polynomial are $\frac{1}{\alpha}, \frac{1}{\beta}$

Quadratic polynomial is of form

$$k\left\{x^2 - \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)x + \frac{1}{\alpha\beta}\right\}$$

$$= k\left\{x^2 - \left(\frac{\alpha+\beta}{\alpha\beta}\right)x + \frac{1}{\alpha\beta}\right\}$$

$$= k\left\{x^2 - \left(\frac{-\frac{b}{a}}{\frac{c}{a}}\right)x + \frac{a}{c}\right\}$$

$$= k\left\{x^2 + \frac{b}{c}x + \frac{a}{c}\right\}$$

$$= \frac{k}{c}\{cx^2 + bx + a\}$$

28. $f(x) = x^3 - 4x^2 - 3x + 12$

As $\sqrt{3}, -\sqrt{3}$ are zeroes of f(x), so $(x - \sqrt{3})$ $(x + \sqrt{3})$ are factors of f(x).

i.e. $(x - \sqrt{3})(x + \sqrt{3}) = x^2 - 3$ is a factor of f(x)

factor of f(x)

$$\begin{array}{r|l}
 & x - 4 \\
x^2 - 3 & x^3 - 4x^2 - 3x + 12 \\
 & \underline{\overset{-}{x^3} \overset{+}{-} 3x} \\
 & -4x^2 + 12 \\
 & \underline{\overset{+}{-} 4x^2 \overset{-}{+} 12} \\
 & 0
\end{array}$$

For third zero, $x - 4 = 0$

$x = 4$

29. $p(x) = 2x^2 + 5x + k$

$$\alpha + \beta = -\frac{5}{1}, \quad \alpha\beta = \frac{k}{2}$$

Given: $\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$

$$\Rightarrow (\alpha + \beta)^2 - \alpha\beta = \frac{21}{4}$$

$$\Rightarrow \frac{25}{4} - \frac{k}{2} = \frac{21}{4}$$

$$\Rightarrow \frac{k}{2} = \frac{25}{4} - \frac{21}{4} = 1$$

$$\Rightarrow k = 2$$

30.

$$\begin{array}{r|l} & x^2 + 2x + 3 \\ \hline x^2 + 5 & x^4 + 2x^3 + 8x^2 + 12x + 18 \\ & x^4 \quad\quad\;\; + 5x^2 \\ & - \quad\quad\quad - \\ \hline & \quad 2x^3 + 3x^2 + 12x + 18 \\ & \quad 2x^3 \quad\quad\;\; + 10x \\ & \quad - \quad\quad\quad\;\; - \\ \hline & \quad\quad 3x^2 + 2x + 18 \\ & \quad\quad 3x^2 \quad\quad\; + 15 \\ & \quad\quad - \quad\quad\quad - \\ \hline & \quad\quad\quad\quad 2x + 3 \\ \hline \end{array}$$

On comparing $2x + 3$ with $px + q$,

we get $p = 2, \quad q = 3$

Section D

31.

$$\begin{array}{r|l} & x^2 - 4x + (8 - k) \\ \hline x^2 - 2x + k & x^4 - 6x^3 + 16x^2 - 25x + 10 \\ & x^4 - 2x^3 + kx^2 \\ & - \quad + \quad\;\; - \\ \hline & \quad - 4x^3 + (16 - k)x^2 - 25x + 10 \\ & \quad - 4x^3 + 8x^2 \quad\quad - 4kx \\ & \quad + \quad\quad\; - \quad\quad\quad\;\; + \\ \hline & \quad\quad (8 - k)x^2 + (-25 + 4k)x + 10 \\ & \quad\quad (8 - k)x^2 - 2(8 - k)x + k(8 - k) \\ \hline & \quad\quad (-9 + 2k)x + (10 - 8k + k^2) \\ \hline \end{array}$$

Remainder $= (-9 + 2k)x + (10 - 8k + k^2)$

$= x + \alpha$

$$\frac{[x^4 - 6x^3 + 16x^2 - 25x + 10]}{x^2 - 2x + k} \text{ and remainder} = x + a$$

On diving the above given equation, we get,

Given the reminder is $(x + a)$

$(4k - 25 + 16 - 2k)x + [10 - k(8 - k)] = x + a$

$(2k - 9)x + [10 - 8k + k^2] = x + a$

On comparing on both sides, we get

$2k - 9 = 1$

$2k = 10$

Therefore, $k = 5$

Also, $10 - 8k + k^2 = a$

$10 - 8(5) - 5^2 = a$

$10 - 40 + 25 = a$

Therefore, $a = -5$

32. $f(x) = x^4 - 6x^3 - 26x^2 + 138x - 35$

Zeroes of $f(x)$ are $2 \pm \sqrt{3}$.

So, $[x - (2 + \sqrt{3})]$, $[x - (2 - \sqrt{3})]$

are factors of $f(x)$

i.e. $[(x - 2) - \sqrt{3}]\,[(x - 2) + \sqrt{3}]$ is a factor of $f(x)$.

i.e. $(x - 2)^2 - (\sqrt{3})^2$ is a factor of $f(x)$

i.e. $x^2 + 4 - 4x - 3$ is a factor of $f(x)$

i.e. $x^2 - 4x + 1$ is a factor of $f(x)$

$$\begin{array}{r|l} & x^2 - 2x - 35 \\ \hline x^2 - 4x + 1 & x^4 - 6x^3 - 26x^2 + 138x - 35 \\ & x^4 - 4x^3 + x^2 \\ & - \quad + \quad\;\; - \\ \hline & \quad - 2x^3 - 27x^2 + 138x \\ & \quad - 2x^3 + 8x^2 - 2x \\ & \quad + \quad\;\; - \quad\;\; + \\ \hline & \quad\quad - 35x^2 + 140x - 35 \\ & \quad\quad - 35x2 + 140x - 35 \\ & \quad\quad + \quad\quad - \quad\quad + \\ \hline & \quad\quad\quad\quad 0 \\ \hline \end{array}$$

For other zeroes

Put $x^2 - 2x - 35 = 0$

$x^2 - 7x + 5x - 35 = 0$

$x(x - 7) + 5(x - 7) = 0$

$$(x + 5)(x - 7) = 0$$
$$x = -5, 7$$

So, other zeroes are –5 and 7.

33. $f(x) = x^3 - 5x^2 - 2x + 24$

Let α, β, γ be the zeroes of f(x).

$$\alpha\beta = 12 \quad \text{...(i)}$$
$$\alpha + \beta + \gamma = 5$$
$$\alpha\beta\gamma = -24 \Rightarrow 12\gamma = -24$$
$$\Rightarrow \gamma = -2$$

Also, $\alpha + \beta + \gamma = 5 \Rightarrow \alpha + \beta - 2 = 5$

$$\alpha + \beta = 7 \quad \text{...(iii)}$$

On solving (i) and (ii), we get

$$\alpha(7 - \alpha) = 12$$
$$7\alpha - \alpha^2 = 12$$
$$\alpha^2 - 7\alpha + 12 = 0$$
$$\alpha^2 - 3\alpha - 4\alpha + 12 = 0$$
$$\alpha(\alpha - 3) - 4(\alpha - 3) = 0$$
$$(\alpha - 3)(\alpha - 4) = 0$$
$$\alpha = 3, 4$$

If $\alpha = 3, \beta = 7 - \alpha = 7 - 3 = 4$

If $\alpha = 4, \beta = 7 - \alpha = 7 - 4 = 3$

So, zeroes of the polynomial are 3, 4 and –2.

34. $f(x) = 2x^4 - 2x^3 - 7x^2 + 3x + 6$

$-\sqrt{\frac{3}{2}}$ and $\sqrt{\frac{3}{2}}$ are zeroes of f(x)

$\left(x + \sqrt{\frac{3}{2}}\right), \left(x - \sqrt{\frac{3}{2}}\right)$ are factors of f(x)

$\left(x + \sqrt{\frac{3}{2}}\right)\left(x - \sqrt{\frac{3}{2}}\right)$ is a factor of f(x)

$\left(x^2 - \frac{3}{2}\right)$ is a factor of f(x)

$(2x^2 - 3)$ is a factor of f(x)

$$\begin{array}{r|l} & x^2 - x - 2 \\ \hline 2x^2 - 3 & 2x^4 - 2x^3 - 7x^2 + 3x + 6 \\ & 2x^4 - 3x^2 \\ & - + \\ \hline & -2x^3 - 4x^2 + 3x + 6 \\ & -2x^3 + 3x \\ & + - \\ \hline & -4x^2 + 6 \\ & -4x^2 + 6 \\ & + - \\ \hline & 0 \\ \hline \end{array}$$

For other zeroes of f(x)

Put $x^2 - x - 2 = 0$

$$x^2 - 2x + x - 2 = 0$$
$$x(x - 2) + 1(x - 2) = 0$$
$$(x + 1)(x - 2) = 0$$
$$x = -1, 2$$

35. $f(x) = 6x^2 + x - 2$

$\alpha + \beta = -\frac{1}{6}, \quad \alpha\beta = -\frac{1}{3}$

(i) $$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$= \frac{\frac{1}{36} + \frac{2}{3}}{-\frac{1}{3}} = \frac{\frac{1+24}{36}}{-\frac{1}{3}}$$

$$= \frac{25}{36} \times -\frac{3}{1} = -\frac{25}{12}$$

(ii) $$2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) = 2\left(\frac{\alpha+\beta}{\alpha\beta}\right)$$

$$= 2\left(\frac{-\frac{1}{6}}{-\frac{1}{3}}\right)$$

$$= 2\left(\frac{1}{2}\right)$$

$$= 1$$

(iii) $\alpha^3 + \beta^3$

$= (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta)$

$= (\alpha + \beta)[(\alpha + \beta)^2 - 3\alpha\beta]$

$= -\frac{1}{6}\left[\frac{1}{36} + 1\right]$

$= -\frac{1}{6}\left[\frac{37}{36}\right]$

$= -\frac{37}{216}$

(iv) $\alpha^3\beta^3 - \alpha^5\beta^5$

$= \alpha^3\beta^3(1 - \alpha^2\beta^2)$

$= \left(-\frac{1}{3}\right)^3\left(1 - \frac{1}{9}\right)$

$= -\frac{1}{27}\left(\frac{9-1}{9}\right)$

$= -\frac{1}{27}\left(\frac{8}{9}\right) = -\frac{8}{243}$

36. (i) Let $p(x) = 8$

$g(x) = 3$

$q(x) = 2$

$r(x) = 2$

deg $p(x)$ = deg $q(x) = 0$

(ii) Let $p(x) = 15$

$g(x) = 4$

$q(x) = 2$

$r(x) = 7$

deg $q(x)$ = deg $r(x) = 0$

(iii) Let $p(x) = 20$

$g(x) = 3$

$r(x) = 2$

$q(x) = 6$

Here, deg $r(x) = 0$

37. Let $f(x) = 2x^3 + x^2 - 5x + 2$

$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{8}\right) + \frac{1}{4} - \frac{5}{2} + 2$

$= \frac{1}{4} + \frac{1}{4} - \frac{5}{2} + 2$

$= \frac{1}{2} + 2 - \frac{5}{2}$

$= \frac{5}{2} - \frac{5}{2} = 0$

So, $\frac{1}{2}$ is a zero of f(x)

$f(1) = 2 + 1 - 5 + 2 = 0$

So, 1 is a zero of f(x)

$f(-2) = 2(-8) + 4 + 10 + 2$

$= -16 + 16$

$= 0$

So, –2 is a zero of f(x).

Sum of zeroes $\frac{-\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$

$= \frac{1}{2} + 1 - 2$ $\quad = -\frac{1}{2}$

$= -\frac{1}{2}$

So, Sum of zeroes = $\frac{-\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$

Sum of product of zeroes taken two at a time $\frac{\text{Coefficient of } x}{\text{Coefficient of } x^3}$

$= \frac{1}{2}(1) + 1(-2) + \frac{1}{2}(-2) = -\frac{5}{2}$

$= \frac{1}{2} - 2 - 1$

$= \frac{1}{2} - 3$

$= -\frac{5}{2}$

So, sum of product of zeroes taken two at a time = $\frac{\text{Coefficient of } x}{\text{Coefficient of } x^3}$

Product of zeroes $\frac{-\text{Constant term}}{\text{Coefficient of } x^3}$

$= \frac{1}{2}(1)(-2)$ $\quad = -\frac{2}{2}$

$= -1$ $\quad = -1$

So, product of zeroes = $\dfrac{-\text{Constant term}}{\text{Coefficient of } x^3}$

Hence, relationship between zeroes and the coefficients is verified.

38. $f(x) = x^3 + 13x^2 + 32x + 20$

$f(x) = (-2)^3 + 13(-2)^2 + 32(-2) + 20$

$= -8 + 52 - 64 + 20$

$= 12 - 12$

$= 0$

$\Rightarrow$ $x + 2$ is a factor of f(x)

$$\begin{array}{r|l} & x^2 + 11x + 10 \\ \hline x+2 & x^3 + 13x^2 + 32x + 20 \\ & x^3 + 2x^2 \\ & - \quad - \\ \hline & 11x^2 + 32x + 20 \\ & 11x^2 + 22x \\ & - \quad - \\ \hline & 10x + 20 \\ & 10x + 20 \\ & - \quad - \\ \hline & 0 \end{array}$$

For other zeroes of f(x),

put $x^2 + 11x + 10 = 0$

$x^2 + 10x + x + 10 = 0$

$x(x + 10) + 1(x + 10) = 0$

$(x + 1)(x + 10) = 0$

$x = -1, -10$

So, zeroes of f(x) are –2, –1, –10

39. $f(x) = ax^2 + bx + c$

$\alpha + \beta = -\dfrac{b}{a}, \quad \alpha\beta = \dfrac{c}{a}$

(i) $\alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha + \beta)$

$= \dfrac{c}{a}\left(-\dfrac{b}{a}\right) = -\dfrac{bc}{a^2}$

(ii) $\alpha^4 + \beta^4 = (\alpha^2)^2 + (\beta^2)^2$

$= [\alpha^2 + \beta^2]^2 - 2\alpha^2\beta^2$

$= [(\alpha + \beta)^2 - 2\alpha\beta]^2 - 2(\alpha\beta)^2$

$= \left(\dfrac{b^2}{a^2} - \dfrac{2c}{a}\right)^2 - \dfrac{2c^2}{a^2}$

$= \dfrac{1}{a^4}\left(b^2 - 2ac\right)^2 - \dfrac{2c^2}{a^2}$

$= \dfrac{1}{a^4}\left(b^4 + 4a^2c^2 - 4ab^2c\right) - \dfrac{2c^2}{a^2}$

$= \dfrac{b^4}{a^4} + \dfrac{4c^2}{a^2} - \dfrac{4b^2c}{a^3} - \dfrac{2c^2}{a^2}$

$= \dfrac{b^4}{a^4} - \dfrac{4b^2c}{a^3} + \dfrac{2c^2}{a^2}$

40. $f(x) = x^3 - 6x^2 + 3x + 10$

$a + (a + b) + (a + 2b) = 6$

$3a + 3b = 6$

$$\boxed{\begin{array}{l} a + b = 2 \\ b = 2 - a \end{array}}$$

$a(a + b)(a + 2b) = -10$

$a(2)(4 - a) = -10$

$2a(4 - a) = -10$

$8a - 2a^2 = -10$

$2a^2 - 8a - 10 = 0$

$a^2 - 4a - 5 = 0$

$a^2 - 5a + a - 5 = 0$

$a(a - 5) + 1(a - 5) = 0$

$(a + 1)(a - 5) = 0$

$a = -1, 5$

For $a = -1$, $b = 2 - a = 2 - (-1) = 3$

For $a = 5$, $b = 2 - a = 2 - 5 = -3$

$a = -1$, $b = 3$

zeroes are $a, a + b, a + 2b$

$= -1, -1 + 3, -1 + 6$

$= -1, 2, 5$

$a = 5, b = -3$

zeroes are $a, a + b, a + 2b$

$= 5, 5 - 3, 5 - 6$

$= 5, 2, -1$

So, zeroes of the given polynomial are –1, 2 and 5.

Chapter 03

Pair of Linear Equations in Two Variables

MULTIPLE CHOICE QUESTIONS

1. (c) The system of equations has no solution if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

i.e. $\frac{1}{2} = \frac{1}{k} \neq \frac{4}{3}$

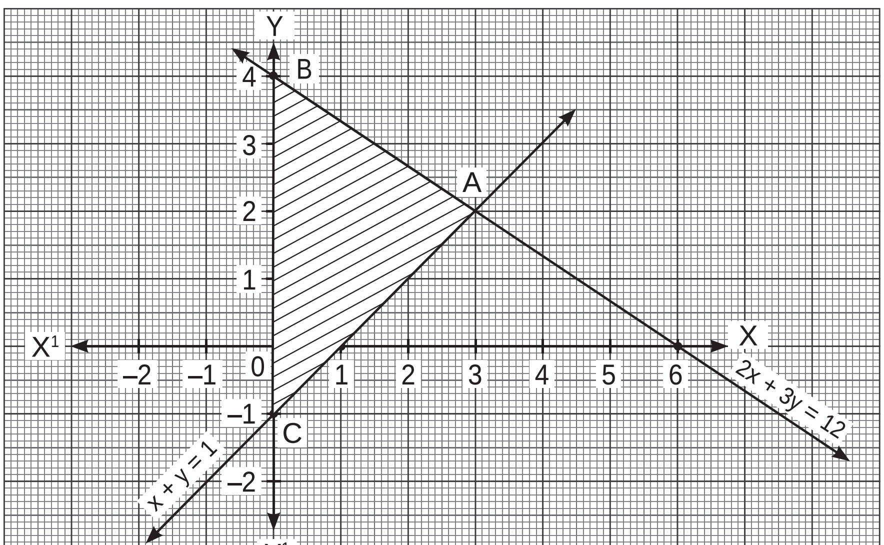
i.e. $k = 2$

2. For no solution, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

i.e $\frac{k}{6} = -\frac{5}{2} \neq \frac{2}{7}$

i.e. $k = -15$

3. (a)



$2x + 3y = 12$

$x - y = 1$

$x = 0$

x	6	0
y	0	4

x	1	0
y	0	–1

area of $\Delta ABC = \frac{1}{2} \times 5 \times 3$

$= \frac{15}{12} = 7.5$ sq. units

4. (c) Let number of coins of ` 1 = x

number of coins of ` 2 = y

$$\therefore \quad x + y = 50$$
$$\underline{-}x \ \underline{+}\ 2y = \underline{-}75$$
$$-y = -25$$
$$y = 25$$

So, $x = 50 - y$

$= 50 - 25$

$= 25$

5. (b) Let x be the tens digit and y be the ones digit.

$\therefore \quad x + y = 9 \quad$ ...(i)

and $10x + y + 27 = 10y + x$

$9x - 9y = -27$

$x - y = -3 \quad$ ....(ii)

On solving (i) and (ii), we get

$$\therefore \quad x + y = 9$$
$$x - y = -3$$
$$2x = 6$$
$$x = 3$$

From (i), $y = 9 - x = 9 - 3 = 6$

So, number is $10x + y$

$= 10(3) + 6 = 36$

WORKSHEET 1

Section A

1. $3x - y + 8 = 0, \quad 6x - ky + 16 = 0$

   The given equations are 3x-y+8=0 and 6x-ky+16 = 0. We have to find the point at which both the equations represent coincident lines.

   For the lines to be coincident,

   Substituting the values, we get

   Either or

   k = 2 or k = 2

   Therefore for k = 2, both the equations represent coincident lines.

2. Let number of girls be x and number of boys be y.

$$x + y = 15 \quad ...(i)$$

$$x = 5 + y \Rightarrow x - y = 5 \quad ...(ii)$$

   On solving equations (i) and (ii), we get

$$x + y = 15$$
$$x - y = 5$$
$$2x = 20$$
$$x = 10$$
$$y = 15 - x = 15 - 10 = 5$$

3. General form of a pair of linear equations in two variables x and y is

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

4. If a pair of linear equations in two variables is consistent, then the lines are either intersecting or coincident.

5. $2x + 3y = 7$

   $8x + (a + b)y = 28$

   Given pair of equations has infinitely many solutions if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{2}{8} = \frac{3}{a+b} = \frac{7}{28}$$

$$a + b = \frac{3 \times \cancel{28}^{4}}{\cancel{7}} = 12$$

$$a + b = 12$$

6. The pair of linear equations has infinitely many solutions if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{10}{20} = \frac{5}{10} = \frac{k-5}{k}$$

$$\left(\frac{5}{10} = \right) \frac{1}{2} = \frac{k-5}{k}$$

$$k = 2k - 10$$

$$10 = k$$

7. $2x + 3y = 7$

   $(a + b)x + (2a - b)y = 21$

   System of equations has infinitely many solutions if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

   i.e. $\frac{2}{a+b} = \frac{3}{2a-b} = \frac{7}{21}$

   $4a - 2b = 3a + 3b$, $\quad 2a - b = \frac{3 \times 21}{7} = 9$

   $a = 56$, $\quad 2a - b = 9$

   $a = 5(1)$ $\quad 2(5) - b = 9$

   $a = 5$ $\quad b = 10 - 9$

   $b = 1$

8. $ax + by = c$

   $lx + my = n$

   $\frac{a}{l} \neq \frac{b}{m}$ will give unique Solution

   $\frac{a}{l} = \frac{b}{m} = \frac{c}{n}$ (infinite solution)

   $\frac{a}{l} = \frac{b}{m} \neq \frac{c}{n}$ (no solution)

   $\frac{a}{l} \neq \frac{b}{m}$ will give unique Solution

Section B

9. The given equation is

$2x + y = 4$

$3y - 2x = 3$

Now, let us find atleast two solutions of each of the above equations, as shown in the following tables.

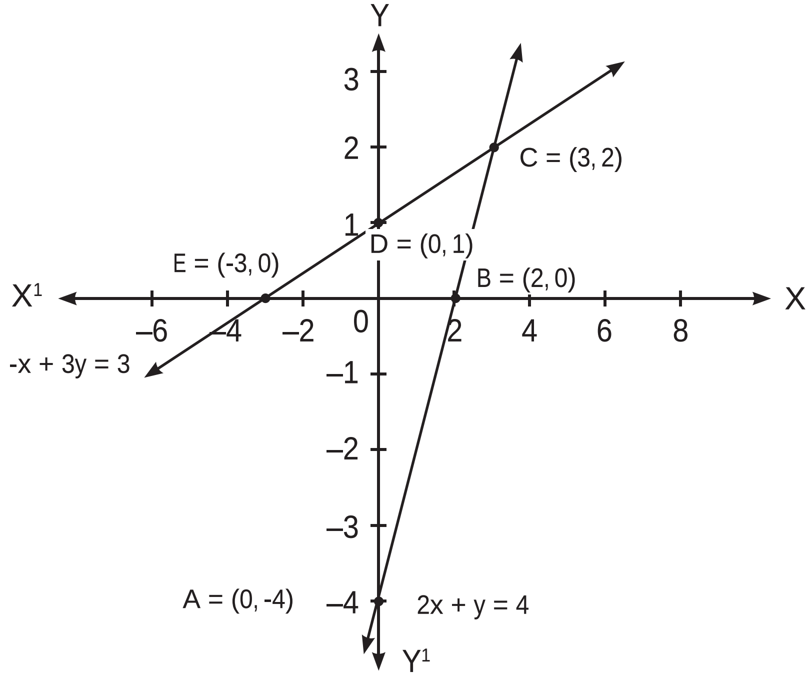
Table for $2x - y = 4$ or $y = 2x - 4$

x	0	2	3
$y = 2x - 4$	–4	0	2

Now table for $3y - 2x = 3$ or

$$y = \frac{x+3}{3}$$

x	0	–3	3
$y = \frac{x+3}{3}$	1	0	2



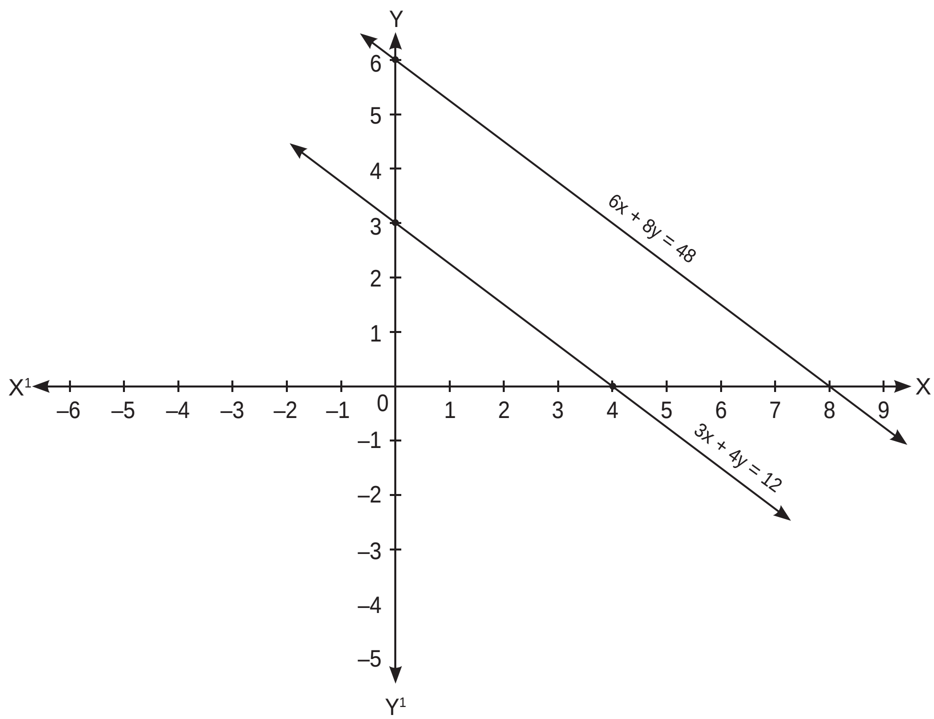
Here, the line intersecting at point C i.e. (3, 2)
The point which intersects at y axis are (0, –4) and D(0, 1)

10. $3x + 4y = 12$

$6x + 8y = 48$

x	4	0
y	0	3

x	8	0
y	0	6



11. (i) $x + 2y = -1$

(ii) $2x - 3y = 12$

From (i), $x = -1 - 2y$

$2(-1 - 2y) - 3y = 12$ (Put in (ii)

$-2 - 4y - 3y = 12$

$-7y = 14$

$y = -2$

So, $x = -1 - 2y$

$= -1 - 2(-2)$
$= -1 + 4$
$= 3$

12. $\frac{2x}{a} + \frac{y}{b} = 2$ ...(i)

$\frac{x}{a} - \frac{y}{b} = 2$ ...(ii)

From (ii), we have $\frac{x}{a} = 4 + \frac{y}{b}$

$$x = a\left(4 + \frac{y}{b}\right) \quad \text{...(iii)}$$

Putting this value of x in (i), we get

$$\frac{2a}{a}\left(4 + \frac{y}{b}\right) + \frac{y}{b} = 2$$

$$2\left(4 + \frac{y}{b}\right) + \frac{y}{b} = 2$$

$$8 + \frac{2y}{b} + \frac{y}{b} = 2$$

$$\frac{3y}{b} = -b$$

$$y = \frac{-6b}{3} = -2b$$

From (iii), $x = a\left(4 + \frac{y}{b}\right)$

$$= a\left(4 - \frac{2b}{b}\right)$$

$$= a(4 - 2)$$

$$= 2a$$

13. $28x + 5y = 9$ ...(i)

$3x + 2y = 4$ ...(ii)

On multiplying (i) by 2 and (ii) by 5, we get

$$56x + 10y = 18$$
$$15x + 10y = 20$$
$$- \quad - \quad -$$
$$41x = -2$$

$$x = -\frac{2}{41}$$

From (i), $28\left(-\frac{2}{41}\right) + 5y = 9$

$$-\frac{56}{41} + 5y = 9$$

$$5y = 9 + \frac{56}{41} = \frac{425}{41}$$

$$y = \frac{85}{41}$$

14. Let $\frac{1}{x} = p$, $\frac{1}{y} = q$

$2p + \frac{2}{3}q = \frac{1}{6} \Rightarrow 12p + 4q = 1$

Other equation becomes $3p + 2q = 0$

On solving equation $12p + 4q = 1$ and $3p + 2q = 0$, we get

$$12p + 4q = 1$$
$$2(3p + 2q = 0)$$
$$12p + 4q = 1$$
$$6p + 4q = 0$$
$$- \quad - \quad -$$
$$6p = 1$$

$$p = \frac{1}{6} = x = \frac{1}{p} = 6$$

From equation $3p + 2q = 0$, we get

$$3\left(\frac{1}{6}\right) + 2q = 0$$

$$2q = -\frac{1}{2}$$

$$q = -\frac{1}{4} \Rightarrow y = -4$$

Now, we need to find a

$$y = ax - 4$$
$$-4 = 6a - 4$$
$$6a = 0$$
$$a = 0$$

15. $2x + y = 35$ (i), $3x + 4y = 65$ (ii)

On multiplying equation (i) by 3 and equation (ii) by 2, we get

$$6x + 3y = 105$$
$$6x + 8y = 130$$
$$- \quad - \quad -$$
$$-5y = -25$$
$$y = 5$$

From (i) $2x + 5 = 35$

$$2x = 30$$
$$x = 15$$

16. For unique solution : $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$$\frac{k}{3} \neq \frac{2}{1}$$
$$k \neq 6$$

For Infinitely many solutions: $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$$\frac{k}{3} = \frac{2}{1} = \frac{5}{2.5}$$
$$k = 6$$

17. $2x + ky = 11$

$5x - 7y = 5$

For no solution: $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$$\frac{2}{5} = \frac{k}{-7} \neq \frac{11}{5}$$
$$5k = -14$$
$$k = \frac{-14}{5}$$

For unique solution:

$$\frac{2}{5} \neq \frac{k}{-7}$$
$$k \neq -\frac{14}{5}$$

18. The given system of equation is

$8x + 5y - 9 = 0$

$kx + 10y - 18 = 0$

The system of equation is of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = 8$, $b_1 = 5$, $c_1 = -9$

And $a_2 = k$, $b_2 = 10$, $c_2 = -18$

For a unique solution, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$
$$\Rightarrow \quad \frac{8}{k} = \frac{5}{10} = \frac{-9}{-18}$$

Now

$$\frac{8}{k} = \frac{5}{10}$$
$$\Rightarrow \quad 8 \times 10 = 5 \times k$$
$$\Rightarrow \quad \frac{8 \times 10}{5} = k$$
$$\Rightarrow \quad k = 8 \times 2 = 16$$

Hence, the given system f equations will have infinitely many solutions, if k = 16

Section C

19. Let the two no be x and y

Therefore $\frac{x}{y} = \frac{5}{6}$

$6 \times x = 5 \times y$

$6 \times x - 5 \times y = 0 \qquad \times 5$

$30 \times x - 25 \times y = 0 \qquad$ ....(i)

Also $\frac{(x-8)}{(y-8)} = \frac{4}{5}$

$5 \times x - 40 = 4 \times y - 32$

$5 \times x - 4 \times y - 8 = 0 \qquad \times 6$

$30 \times x - 24 \times y - 48 = 0 \qquad$ ....(ii)

Subtracting (i) from (ii)

$y - 48 = 0$

$y = 48$

Putting y = 48 in $\frac{x}{y} = \frac{5}{6}$

$$\frac{x}{48} = \frac{5}{6}$$
$$x = 5 \times \frac{48}{6}$$
$$x = 5 \times 8$$
$$x = 40$$

Therefore the two numbers are 40 and 48.

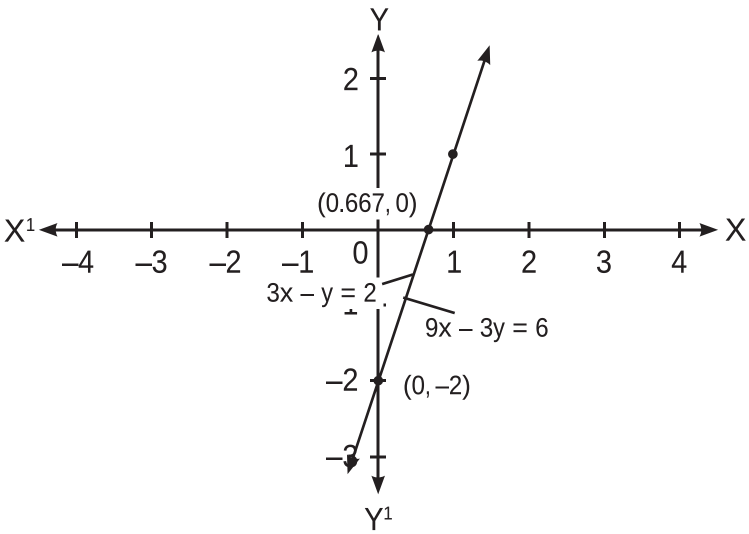
20. $\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$

Thus this equation has infinite solution

x = 0 and y = –2

x = 1 and y = 1

Both lines overlap, thus they are having infinite solutions.



21. $\frac{5}{x+y} - \frac{2}{x-y} = -1, \quad \frac{15}{x+y} + \frac{7}{x-y} = 10$

Let $\frac{1}{x+y} = p$ and $\frac{1}{x-y} = q$

5p – 2q = – 1, 15p + 7q = 10

Using elimination method, we get

$$3(5p - 2q) = -1) \Rightarrow \begin{array}{r} 15p - 6q = -3 \\ 15p + 7q = 10 \\ - \quad - \quad\quad - \\ \hline -13q = -13 \\ q = 1 \end{array}$$

$\boxed{x - y = 1}$

From equation 5p – 2q = –1, we get

$$5p - 2(1) = -1$$
$$5p = 1$$
$$p = \frac{1}{5}$$

$\boxed{x + y = 5}$

On solving equations x + y = 5 and x – y = 1, we get

$$\begin{array}{r} x - y = 1 \\ x + y = 5 \\ \hline 2x = 6 \end{array}$$

$$x = 3 \Rightarrow y = 5 - 3 = 2$$

22. x – 4 = 0

$\Rightarrow$ x = 4

$f(x) = x^3 + ax^2 + 2bx - 24$

$f(4) = 4^3 + a.4^2 + 2.b.4 - 24 = 0$

$\Rightarrow$ 64 + 16a + 8b = 24

$\Rightarrow$ 16a + 8b = 24 – 64

$\Rightarrow$ 2a + 1b = 3 – 8

$\Rightarrow$ 2a + b = –5 ....(i)

a – b = 8 ....(ii)

Solving eq.(i) & (ii)

$$\begin{array}{r} 2a + b = -5 \\ a - b = 8 \\ \hline 3a = 3 \end{array}$$

$\Rightarrow$ a = 1

Substituting the value of a in eq.(ii)

1 – b = 8

$\Rightarrow$ b = –7

23. Let number of rows be x and number of students in each row be y. So, total number of students = xy

According to question,

$$(y + 3)(x - 1) = xy$$
$$xy + 3x - y - 3 = xy$$
$$3x - y = 3 \quad ...(i)$$

Again, $(y - 3)(x + 2) = xy$

$$xy + 2y - 3x - 6 = xy$$

$$-3x + 2y = 6 \quad ...(ii)$$

On solving equations (i) and (ii), we get

$$2(3x - y = 3) \Rightarrow \quad 6x - 2y = 6$$
$$-3x + 2y = 6$$
$$3x = 12$$
$$x = 4$$

From (i) $\quad y = 3x - 3$
$$= 12 - 3$$
$$= 9$$

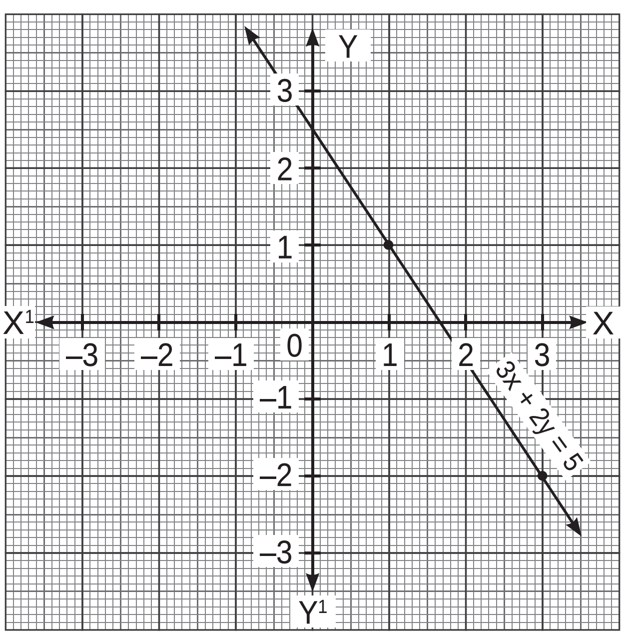
So, Total number of students = xy
$$= 4(9)$$
$$= 36$$

24. $\quad 3x + 2y = 5$
$$3x = 5 - 2y$$
$$x = \frac{5-2y}{3}$$

To check : (1, 1) is a point on the 3x + 2y = 5
$$\text{LHS} = 3x + 2y$$
$$= 3(1) + 2(1)$$
$$= 5$$
$$= \text{RHS}$$

So, (1, 1) is a point on the line 3x + 2y = 5

x	1	3
y	1	–2



25. Let digit at ten's place be x and digit at unit's place be y

So, number = 10x + y

According to question,
$$x + y = 5 \quad ...(i)$$
$$10y + x = 10x + y + 9$$
$$0 = 9x - 9y + 9$$
$$x - y = -1 \quad ...(ii)$$

On solving (i) and (ii), we get
$$x + y = 5$$
$$x - y = -1$$
$$2x = 4$$
$$x = 2$$

From (i), $\quad y = 5 - 2 = 3$

So, $\quad$ number $= 10x + y$
$$= 10(2) + 3$$
$$= 23$$

26. Let the adjacent angle be x.
$$\text{Other angle} = \frac{4}{5}x$$

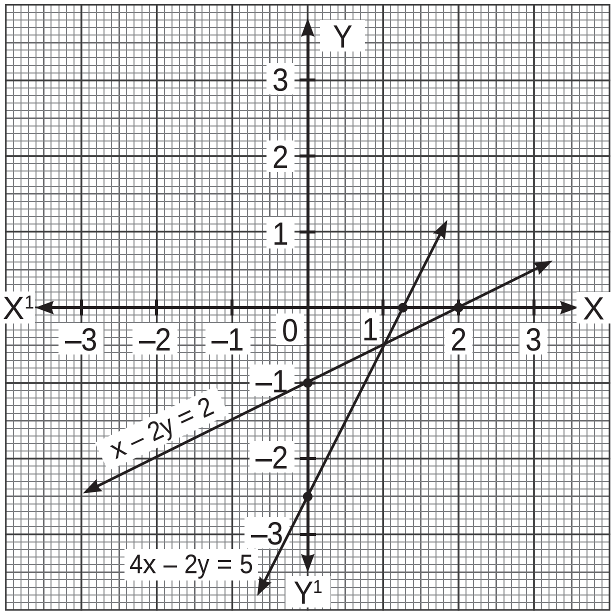
As sum of adjacent angles of a parallelogram is 180°,
$$x + \frac{4}{5}x = 180$$
$$\frac{9x}{5} = 180$$
$$x = \frac{\cancel{180}^{20} \times 5}{\cancel{9}} = 100°$$

Angles are x, $\frac{4}{5}$x
$$= 100, \frac{4}{5}(100)$$
$$= 100, 80$$

27. (i) $\quad x - 2y = 2, \qquad 4x - 2y = 5$

x	0	2
y	–1	0

x	0	1.25
y	–2.5	0

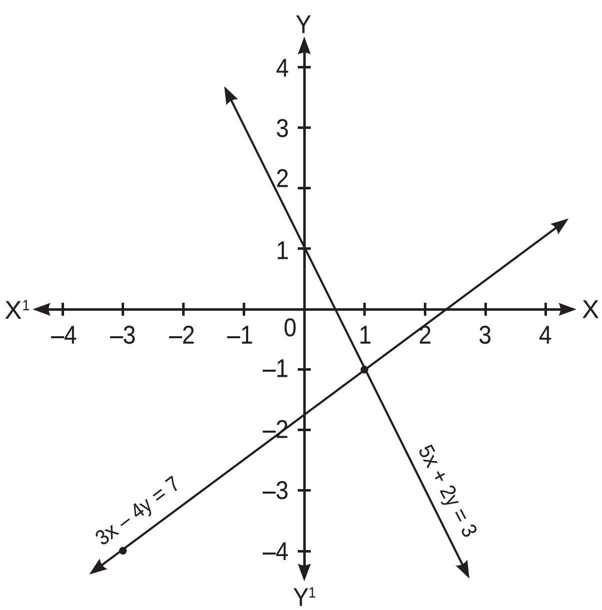


As the lines are intersecting, so the system of equations has unique solution and hence, consistent.

(ii) $3x - 4y = 7$, $5x + 2y = 3$

x	–3	1
y	–4	–1

x	1	2
y	–1	–3.5



28. In ΔABC,

$$\angle A + \angle B + \angle C = 180° \text{ (Angle sum property)}$$

$$x + 3x + y = 180°$$

$$4x + y = 180° \quad ...(i)$$

Given: $3y - 5x = 30 \quad ...(ii)$

From (i), $y = 180 - 4x$

So, eq^n (ii) becomes $3(180 - 4x) - 5x = 30$

$$540 - 12x - 5x = 30$$

$$17x = 510$$

$$x = 30$$

From (i), $y = 180 - 4(30)$

$$= 180 - 120$$

$$= 60°$$

So, $\angle A = x = 30°$

$$\angle B = 3x = 90°$$

$$\angle C = y = 60°$$

In ΔABC, $\angle B = 90°$, so it is a right angled triangle

Section D

29. Let speed of boat in still water be x km/hr and that of stream be y km/hr.

So, speed of boat upstream = (x – y) km/hr

Speed of boat downstream = (x + y) km/hr

According to question,

$$\frac{32}{x-y} + \frac{36}{x+y} = 7$$

$$\frac{40}{x-y} + \frac{48}{x+y} = 9$$

Let $\frac{1}{x-y} = p$, $\frac{1}{x+y} = q$

So, we get equations as

$$32p + 36q = 7 \quad ...(i)$$

$$40p + 48q = 9 \quad ...(ii)$$

On multiplying (i) by 5 and (ii) by 4, we get

$$160p + 180q = 35$$

$$\underset{-}{}160p \underset{-}{+} 192q = \underset{-}{}36$$

$$-\ 12q = -1$$

$$q = \frac{1}{12}$$

i.e. $x + y = 12$ ...(iii)

From (i), $32p + 36\left(\frac{1}{12}\right) = 7$

$$32p = 7 - 3 = 4$$

$$p = \frac{1}{8}$$

$$x - y = 8 \quad ...(iv)$$

On solving (iii) and (iv), we get

$$x = 10$$

$$y = 2$$

Speed of boat in still water = 10 km/hr

Speed of stream = 2 km/hr

30. $ax + by = 1$ ...(i)

$$bx + ay = \frac{(a+b)^2}{a^2+b^2} - 1 \quad ...(ii)$$

$$= \frac{a^2+b^2+2ab-a^2-b^2}{a^2+b^2}$$

$$bx + ay = \frac{2ab}{a^2+b^2} \quad ...(iii)$$

On multiplying (i) by b and (iii) by a, we get

$$abx + b^2y = b$$

$$abx + a^2y = \frac{2a^2b}{a^2+b^2}$$

$$-\quad -\quad -$$

$$y(b^2 - a^2) = b - \frac{2a^2b}{a^2+b^2}$$

$$y(b^2 - a^2) = \frac{a^2b + b^3 - 2a^2b}{a^2+b^2}$$

$$= \frac{b^3 - a^2b}{a^2+b^2}$$

$$= \frac{b(b^2 - a^2)}{a^2+b^2}$$

$$\therefore \quad y = \frac{b}{a^2+b^2}$$

From (i), $ax + b\left(\frac{b}{a^2+b^2}\right) = 1$

$$ax = 1 - \frac{b^2}{a^2+b^2}$$

$$= \frac{a^2}{a^2+b^2}$$

$$ax = \frac{a^2}{a^2+b^2}$$

$$x = \frac{a}{a^2+b^2}$$

31. Let speed of X be x km/hr and that of Y be y km/hr

Time taken by X to walk 30 km

$$= \frac{30}{x} \text{ hours}$$

Time taken by Y to walk 30 km

$$= \frac{30}{y} \text{ hours}$$

According to question

$$\frac{30}{x} = \frac{30}{y} + 3$$

$$\frac{30}{x} - \frac{30}{y} = 3$$

$$\frac{1}{x} - \frac{1}{y} = \frac{1}{10} \quad ...(i)$$

Also, $\frac{30}{2x} = \frac{30}{y} - \frac{3}{2}$

$$\frac{15}{x} = \frac{30}{y} - \frac{3}{2}$$

$$\frac{15}{x} - \frac{30}{y} = \frac{-3}{2}$$

$$\frac{1}{x} - \frac{2}{y} = -\frac{1}{10} \quad ...(ii)$$

Let $\frac{1}{x} = p$, $\frac{1}{y} = q$

So, equations (i) and (ii) become

$p - q = \frac{1}{10} \Rightarrow 10p - 10q = 1$ ...(iii)

and $p - 2q = -\frac{1}{10} \Rightarrow 10p - 20q = 1$ ...(iv)

On solving equations (iii) and (iv), we get

$$10p - 10q = 1$$

$$\underset{-}{10p} \underset{+}{-} 20q = \underset{+}{-1}$$

$$10q = 2$$

$$q = \frac{1}{5} \Rightarrow y = 5$$

From (iii), we get $10p - 10\left(\frac{1}{5}\right) = 1$

$$10p = 1 + 2 = 3$$

$$p = \frac{3}{10}$$

$$x = \frac{10}{3}$$

So, Speed of X = $\frac{10}{3}$ km/hr

Speed of Y = 5 km/hr

32. $a(x + y) + b(x - y) = a^2 - ab + b^2$ ...(i)

$a(x + y) - b(x - y) = a^2 - ab + b^2$ ...(ii)

Let $x + y = p$ and $x - y = q$

So, equations (i) and (ii) becomes

$ap + bq = a^2 - ab + b^2$ ...(iii)

$ap - bq = a^2 + ab + b^2$ ...(iv)

On adding (iii) and (iv) we get,

$$2ap = 2(a^2 + b^2)$$

$$p = \frac{1}{a}(a^2 + b^2)$$

From equation (iii),

$$a\frac{1}{a}(a^2 + b^2) + bq = a^2 - ab + b^2$$

$$a^2 + b^2 + bq = a^2 - ab + b^2$$

$$bq = -ab$$

$$q = -a$$

So,

$$x + y = \frac{1}{a}(a^2 + b^2)$$

$$x - y = -a$$

$$2x = \frac{1}{a}(a^2 + b^2) - a$$

$$2x = a + \frac{b^2}{a} - a = \frac{b^2}{a}$$

$$\boxed{x = \frac{b^2}{2a}}$$

So, $y = x + a = \frac{b^2}{2a} + a = \frac{b^2 + 2a^2}{2a}$

33. Let $\frac{1}{2x + 3y} = p$ and $\frac{1}{3x - 2y} = q$

So, equations become

$$\frac{1}{2}p + \frac{12}{7}q = \frac{1}{2}$$

and $7p + 4q = 2$

i.e. $7p + 24q = 7$ ....(i)

and $7p + 4q = 2$ .... (ii)

On subtracting (ii) from (i), we get

$$20q = 5 \Rightarrow q = \frac{1}{4}$$

From (i), $7p + 24\left(\frac{1}{4}\right) = 7$

$$7p = 1$$

$$p = \frac{1}{7}$$

So, we get $2x + 3y = 7$ ...(iii)

$3x - 2y = 4$ ...(iv)

On multiplying (iii) by 3 and (iv) by 2 and subtracting, we get

$$\begin{aligned} 6x + 9y &= 21 \\ 6x - 4y &= 8 \\ -\ \ +\ \ \ \ &\ \ - \\ \hline 13y &= 13 \\ y &= 1 \end{aligned}$$

From (iii)

$$2x + 3(1) = 7$$

$$2x = 4$$

$$x = 2$$

34.

$$kx - y = 2$$

$$6x - 2y = 3$$

(i) For unique solution : $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$$\frac{k}{6} \neq \frac{-1}{-2}$$

$$k \neq 3$$

(ii) For no solution : $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$$\frac{k}{6} = \frac{-1}{-2} \neq \frac{2}{3}$$

$$k = 3$$

The system has infinitely many

solutions, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

i.e. $\frac{k}{6} = \frac{-1}{-2} = \frac{2}{3}$

Clearly, $\frac{-1}{-2} \neq \frac{2}{3}$,

So, there is no value of k for which the given system of equations has infinitely many solutions.

35. Income is 8a and 7a expenditure is 19b and 16b

Saving is 1250

$8a - 19b = 1250$

$7a - 16b = 1250$

$19 \times 7b = 1250 \times 7$

$16 \times 8b = 1250 \times 8$

$133b - 128b = 1250$

$5b = 1250$

$b = 250$

$a = 750$

Income of x = 6000

Income of y = 5250

36. $\frac{2}{p+q} = \frac{3}{2p-q} = \frac{7}{21}$

$\frac{2}{p+q} = \frac{3}{2p-q} = \frac{1}{3}$

$\frac{2}{p+q} = \frac{1}{3}$ and $\frac{3}{2p-q} = \frac{1}{3}$

$p+q = 6$ and $2p - q = 9$

$(p + q) + (2p - q) = 6 + 9$

$3p = 15$

$p = 5$

Put $p = 5$ in $p + q = 6$ or $2p - q = 9$, for getting the value of q.

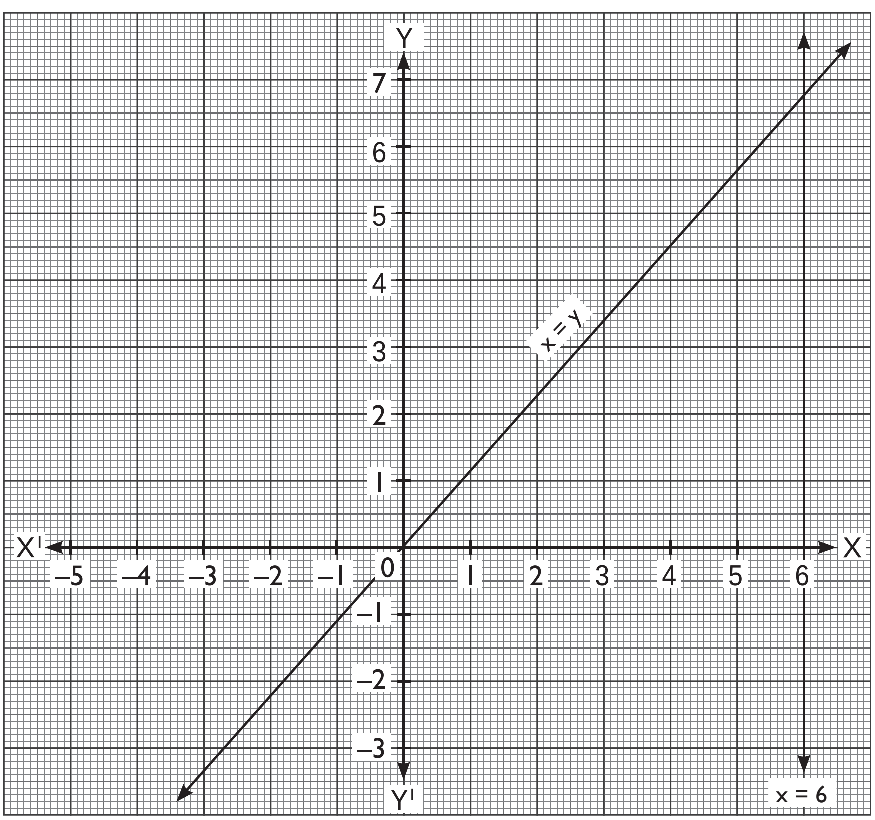
$q = 1$.

Given system of equations will have infinitely many solutions, if $p = 5$ and $q = 1$.

WORKSHEET 2

Section A

1.



Area of $\Delta = \frac{1}{2} \times 6 \times 6$

$= 18$ sq. units

2. The system of equations has no solution if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

i.e. $\frac{1}{3} = \frac{2}{k} \neq \frac{5}{15}$

$k = 6$

3. $3x + y = 1$ and

$(2k - 1)x + (k - 1)y = 2k + 1$

Inconsistent $\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow \frac{3}{(2k-3)} \neq \frac{10}{(k-1)}$

$\Rightarrow 3k - 1 \neq 2k - 1$

$\Rightarrow 3k - 2k \neq -1 + 1$

$\Rightarrow k \neq 1$

4. The system of equations represent intersecting lines if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\frac{2}{k} \neq \frac{5}{7}$

$k \neq \frac{14}{5}$

5. The system of equations has a unique solution if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\frac{k}{6} \neq \frac{-1}{-2}$

$k \neq 3$

6. $x + ky = 0$ $\quad a_1 = 1, \; b_1 = k$

$2x - y = 0$ $\quad a_2 = 2, \; b_2 = -1$

They have unique solution when

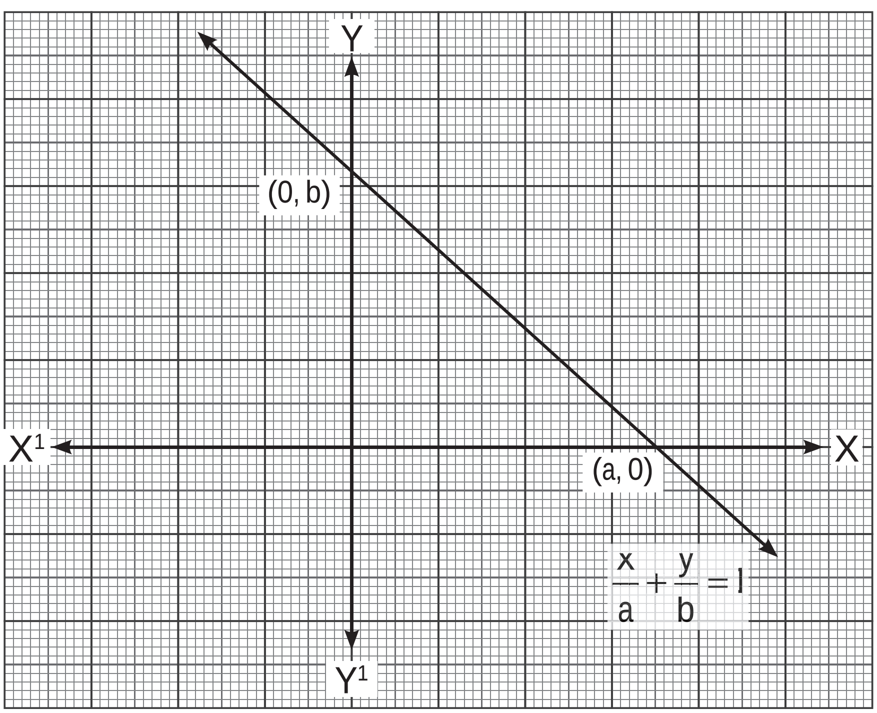
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

i.e. $\frac{1}{2} \neq \frac{k}{-1}$

i.e. $k \neq \frac{-1}{2}$

It means for all values of k except $k = \frac{-1}{2}$, the equation will have unique solution.

7.



Area of triangle $= \frac{1}{2} \times a \times b$

$= \frac{ab}{2}$

8. As (3, a) lies on line $2x - 3y = 5$

$2(3) - 3(a) = 5$

$6 - 3a = 5$

$3a = 1$

$a = \frac{1}{3}$

9. $X + 2y - 8 = 0$

$2x + 4y - 16 = 0$

Here, $a_1 = 1, \; b_1 = 2, c_1 = -8$

$a_2 = 2, b_2 = 4, c_2 = -16$

so, $\frac{a_1}{a_2} = \frac{1}{2}$

$\frac{b_1}{b_2} = \frac{2}{4} = \frac{1}{2}$

$\frac{c_1}{c_2} = \frac{-8}{-16} = \frac{1}{2}$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, the pair of linear equations has infinitely many solutions.

10. $x + y = 14$

$x = 14 - y$ [1]

Now

$x - y = 4$

$x = 4 + y$ [2]

From [1] & [2]

$14 - y = 4 + y$

$10 = 2y$

$y = 5$

Now	&
$x + y = 14$	$x - y = 4$
$x + 5 = 14$	$x - 5 = 4$
$x = 9$	$x = 9$

Hence in both cases value of x and y r same so it is consistent

Section B

11. $-4x + y = 1$ ...(i)

$6x - 5y = 9$ ...(ii)

On multiplying eq^n (i) by 5 and adding both the equations, we get

$$5(-4x + y) + 6x - 5y = 5 + 9$$
$$-20x + 5y + 6x - 5y = 14$$
$$-14x = 14$$
$$x = -1$$

From (i), $y = 1 + 4x = 1 - 4$

$$y = -3$$

12. Given,

$2x - 3y + 6 = 0$ ....(i)

$4x - 5y + 2 = 0$ ....(ii)

From eq. (i) and (ii) we have,

$a_1 = 2, \ b_1 = -3, \ c_1 = -6$

$a_2 = 4, \ b_2 = -5, c_2 = 2$

Now, $\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$

$\frac{b_1}{b_2} = \frac{-3}{-5} = \frac{3}{5}$

$\frac{c_1}{c_2} = \frac{6}{2} = 3$

Since, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\therefore$ The given equation will have a unique solution and the equation will intersect at a point.

13. Given,

$2x = 5y + 4$

$2x - 5y = 4$ ....(i)

$3x - 2y + 16 = 0$

$-3x + 2y = 16$ ......(ii)

Multiplying (i) by 3 and (ii) by 2, we get

$6x - 15y = 12$ .....(iii)

$-6x + 4y = 32$ ......(iv)

Adding (iii) and (iv) we get

$11y = 44$

$y = 4$

Substituting the value of y in (i) we get

$$2x - 5y = 4$$
$$2x - 5 \times 4 = 4$$
$$2x - 20 = 4$$
$$2x = 24$$
$$x = 12$$

Hence, $x = 12, \ y = 4$

14. For unique solution, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

i.e. $\frac{6}{k} \neq \frac{2}{1}$

$k \neq 3$

For infinitely many solutions, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$$\frac{6}{k} = \frac{2}{1} = \frac{3}{b_2}$$

Clearly $\frac{2}{1} \neq \frac{3}{2}$. so, there does not exist any values of k for which the system of equations has infinitely many solutions.

15. $99x + 101y = 499$ ...(i)

$101x + 99y = 501$ ...(ii)

On subtracting (i) from (ii), we get

$2x - 2y = 2$

$x - y = 1$ ...(iii)

On adding (i) and (ii), we get

$200x + 200y = 1000$

$x + y = 5$ ...(iv)

On adding (iii) and (iv), we get

$2x = 6$

$x = 3$

From (iv), $y = 5 - x$

$= 5 - 3$

$= 2$

16. The system of equations has infinite solutions if

$x + (k + 1)y = \underline{5}$

$(k + 1)x + 9y = (8k - 1)$

$\Rightarrow$ For infinitely many solutions

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{1}{k+1} = \frac{k+1}{9} = \frac{5}{8k-1}$

$\Rightarrow \frac{1}{k+1} = \frac{k+1}{9}$

$\Rightarrow (k + 1)^2 = 9$

$\Rightarrow k + 1 = \pm\sqrt{9}$

$\Rightarrow k + 1 = \pm 3$

Case - 1	Case - 2
$\Rightarrow k + 1 = +3$	$\Rightarrow k + 1 = -3$
$\Rightarrow k = 2$	$\Rightarrow k = -4$

17. Let the numerator be x and denominator be y.

So, fraction $= \frac{x}{y}$

According to question,

$\frac{x+1}{y+1} = \frac{7}{8}$

$8x + 8 = 7y + 7$

$8x - 7y = -1$ ...(i)

Again, $\frac{x-1}{y-1} = \frac{6}{7}$

$7x - 7 = 6y - 6$

$7x - 6y = 1$ ...(ii)

On multiplying (i) by 7 and (ii) by 8, we get,

$56x - 49y = -7$

$56x - 48y = 8$

$- \quad + \quad -$

$-y = -15$

$y = 15$

From (i), $8x - 7(15) = -1$

$8x = -1 + 105 = 104$

$x = \frac{104}{8} = 13$

18. a = 8 and b = 5

Step-by-step explanation:

As, $a_1 = 3$, $b_1 = -a - 1$, $c_1 = 2b - 1$

$a_2 = 5$, $b_2 = 1 - 2a$, $c_2 = 3b$

As, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Part I

$\frac{a_1}{a_2} = \frac{b_1}{b_2}$

$\frac{3}{5} = \frac{-a-1}{1-2a}$

$5(-a-1) = 3(1-2a)$

$-5a - 5 = 3 - 6a$

$-5 = 3 - a$

$-8 = -a$

Cancelling the minus sign from both the sides

We get;

$8 = a$

Part II

$\frac{a_1}{a_2} = \frac{c_1}{c_2}$

$\frac{3}{5} = \frac{2b-1}{3b}$

On cross multiplication

$9b = 10b - 5$

$-b = -5$

By cancelling minus sign from both the sides

We get,

$b = 5$

Therefore, a = 8 and b = 5

19. $2x - 3y + 6 = 0$ ...(i)

$4x - 5y + 2 = 0$ ...(ii)

Here, $\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$

$\frac{b_1}{b_2} = \frac{-3}{-5} = \frac{3}{5}$

$\frac{c_1}{c_2} = \frac{6}{2} = 3$

As $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, so, the system has a unique solution.

On multiplying (i) by 2 and subtracting (ii) from (i), we get

$$\begin{array}{r} 4x - 6y + 12 = 0 \\ 4x - 5y + 2 = 0 \\ - \quad + \quad - \quad \\ \hline -y = -10 \end{array}$$

$y = 10$

From (i), $2x - 3(10) + 6 = 0$

$2x - 24 = 0$

$x = 12$

20. $\frac{x}{10} + \frac{y}{5} + 1 = 15$

$\frac{x}{10} + \frac{y}{5} = 14$

$2x + y = 140$ ...(i)

Again, $\frac{x}{8} + \frac{y}{6} = 15$

$\frac{3x+4y}{24} = 15$

$3x + 4y = 360$ ...(ii)

From (i), $y = 140 - 2x$

On putting this value of y in (ii), we get

$3x + 4(140 - 2x) = 360$

$3x + 560 - 8x = 360$

$-5x = -200$

$x = 40$

So, $y = 140 - 2x$

$= 140 - 2(40)$

$= 140 - 80$

$= 60$

Section C

21. Let the fixed charge be `x and cost of food per day be `y.

According to question,

$x + 20y = 3000$ (i)

$x + 25y = 3500$ (ii)

$$\begin{array}{r} x + 20y = 3000 \\ x + 25y = 3500 \\ - \quad + \quad - \quad \\ \hline -5y = -500 \end{array}$$

$y = 100$

From (i), we get $x = 3000 - 20(100)$

$= 3000 - 2000$

$= 1000$

So, fixed charge = `1000

Cost of food per day = `100

22. $x + y = 1$

$2x - 3y = 11$

According to cross multiplication method,

$$\frac{x}{-11-3} = \frac{y}{-2+11} = \frac{1}{-3-2}$$

$$\frac{x}{-14} = \frac{y}{9} = \frac{1}{-5}$$

$$\frac{x}{-14} = \frac{1}{-5}, \quad \frac{y}{9} = \frac{1}{-5}$$

$$x = \frac{14}{5}, \quad y = \frac{-9}{5}$$

23. (i) $5x + 6y = 15$

As $\frac{4}{5} \neq \frac{-5}{6}\left(\frac{a_1}{a_2} \neq \frac{b_1}{b_2}\right)$

(ii) $8x - 10y = 30$

As $\frac{4}{8} = \frac{-5}{-10} \neq \frac{10}{30}\left(\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}\right)$

(iii) $8x - 10y = 20$

As $\frac{4}{8} = \frac{-5}{-10} = \frac{10}{20}\left(\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}\right)$

24. For infinite solution

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{2}{4} = \frac{a-4}{a-1} = \frac{-2b+1}{-5b-1}$$

$$2(a-1) = 4(a-4)$$

$$2a - 2 - 4a + 16 = 0$$

$$-2a = -14$$

$$a = 7,$$

$$2(-5b + 1) = 4(-2b - 1)$$

$$-10b + 8b + 2 + 4 = 0$$

$$-2b = -6$$

$$b = 3$$

25. Let $\frac{1}{x-1} = p$ and $\frac{1}{y-2} = q$

So, equations become

$$5p + q = 2 \quad \text{(i)}$$

$$6p - 3q = 1 \quad \text{(ii)}$$

On multiplying (i) by 3 and subtracting equations (i) and (ii), we get

$$15p + 3q = 6$$

$$6p - 3q = 1$$

$$21p = 7$$

$$p = \frac{1}{3}$$

$\therefore \quad x - 1 = 3 \Rightarrow x = 4$

From eq^n (i), $q = 2 - 5p$

$$= 2 - 5\left(\frac{1}{3}\right)$$

$$= \frac{1}{3}$$

$\therefore \quad y - 2 = 3 \Rightarrow y = 5$

26. Let $\frac{1}{x} = p$ and $\frac{1}{y} = q$

So, equations become

$$p - 4q = 2$$

$$p + 3q = 9$$

$$-7q = -7$$

$$q = 1$$

From eq^n $p - 4q = 2$, we get

$$p = 2 + 4(1)$$

$$= 6$$

So, $x = \frac{1}{6}$, $y = 1$

27. Let father's age be x years and son's age be y years.

According to question,

$$2y + x = 70 \quad \text{(i)}$$

$$2x + y = 95 \quad \text{(ii)}$$

From (i), $x = 70 - 2y$

On putting value of x in (ii), we get

$$2(70 - 2y) + y = 95$$

$$140 - 4y + y = 95$$

$$3y = 45$$

$$y = 15$$

So, $x = 70 - 2y$

$$= 70 - 2(15)$$
$$= 70 - 30$$
$$= 40$$

So, age of father = 40 years

age of son = 15 years

28. Let speed of train be x km/hr and speed of car be y km/hr.

According to question,

$$\frac{160}{x} + \frac{600}{y} = 8$$
$$\frac{240}{x} + \frac{520}{y} = \frac{41}{5}$$

Let $\frac{1}{x} = p$, $\frac{1}{y} = q$

So, we get equations as

$$160p + 600q = 8 \quad ...(i)$$
$$1200p + 2600q = 41 \quad ...(ii)$$

On multiplying (i) by 30 and (ii) by 4, we get

$$4800p + 18000q = 240$$
$$4800p + 10400q = 164$$
$$- \quad - \quad -$$
$$7600q = 76$$
$$q = \frac{76}{7600} = \frac{1}{100}$$

i.e. y = 100

From (i), we get

$$160p + 600\left(\frac{1}{100}\right) = 8$$
$$160p + 6 = 8$$
$$160p = 2$$
$$p = \frac{1}{80}$$

i.e. x = 80

So, Speed of train = 80 km/hr

Speed of car = 100 km/hr

29. Let time taken by one man alone be x days.

Let time taken by one boy alone be y days.

According to question,

$$\frac{8}{x} + \frac{12}{y} = \frac{1}{10}$$
$$\frac{6}{x} + \frac{8}{y} = \frac{1}{14}$$

Let $\frac{1}{x} = p$, and $\frac{1}{y} = q$

So, we get equations as

$$8p + 12q = \frac{1}{10}$$
$$80p + 120q = 1 \quad (i)$$

Another equation becomes,

$$6p + 8q = \frac{1}{14}$$
$$84p + 112q = 1 \quad (ii)$$

On multiplying (i) by 21 and (ii) by 20, we get

$$1680p + 2520q = 21$$
$$1680p + 2240q = 20$$
$$- \quad - \quad -$$
$$280q = 1$$
$$q = \frac{1}{280}$$

So, y = 280

From (i),

$$80p + 120\left(\frac{1}{280}\right) = 1$$
$$80p + \frac{3}{7} = 1$$
$$80p = 1 - \frac{3}{7} = \frac{4}{7}$$
$$p = \frac{1}{140}$$

So, x = 140

∴ A man can complete the work in 140 days and a boy can complete the work in 280 days.

30. Let father's age = x years

Sum of ages of 2 children = y years

According to question,

$$x = 2y \quad (i)$$

and

$$x + 20 = y + 20 + 20$$
$$x - y = 20 \quad (ii)$$

On putting (i) in (ii), we get

$$2y - y = 20$$
$$y = 20$$
$$\therefore \quad x = 2y = 40$$
So, father's age = 40 years

Section D

31. Let Speed of car A = x km/hr

Speed of car B = y km/hr

According to question,

$$\frac{4}{3}x + \frac{4}{3}y = 80$$
$$x + y = 60 \qquad \text{(i)}$$

$$8x - 8y = 80$$
$$x - y = 10 \qquad \text{(ii)}$$

On adding (i) and (ii), we get

$$2x = 70$$
$$x = 35$$

From (i), $y = 60 - x$

$= 60 - 35$

$= 25$

From (i), $y = 60 - x$

$= 60 - 35$

$= 25$

So, Speed of car A = 35 km/hr

Speed of car B = 25 km/hr

32. Let cost of one chair be \$x and cost of one table be \$y.

According to question,

$$4x + 3y = 2100 \qquad \text{(i)}$$
$$5x + 2y = 1750 \qquad \text{(ii)}$$

On multiplying eq^n (i) by 5 and (ii) by 4, we get

$$20x + 15y = 10500$$
$$20x + 8y = 7000$$
$$- \quad - \quad -$$
$$7y = 3500$$

$$y = \frac{3500}{7} = 500$$

From (i), $4x + 3(500) = 2100$

$4x = 2100 - 1500$

$4x = 600$

$x = 150$

Cost of one chair = \$ 150

Cost of one table = \$ 500

Therefore,

Cost of five chairs = 5×150

= \$ 750

Cost of eight tables = 8×500

= \$ 4000

33. Let father's age = x years

Son's age = y years

According to question,

$$x - 10 = 12(y - 10)$$

i.e. $x - 12y = -110 \qquad$ (i)

For another eq^n,

$$x + 10 = 2(y + 10)$$
$$x - 2y = 10 \qquad \text{(ii)}$$

On subtracting eq^n (ii) from (i), we get

$$x - 12y = -110$$
$$x - 2y = 10$$
$$- \quad + \quad -$$
$$-10y = -120$$
$$y = 12$$

From (ii), $x = 10 + 2y$

$= 10 + 24$

$= 34$

So, Father's age = 34 years

Son's age = 12 years

34. Perimeter of ABCDE = 21 cm

i.e. AB + BC + CD + DE + AE = 21

$$3 + x - y + x + y + x - y + 3 = 21$$
$$3x - y = 15 \qquad \text{(i)}$$

As BE || CD and BC || DE,

BCDE is a parallelogram

$\therefore$ BE = CD (opposite sides of parallelogram)

i.e. $x + y = 5$ (ii)

On adding equations (i) and (ii), we get

$$4x = 20$$
$$x = 5$$

from (i), $3(5) - y = 15$

$$y = 0$$

So, $BC = x - y = 5 - 0 = 5$ cm

$CD = x + y = 5 + 0 = 5$ cm

$DE = x - y = 5 - 0 = 5$ cm

$BE = 5$ cm

So, perimeter of quadrilateral BCDE

= 4 × 5 (perimeter = 4 × side) = 20 cm

35. Let $\frac{1}{x} = p$ and $\frac{1}{y} = q$.

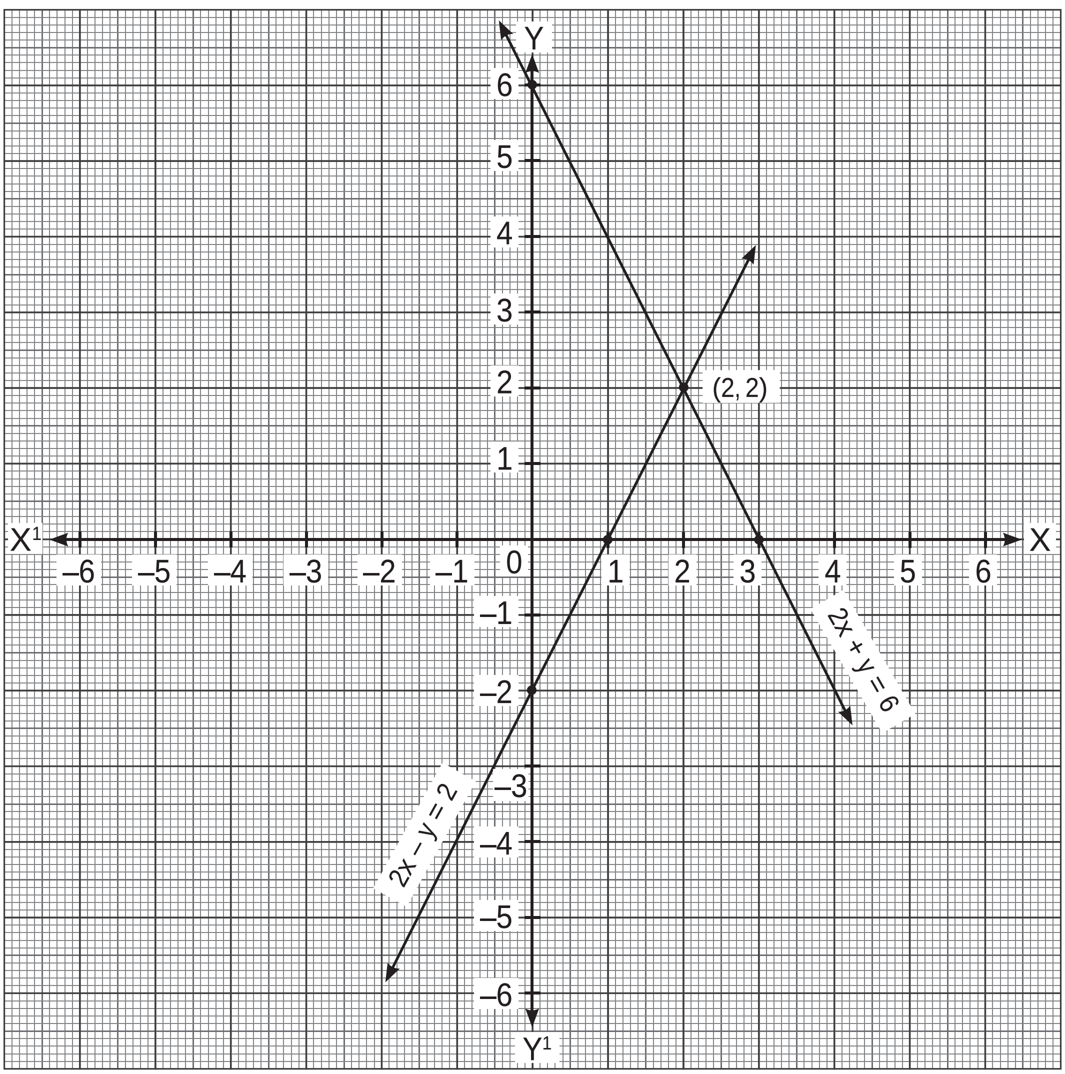
So, equations become

$$ap - bq = 0$$
$$ab^2p + a^2bq = a^2 + b^2$$
$$\frac{p}{a^2b + b^3 - 0} = \frac{q}{0 + a^3 + ab^2} = \frac{1}{a^3b + ab^3}$$
$$\frac{p}{b(a^2 + b^2)} = \frac{q}{a(a^2 + b^2)} = \frac{1}{ab(a^2 + b^2)}$$
$$\frac{p}{b(a^2 + b^2)} = \frac{1}{ab(a^2 + b^2)}$$
$$\frac{q}{a(a^2 + b^2)} = \frac{1}{ab(a^2 + b^2)}$$

$p = \frac{1}{a}$ $q = \frac{1}{b}$

$\therefore$ $x = a$ $y = b$

36.



2x + y = 6,

x	3	0
y	0	6

2x – y = 2

x	0	1
y	–2	0

As the equations intersect at point (2, 2), so, (2, 2) is a solution of given set of equations.

Area of triangle formed by lines representing these equations with the x – axis = $\frac{1}{2} \times 2 \times 2 =$ 2 sq units.

Area of triangle formed by lines representing these equations with the y – axis = $\frac{1}{2} \times 8 \times 2 =$ 8 sq units.

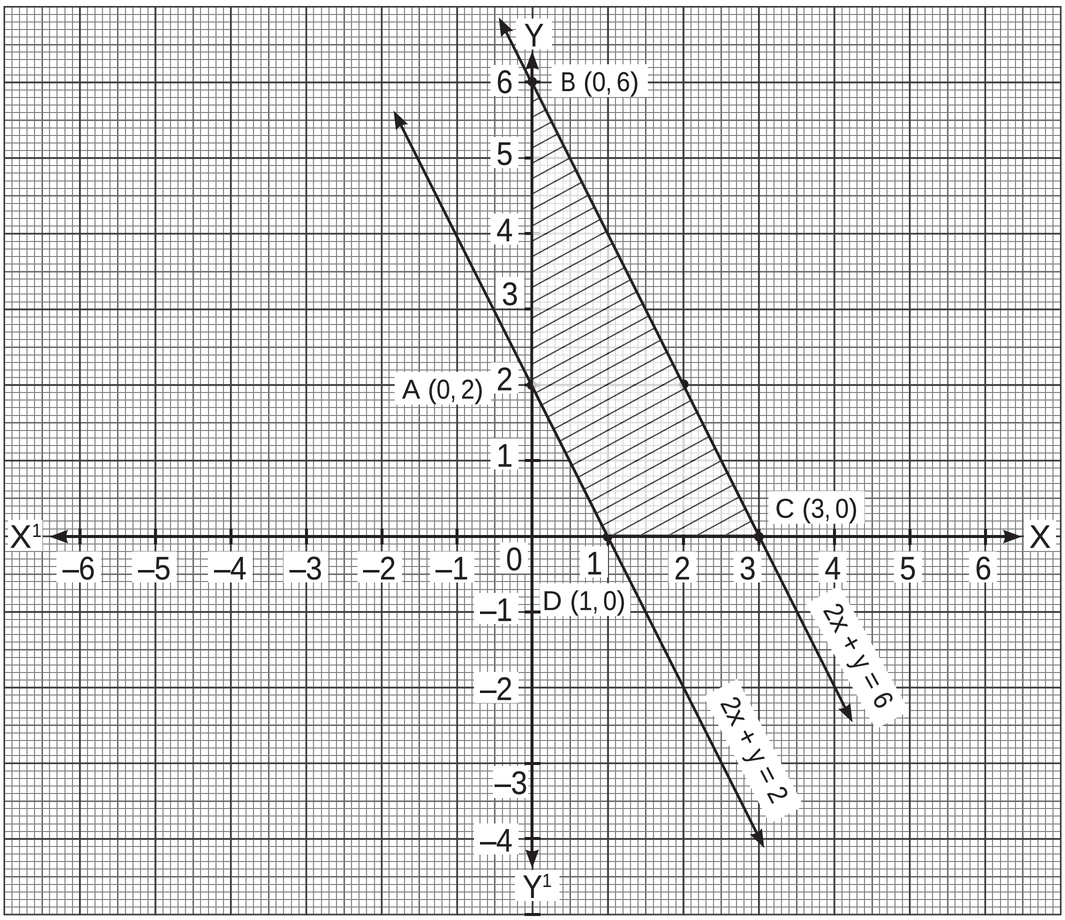
So, Ratio = $\frac{2}{8} = \frac{1}{4}$.

37. $2x + y = 2$

x	0	1
y	2	0

$2x + y = 6$

x	0	3
y	6	0



Vertices of trapezium are A(0, 2), B(0, 6), C(3, 0) and D(1, 0).

Area of trapezium ABCD = area of ΔBOC – area of ΔAOD

$= \frac{1}{2} \times 3 \times 6 - \frac{1}{2} \times 1 \times 2 = 4.5 - 1 = 3.5$ sq. units

38. Let the numerator be x and denominator be y
According to question,

$$y = 5 + 2x$$

$$-2x + y = 5 \qquad \text{(i)}$$

For the other equation,

$$\frac{x-1}{y-1} = \frac{3}{8}$$

$$8x - 8 = 3y - 3$$

$$8x - 3y = 5 \qquad \text{(ii)}$$

From (i), $y = 5 + 2x$

On putting this value of y in (ii), we get

$$8x - 3(5 + 2x) = 5$$
$$8x - 15 - 6x = 5$$
$$2x = 20$$
$$x = 10$$

So, $y = 5 + 2(10)$

$= 25$

So, Fraction $= \dfrac{x}{y} = \dfrac{10}{25}$

39. $mx - ny = m^2 + n^2$

$x + y = 2m$

$$\frac{x}{2mn + m^2 + n^2} = \frac{y}{-m^2 - n^2 + 2m^2} = \frac{1}{m+n}$$

$$\frac{x}{(m+n)^2} = \frac{y}{m^2 - n^2} = \frac{1}{m+n}$$

$$\frac{x}{(m+n)^2} = \frac{1}{m+n}$$

$$x = \frac{(m+n)^2}{m+n}$$

$= m + n$

$$\frac{y}{m^2 - n^2} = \frac{1}{m+n}$$

$$y = \frac{m^2 - n^2}{m+n}$$

$= m - n$

40. $(a - b)x + (a + b)y = a^2 - 2ab - b^2$

$(a + b)x + (a + b)y = a^2 - b^2$

$$\frac{x}{(a+b)(-a^2-b^2) - (a+b)(-a^2+2ab+b^2)} = \frac{y}{(a+b)(-a^2+2ab+b^2) - (a-b)(-a^2-b^2)} = \frac{1}{(a-b)(a+b) - (a+b)^2}$$

$$\frac{x}{-a^3 - ab^2 - a^2b - b^3 + a^3 - 2a^2b - ab^2 + a^2b - 2ab^2 - b^3} = \frac{y}{-a^3 + 2a^2b + ab^2 - a^2b + 2ab^2 + b^3 + a^3 + ab^2 - a^2b - b^3} = \frac{1}{a^2 - b^2 - a^2 - b^2 - 2ab}$$

$$\frac{x}{-2b^3 - 2a^2b - 4ab^2} = \frac{y}{4ab^2} = \frac{1}{-2b^2 - 2ab}$$

$$x = \frac{-2b^3 - 2a^2b - 4ab^2}{-2b^2 - 2ab}$$

$$= \frac{-2b(b^2 + a^2 + 2ab)}{-2b(b+a)}$$

$= a + b$

Also, $\dfrac{y}{4ab^2} = \dfrac{1}{-2b(a+b)}$

$$y = \frac{-2ab}{a+b}$$

Chapter 04

Quadratic Equations

MULTIPLE CHOICE QUESTIONS

1. (b) As $x = -\frac{1}{2}$ is a solution of $3x^2 + 2kx - 3 = 0$

$$\therefore \quad 3\left(-\frac{1}{2}\right)^2 + 2k\left(-\frac{1}{2}\right) - 3 = 0$$

$$\frac{3}{4} - k - 3 = 0$$

$$k = \frac{3}{4} - 3 = \frac{3-12}{4} = \frac{-9}{4}$$

2. (b) Equation has no real roots if $D < 0$

i.e. $b^2 - 4ac < 0$

i.e. $b^2 - 4(1)(1) < 0$

i.e. $b^2 - 4 < 0$

i.e. $(b + 2)(b - 2) < 0$

i.e. $-2 < b < 2$

3. (d) let α, β be the roots then $\alpha\beta = 3$

$\alpha\beta = 3$

$(1)\,\beta = 3 \qquad (\because \alpha = 1)$

$\alpha\beta = 3$

4. (a) $3\sqrt{3}x^2 + 10x + \sqrt{3} = 0$

$D = b^2 - 4ac$

$= (10)^2 - 4(3\sqrt{3}/\sqrt{3})$

$= 100 - 36$

$= 64$

5. (a) $x^2 - px - q = 0$

As p is the root

$\therefore \quad p^2 - p^2 + q = 0$

$\Rightarrow \quad q = 0$

Also, q is a root

$\therefore \quad q^2 - pq + q = 0$

$q(q - p + 1) = 0$

$q = 0$ or $q = p - 1$

$\therefore \quad q = p - 1$

$\Rightarrow \quad p = q + 1$

$= 0 + 1$

$= 1$

So, $p = 1, q = 0$

WORKSHEET 1

Section A

1. $x^2 - 7x + 12$

$x^2 - 3x - 4x + 12$

$x(x - 3) - 4(x - 3)$

$(x - 3)(x - 4)$

2. $2x^2 + 3x - 4 = 0$

$b^2 - 4ac = 9 - 4(2)(-4)$

$= 9 + 32$

$= 41 > 0$

As $b^2 - 4ac > 0$

$\Rightarrow$ The equation has real and distinct roots.

3. $3x^2 + 13x + 14 = 0$

LHS $= 3x^2 + 13x + 14$

$= 3(-2)^2 + 13(-2) + 14 \qquad$ (put $x = -2$)

$= 12 - 26 + 14$

$= 0$

$= \text{RHS}$

So, $x = -2$ is a root of $3x^2 + 13x + 4 = 0$

4. $x^2 - 3x - 1 = 0$

LHS $= x^2 - 3x - 1$

$= 1^2 - 3(1) - 1$ (Put $x = 1$)

$= 1 - 3 - 1$

$= -3 \neq \text{RHS} (= 0)$

So, $x = 1$ is not a solution of equation $x^2 - 3x - 1 = 0$

5. $x^2 - 3x - 10 = 0$

$D = b^2 - 4ac$

$= (-3)^2 - 4(1)(-10)$

$= 9 + 40$

$= 49$

6. Let $\sqrt{6+\sqrt{6+\sqrt{6+\ldots}}} = x$

$\sqrt{6+x} = x$

On squaring both sides, we get

$6 + x = x^2$

$x^2 - x - 6 = 0$

$x^2 - 3x + 2x - 6 = 0$

$x(x - 3) + 2(x - 3) = 0$

$(x - 3)(x + 2) = 0$

$x = 3, x = -2$

As value of $\sqrt{6+\sqrt{6+\sqrt{6+\ldots}}}$ cannot be negative, so, $x = 3$

7. $3x^2 - kx + 38 = 0$

The quadratic equation has equal roots

if $D = 0$

i.e. $b^2 - 4ac = 0$

i.e. $k^2 - 4(3)(38) = 0$

$k^2 - 456 = 0$

$k^2 = 456$

$k = \pm 2\sqrt{114}$

8. $bx^2 - 2\sqrt{ac}\, x + 6 = 0$

The equation has equal roots if discriminant $= 0$

i.e. $\left(2\sqrt{ac}\right)^2 - 4(b)(b) = 0$

$4ac - 4b^2 = 0$

$b^2 = ac$

Section B

9. Using quadratic formula,

General form $ax^2 + bx + c = 0$

$D = b^2 - 4ac$

Solution is $x = \dfrac{-b \pm \sqrt{D}}{2a}$

Equation is $16x^2 - 24x - 1 = 0$

Where, $a = 16, b = -24, c = -71$

$D = b^2 - 4a$

$D = (-24)^2 - 4(16)(-1)$

$D = 576 + 64$

$D = 640$

Solution is $x = \dfrac{-b \pm \sqrt{D}}{2a}$

$x = \dfrac{-(-24) \pm \sqrt{640}}{2(16)}$

$x = \dfrac{24 \pm 8\sqrt{10}}{16}$

$x = \dfrac{3 \pm \sqrt{10}}{4}$

Therefore, the roots are $x = \dfrac{3+\sqrt{10}}{4}, \dfrac{3-\sqrt{10}}{4}$

10. $\dfrac{1}{x-2} + \dfrac{2}{x-1} = \dfrac{6}{x}$

$\dfrac{x-1+2x-4}{(x-1)(x-2)} = \dfrac{6}{x}$

$x(3x - 5) = 6(x - 1)(x - 2)$

$$3x^2 - 5x = 6(x^2 - 3x + 2)$$
$$3x^2 - 5x = 6x^2 - 18x + 12$$
$$0 = 3x^2 - 13x + 12$$
$$0 = 3x^2 - 9x - 4x + 12$$
$$0 = 3x(x - 3) - 4(x - 3)$$
$$0 = (3x - 4)(x - 3)$$
$$3x - 4 = 0, \qquad x - 3 = 0$$
$$x = \frac{4}{3}, \qquad x = 3$$

11. $x^2 - 2ax + a^2 - b^2 = 0$

$x^2 + [(-a - b) + (-a + b)]x + (a + b)(a - b) = 0$

$x^2 - (a + b)x - (a - b)x + (a + b)(a - b) = 0$

$x[x - (a + b)] - (a - b)[x - (a + b)] = 0$

$[x - (a - b)][x - (a + b)] = 0$

$x - (a - b) = 0$ or $x - (a + b) = 0$

$x = a - b$ or $x = a + b$

12. $4x^2 - 4a^2x + (a^4 - b^4) = 0$

$4x^2 + [-2(a^2 - b^2) - 2(a^2 + b^2)]x + (a^4 - b^4) = 0$

$4x^2 - 2(a^2 - b^2)x - 2(a^2 + b^2)x + (a^2 - b^2)(a^2 + b^2) = 0$

$2x[2x - a^2 - b^2] - (a^2 + b^2)(2x - a^2 - b^2) = 0$

$[2x - (a^2 - b^2)][2x - (a^2 + b^2)] = 0$

$2x - (a^2 - b^2) = 0$ or $2x(a^2 + b^2) = 0$

$x = \frac{-b^2 + a^2}{2}$ or $x = \frac{a^2 + b^2}{2}$

13. Given

$(k - 12)x^2 - 2(k - 12)x + 2 = 0$

Comparing with $ax^2 + bx + c = 0$ we get :-

$a = k - 12,\ b = -2(k - 12), c = 2$

The equation has equal roots!

So $b^2 = 4ac$

$\Rightarrow (-2(k - 12))^2 = 2(k - 12)$

$\Rightarrow (-2)^2(k - 12)^2 = 2(k - 12)$

$\Rightarrow 2(k^2 + 144 - 24k) = k - 12$

$\Rightarrow 2k^2 + 288 - 48k = k - 12$

$\Rightarrow 2k^2 - 49k + 300 = 0$

$\Rightarrow 2k^2 - 25k - 24k + 300 = 0$

$\Rightarrow k(2k - 25) - 12(2k - 25) = 0$

$\Rightarrow (2k - 25)(k - 12) = 0$

Either :

$k - 12 = 0$

$\Rightarrow k = 12$

or :

$2k - 25 = 0$

$\Rightarrow 2k = 25$

$\Rightarrow k = \frac{25}{2} = 12.5$

The value of k is 12 or 12.5

14. $12abx^2 - (9a^2 - 8b^2)x - 6ab = 0$

$12abx^2 + (-9a^2 + 8b^2)x - 6ab = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-9a^2+8b^2) \pm \sqrt{(-9a^2+8b^2)^2 - 4(12ab)(-6ab)}}{2(12ab)}$$

$$x = \frac{-(-9a^2+8b^2) + \sqrt{(-9a^2+8b^2)^2 - 4(12ab)(-6ab)}}{2(12ab)}$$

$$\Rightarrow x = \frac{3a}{4b}$$

$$x = \frac{-(-9a^2+8b^2) - \sqrt{(-9a^2+8b^2)^2 - 4(12ab)(-6ab)}}{2(12ab)}$$

$$\Rightarrow x = \frac{-2b}{3a}$$

15. Let the two numbers be x and 16 – x.

According to question,

$$\frac{1}{x} + \frac{1}{16 - x} = \frac{1}{3}$$
$$\frac{16 - x + x}{x(16 - x)} = \frac{1}{3}$$
$$48 = 16x - x^2$$
$$x^2 - 16x + 48 = 0$$
$$x^2 - 12x - 4x + 48 = 0$$

$$x(x-12)-4(x-12)=0$$
$$(x-4)(x-12) = 0$$
$$x = 4, 12$$

If $x = 4$, Other number $= 16 - 4 = 12$

if $x = 12$, Other number $= 16 - 12 = 4$

16.
$$x+\frac{1}{x} = 11\frac{1}{11}$$
$$\frac{x^2+1}{x} = \frac{122}{11}$$
$$11(x^2+1) = 122x$$
$$11x^2-122x+11 = 0$$
$$11x^2-x-121x+11 = 0$$
$$x(11x-1)-11(11x-1) = 0$$
$$(11x-1)(x-11) = 0$$
$$11x-1=0 \quad \text{or} \quad x-11 = 0$$
$$x=\frac{1}{11} \quad \text{or} \quad x = 11$$

Section C

17. Let D_1 and D_2 be the discriminants of equations $x^2 + 2cx + ab = 0$ and $x^2 - 2(a + b)x + a^2 + b^2 + 2c^2 = 0$ respectively.

$$x^2+2cx+ab=0$$
$$D_1 = (2c)^2 - 4(1)(ab)$$
$$= 4c^2 - 4ab$$
$$= 4(c^2 - ab)$$

As roots are real and unequal,

so $D_1 > 0$

$c^2 - ab > 0$ (i)

$$x^2-2(a+b)x+a^2+b^2+2c^2=0$$
$$D_2 = 4(a+b)^2 - 4(1)(a^2+b^2+2c^2)$$
$$= 8ab - 8c^2$$
$$= -8(c^2-ab) < 0 \text{ [From (i)]}$$

So, the given equation has no real roots.

18.
$$\frac{1}{x+4}-\frac{1}{x-7} = \frac{11}{30}$$
$$\frac{x-7-x-4}{(x+4)(x-7)} = \frac{11}{30}$$
$$\frac{-11}{x^2-3x-28} = \frac{11}{30}$$
$$\frac{-1}{x^2-3x-28} = \frac{1}{30}$$
$$x^2-3x-28+30 = 0$$
$$x^2-3x+2 = 0$$
$$x^2-2x-x+2 = 0$$
$$x(x-2)-1(x-2) = 0$$
$$(x-1)(x-2) = 0$$
$$x = 1, 2$$

19. Le the smaller side and larger side be x cm and y cm respectively.

Hypotenuse $= 3\sqrt{5}$ cm

So, $x^2 + y^2 = \left(3\sqrt{5}\right)^2$

$x^2 + y^2 = 45$ (i)

If smaller side is tripled and the larger side is doubled,

$(3x)^2 + (2y)^2 = (15)^2$

$9x^2 + 4y^2 = 225$ (ii)

From (i), $x^2 = 45 - y^2$

So, we get $9(45 - y^2) + 4y^2 = 225$

$$405-9y^2+4y^2 = 225$$
$$5y^2 = 180$$
$$y^2 = \frac{180}{5}=36$$
$$y = \pm 6$$

For $y = -6$, $x^2 = 45 - 36 = 9$

$x = \pm 3$

For $y = 6$, $x^2 = 45 - 36 = 9$

$x = \pm 3$

As length cannot be negative,

So, $y = -6$, $x = -3$ rejected

$\therefore \quad x = 3, \quad y = 6$

Length of smaller side = 3 cm

Length of larger side = 6 cm

20. As x = – 2 is a root of equation

$3x^2 + 7x + p = 0$, we have

$$3(-2)^2 + 7(-2) + p = 0$$
$$12 - 14 + p = 0$$
$$p = 2$$
$$x^2 + k(4x + k - 1) + p = 0$$
$$x^2 + k(4x + k - 1) + 2 = 0 \quad \text{(Put } p = 2)$$
$$x^2 + (4k)x + k^2 - k + 2 = 0$$

As roots are equal,

$$\text{Discriminant (D)} = 0$$
$$(4k)^2 - 4(k^2 - k + 2) = 0$$
$$16k^2 - 4k^2 + 4k - 8 = 0$$
$$12k^2 + 4k - 8 = 0$$
$$3k^2 + k - 2 = 0$$
$$3k^2 + 3k - 2k - 2 = 0$$
$$3k(k + 1) - 2(k + 1) = 0$$
$$(3k - 2)(k + 1) = 0$$
$$3k - 2 = 0 \quad \text{or} \quad k + 1 = 0$$
$$k = \frac{2}{3} \quad \text{or} \quad k = -1$$

21. $x^2(a^2 + b^2) + 2(ac + bd)x + (c^2 + d^2) = 0$

Consider

Discriminant (D)

$= 4(ac + bd)^2 - 4(a^2 + b^2)(c^2 + d^2)$

$= 4(a^2c^2 + b^2d^2 + 2abcd) - 4(a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2)$

$= 8abcd - 4a^2d^2 - 4b^2c^2$

$= -4[(ad)^2 + (bc)^2 - 2abcd)$

$= -4(ad - bc)^2$

≤ 0

For no real roots, D < 0

i.e. $D \neq 0$ i.e. $ad \neq bc$

22. As 2 is a root of the quadratic equation $3x^2 + px - 8 = 0$,

$$3(2)^2 + p(2) - 8 = 0$$
$$12 + 2p - 8 = 0$$
$$2p = -4$$
$$p = -2$$

$\therefore$ Other equation becomes

$$4x^2 - 2(-2)x + k = 0$$
$$4x^2 + 4x + k = 0$$

As roots are equal,

$$\text{Discriminant (D)} = 0$$

i.e. $16 - 4(4)(k) = 0$

$$16 - 16k = 0$$
$$k = 1$$

23. $(x - a)(x - b) + (x - b)(x - c) + (x - c)(x - a) = 0$

$\Rightarrow x^2 - bx - ax + ab + x^2 - cx - bx + bc + x^2 - ax - cx + ac = 0$

$\Rightarrow 3x^2 - 2bx - 2ax - 2cx + ab + bc + ca = 0$

$\Rightarrow 3x^2 - 2(a + b + c)x + (ab + bc + ca) = 0$

Discriminant (D)

$= 4(a + b + c)^2 - 12(ab + bc + ca)$

$= 4(a^2 + b^2 + c^2 + 2ab + 2bc + 2ac - 3ab - 3bc - 3ca)$

$= 4(a^2 + b^2 + c^2 - ab - bc - ca)$

$= 2(2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ac)$

$= 2[(a - b)^2 + (b - c)^2 + (a - c)^2]$

$D = 2[(a - b)^2 + (b - c)^2 + (a - c)^2] \geq 0$

As $D \geq 0$, so roots are real.

Roots are equal if D = 0

i.e. $2[(a - b)^2 + (b - c)^2 + (a - c)^2] = 0$

i.e. $a - b = 0, \quad b - c = 0, \quad a - c = 0,$

$a = b, \quad b = c, \quad a = c$

i.e. $a = b = c$.

24. Let the two numbers be x and y such that $x > y$.

$$x - y = 3 \quad \text{(i)}$$

Also, $$\frac{1}{y} - \frac{1}{x} = \frac{3}{28} \quad \text{(ii)}$$

From (i), $x = 3 + y$

Putting in (ii), we get

$$\frac{1}{y} - \frac{1}{3+y} = \frac{3}{28}$$

$$\frac{3+y-y}{y(3+y)} = \frac{3}{28}$$

$$\frac{3}{y(3+y)} = \frac{3}{28}$$

$$28 = y^2 + 3y$$

$$y^2 + 3y - 28 = 0$$

$$y^2 + 7y - 4y - 28 = 0$$

$$y(y + 7) - 4(y + 7) = 0$$

$$(y - 4)(y + 7) = 0$$

$$y = 4, -7$$

As y is a natural number,

$y = -7$ is rejected

So, $y = 4$

$\therefore \quad x = 3 + y = 7$

Section D

25. $$\frac{x-2}{x-3} + \frac{x-4}{x-5} = \frac{10}{3}$$

$$\frac{(x-2)(x-5)+(x-3)(x-4)}{(x-3)(x-5)} = \frac{10}{3}$$

$$\frac{x^2-7x+10+x^2-7x+12}{x^2-8x+15} = \frac{10}{3}$$

$$\frac{2x^2-14x+22}{x^2-8x+15} = \frac{10}{3}$$

$$\frac{x^2-7x+11}{x^2-8x+15} = \frac{5}{3}$$

$$3x^2 - 21x + 33 = 5x^2 - 40x + 75$$

$$0 = 2x^2 - 19x + 42$$

$$0 = 2x^2 - 12x - 7x + 42$$

$$0 = 2x(x - 6) - 7(x - 6)$$

$$0 = (2x - 7)(x - 6)$$

$$(2x - 7) \quad (x - 6) = 0$$

$$2x - 7 = 0 \quad \text{or} \quad x - 6 = 0$$

$$x = \frac{7}{2} \quad \text{or} \quad x = 6$$

26. Let speed of stream be x km/hr

Speed of boat in still water = 18 km/hr

So, Speed of boat downstream = (18 + x) km/hr

Speed of boat upstream = (18 – x) km/hr

According to equation,

$$\frac{24}{18-x} = \frac{24}{18+x} + 1 \qquad \text{up} = \text{D} + 1$$

$$\frac{24}{18-x} - \frac{24}{18+x} = 1$$

$$\frac{18+x-18+x}{(18-x)(18+x)} = \frac{1}{24}$$

$$\frac{2x}{324-x^2} = \frac{1}{24}$$

$$324 - x^2 = 48x$$

$$x^2 + 48x - 324 = 0$$

$$x^2 + 54x - 6x - 324 = 0$$

$$x(x + 54) - 6(x + 54) = 0$$

$$(x - 6)(x + 54) = 0$$

$$x = 6, \; -54$$

As speed cannot be negative,

$x = -54$ is rejected.

So, $x = 6$

$\therefore$ Speed of stream = 6 km/hr

27. $$3\left(\frac{3x-1}{2x+3}\right)-2\left(\frac{2x+3}{3x-1}\right) = 5$$

Let $\frac{3x-1}{2x+3} = y$

So, equation becomes

$$3y - \frac{2}{y} = 5$$
$$3y^2 - 2 = 5y$$
$$3y^2 - 5y - 2 = 0$$
$$3y^2 - 6y + y - 2 = 0$$
$$3y(y-2) + 1(y-2) = 0$$
$$(3y+1)(y-2) = 0$$

$3y + 1 = 0$ or $y - 2 = 0$

$y = -\frac{1}{3}$ or $y = 2$

$y = -\frac{1}{3}$	$y = 2$
$\frac{3x-1}{2x+3} = -\frac{1}{3}$	$\frac{3x-1}{2x+3} = 2$
$9x - 3 = -2x - 3$	$3x - 1 = 4x + 6$
$11x = 0$	$x = -7$
$x = 0$	

28. Let original speed of the aircraft be x km/hr.

be x km/hr.

∴ New speed = (x – 200) km/hr.

Duration of flight at original speed

$$= \frac{600}{x} \text{ hours}$$

Duration of flight at reduced speed

$$= \frac{600}{x-200} \text{ hours}$$

According to question,

$$\frac{600}{x-200} = \frac{1}{2}+\frac{600}{x}$$
$$\frac{600}{x-200}-\frac{600}{x} = \frac{1}{2}$$
$$\frac{1}{x-200}-\frac{1}{x} = \frac{1}{1200}$$
$$\frac{x-x+200}{x(x-200)} = \frac{1}{1200}$$
$$x^2 - 200x = 240000$$
$$x^2 - 200x = 240000 = 0$$
$$x^2 - 600x + 400x - 240000 = 0$$
$$x(x-600) + 400(x-600) = 0$$
$$(x+400)(x-600) = 0$$
$$x = -400 \quad \text{or } x = 600$$

As x, being speed of aircraft can't be negative.

So, x = 600

∴ Original speed of aircraft = 600 km/hr

Duration of flight = $\frac{600}{600} = 1$ hour

29. Let the usual speed of plane be x km / hr

∴ Time taken = $\frac{1500}{x}$ hours

New speed = x + 250 km / hr

∴ Time taken = $\frac{1500}{x+250}$ km / hr

According to question,

$$\frac{1500}{x+250} = \frac{1500}{x} - \frac{1}{2}$$
$$\frac{1500}{x} - \frac{1500}{x+250} = \frac{1}{2}$$
$$\frac{1}{x} - \frac{1}{x+250} = \frac{1}{3000}$$
$$\frac{x+250-x}{x(x+250)} = \frac{1}{3000}$$
$$x^2 + 250x = 750000$$

$x^2 + 250x - 750000 = 0$

$x^2 + 1000x - 750x - 750000 = 0$

$x(x+1000) - 750(x+1000) = 0$

$(x-750)(x+1000) = 0$

$x = 750$ or $x = -1000$

Now, x being the speed of plane cannot be negative,

$x = -1000$ is rejected

So, $x = 750$

$\therefore$ Speed of plane = 750 km/hr

30. Let total number of camels be x.

According to question

$$\frac{1}{4}x + 2\sqrt{x} + 15 = x$$
$$2\sqrt{x} + 15 = x - \frac{x}{4}$$
$$2\sqrt{x} + 15 = \frac{3x}{4}$$
$$8\sqrt{x} + 60 = 3x$$
$$3x - 8\sqrt{x} - 60 = 0$$
$$3\left(\sqrt{x}\right)^2 - 8\sqrt{x} - 60 = 0$$

Let $\sqrt{x} = y$

$$3y^2 - 8y - 60 = 0$$
$$3y^2 - 18y + 10y - 60 = 0$$
$$3y(y-6) + 10(y-6) = 0$$
$$(3y+10)(y-6) = 0$$
$$y = -\frac{10}{3} \quad \text{or} \quad y = 6$$

Now,

$y = -\frac{10}{3}$ is rejected as number of camels can not be negative,

So, $y = 6$

i.e. $\sqrt{x} = 6$

$\therefore$ $x = 36$

So, total number of camels = 36

31. Let Varun's age be x years and Nihal's age be y years.

According to question.

$$x - 7 = 5(y-7)^2$$
$$x - 7 = 5(y-7)^2 \qquad \text{(i)}$$

For second equation,

$$y + 3 = \frac{2}{5}(x+3)$$
$$5y + 15 = 2x + 6$$
$$2x - 5y = 9 \qquad \text{(ii)}$$

From (ii), $x = \frac{9+5y}{2}$

Putting in (i), we get

$$\frac{9+5y}{2} - 7 = 5(y-7)^2$$
$$9 + 5y - 14 = 10(y^2 + 49 - 14y)$$
$$5y - 5 = 10(y^2 + 49 - 14y)$$
$$y - 1 = 2(y^2 + 49 - 14y)$$
$$y - 1 = 2y^2 + 98 - 28y$$
$$2y^2 - 29y + 99 = 0$$
$$y = \frac{29 \pm \sqrt{841 - 8(99)}}{4}$$
$$y = \frac{29 \pm \sqrt{49}}{4}$$
$$y = \frac{29 \pm 7}{4}$$
$$y = \frac{29+7}{4}, \qquad y = \frac{29-7}{4}$$
$$y = 9, \qquad y = \frac{11}{2}$$

Now, $y = \frac{11}{2}$ is rejected

So, $y = 9$

$\therefore$ Nihal's age = 9 years

$$\text{Varun's age} = \frac{9+5y}{2} = \frac{9+45}{2} = 27 \text{ years}$$

32.
$$\frac{1}{a+b+x} - \frac{1}{x} = \frac{1}{a} + \frac{1}{b}$$
$$\frac{x-(a+b+x)}{x(a+b+x)} = \frac{a+b}{ab}$$

$$\frac{-(a+b)}{x(a+b+x)} = \frac{a+b}{ab}$$

$$\frac{-1}{x(a+b+x)} = \frac{1}{ab}$$

$$x(a+b+x) + ab = 0$$

$$xa + xb + x^2 + ab = 0$$

$$x^2 + xa + xb + ab = 0$$

$$x(x+a) + b(x+a) = 0$$

$$(x+a)(x+b) = 0$$

$$x = -a \quad \text{or} \quad x = -b$$

WORKSHEET 2

Section A

1. $$\text{LHS} = x^2 - 3\sqrt{3}x + 6$$
$$= (-2\sqrt{3})^2 - 3\sqrt{3}(-2\sqrt{3}) + 6$$
$$= 12 + 18 + 6$$
$$= 36$$
$$\neq \text{RHS } (= 0)$$

So, $x = -2\sqrt{3}$ is not a solution of the given equation.

2. As $x = -\frac{1}{2}$ is a solution of $3x^2 + 2kx - 3 = 0$,

$$3\left(-\frac{1}{2}\right)^2 + 2k\left(-\frac{1}{2}\right) - 3 = 0$$

$$\frac{3}{4} - K - 3 = 0$$

$$K = \frac{3}{4} - 3 = \frac{-9}{4}$$

3. Let the two consecutive positive integers be x, x + 1.

According to question,

$$x(x+1) = 240$$

$$x^2 + x - 240 = 0$$

4. $$x^2 + 6x + 5 = 0$$

$$x^2 + 5x + x + 5 = 0$$

$$x(x+5) + 1(x+5) = 0$$

$$(x+1)(x+5) = 0$$

$$x + 1 = 0 \quad \text{or} \quad x + 5 = 0$$

$$x = -1 \quad \text{or} \quad x = -5$$

5. $$x + \frac{2}{x} = 3$$

$$x^2 + 2 = 3x$$

$$x^2 - 3x + 2 = 0$$

$$x^2 - 2x - x + 2 = 0$$

$$x(x-2) - 1(x-2) = 0$$

$$(x-1)(x-2) = 0$$

$$x = 1, 2$$

6. $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$

$$\text{Discriminant} = (-2\sqrt{2})^2 - 4(\sqrt{3})(-2\sqrt{3})$$
$$= 8 + 24$$
$$= 32$$

7. $2x^2 + 5\sqrt{3}x + 6 = 0$

$$\text{Discriminant (D)} = (5\sqrt{3})^2 - 4(2)(6)$$
$$= 75 - 48$$
$$= 27 > 0$$

So, the given equation has real roots.

8. $$abx^2 + (b^2 - ac)x - bc = 0$$

$$abx^2 + b^2x - acx - bc = 0$$

$$bx(ax+b) - c(ax+b) = 0$$

$$(bx-c)(ax+b) = 0$$

$$bx - c = 0 \quad \text{or} \quad ax + b = 0$$

$$x = \frac{c}{b} \quad \text{or} \quad x = -\frac{b}{a}$$

9. Compare given Quadratic equation $2x^2 - kx + k = 0$ with $ax^2 + bx + c = 0$, we get

$a = 2, b = -k, c = k$

Discriminant (D) = 0

[Given roots are equal]

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow (-k)^2 - 4 \times 2 \times k = 0$

$\Rightarrow k^2 - 8k = 0$

$\Rightarrow k(k - 8) = 0$

$\Rightarrow k = 0$ or $k = 8$

10. $$x^2 + \left(a + \frac{1}{a}\right)x + 1 = 0$$

$$ax^2 + (a^2 + 1)x + a = 0$$

$$ax^2 + a^2x + x + a = 0$$

$$ax(x + a) + 1(x + a) = 0$$

$$(ax + 1)(x + a) = 0$$

$$ax + 1 = 0 \quad \text{or} \quad x + a = 0$$

$$x = -\frac{1}{a} \quad \text{or} \quad x = -a$$

Section B

11. As $x = \frac{2}{3}$ is a root of equation

$$ax^2 + 7x + b = 0$$

$$a\left(\frac{2}{3}\right)^2 + 7\left(\frac{2}{3}\right) + b = 0$$

$$\frac{4}{9}a + \frac{14}{3} + b = 0$$

$$\frac{4a + 42 + 9b}{9} = 0$$

$$4a + 9b = -42 \qquad \text{(i)}$$

As $x = -3$ is a root of equation

$$ax^2 + 7x + b = 0$$

$$9a - 21 + b = 0$$

$$9a + b = 21 \qquad \text{(ii)}$$

From (ii), $b = 21 - 9a$

Putting in (i), we get

$$4a + 9(21 - 9a) = -42$$

$$4a + 189 - 81a = -42$$

$$189 + 42 = 81a - 4a$$

$$231 = 77a$$

$$a = 3$$

So, $b = 21 - 9(3)$

$= 21 - 27$

$= -6$

12. As -5 is a root of equation

$$px^2 + px + k = 0$$

$$p(-5)^2 + p(-5) + k = 0$$

$$25p - 5p + k = 0$$

$$20p + k = 0 \qquad \text{(i)}$$

Also, as equation has equal roots,

$$\text{Discriminant} = 0$$

$$p^2 - 4pk = 0$$

$$p(p - 4k) = 0$$

$$p = 0 \quad \text{or} \quad p = 4k$$

if $p = 0$, $20(0) + k = 0$

$$k = 0$$

if $p = 4k$, $20(4k) + k = 0$

$$k = 0$$

13. $$\sqrt{2x + 9} + x = 13$$

$$\sqrt{2x + 9} = 13 - x$$

Squaring both sides

$$2x + 9 = 169 + x^2 - 26x$$

$$x^2 - 28x + 160 = 0$$

$$x^2 - 20x - 8x + 160 = 0$$

$$x(x - 20) - 8(x - 20) = 0$$

$$(x - 8)(x - 20) = 0$$

$$x = 8 \quad \text{or} \quad x = 20$$

If $x = 20$

LHS $= \sqrt{40 + 9} + 20 = 27 \neq$ RHS $(= 13)$

So, $x = 20$ is rejected

If x = 8,

$$\text{LHS} = \sqrt{16+9} + 8 = 5 + 8 = 13 = \text{RHS}$$

Therefore, x = 8

14. $9x^2 - 6b^2x - (a^4 - b^4) = 0$

$9x^2 + [-3(b^2 - a^2) - 3(b^2 + a^2)]x + (-a^4 + b^4) = 0$

$9x^2 - 3(b^2 - a^2)x - 3(b^2 + a^2)x + (a^2 + b^2)(-a^2 + b^2) = 0$

$3x[3x - (b^2 - a^2)] - (a^2 + b^2)[3x - (b^2 - a^2)] = 0$

$[3x - (a^2 + b^2)][3x - (b^2 - a^2)] = 0$

$x = \dfrac{a^2 + b^2}{3}$ or $x = \dfrac{b^2 - a^2}{3}$

15.
$$\frac{4}{x} - 3 = \frac{5}{2x+3}$$
$$\frac{4-3x}{x} = \frac{5}{2x+3}$$
$$(4 - 3x)(2x + 3) = 5x$$
$$8x + 12 - 6x^2 - 9x = 5x$$
$$6x^2 + 6x - 12 = 0$$
$$x^2 + x - 2 = 0$$
$$x^2 + 2x - x - 2 = 0$$
$$x(x + 2) - 1(x + 2) = 0$$
$$(x - 1)(x + 2) = 0$$
$$x = 1,\ -2$$

16.
$$\sqrt{2}y^2 + 7y + 5\sqrt{2} = 0$$
$$\sqrt{2}y^2 + 2y + 5y + 5\sqrt{2} = 0$$
$$\sqrt{2}y(y + \sqrt{2}) + 5(y + \sqrt{2}) = 0$$
$$(y + \sqrt{2})(\sqrt{2}y + 5) = 0$$
$$y + \sqrt{2} = 0 \quad \text{or} \quad \sqrt{2}y + 5 = 0$$
$$y = -\sqrt{2} \quad \text{or} \quad y = -\frac{5}{\sqrt{2}}$$

17. Roots of the equation are equal if Discriminant (D) = 0

$$mx(6x + 10) + 25 = 0$$
$$6mx^2 + 10mx + 25 = 0$$
$$D = 0$$
$$(10m)^2 - 4(6m)25) = 0$$
$$100m^2 - 600m = 0$$
$$100m(m - 6) = 0$$
$$m = 0, 6$$

For m = 0, equation will become 25 = 0, which is not possible

So, m = 6

18. Given 4 roots 3x square + 5x – 2 root 3

The above given education can be written as under:

$$4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$$
$$4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3} = 0$$
$$4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2) = 0$$
$$(\sqrt{3}x + 2)(4x - \sqrt{3}) = 0$$
$$(\sqrt{3}x + 2) = 0 \quad \text{or} \quad (4x - \sqrt{3}) = 0$$
$$x = \frac{2}{\sqrt{3}} \quad \text{or} \quad x = \frac{\sqrt{3}}{4}$$

19. Let x be the side of square.

So, area of square = x^2

Number of students = $x^2 + 24$

If side of a square is increased by one student, side = x + 1

So, number of students = $(x + 1)^2 - 25$

According to question,

$$x^2 + 24 = (x + 1)^2 - 25$$
$$x^2 + 24 = x^2 + 1 + 2x - 25$$
$$48 = 2x$$
$$x = 24$$

$\therefore$ Number of students $= x^2 + 24$

$= (24)^2 + 24$

$= 576 + 24$

$= 600$

20. $4x^2 + 3x + 5 = 0$

$\Rightarrow x^2 + \frac{3}{4}x + \frac{5}{4} =$

$\Rightarrow x^2 + \frac{3}{4}x + \left(\frac{3}{8}\right)^2 = \frac{-5}{4} + \frac{9}{64}$

$\Rightarrow \left(x + \frac{3}{8}\right)^2 = \frac{-71}{64}$

$\Rightarrow x + \frac{3}{8} = \sqrt{\frac{-71}{64}}$ not a real no.

Hence, QE has no real roots.

Section C

21. $(x - 5)(x - 6) = \frac{25}{(24)^2}$

$x^2 - 11x + 30 = \frac{25}{576}$

$x^2 - 11x + 30 - \frac{25}{576} = 0$

$x^2 - 11x + \frac{17255}{576} = 0$

$576x^2 - 6336x + 17255 = 0$

$576x^2 - 2856x - 3480x + 17255 = 0$

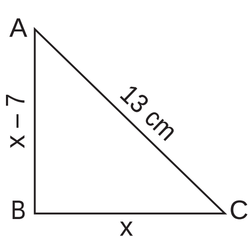
$24x(24x - 119) - 145(24x - 119) = 0$

$(24x - 145)(24x - 119) = 0$

$24x - 145 = 0$ or $24x - 119 = 0$

$x = \frac{145}{24}$ or $x = \frac{119}{24}$

22.



Let the base of Δ ABC=x cm

$\therefore$ Altitude of Δ ABC $= (x - 7)$ cm

We know that,

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

$AC^2 = AB^2 + BC^2$

$(13)^2 = (x - 7)^2 + x^2$

$169 = x^2 + 49 - 14x + x^2$

$2x^2 - 14x - 120 = 0$

$x^2 - 7x - 60 = 0$

$x^2 - 12x + 5x - 60 = 0$

$x(x - 12) + 5(x - 12) = 0$

$(x + 5)(x - 12) = 0$

$x = -5, 12$

Since, side cannot be negative,

So, $x = -5$ is rejected

$\therefore$ $x = 12$

$BC = x = 12$ cm

$AB = x - 7 = 12 - 7 = 5$ cm

23. $(a - b)x^2 + (b - c)x + (c - a) = 0$

As roots of equation are equal,

Discriminant (D) = 0

$(b - c)^2 - 4(a - b)(c - a) = 0$

$(b^2 + c^2 - 2bc) - 4(ac - a^2 - bc + ab) = 0$

$\Rightarrow 4a^2 + b^2 + c^2 - 4ac + 2bc - 4ab = 0$

$\Rightarrow (2a)^2 + b^2 + c^2 - 4ac + 2bc - 4ab = 0$

$\Rightarrow (-2a + b + c)^2 = 0$

$\Rightarrow -2a + b + c = 0$

$\Rightarrow 2a = b + c$

24. Let the sides of two squares be x and y

Area of square with side $x = x^2$

Area of square with side $y = y^2$

Perimeter of square with side $x = 4x$

Perimeter of square with side $y = 4y$

According to question,

$$x^2 + y^2 = 468 \quad \text{(i)}$$
$$4x - 4y = 24$$
$$\text{i.e.} \quad x - y = 6 \quad \text{(ii)}$$

From (ii), $x = 6 + y$

On putting in (i), we get

$$(6 + y)^2 + y^2 = 468$$
$$36 + y^2 + 12y + y^2 = 468$$
$$2y^2 + 12y - 432 = 0$$
$$y^2 + 6y - 216 = 0$$
$$y^2 - 12y + 18y - 216 = 0$$
$$y(y - 12) + 18(y - 12) = 0$$
$$(y - 12)(y + 18) = 0$$
$$y = 12, -18$$

As side cannot be negative,

$y = -18$ is rejected

$\therefore \quad y = 12$

So, $x = 6 + y$

$= 6 + 12$

$= 18$

So, sides of two squares are 12m and 18m respectively.

25.
$$a^2x^2 - 3abx + 2b^2 = 0$$
$$(ax)^2 - 2\left(\frac{3}{2}\right)abx + 2b^2 = 0$$
$$(ax)^2 - 2ax\left(\frac{3b}{2}\right) + 2b^2 = 0$$
$$(ax)^2 - 2ax\left(\frac{3b}{2}\right) + \left(\frac{3b}{2}\right)^2 + 2b^2 - \left(\frac{3b}{2}\right)^2 = 0$$
$$\left(ax - \frac{3b}{2}\right)^2 + 2b^2 - \frac{9}{4}b^2 = 0$$
$$\left(ax - \frac{3b}{2}\right)^2 - \frac{b^2}{4} = 0$$
$$\left(ax - \frac{3b}{2}\right)^2 = \frac{b^2}{4}$$
$$ax - \frac{3b}{2} = \pm\frac{b}{2}$$

$$ax - \frac{3b}{2} = \frac{b}{2} \qquad ax - \frac{3b}{2} = -\frac{b}{2}$$
$$ax = \frac{4b}{2} = 2b \qquad ax = -\frac{b}{2} + \frac{3b}{2} = b$$
$$x = \frac{2b}{a} \qquad x = \frac{b}{a}$$

26. $$\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0$$

$$\Rightarrow \quad \frac{2x(2x+3) + (x-3) + 3x + 9}{(x-3)(2x+3)} = 0$$
$$\Rightarrow \quad 2x(2x + 3) + (x - 3) + 3x + 9 = 0$$
$$\Rightarrow \quad 4x^2 + 6x + x - 3 + 3x + 9 = 0$$
$$\Rightarrow \quad 4x^2 + 10x + 6 = 0$$
$$\Rightarrow \quad 4x^2 + 4x + 6x + 6 = 0$$
$$\Rightarrow \quad 4x(x + 1) + 6(x + 1) = 0$$
$$\Rightarrow \quad (4x + 6)(x + 1) = 0$$
$$\Rightarrow \quad 4x + 6 = 0 \quad \text{or} \quad x + 1 = 0$$
$$\Rightarrow \quad x = -\frac{3}{2} \quad \text{or} \quad x = -1$$

27. Let the three consecutive natural numbers be $x - 1$, x and $x + 1$.

According to equation,

$$x^2 = [(x + 1)^2 - (x - 1)^2] + 60$$
$$x^2 = x^2 + 1 + 2x - x^2 - 1 + 2x + 60$$
$$x^2 = 4x + 60$$
$$x^2 - 4x - 60 = 0$$
$$x^2 - 10x + 6x - 60 = 0$$
$$x(x - 10) + 6(x - 10) = 0$$
$$(x + 6)(x - 10) = 0$$
$$x = -6 \text{ or } 10$$

As x is a natural number,

$x = -6$ or 10

As x is a natural number,

$x = -6$ is rejected

So, $x = 10$

$\therefore$ The three numbers 9, 10, 11.

28. Let the time taken by smaller tap to fill tank completely = x hours

∴ Time taken by larger tap to fill tank completely = x – 8 hours

According to question,

$$\frac{1}{x} + \frac{1}{x-8} = \frac{5}{48}$$

$$\frac{x-8+x}{x(x-8)} = \frac{5}{48}$$

$$\frac{2x-8}{x(x-8)} = \frac{5}{48}$$

$$48(2x-8) = 5x(x-8)$$

$$96x - 384 = 5x^2 - 40x$$

$$5x^2 - 136x + 384 = 0$$

$$5x^2 - 16x - 120x + 384 = 0$$

$$x(5x-16) - 24(5x-16) = 0$$

$$(x-24)(5x-16) = 0$$

$$x = 24 \quad \text{or} \quad \frac{16}{5}$$

For x = 24

Time taken by smaller tap = 24 hours

Tap taken by larger tap = x – 8

= 24 – 8

= 16 hours

For $x = \frac{16}{5}$

Time taken by larger pipe = x – 8

$$= \frac{16}{5} - 8$$

$$= -\frac{24}{5}$$

Since time cannot be negative,

$x = \frac{16}{5}$ is rejected.

29. $$9x^2 - 63x - 162 = 0$$

$$\text{Discriminant (D)} = (-63)^2 - 4(9)(-162)$$

$$= 3969 + 5832$$

$$= 9801$$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{63 \pm \sqrt{9801}}{18}$$

$$= \frac{63 \pm 99}{18}$$

$$x = \frac{63+99}{18} \quad \text{or} \quad x = \frac{63-99}{18}$$

$$x = 9 \quad \text{or} \quad x = -2$$

30. Let the larger part be x.

∴ Smaller part = 16 – x

According to question,

$$2(x)^2 = (16-x)^2 + 164$$

$$2x^2 = 256 + x^2 - 32x + 164$$

$$x^2 + 32x - 420 = 0$$

$$x^2 + 42x - 10x - 420 = 0$$

$$x(x+42) - 10(x+42) = 0$$

$$(x-10)(x+42) = 0$$

$$x = 10 \quad \text{or} \quad -42$$

x = – 42 is rejected as x < 0.

∴ x = 10

So, the required parts are 10 and 6.

Section D

31. $$\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$$

$$\frac{1}{2a+b+2x} - \frac{1}{2x} = \frac{1}{b} + \frac{1}{2a}$$

$$\frac{2x-2a-b-2x}{2x(2a+b+2x)} = \frac{2a+b}{2ab}$$

$$\frac{-2a-b}{2x(2a+b+2x)} = \frac{2a+b}{2ab}$$

$$\frac{-1}{2x(2a+b+2x)} = \frac{1}{2ab}$$

$$\frac{-1}{x(2a+b+2x)} = \frac{1}{ab}$$

$$x(2a+b+2x) + ab = 0$$

$$2x^2 + 2ax + bx + ab = 0$$

$$2x(x + a) + b(x + a) = 0$$

$$(2x + b)(x + a) = 0$$

$$x = \frac{-b}{2}, -a$$

32. Let number of books = x

$\therefore$ Cost of each book = $\frac{80}{x}$

According to question,

$$\frac{80}{x+4} = \frac{80}{x} - 1$$

$$\frac{80}{x} - \frac{80}{x+4} = 1$$

$$\frac{1}{x} - \frac{1}{x+4} = \frac{1}{80}$$

$$\frac{x+4-x}{x(x+4)} = \frac{1}{80}$$

$$\frac{1}{x(x+4)} = \frac{1}{320}$$

$$x^2 + 4x - 320 = 0$$

$$x^2 - 16x + 20x - 320 = 0$$

$$x(x - 16) + 20(x - 16) = 0$$

$$(x - 16)(x + 20) = 0$$

$x = 16$ or $x = -20$

Since, number of books cannot be negative, $x = 16$

So, number of books = 16

33. Let original duration of flight = x hours

Speed of an aircraft = $\frac{2800}{x}$ km/hr

If time increased by 30 minutes

i.e. $\frac{1}{2}$ hour, speed = $\frac{2800}{x+\frac{1}{2}}$

According to question,

$$\frac{2800}{x+\frac{1}{2}} = \frac{2800}{x} - 100$$

$$\frac{2800}{x} - \frac{2800}{\frac{2x+1}{2}} = 100$$

$$\frac{2800}{x} - \frac{5600}{2x+1} = 100$$

$$\frac{28}{x} - \frac{56}{2x+1} = 1$$

$$\frac{1}{x} - \frac{2}{2x+1} = \frac{1}{28}$$

$$\frac{2x+1-2x}{x(2x+1)} = \frac{1}{28}$$

$$\frac{1}{x(2x+1)} = \frac{1}{28}$$

$$2x^2 + x - 28 = 0$$

$$2x^2 + 8x - 7x - 28 = 0$$

$$2x(x + 4) - 7(x + 4) = 0$$

$$(x + 4)(2x - 7) = 0$$

$$x = -4, \frac{7}{2}$$

Since, time cannot be negative,

$$x = \frac{7}{2} = 3.5 \text{ hours}$$

34. Let speed of stream = x km/hr

Speed of boat in still water = 20 km/hr

Speed of boat upstream = (20 – x) km/hr

Speed of boat downstream = (20 + x) km/hr

According to equation,

$$\frac{48}{20-x} = \frac{48}{20+x} + 1$$

$$\frac{1}{20-x} - \frac{1}{20+x} = \frac{1}{48}$$

$$\frac{20+x-20+x}{(20-x)(20+x)} = \frac{1}{48}$$

$$\frac{2x}{(20-x)(20+x)} = \frac{1}{48}$$

$$96x = 400 - x^2$$

$$x^2 + 96x - 400 = 0$$

$$x^2 + 100x - 4x - 400 = 0$$
$$x(x + 100) - 4(x + 100) = 0$$
$$(x - 4)(x + 100) = 0$$
$$x = 4, -100$$

Being the speed, x can not be negative.

So, $x = -100$ is rejected

$\therefore$ $x = 4$

Speed of stream = 4 km/hr

35. $$\frac{1}{2x-3} + \frac{1}{x-5} = 1$$
$$\frac{x-5+2x-3}{(2x-3)(x-5)} = 1$$
$$\frac{3x-8}{2x^2-10x-3x+15} = 1$$
$$2x^2 - 13x + 15 = 3x - 8$$
$$2x^2 - 16x + 23 = 0$$
$$\text{Discriminant (D)} = (-16)^2 - 4(2)(23)$$
$$= 256 - 184$$
$$= 72$$
$$x = \frac{-b \pm \sqrt{D}}{2a}$$
$$= \frac{16 \pm \sqrt{72}}{4}$$
$$= \frac{16 \pm 6\sqrt{2}}{4}$$
$$= \frac{8 \pm 3\sqrt{2}}{2}$$

36. Let present age of sister be x years

$\therefore$ age of girl = 2x years

According to question,
$$(x + 4)(2x + 4) = 160$$
$$2x^2 + 12x + 16 - 160 = 0$$
$$2x^2 + 12x - 144 = 0$$
$$x^2 + 6x - 72 = 0$$
$$x^2 + 12x - 6x - 72 = 0$$
$$x(x + 12) - 6(x + 12) = 0$$
$$(x - 6)(x + 12) = 0$$
$$x = 6, -12$$

As age cannot be negative,
$$x = -12 \text{ is rejected}$$
$$\therefore \quad x = 6$$

Age of sister = 6 years

Age of girl = 2x

= 2 (6)

= 12 years

37. Let number of articles be x

$\therefore$ Cost of production of each article = 2x + 3

According to question,
$$x(2x + 3) = 90$$
$$2x^2 + 3x - 90 = 0$$
$$2x^2 - 12x + 15x - 90 = 0$$
$$2x(x - 6) + 15(x - 6) = 0$$
$$(2x + 15)(x - 6) = 0$$
$$x = \frac{-15}{2} \quad \text{or} \quad x = 6$$

Being number of articles, x cannot be negative.

$\therefore$ $x = 6$

Number of articles = 6

Cost of production of each article

= 2x + 3

= 12 + 3

= $ 15

38. Let Shefali's marks in English be x..

$\therefore$ Shefali's marks in Mathematics = 30 – x

According to question,
$$(30 - x + 2)(x - 3) = 210$$

$$(32 - x)(x - 3) = 210$$
$$32x - 96 - x^2 + 3x = 210$$
$$x^2 - 35x + 306 = 0$$
$$x^2 - 17x - 18x + 306 = 0$$
$$x(x - 17) - 18(x - 17) = 0$$
$$(x - 17)(x - 18) = 0$$
$$x = 17 \quad \text{or} \quad x = 18$$

If $x = 17$

Shefali's marks in English = 17

Shefali's marks in Mathematics = 30 – 17 = 13

If $x = 18$

Shefali's marks in English = 18

Shefali's marks in Mathematics = 30 – 18

= 12

39. Let speed of train = x km/hr

Distance covered = 360 km

So, time taken = $\frac{360}{x}$

According to question,

$$\frac{360}{x+5} = \frac{360}{x} - 1$$
$$\frac{360}{x} - \frac{360}{x+5} = 1$$
$$\frac{x+5-x}{x(x+5)} = \frac{1}{360}$$
$$\frac{5}{x(x+5)} = \frac{1}{360}$$
$$x^2 + 5x - 1800 = 0$$
$$x^2 - 40x + 45x - 1800 = 0$$
$$x(x - 40) + 45(x - 40) = 0$$
$$(x - 40)(x + 45) = 0$$
$$x = 40, -45$$

Being speed of train, $x = -45$ is rejected.

∴ Speed of train = 40 km/hr

40. Let breadth of rectangular mango grove = x m

∴ Length = 2x

According to question,

$$2x(x) = 800$$
$$2x^2 = 800$$
$$x^2 = 400$$
$$x = \pm 20$$

Being a dimension, x cannot be negative.

∴ $x = 20$

So, Breadth = 20 m

Length = 2x = 40 m

Chapter 05 Arithmetic Progressions

MULTIPLE CHOICE QUESTIONS

1. (b) $a_n = 3n + 7$

$a_{n+1} = 3(n+1) + 7 = 3n + 10$

So, $d = a_{n+1} - a_n$

$= 3n + 10 - 3n - 7$

$= 3$

2. (c) $a = 1, a_n = 11$

$S_n = 36$

We know that $S_n = \frac{n}{2}(a + a_n)$

$36 = \frac{n}{2}(1 + 11)$

$n = \frac{36 \times 2}{12}$

$= 6$

3. (b) $S_n = 2n^2 + 5n$

$a_n = S_n - (S_{n-1})$

$= (2n^2 + 5n) - [2(n-1)^2 + 5(n-1)]$

$= 2n^2 + 5n - 2n^2 - 2 + 4n - 5n + 5$

$= 4n + 3$

4. (d) We can write reverse AP as

185, ..., 13, 9, 5

Such that $a = 185, d = -4$

So, $a_9 = 185 + (9-1)(-4)$

$= 185 - 32$

$= 153$

5. (a) 18, a, b, – 3 are in AP.

$\therefore \quad a - 18 = b - a = -3 - b$

$a - 18 = b - a$

$2a - b = 18 \quad$ (i)

$b - a = -3 - b$

$a - 2b = 3 \quad$ (ii)

Solving (i) and (ii), we get

$a = 11, b = 4$

So, $a + b = 11 + 4$

$= 15$

WORKSHEET 1

Section A

1. k + 9, 2k – 1 and 2k + 7 are in A.P. if

$(2k - 1) - (k + 9) = (2k + 7) - (2k - 1)$

$k - 10 = 8$

$k = 18$

2. $S_n = 3n^2 + 5n$

$S_{20} = 3(20)^2 + 5(20)$

$= 3(400) + 100$

$= 1200 + 100$

$= 1300$

3. Consider AP : 2, 4, 6, 8, ..., n

Here $a = 2, d = 2$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[4 + (n-1)^2]$

$= \frac{n}{2}[2n + 2)$

$= n(n + 1)$

4. A.P.: $-5, -\frac{5}{2}, 0, \frac{5}{2}, \ldots$.

$a_n = a + (n-1)d, d = -\frac{5}{2} + 5 = \frac{5}{2}$

$\therefore a_{25} = -5 + (25-1)\frac{5}{2}$

$= -5 + 24\left(\frac{5}{2}\right)$

$= -5 + 60$

$= 55$

5. $S_p = ap^2 + bp$

$a_p = S_p - S_{p-1}$

$= (ap^2 + bp) - [a(p-1)^2 + b(p-1)]$

$= ap^2 + bp - [ap^2 + a - 2ap + bp - b]$

$= ap^2 + bp - ap^2 - a + 2ap - bp + b$

$= 2ap - a + b$

$\therefore \quad a_{p+1} = 2a(p+1) - a + b$

$= 2ap + 2a - a + b$

$= 2ap + a + b$

So, $d = a_{p+1} - a_p$

$= 2ap + a + b - 2ap + a - b$

$= 2a$

6. $a_n = n^2 + 1$

$a_1 = 1 + 1 = 2$

$a_2 = 2^2 + 1 = 5$

7. $a_n = \frac{3n-2}{4n+5}$

$a_1 = \frac{3-2}{4+5} = \frac{1}{9}, \quad a_2 = \frac{6-2}{8+5} = \frac{4}{13}$

$a_3 = \frac{9-2}{17} = \frac{7}{17}$

So, sequence is $\frac{1}{9}, \frac{4}{13}, \frac{7}{17}, \ldots$

8. $a_n = 3n - 2$

$a_1 = 3 - 2 = 4$

$a_2 = 6 - 2 = 4$

$a_3 = 9 - 2 = 7$ so on

So, sequence is 1, 4, 7, ...

$a_2 - a_1 = 4 - 1 = 3$

$a_3 - a_2 = 7 - 4 = 3$

As difference between the terms is same, so, the given sequence is in A.P.

$a_{10} = 30 - 2 = 28$

9. A.P : 18, 16, 14, ...

$S_n = 0$

$\frac{n}{2}[2a + (n-1)d] = 0$

$\frac{n}{2}[36 + (n-1)(-2)] = 0$

$n(36 - 2n + 2) = 0$

$n(38 - 2n) = 0$

$n = \frac{38}{2} = 19$

10. $a_4 = 0 \Rightarrow a + 3d = 0 \Rightarrow a = -3d$

To prove : $a_{25} = 3a_{11}$

Consider $a_{25} = a + (25-1)d$

$= a + 24d$

$= -3d + 24d$

$= 21d$

$a_{11} = a + 10d$

$= -3d + 10d$

$= 7d$

So, $a_{25} = 3a_{11}$

11. A. P: 6, 13, 20, ..., 216

$a_n = a + (n-1)d$

$216 = 6 + (n-1)7$

$210 = 7(n-1)$

$30 = n - 1$

$n = 31$

So, 216 is 31^{st} term of an A.P.

So, 16^{th} term is the middle term

$a_{16} = 6 + (16 - 1)\, 7$
$= 6 + 7\,(15)$
$= 6 + 105$
$= 111$

12. Consider 9, 12, 15, 18, ...

$a_2 - a_1 = 12 - 9 = 3$
$a_3 - a_2 = 15 - 12 = 3$
$a_4 - a_3 = 18 - 15 = 3$

As difference between the terms is same,

So, the terms are in A.P.

$a_{16} = a + 15d$
$= 9 + 15\,(3)$
$= 9 + 45$
$= 54$

$a_n = a + (n - 1)\, d$
$= 9 + (n - 1)\, 3$
$= 9 + 3n - 3$
$= 3n + 6$

13. $S_5 + S_7 = 167$

$\frac{5}{2}\,[2a + (5 - 1)\, d] + \frac{7}{2}\,[2a + (7 - 1)\, d] = 167$

$\Rightarrow \quad 5a + \frac{5}{2}\,(4d) + 7a + \frac{7}{2}\,(6d) = 167$

$\Rightarrow \quad 5a + 10d + 7a + 21d = 167$

$\Rightarrow \quad 12a + 31d = 167 \qquad$ (i)

$S_{10} = 235$

$\frac{10}{2}\,[2a + (10 - 1)\, d] = 235$

$2a + 9d = 47 \qquad$ (ii)

Multiplying equation (ii) by 6 and subtracting (ii) from (i), we get

$(12a + 54d) - (12a + 31d)$
$= 282 - 167$

$23d = 115$

$d = \frac{115}{23} = 5$

From (ii), $2a + 9(5) = 47$

$2a + 45 = 47$

$a = 1$

So, AP is a, a + d, a + 2d, ...

i.e. 1, 6, 11, ...

14. A.P.: 5, 15, 25, ...

Let n^{th} term of an AP be 130 more than its 31^{st} term

i.e. $a_n = 130 + a_{31}$

$5 + (n - 1)\, 10 = 130 + 5 + (31 - 1)\, 10$

$5 + 10n - 10 = 135 + 300$

$10n = 435 + 5$

$10n = 440$

$n = 44$

So, 44^{th} term of an AP is 130 more than its 31^{st} term

15. $a_s + a_9 = 72$

$a + 4d + a + 8d = 72$

$2a + 12d = 72$

$a + 6d = 36 \qquad$ (i)

$a_7 + a_{12} = 97$

$a + 6d + a + 11d = 97$

$2a + 17d = 97 \qquad$ (ii)

On multiplying (i) by 2 and subtracting (ii) from (i), we get

$(2a + 17d) - (2a + 12d) = 97 - 72$

$5d = 25$

$d = 5$

From (i), $a = 36 - 6\,(5)$

$= 36 - 30 = 6$

$a = 6$

So, AP is a, a + d, a + 2d, ...

i.e. 6, 11, 16, ...

16. Consider AP : 7, 14, 21, ..., 497

$$a_n = a + (n-1)d$$
$$497 = 7 + (n-1)7$$
$$497 - 7 = 7(n-1)$$
$$70 = \frac{490}{7} = n - 1$$
$$n = 71$$

17. $S_7 = 49$

$$\frac{7}{2}[2a + 6d] = 49$$
$$2a + 6d = 14$$
$$a + 3d = 7 \quad \text{(i)}$$

Also, $S_{17} = 289$

$$\frac{17}{2}[2a + 16d] = 289$$
$$2a + 16d = 34$$
$$a + 8d = 17 \quad \text{(ii)}$$

On subtracting (ii) from (i), we get

$$(a + 3d) - (a + 8d) = 7 - 17$$
$$-5d = -10$$
$$d = 2$$

From (i), $a = 7 - 3d$

$$= 7 - 6$$
$$= 1$$

So, $S_n = \frac{n}{2}[2a + (n-1)d]$

$$= \frac{n}{2}[2 + (n-1)2]$$
$$= n[1 + (n-1)]$$
$$= n^2$$

18. Let Sn and S_n^1 be sum of a terms of two A.P.

$$\frac{S_n}{S_n^1} = \frac{7n+1}{4n+27}$$

$$\frac{\frac{n}{2}\left[2a+(n-1)d\right]}{\frac{n}{2}\left[2a^1+(n-1)d^1\right]} = \frac{7n+1}{4n+27}$$

$$\frac{2a+(n-1)d}{2a^1+(n-1)d^1} = \frac{7n+1}{4n+27}$$

$$\frac{a+\left(\frac{n-1}{2}\right)d}{a^1+\left(\frac{n-1}{2}\right)d^1} = \frac{7n+1}{4n+27}$$

$$\frac{a+(m-1)d}{a^1+(m-1)d^1} = \frac{7(2m-1)+1}{4(2m-1)+27}$$

$$\frac{a_m}{a_m^1} = \frac{14m-6}{8m+23}$$

19. Let the digits be a – d, a, a + d

$$a - d + a + a + d = 15$$
$$3a = 15$$
$$a = 5$$

Also, $100(a-d) + 10a + a + d$

$$= [100(a+d) + 10a + a - d] - 594$$

$\therefore \quad 100a - 100d + 11a + d$

$$= 100a + 100d + 11a - d - 594$$
$$0 = 200d - 2d - 594$$
$$198d = 594$$
$$d = 3$$

So, number $= 100(a+d)\,10a + a - d$

$$= 100(8) + 50 + 2$$
$$= 852$$

20. $a_p = q \Rightarrow a + (p-1)d = q \quad \text{(i)}$

$a_q = p \Rightarrow a + (q-1)d = p \quad \text{(ii)}$

On subtracting (ii) from (i), we get

$$(p-1)d - (q-1)d = q - p$$
$$d[p - 1 - q + 1] = q - p$$
$$d = \frac{q-p}{p-q} = -1$$

From (i), $a + (p-1)(-1) = q$

$$a = p - 1 + q$$

So, $a_n = a + (n-1)d$

$= (p - 1 + q) + (n - 1)(-1)$

$= p - 1 + q - n + 1$

$= p + q - n$

21. Here, $a2 - a1 = 19\frac{1}{4} - 20$

$= \frac{77}{4} - 20$

$= \frac{77 - 80}{4} = -\frac{3}{4}$

$a_3 - a_2 = 18\frac{1}{2} - 19\frac{1}{4}$

$= \frac{37}{2} - \frac{77}{4}$

$= \frac{74 - 77}{4}$

$= \frac{-3}{4}$

as $a_3 - a_2 = a_2 - a_1$

i.e._difference between the terms is same, so, the given sequence forms an A.P.

Here, $a = 20, d = \frac{-3}{4}$

$a_n < 0$

$a + (n - 1) d < 0$

$20 + (n - 1)\left(\frac{-3}{4}\right) < 0$

$\frac{-3}{4}(n - 1) < -20$

$n - 1 > -20\left(\frac{-4}{3}\right)$

$n - 1 > \frac{80}{3}$

$n > \frac{80}{3} + 1 = \frac{83}{3} = 27.67$

So, $n = 28$

So, a_{28} is the first negative term

22. For S_1, $a = 1$, $d = 1$

So, $S_1 = \frac{n}{2}[2 + (n - 1)(1)]$

$\frac{n}{2}(n + 1)$

For S_2, $a = 1$, $d = 2$

So, $S_2 = \frac{n}{2}[2 + (n - 1)(2)]$

$= n[1 + n - 1]$

$= n^2$

For S_3, $a = 1$, $d = 3$

$S3 = \frac{n}{2}[2 + (n - 1)3]$

$= \frac{n}{2}[3n - 1]$

Consider $S_1 + S_3 = \frac{n}{2}(n + 1) + \frac{n}{2}(3n - 1)$

$= \frac{n^2}{2} + \frac{n}{2} + \frac{3n^2}{2} - \frac{n}{2}$

$= \frac{4n^2}{2}$

$= 2n^2$

$= 2S_2$

23. A.P.: a, 7, b, 23, c

As the terms are in A.P.,

$7 - a = b - 7 = 23 - b = c - 23$

As $7 - a = b - 7$

$a + b = 14$ (i)

As $b - 7 = 23 - b$

$2b = 30$

$b = 15$

From (i), $a = 14 - b = 14 - 15$

$a = -1$

As $23 - b = c - 23$

$23 - 15 = c - 23$

$c = 31$

24. Let the four parts be

$a - 3d, a - d, a + d, a + 3d$ such that

$a - 3d + a - d + a + d + a + 3d = 32$

$4a = 32$

$a = 8$

Also, $\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{7}{15}$

i.e. $\frac{(8-3d)(8+3d)}{(8-d)(8+d)} = \frac{7}{15}$

$$\frac{64-9d^2}{64-d^2} = \frac{7}{15}$$

$$960 - 135d^2 = 448 - 7d^2$$

$$512 = 128\,d^2$$

$$d^2 = 4$$

$$d = \pm 2$$

For a = 8, d = 2

the four parts are

a – 3d, a – d, a + d, a + 3d

i.e. 8 – 6, 8 – 2, 8 + 2, 8 + 6

i.e. 2, 6, 10, 14

For a = 8, d = – 2

the four parts are

a – 3d, a – d, a + d, a + 3d

i.e. 8 + 6, 8 + 2, 8 – 2, 8 – 2, 8 – 6

i.e. 14, 10, 6 and 2

25. Let the policeman catches the thief in n minutes.

Uniform speed of thief = 100 m/min

As after are minute a policeman runs after the thief to catch him.

So, distance travelled by thief.

= 100 (n + 1) minutes

Given that speed of policeman increases by 10m/min.

speed of policeman forms an AP:

100 m/min, 110 m/min, 120 m/min, ...

So, distance travelled by policeman

$= S_n$

$= \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[200 + (n-1)10]$

$= n[100 + 5n - 5]$

$= n(95 + 5n)$

Distance travelled by thief

= Distance travelled by policeman

$$100(n+1) = n(95+5n)$$

$$100n + 100 = 95n + 5n^2$$

$$5n^2 - 5n - 100 = 0$$

$$n^2 - n - 20 = 0$$

$$n^2 - 5n + 4n - 20 = 0$$

$$n(n-5) + 4(n-5) = 0$$

$$(n+4)(n-5) = 0$$

$$n = -4 \text{ or } n = 5$$

As n cannot be negative, n = 5

26. Consider the sequence formed by all three digit numbers which leaves a remainder 3, when divided by 4: 103, 107, 111, 115, ..., 999.

The above sequence forms an A.P. with a = 103 and common difference d = 4

$$a_n = a + (n-1)d$$

$$999 = 103 + (n-1)4$$

$$4(n-1) = 999 - 103$$

$$4(n-1) = 896$$

$$n - 1 = 224$$

$$n = 225$$

The middle term is $\left(\frac{n+1}{2}\right)^{th}$ term

i.e. $\left(\frac{225+1}{2}\right) = 113^{th}$ term

$$a_{113} = 103 + (113-1)4$$

$$= 103 + 112(4)$$

$$= 103 + 448$$

$$= 551$$

Sum of all terms before middle term

$= S_{112}$

$= \frac{112}{2}$ [2 (103) + (112 – 1) 4]

= 56 [206 + 144]

= 56 (650)

= 36,400

$S_{225} = \frac{225}{2}$ [2 (103) + (225 – 1) 4]

$= \frac{225}{2}$ [206 + 896]

$= \frac{225}{2}(1102)$

= 123975

So, sum of terms after the middle term

$= 123975 - (S_{112} + 551)$

= 123975 – 36400 – 551

= 87024

27. Given: $S_m = S_n$

To prove: $S_{m+n} = 0$

$S_m = \frac{m}{2}[2a + (m-1)d]$

$S_n = \frac{n}{2}[2a + (n-1)d]$

As $S_m = S_n$

$\therefore \frac{m}{2}[2a + (m-1)d] = \frac{n}{2}[2a + (n-1)d]$

$m[2a + (m-1)d] = n[2a + (n-1)d]$

$2am + md(m-1)] = 2na + nd(n-1)$

$2am + m^2d - md = 2an + n^2d - nd$

$2am - 2an + m^2d - n^2d - md + nd = 0$

$2a(m-n) + d(m^2 - n^2) - d(m-n) = 0$

$(m-n)[2a + (m+n)d - d) = 0$

$(m-n)[2a + (m+n-1)d] = 0$

As $m \neq n$, $2a + (m+n-1)d = 0$

Consider

$S_{m+n} = \frac{m+n}{2}[2a + (m+n-1)d]$

$= \left(\frac{m+n}{2}\right)(0)$

$= 0$

So, sum of its (m + n) terms is zero.

28. AP = –12, –, –6, ... , 21

If 1 is added to each term,

A.P. becomes –12 + 1, –9 + 1, –6 + 1, ..., 21 + 1

i.e. – 11, –8, –5, ..., 22

We know that

$a_n = a + (n-1)d$

$22 = -11 + (n-1)(3)$

$\frac{33}{3} = n - 1$

$n = 12$

$S_{12} = \frac{12}{2}[2(-11) + (12-1)3]$

$= 6[-22 + 33]$

$= 6(11)$

$= 66$

29. Let the prizes be a, a – 20, a – 40, ...

$S_{10} = 1600$

$\frac{10}{2}[2a + (10-1)(-20)] = 1600$

$5(2a - 180) = 1600$

$2a - 180 = 320$

$2a = 500$

$a = 250$

So, the prize are 250, 230, 210, 190, 170, 150, 130, 110, 90.

30. First term = a

Second term = b

last term (a_n) = c

To prove: $S_n = \frac{(a+c)(b+c-2a)}{2(b-a)}$

Solution: $d = b - a$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

or

$$= \frac{n}{2}[a + a_n]$$

$$= \frac{n}{2}[a + c] \quad \text{(i)}$$

We know that $a_n = c$

i.e. $a + (n-1)(b-a) = c$

$$(n-1) = \frac{c-a}{b-a}$$

$$n = \frac{c-a}{b-a} + 1$$

$$= \frac{c+b-2a}{b-a} \quad \text{(ii)}$$

On putting (ii) in (i), we get

$$S_n = \frac{1}{2}(a+c)\left(\frac{c+b-2a}{b-a}\right)$$

$$= \frac{(a+c)(b+c-2a)}{2(b-a)}$$

31. Given that Raghav buys a shop for 120000.

He pays half of the amount in cash

$$= \frac{1}{2} \times 120000$$

$= 60000.$

Balance amount to be paid = 120000 – 60000

= 60000.

Given that amount of each installment = 5000.

He agrees to pay the balance in 12 annual installments with interest of 12%.

1. Amount of the first installment

$$= 5000 + \frac{12}{100} \times 60000$$

$= 5000 + 600 \times 12$

$= 5000 + 7200$

$= 12200.$

2. Amount of the second installment

$$= 5000 + \frac{12}{100} \times (60000 - 5000)$$

$$= 5000 + \frac{12}{100} \times 55000$$

$= 5000 + 550 \times 12$

$= 5000 + 6600$

$= 11600.$

So, the amount paid for installment is 12200, 11600.......It forms an AP.

The 1st term a = 12200

Common Difference d = 11600 – 12200

= –600.

The number of terms n = 12.

We know that sum of n terms

$$= \frac{n}{2}(2a + (n-1) \times d)$$

Therefore the total cost of the shop

$$= 60000 + \frac{12}{2}(2(12200) + (12-1) \times (-600))$$

$= 60000 + 6\ (24400 - 6600)$

$= 60000 + 6 \times 17800$

$= 60000 + 106800$

$= 166800.$

The total cost of the shop = 166800.

32. 4^{th} term = a + 3d

8^{th} term = a + 7d

Sum of the 4th term and 8th term

= a + 3d + a + 7d = 24.

$\Rightarrow 2a + 10d = 24$

Take 2 common from the equation......

a + 5d = 12................(1)

Sum of 6th term and 10^{th} term = 44

$\Rightarrow a + 5d + a + 9d = 44$

$2a + 14d = 44$

Take 2 common from equation....

a + 7d = 22.................(2)

By Elimination Method:-

$a + 5d = 12$

$a + 7d = 22$

$2d = 10$

$d = 5$

Substitute d = 5 in eq (1)

$a + 5d = 12$

$a + 5(5) = 12$

$a = 12 - 25$

$a = -13$

- The 1st term 3 term are –13, –13 + 5, –13 + 10
- –13, –8, –3.

WORKSHEET 2

Section A

1. Consider an AP: 12, 18, 24, ..., 96

$$a_n = a + (n-1)\,d$$
$$96 = 12 + (n-1)\,6$$
$$96 - 12 = 6\,(n-1)$$
$$n - 1 = \frac{84}{6}$$
$$n - 1 = 14$$
$$n = 15$$

2. $S_q = 2q + 3q^2$

$$S_{q-1} = 2\,(q-1) + 3\,(q-1)^2$$
$$= 2q - 2 + 3q^2 + 3 - 6q$$
$$= 3q^2 - 4q + 1$$
$$a_q = s_q - s_{q-1}$$
$$= 2q + 3q^2 - 3q^2 + 4q - 1$$
$$= 6q - 1$$
$$a_{q+1} = 6q + 6 - 1 = 6q + 5$$
$$\therefore\ d = a_{q+1} - a_q = 6q + 5 - 6q + 1$$
$$= 6$$

3. Consider AP: 1, 3, 5, 7, ..., n

with a = 1, d = 2

$$Sn = \frac{n}{2}\,[2a + (n-1)\,d]$$
$$= \frac{n}{2}\,[2 + (n-1)\,2]$$
$$= n\,[1 + n - 1)$$
$$= n^2$$

4. As terms are in AP,

$$13 - (2p + 1) = (5p - 3) - 13$$
$$13 - 2p - 1 = 5p - 3 - 13$$
$$12 + 16 = 7p$$
$$7p = 28$$
$$p = 4$$

5. First term = a

Second term = b

Last term (a_n) = 2a

Common difference (d) = b – a

$$S_n = \frac{n}{2}\,[2a + (n - 12d] \text{ or } \frac{n}{2}\,[a + a_n]$$
$$S_n = \frac{n}{2}\,[a + 2a]$$
$$= \frac{3a}{2}\,n \qquad \text{(i)}$$

As $a_n = 2a$

$$a + (n-1)\,d = 2a$$
$$a + (n-1)\,(b-a) = 2a$$
$$(n-1)\,(b-a) = a$$
$$n - 1 = \frac{a}{b-a}$$
$$n = \frac{a}{b-a} + 1$$
$$n = \frac{b}{b-a} \qquad \text{(ii)}$$

On putting (ii) in (i), we get

$$S_n = \frac{3a}{2}\,\frac{b}{(b-a)}$$
$$= \frac{3ab}{2(b-a)}$$

6. $\frac{1}{m}, \frac{1+m}{m}, \frac{1+2m}{m}, ...$

Here, $a = \frac{1}{m}$

$$d = \frac{1+m}{m} - \frac{1}{m} = \frac{1}{m} + 1 - \frac{1}{m} = 1$$

$$a_n = a + (n-1)d$$

$$= \frac{1}{m} + (n-1)1$$

$$= \frac{1}{m} + n - 1$$

7. Let 184 be the n^{th} term of

$$AP = 3, 7, 11, ... \text{ with}$$

$$a = 3$$

$$d = 7 - 3 = 4$$

$$a_n = a + (n-1)d$$

$$184 = 3 + (n-1)4$$

$$184 - 3 = 4(n-1)$$

$$n - 1 = \frac{181}{4}$$

$n = \frac{185}{4}$ which is not a natural number.

So, 184 is not a term of AP: 3, 7, 11, ...

8. Consider AP: 254, ..., 14, 9, 4

with $a = 254$

$$d = 9 - 14 = -5$$

So, $a_{10} = 254 + (10-1)(-5)$

$$= 254 - 45$$

$$= 209$$

9. $a_1 = 4$

$$a_n = 4a_{n-1} + 3, n > 1$$

$$a_2 = 4a_1 + 3 = 16 + 3 = 19$$

$$a_3 = 4a_2 + 3 = 4(19) + 3 = 79$$

$$a_4 = 4a_3 + 3 = 4(79) + 3 = 319$$

$$a_5 = 4a_4 + 3 = 4(319) + 3 = 1279$$

$$a_6 = 4a_5 + 3 = 4(1279) + 3$$

$$= 5119.$$

10. $a_n = 4n + 5$

$$a_1 = 4 + 5 = 9$$

$$a_2 = 4(2) + 5 = 13$$

$$a_3 = 4(3) + 5 = 17$$

$$a_4 = 4(4) + 5 = 21$$

$$a_2 - a_1 = 13 - 9 = 4$$

$$a_3 - a_2 = 17 - 13 = 4$$

$$a_4 - a_3 = 21 - 17 = 4$$

As difference between the terms is same, the sequence defined by $a_n = 4n + 5$ is an A.P. such that d = 4.

11. A.P : 27, 24, 21, ...

Let sum of n terms of the A.P. be 0.

Here, first term (a) = 27

Common difference (d) = 24 – 27

= –3

$$S_n = 0$$

$$\frac{n}{2}[2a + (n-1)d] = 0$$

$$\frac{n}{2}[54 + (n-1)(-3)] = 0$$

$$n(54 - 3n + 3) = 0$$

$$n(18 - n + 1) = 0$$

$$18 - n + 1 = 0$$

$$n = 19$$

So, sum of 19 terms is 0.

12. $\frac{S_m}{S_n} = \frac{m^2}{n^2}$

To prove : $\frac{a_m}{a_n} = \frac{2m-1}{2n-1}$

As $\frac{S_m}{S_n} = \frac{m^2}{n^2}$

$$\therefore \frac{\frac{m}{2}\left[2a + (m-1)d\right]}{\frac{n}{2}\left[2a + (n-1)d\right]} = \frac{m^2}{n^2}$$

$$\frac{2a+(m-1)d}{2a+(n-1)d} = \frac{m}{n}$$

$$\frac{a+\frac{(m-1)}{2}d}{a+\frac{(n-1)}{2}d} = \frac{m}{n}$$

On replacing m by 2m – 1 and n by 2n – 1 on both sides of equation, we get

$$\frac{a+(m-1)d}{a+(n-1)d} = \frac{2m-1}{2n-1}$$

13. $S_n = \frac{3n^2}{2} + \frac{5n}{2}$

We know that $a_n = S_n - S_{n-1}$

So, $a_{25} = S_{25} - S_{24}$

$$= \left[\frac{3}{2}(625+\frac{5}{2}(25)\right] - \left[\frac{3}{2}(576)+\frac{5}{2}(24)\right]$$

$$= \frac{1875}{2} + \frac{125}{2} - \frac{1728}{2} - \frac{120}{2}$$

$$= \frac{1875+125-1728-120}{2}$$

= 76

14. We know

$M = \frac{(n+1)}{2}$ th observation for n = odd

Therefore the 6th term of this AP is 30

Therefore A6 = 30

a + 5d = 30

Therefore we need to find S11

Therefore S11 = $\frac{n}{2}$ (2a + (n – 1)d)

= $\frac{11}{2}$ (2a + 10d)

= $\frac{11}{2}$ (2(a + 5d)

Replacing value

= $\frac{11}{2}$ (2(30))

= 11 × 30

= 330

15. A = – 7

d = 5

T18 = a + 17d

= – 7 + 17 × 5

= – 7 + 85

= 78

General term = a + (n – 1)d

= – 7 + (n – 1) 5

= – 7 + 5n – 5

= 5n – 12

16. $a_{10} = 52$

∴ a + 9d = 52 (i)

$a_{17} = 20 + a_{13}$

a + 16d = 20 + a + 12d

4d = 20

d = 5

From (i), a + 9 (5) = 52

a + 45 = 52

a = 7

So, AP is a, a + d, a + 2d, ...

i.e. 7, 12, 17, ...

17. $a_9 = -32$

a + 8d = –32 (i)

Also, $a_{11} + a_{13} = -94$

a + 10d + a + 12d = –94

2a + 22d = –94

a + 11d = –47 (ii)

On subtracting (i) from (ii), we get

a + 11d – a – 8d = –47 + 32

3d = –15

d = –5

From (i), a = –32 – 8d

$= -32 - 8(-5)$

$= -32 + 40$

$= 8$

18. $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{30} = \frac{30}{2}[2a + 29d] \Rightarrow S_{30} = 30a + 435d$.(i)

$S_{20} = \frac{20}{2}[2a + 19d] \Rightarrow S_{20} = 20a + 190d$

$S_{10} = \frac{10}{2}[2a + 9d] \Rightarrow S_{10} = 10a + 45d$

$3(S_{20} - S_{10}) = 3[20a + 190d - 10a - 45d)$

$= 3[10a + 145d]$

$= 30a + 435d = S_{30}$ [From (i)]

Hence, $S_{30} = 3(S_{20} - S_{10})$ Hence proved.

19. $a_{14} = 2a_8$

$a + 13d = 2[a + 7d]$

$a + 13d = 2a + 14d$

$-d = a$

$a_6 = -8$

$a + 5d = -8$

$-d + 5d = -8$ (As $a = -d$)

$4d = -8$

$d = -2$

So, $a = -d = 2$

We know that $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore S_{20} = \frac{20}{2}[2(2) + (20-1)(-2)]$

$= 10[4 - 38]$

$= 10(-34)$

$= -340$

20. First term (a) $= 7$

Last term (a_n) $= 49$

$S_n = 420$

We know that $S_n = \frac{n}{2}[a + a_n]$

$420 = \frac{n}{2}[7 + 49]$

$n = \frac{420}{28} = 15$

Now, $a_n = 49$

$a + (n-1)d = 49$

$7 + (15-1)d = 49$

$14d = 42$

$d = 3.$

21. $a_2 + a_7 = 30$

$a + d + a + 6d = 30$

$2a + 7d = 30$ (i)

Also, $a_{15} = 2a_8 - 1$

$a + 14d = 2[a + 7d] - 1$

$a + 14d = 2a + 14d - 1$

$0 = a - 1$

$a = 1$

From (i), $2(1) + 7d = 30$

$7d = 28$

$d = 4$

So, A.P. is a, a + d, a + 2d, ...

i.e. 1, 5, 9, ...

22. AP : $18, 15\frac{1}{2}, 13, ..., -49\frac{1}{2}$

i.e. $18, \frac{31}{2}, 13, ..., -\frac{99}{2}$

Here, first term (a) = 18

Common difference (d) $= \frac{31}{2} - 18$

$= \frac{31-36}{2} = -\frac{5}{2}$

Last term (a_n) $= -\frac{99}{2}$

$a + (n-1)d = -\frac{99}{2}$

$$18 + (n-1)\left(-\frac{5}{2}\right) = -\frac{99}{2}$$

$$-\frac{5}{2}(n-1) = -\frac{99}{2} - 18$$

$$-\frac{5}{2}(n-1) = \frac{-99-36}{2}$$

$$-\frac{5}{2}(n-1) = -\frac{135}{2}$$

$$n - 1 = -\frac{135}{2} \times \frac{2}{-5} = 27$$

$$n = 28$$

So, number of terms (n) = 28

We know that $S_n = \frac{n}{2}[2a + (n-1)d]$

$$S_{28} = \frac{28}{2}\left[36 + (28-1)\left(\frac{-5}{2}\right)\right]$$

$$= 14\left[36 - \frac{135}{2}\right]$$

$$= \left(\frac{72-135}{2}\right)$$

$$= 7(-63)$$

$$= -441$$

23. $a_n = -4n + 15$

$a_1 = -4 + 15 = 11$

$a_2 = -4(2) + 15 = -8 + 15 = 7$

$a_3 = -12 + 15 = 3.$

So, First term (a) = 11

Common difference (d) = 7 – 11 = – 4

We know that

$$S_n = \frac{n}{2}[2a + (n - 11d]$$

$$S_{20} = \frac{20}{2}[2(11) + (20-1)(-4)]$$

$$= 10[22 - 76]$$

$$= 10(-54)$$

$$= -540$$

24. $a_8 = 31$

$a + 7d = 31$ (i)

$a_{15} = a_{11} + 16$

$a + 14d = a + 10d + 16$

$4d = 16$

$d = 4$ (ii)

From (i), $a + 28 = 31$

$a = 3$

So, A.P. is a, a + d, a + 2d, ...

i.e. 3, 7, 11, ...

25. $a_{15} = 3 + 2a_7$

$a + 14d = 3 + 2(a + 6d)$

$a + 14d = 3 + 2a + 12d$

$0 = a - 2d + 3$ (i)

Also, $a_{10} = 41$

$a + 9d = 41$ (ii)

On subtracting (i) from (ii), we get

$a + 9d - a + 2d = 41 + 3$

$11d = 44$

$d = 4$

From (ii), $a + 9(4) = 41$

$a = 41 - 36$

$= 5$

We know that $a_n = a + (n-1)d$

$= 5 + (n-1)4$

$= 4n + 1$

26. Consider an AP = 504, 511, 518,...., 896

Here, first term (a) = 504

Common difference (d) = 511 – 504 = 7

Last term (a_n) = 896

As $a_n = 896$

$a + (n-1)d = 896$

$$504 + (n-1)7 = 896$$
$$7(n-1) = 392$$
$$n-1 = 56$$
$$n = 57$$

We know that $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_{57} = \frac{57}{2}[2(504 + (57-1)7]$

$= \frac{57}{2}[1008 + 392]$

$= 39900$

27. First term (a) = 5

Let d be the common difference

$S_4 = \frac{1}{2}[S_8 - S_4]$

i.e. $\frac{4}{2}[2a + (4-1)d]$

$= \frac{1}{2}\left\{\frac{8}{2}\left[2a + (8-1)d\right] - \frac{4}{2}\left[2a + (4-1)d\right]\right\}$

i.e. $2(2a + 3d) = 2(2a + 7d) - (2a + 3d)$

$4a + 6d = 4a + 14d - 2a - 3d$

$4a + 6d = 2a + 11d$

$2a = 5d$

$d = \frac{2a}{5} = \frac{2}{5}(5) = 2$

So, common difference (d) = 2

28. A.P: 3, 9, 15, ..., 99

Here, first term (a) = 3

Common difference (d) = 9 – 3

= 6

Last term (a_n) = 99

$a + (n-1)d = 99$

$3 + (n-1)6 = 99$

$6(n-1) = 96$

$n-1 = 16$

$n = 17$

We know that $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{17} = \frac{17}{2}[6 + (17-1)6]$

$= \frac{17}{2}[6 + 96]$

$= 867$

29. First term (a) = 8

Last term (a_n) = 350

Common difference (d) = 9

As $a_n = 350$

$a + (n-1)d = 350$

$8 + (n-1)9 = 350$

$9(n-1) = 342$

$n-1 = \frac{342}{9} = \frac{114}{3} = 38$

$n-1 = 38$

$n = 39$

We know that

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{39} = \frac{39}{2}[16 + (39-1)9]$

$= \frac{39}{2}[16 + 342]$

$= 6981$

30. Let the first term of an AP be 'a' and common difference be 'd'.

$S_{10} = -150$

$\frac{11}{2}[2a + (10-1)d] = -150$

$5(2a + 9d) = -150$

$2a + 9d = -30 \quad$ (i)

Also, $S_{20} - S_{10} = -550$

$\frac{20}{2}(2a + 19d) - \frac{10}{2}(2a + 9d) = -550$

$10(2a + 19d) - 5(2a + 9d) = -550$

$$2(2a + 19d) - (2a + 9d) = -110$$
$$4a + 38d - 2a - 9d = -110$$
$$2a + 29d = -110 \qquad \text{(ii)}$$

On subtracting (ii) from (i), we get

$$2a + 9d - 2a - 29d = -30 + 110$$
$$-20d = 80$$
$$d = -4$$

From (i), $2a + 9(-4) = -30$

$$2a = -30 + 36$$
$$2a = 6$$
$$a = 3$$

So, A.P. is a, a + d, a + 2d, ...

i.e. 3, 3 – 4, 3 – 8, ...

i.e. 3, –1, –5, ...

Section D

31. Let A, D be first term and common difference respectively.

$$S_p = a \Rightarrow \frac{p}{2}[2A + (p-1)D] = a$$

$$S_q = b \Rightarrow \frac{q}{2}[2A + (q-1)D] = b$$

$$S_r = c \Rightarrow \frac{r}{2}[2A + (r-1)D] = c$$

Consider

$$\frac{a}{p}(q-r) + \frac{b}{q}(r-p) + \frac{c}{r}(p-q)$$

$$= \frac{1}{p}\frac{p}{2}[2A + (p-1)D](q-r) + \frac{1}{q}\frac{q}{2}[2A + (q-1)D)(r-p) + \frac{1}{r}\frac{r}{2}[2A + (r-1)D](p-q)$$

$$= \frac{1}{2}[2A + (p-1)D](q-r) + \frac{1}{2}[2A + (q-1)D](r-p) + \frac{1}{2}[2A + (r-1)D](p-q)$$

$$= [A(q-r) + A(r-p) + A(p-q)] + \frac{D}{2}[(p-1)(q-r) + (q-1)(r-p) + (r-1)(p-q)]$$

$$= A(q - r + r - p + p - r) + \frac{D}{2}[(p-1)(q-r) + (q-1)(r-p) + (r-1)(p-q)]$$

$$= 0 + \frac{D}{2}[pq - pr - q + r + qr - qp - r + p + rp - rq - p + q]$$

$$= 0 + 0$$

$$= 0$$

32. Let a and d be the first term and common term of an A.P.

$$a_m = \frac{1}{n}$$

$$a + (m-1)d = \frac{1}{n} \qquad \text{(i)}$$

Also, $a_n = \frac{1}{m}$

$$a + (n-1)d = \frac{1}{m} \qquad \text{(ii)}$$

On subtracting (i) from (ii), we get

$$a + (n-1)d - a - (m-1)d = \frac{1}{m} - \frac{1}{n}$$

$$d(n - 1 - m + 1) = \frac{n-m}{mn}$$

$$d(n-m) = \frac{n-m}{mn}$$

$$d = \frac{1}{mn}$$

From (i), $a + (m-1)\frac{1}{mn} = \frac{1}{n}$

$$a + (m-1)\frac{1}{mn} = \frac{1}{n}$$

$$a + \frac{1}{n} - \frac{1}{mn} = \frac{1}{n}$$

$$a = \frac{1}{mn}$$

Consider

$$S_{mn} = \frac{mn}{2}\left[\frac{2}{mn} + (mn-1)\frac{1}{mn}\right]$$

$$= \frac{mn}{2}\left[\frac{2}{mn}+1-\frac{1}{mn}\right]$$

$$= \frac{mn}{2}\left[\frac{1}{mn}+1\right]$$

$$= \frac{mn}{2}\left[\frac{mn+1}{mn}\right]$$

$$= \frac{1}{2}(mn + 1)$$

33. Length of each step = 50 m

Width of each step $= \frac{1}{2}$ m

Height of first step $= \frac{1}{4}$ m

Height of second step $= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$ m

Height of third step $= \frac{3}{4}$ m so on.

Volume of concrete required to build the first step $(V_1) = 50 \times \frac{1}{2} \times \frac{1}{4}$ m^3

Volume of concrete required to build the second step $(V_2) = 50 \times \frac{1}{2} \times \left(2\times\frac{1}{4}\right)$

$(V_3) = 50 \times \frac{1}{2} \times \frac{3}{4}$ m^3 and so on.

Total volume of concrete

$$= V_1 + V_2 + V_3 + \dots + V_{15}$$

$$= \left(50\times\frac{1}{2}\times\frac{1}{4}\right)+\left[50\times\frac{1}{2}\times\left(2\times\frac{1}{4}\right)\right]$$

$$+\left(50\times\frac{1}{2}\times3\times\frac{1}{4}\right)+\dots+\left[50\times\frac{1}{2}\times\left(15\times\frac{1}{4}\right)\right]$$

$$= \left(50\times\frac{1}{2}\right)\left[\frac{1}{4}+\frac{2}{4}+\frac{3}{4}+\dots+\frac{15}{4}\right]$$

$$= 25\left[\frac{1}{4}+\frac{2}{4}+\frac{3}{4}+\dots+\frac{15}{4}\right]m^3$$

$$= \frac{25}{4}(1 + 2 + \dots + 15)\ m^3$$

$$= \frac{25}{4} \times \frac{15}{2}(1 + 15) = 750\ m^3$$

34. Let the first term and common difference of an A.P. be a and d respectively.
Let S and S^1 be the sum of odd terms and even terms of A.P.

$$S = a_1 + a_3 + a_5 + \dots + a_{2n+1}$$

$$= \frac{n+1}{2}(a_1 + a_{2n+1})$$

$$= \frac{n+1}{2}[a + a + (2n + 1 - 1) d]$$

$$= (n + 1)(a + nd)$$

$$S^1 = a_2 + a_4 + a_6 + \dots + a_{2n}$$

$$S^1 = \frac{n}{2}[2a + 2nd]$$

$$= n(a + nd)$$

Consider $\frac{S}{S^1} = \frac{(n+1)(a+nd)}{n(a+nd)}$

$$= \frac{n+1}{n}$$

35. Consider 1, 2, 3, ..., 999, 1000

This sequence forms an AP with first term (a) = 1 and common difference (d) = 1

We know that

$$S_n = \frac{n}{2}[2a + (n - 1) d]$$

$$S_{1000} = \frac{1000}{2}[2 + (1000 - 1) 1]$$

$$= 500 (2 + 999)$$

$$= 500 (1001)$$

$$= 500500$$

Now consider list of numbers divisible by 2: 2, 4, 6, 8, ..., 1000

This sequence also forms an AP with a = 2, d = 2, $n = \frac{1000}{2} = 500$

$$S_{500} = \frac{500}{2}[2 (2) + (500 - 1) 2]$$

$$= 250 (4 + 499 (2)]$$

$$= 250500$$

Again, consider list of numbers divisible by 5: 5, 10, 15, ..., 1000

Here, $a = 5, d = 5, n = \frac{1000}{5} = 200$

$$S_{200} = \frac{200}{2}[10 + (200 - 1) 5]$$
$$= 100 [10 + 5 (199)]$$
$$= 100500$$

Now, we will consider list of numbers divisible by both 2 and 5 i.e. 2 × 5 = 10

10, 20, 30, ..., 1000

This list of numbers form an AP with

$a = 10, d = 10, n = \frac{1000}{10} = 100$

$$S_{100} = \frac{100}{2}[20 + (100 - 1) 10]$$
$$= 50 (20 + 990)$$
$$= 50500$$

Therefore, sum of numbers which are either divisible by 2 or 5

$$= S_{200} + S_{500} - S_{100}$$
$$= 100500 + 250500 - 50500$$
$$= 300500$$

So, sum of numbers from 1 to 1000 that are neither divisible by 2 nor by 5

2 nor by S $= S_{1000} - 300500$
$$= 500500 - 300500$$
$$= 200000$$

36. Suppose the work is completed in n days

Consider an AP: 150, 146, 142, ...

Here, First term (a) = 150

Common difference (d) = – 4

Total number of workers who worked all the n days. = S_n

$$= \frac{n}{2}[2 (150) + (n - 1) (-4)]$$
$$= \frac{n}{2}(300 - 4n + 4)$$
$$= \frac{n}{2}[304 - 4n]$$
$$= n (152 - 2n)$$

If the workers did not drop,

work would have been finished in (n – 8) days such that 150 workers work on each day.

∴ Total number of workers who worked all the n days = 150 (n – 8)

∴
$$n(152 - 2n) = 150 (n - 8)$$
$$152 n - 2n^2 = 150n - 1200$$
$$152n - 150n = 2n^2 - 1200$$
$$2n^2 - 2n - 1200 = 0$$
$$n^2 - n - 600 = 0$$
$$n^2 - 25n + 24n - 600 = 0$$
$$n (n - 25) + 24 (n - 25) = 0$$
$$(n + 24) (n - 25) = 0$$
$$n = -24, n = 25$$

Being the number of days, n cannot be negative, so, n = 25

∴ Work would be completed in 25 days

37. Consider the sequence: 200, 250, 300, ...

This sequence form an AP with first term (a) = 200 and common difference (d) = 50

We know that

$$S_n = \frac{n}{2}[2a + (n - 1) d]$$
$$S_{30} = \frac{30}{2}[2 (200) + (30 - 1) 50]$$
$$= 15 [400 + 1450]$$
$$= 27,750$$

∴ The contractor has to pay $ 27,750 as penalty, if he has delayed the work by 30 days.

38. Consider AP: 20, 19, 18, ...

Here, First term (a) = 20

Common difference (d) = – 1

Let 200 logs be placed in n rows

$\therefore \; S_n = 200$

$$\frac{n}{2}[2(20) + (17-1)(-1)] = 200$$

$$\frac{n}{2}[40 - n + 1] = 200$$

$$n(41 - n) = 400$$

$$-n^2 + 41n - 400 = 0$$

$$n^2 - 41n + 400 = 0$$

$$n^2 - 16n - 25n + 400 = 0$$

$$n(n-16) - 25(n-16) = 0$$

$$(n-16)(n-25) = 0$$

$$n = 16 \text{ or } 25$$

If $n = 25$,

$a_{25} = 20 + (25-1)(-1)$

$= 20 - 24$

$= -4$ not possible

So, $n = 16$

So, 200 logs are placed in 16 rows.

$a_{16} = 20 + (16-1)(-1)$

$= 20 - 15 = 5$

So, there are 5 logs in the top row.

39. Given : a^2, b^2, c^2 are in A.P.

To prove : $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P.

$\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in AP

if $$\frac{1}{c+a} - \frac{1}{b+c} = \frac{1}{a+b} - \frac{1}{c+a}$$

i.e. $$\frac{(b+c)-(c+a)}{(b+c)(a+c)} = \frac{(a+c)-(a+b)}{(a+b)(a+c)}$$

i.e. $$\frac{b+c-c-a}{(b+c)(a+c)} = \frac{a+c-a-b}{(a+b)(a+c)}$$

i.e. $$\frac{b-a}{(b+c)(a+c)} = \frac{c-b}{(a+b)(a+c)}$$

i.e. $$\frac{b-a}{b+c} = \frac{c-b}{a+b}$$

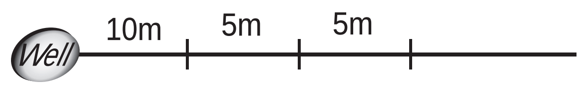
i.e. $(b-a)(a+b) = (c-b)(b+c)$

i.e. $ab + b^2 - a^2 - ab = bc + c^2 - b^2 - bc$

i.e. $b^2 - a^2 = c^2 - b^2$

$\therefore \; a^2, b^2, c^2$ are in A.P.

40.



Distance covered by gardener to water 1st tree and return to the initial position

$= 10 \text{ m} + 10 \text{ m} = 20 \text{ m}$

Distance covered by gardener to water 2nd tree and return to initial position

$= 15 \text{ m} + 15 \text{ m} = 30 \text{ m}$

Distance covered by gardener to water 3rd tree and return to initial position.

$= 20 \text{ m} + 20 \text{ m} = 40 \text{ m}$

So, we get an AP: 20, 30, 40, ...

With first term (a) = 20

difference (d) = 10

Total distance covered by the gardener

$= S_{25}$

$= \frac{25}{2}[2(20) + (25-1)10]$

$= \frac{25}{2}[40 + 240]$

$= \frac{25}{2} \times 280$

$= 25 \times 140$

$= 3500 \text{ m}$

$\therefore$ Total distance covered by the gardener to water all trees = 3500 m

Chapter 06

Triangles

MULTIPLE CHOICE QUESTIONS

1. Δ ABC ~ Δ PQR

$$\therefore \frac{ar\Delta ABC}{ar\Delta PQR} = \left(\frac{BC}{QR}\right)^2$$

$$\Rightarrow \quad \frac{9}{16} = \frac{(4.5)^2}{(QR)^2}$$

$$\Rightarrow \quad (QR)^2 = \frac{4.5\times 4.5\times 16}{9}$$

$$\Rightarrow \quad QR = \frac{4.5\times 4}{3}$$

$= 1.5 \times 4$

$= 6$ cm

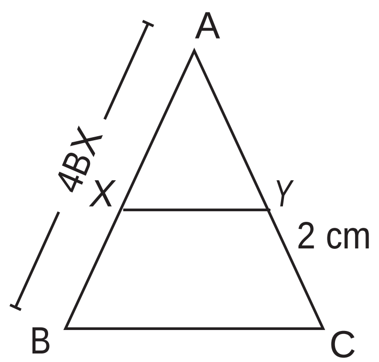
2. We know that ratio of area of two similar triangles is equal to square of ratio of their Corresponding sides (say x and y)

$$\Rightarrow \frac{9}{16} = \left(\frac{x}{y}\right)^2$$

$$\Rightarrow \frac{x}{y} = \frac{3}{4}$$

Option (a)

3.



AS $XY \parallel BC$, so by basic proportionality theorem

$$\frac{AX}{BX} = \frac{AY}{YC}$$

$$\frac{AX}{BX} + 1 = \frac{AY}{YC} + 1$$

$$\frac{AB}{BX} = \frac{AC}{CY}$$

$$\frac{4BX}{BX} = \frac{AC}{2} \quad (\because AB = 4BX)$$

$$4 = \frac{AC}{2} 25$$

AC = 8 cm

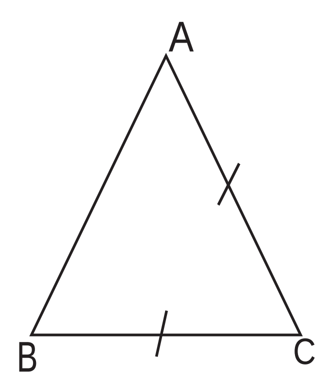
So, AY = AC – CY

= 8 – 2

= 6 cm

Option (d)

4.



$AB^2 = 2\,AC^2$

$= AC^2 + AC^2$

$= AC^2 + BC^2 +$

[$\therefore$ AC = BC]

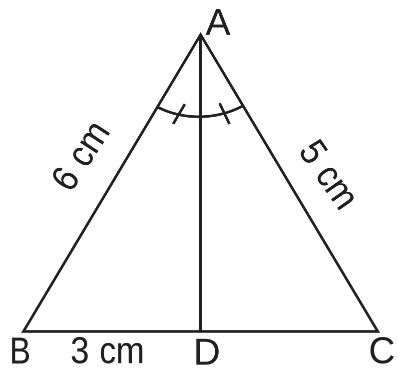
$\therefore$ $AB^2 = Ac^2 + BC^2$

$\therefore$ ΔABC is a triangle right angled at C

i.e. $\angle C = 90^o$

Option (c)

5.



AS AD bisects $\angle$BAC

$\therefore \quad \frac{AB}{BD} = \frac{AC}{CD}$

[By internal angle bisector theorem]

$\Rightarrow \quad \frac{6}{3} = \frac{5}{CD}$

$\Rightarrow \quad CD = \frac{3\times5}{6} = 2.5$ cm

Option (b)

WORKSHEET 1

Section A

1. Δ ABC ~ Δ DEF

In Δ ABC, $\angle A + \angle B + \angle C = 180^o$

$57^o + \angle B\ 73^o = 180^o$

$\angle B + 130^o = 180^o$

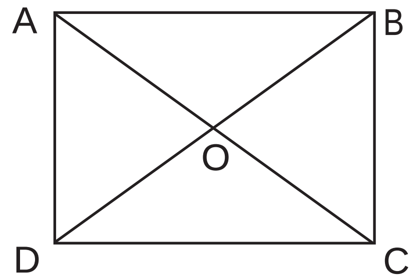
$\angle B = 108^o - 130^o$

$= 50^o$

$\therefore$ $\angle E = \angle B = 50^o$

[Corresponding angles of similar triangles are equal.]

2.



AC = 30 cm

BD = 40 cm

$OA = OC = \frac{1}{2} AC = 15$ cm

$OB = OD = \frac{1}{2} BD = 20$ cm

In Δ AOB, $\angle AOB = 90^O$

(Diagonals of rhombus bisect each other at 90^o)

$AB^2 = AO^2 + OB^2$ (Pythagoras theorem)

$= (15^2) + (20^2)$

$= 225 + 400$

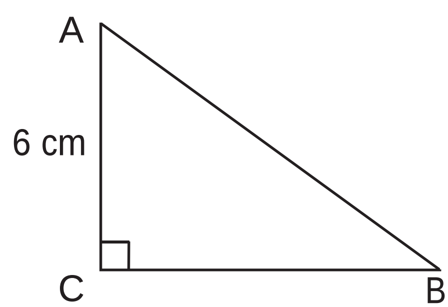
$= 625$

AB = 25 cm

$\therefore$ AB = BC = CA = AD = 25 cm

(All sides of rhombus are equal)

3.



In Δ ABC,

AC = BC = 6 cm (AS Δ ABC is isosceles)

Also, $\angle C = 90^o$

$\therefore$ $AB^2 = AC^2 + BC^2$ (Pythagoras theorem)

$= 6^2 + 6^2$

$= 36 + 36$

$AB^2 = 72$

$AB = 6\sqrt{2}$ cm

4. As, $\Delta DEF \sim \Delta ABC$

$$\frac{DE}{AB} = \frac{EF}{BC} = \frac{DF}{AC}$$

$$\frac{DE}{3} = \frac{4}{2} = \frac{DF}{2.5}$$

$$\frac{DE}{3} = \frac{4}{2}$$

$$DE = \frac{12}{2}$$

= 6 cm

$$\frac{4}{2} = \frac{DF}{2.5}$$

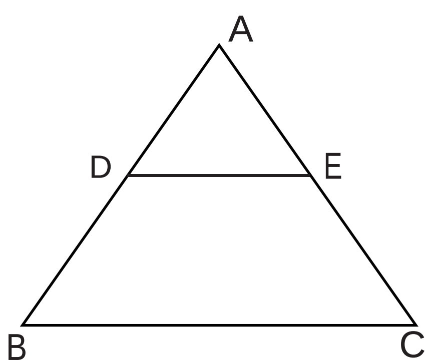
$$DF = \frac{4 \times 2.5}{2}$$

= 5 cm

Perimeter of Δ DEF = DF + EF + DF

= 6 + 4 + 5 = 15 cm

5.



Let AE = x cm

$\therefore$ CE = AC – AE = 5.6 – x cm

As DE II BC,

$$\frac{AD}{DB} = \frac{AE}{CE}$$

(By Basic proportionality theorem)

$$\frac{3}{5} = \frac{x}{5.6 - x}$$

5x = 3 (5.6 -x)

5x = 16.8 - 3x

8x = 16.8

x = 2.1 cm

$\therefore$ AE = x =2.1 cm

6. We know that ratio of the areas of two similar triangles is equal to the square of their altitudes.

$\therefore$ Ratio of areas = $\left(\frac{2}{3}\right)^2$ = 4.9

7. Given: abc~def

Find: Area of def

$$\frac{\text{Area of def}}{\text{Area of abc}} = \frac{\text{ef}}{\text{Bc}}$$

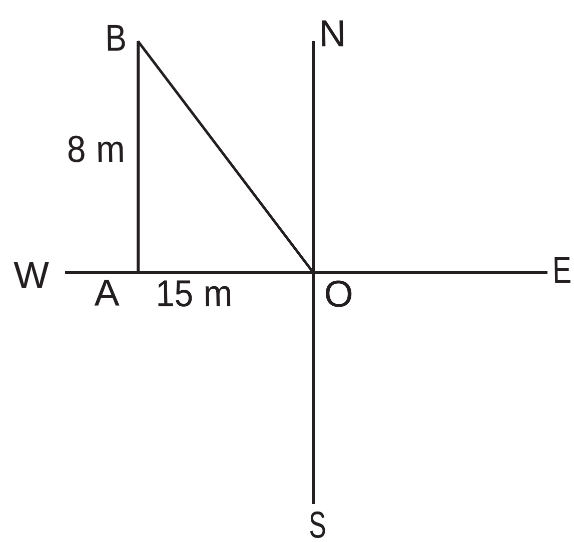
$$\frac{\text{Area of def}}{54} = \frac{4}{3}$$

Area of def = $4 \times \frac{54}{3}$

Area of def = 4 × 18

Area of def = 72cm

8.



In ΔBAO, $\angle BAO = 90^o$

$OB^2 = AB^2 + AO^2$ (Pythagoras theorem)

$= 8^2 + 15^2$

= 64 + 225

= 289

$\therefore$ OB = 17m

Section B

9. $\Delta ABC \sim \Delta DEF$,

$$\frac{ar\Delta ABC}{ar\Delta DEF} = \frac{BC^2}{EF^2}$$

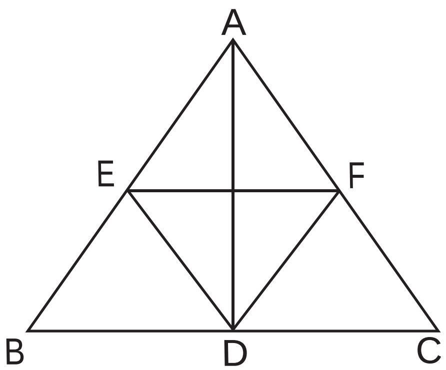
(In two similar triangles, the ratio of their areas is the square of ratio of their sides)

$$\frac{64}{121} = \frac{BC^2}{(15.4)^2}$$

$$(BC)^2 = \frac{64}{121} \times 15.4 \times 15.4$$

$\therefore$ BC = 11.2 cm

10.



In ΔADB, DE is bisector of $\angle$ ADB

$$\frac{BD}{BE} = \frac{AD}{AE}$$

i.e. $\frac{BD}{AD} = \frac{BE}{AE}$ (i)

In ΔADB, DF is bisector of $\angle$ADC

$$\frac{CD}{CF} = \frac{AD}{AF}$$

i.e. $\frac{CD}{AD} = \frac{CF}{AF}$

$\frac{BD}{AD} = \frac{CF}{AF}$ (ii)

(As AD is median $\therefore$ BD = CD)

From (i) and (ii), we get

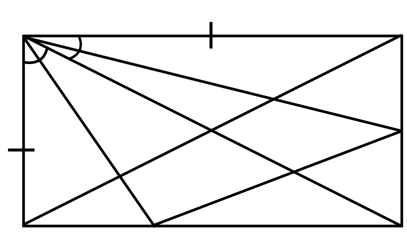
$$\frac{BE}{AE} = \frac{CF}{AF}$$

$$\frac{AE}{BE} = \frac{AF}{CF}$$

So, by converse of Basic proportionality theorem

EF II BC

11.



In Δ ADC, AF bisects $\angle$ DAC

$$\therefore \frac{CF}{DF} = \frac{AC}{AD}$$

$= \frac{AC}{AB}$ (i) (As AB = AD)

In ΔABC, AE bisects $\angle$ BAC

$\frac{CE}{BE} = \frac{AC}{AB}$ (ii)

From (i), (ii)

$$\frac{CF}{DF} = \frac{CE}{BE}$$

$\therefore$ EF II BD

(By converse of Basic proportionaly theorem)

12. In ΔAOB ~ ΔCOD

$\angle$AOB = $\angle$COD (Vertically opposite angles)

$\frac{AO}{OC} = \frac{BO}{DO}$ (Given)

$\therefore$ ΔAOB ~ ΔAOB (SAS)

So, $\frac{AO}{OC} = \frac{BO}{OD} = \frac{AB}{DC}$

(Corresponding sides of similar triangles are proportional)

$$\frac{1}{2} = \frac{5}{CD}$$

CD = 10 cm

13. In Δ KPN and Δ KLM,

$\angle$ K = $\angle$ K (Common)

$\angle$ KNP = $\angle$ KML = 46° (Given)

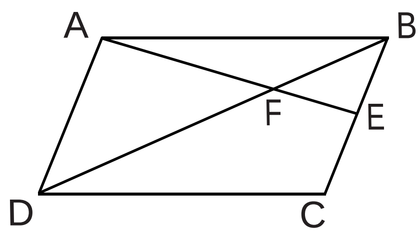
$\therefore$ ΔKPN ~ ΔKLM (AA similarity criterion)

$$\frac{KP}{KL} = \frac{PN}{LM} = \frac{KN}{KM}$$

$$\frac{x}{a} = \frac{c}{b+c}$$

$$x = \frac{ac}{b+c}$$

14.



In ΔAFD and ΔBEF

(Alternate intetior angles)

$\angle AFD = \angle BFE$

(Vertically opposite angles)

$\therefore$ $\Delta AFD \sim \Delta EFB$

So, $\frac{EF}{FA} = \frac{FB}{DF}$

(Corresponding sides of similar triangles are proportional.)

DF x EF = FB x FA

15. As DE II AC, So in ΔABC

$\frac{BC}{CP} = \frac{BE}{EC}$ (i)

(Basic proportionality theorem)

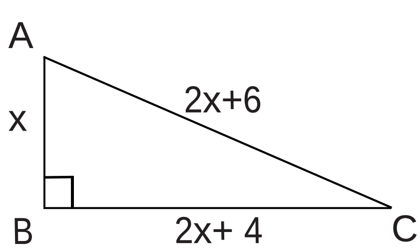
Also, $\frac{BE}{EC} = \frac{BC}{CP}$ (ii) (Given)

From (i), (ii), we have $\frac{BD}{AD} = \frac{BC}{CP}$

$\therefore$ DC II AP

(By converse of Basic proportionality theorem.)

16.



Let the shorter side be x m

$\therefore$ Hypotenuse = 2x + 6

Also, Third side = 2x + 6 -2

= 2x + 4

In ΔABC, $AC^2 = AB^2 + BC^2$

$(2x + 6)^2 = x^2 + (2x + 4)^2$

$4x^2 + 36 + 24x = x^2 + 4x^2 + 16 + 16x$

$O = x^2 - 20 - 8x$

$x^2 - 8x - 20 = 0$

$x^2 - 10x + 2 (x^2 - 10) = 0$

$(x^2 - 10) (x - 2) = 0$

$x = 10, -2$

Being a side, x = – 2 is rejected

$\therefore$ x = 10

So, AB = 10 m

BC = 2x + 4 = 24 m

AC = 2x + 6 = 26 m

17. We know that diagonals of rhombus bisect each other at 90^o.

Let AC = 24 cm

BD = 10 cm

AO = OC = $\frac{1}{2}$ AC = 12 cm

BO = OD = $\frac{1}{2}$ BD = 5 CM

In ΔAOB,

$AB^2 = BO^2 + AO^2$

$= 5^2 + 12^2$

= 25 + 144

= 169

$\therefore$ AB = 13 cm

As all sides of rhombus are equal,

AB = BC = CD = AD = 13 cm

18. In ΔABC, DE II BC

$\frac{AD}{DB} = \frac{AE}{EC}$

[Basic proportionality theorem]

$$\frac{x}{x-2} = \frac{x-2}{x-1}$$

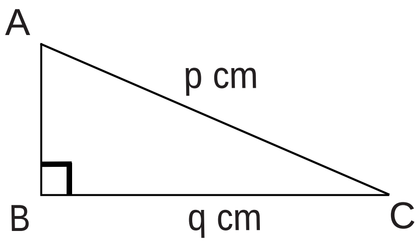
$$x(x-1) = (x-2)(x+2)$$

$$x2 - x = x2 - 4$$

$$x = 4$$

Section C

19.



In Δ ABC, $AC^2 = AB^2 + BC^2$

$P^2 = AB^2 + q^2$

$AB^2 = p^2 - q^2$

$= (p - q)\ (p + q)$

$= 1\ (p + q)$

$AB^2 = p + q$

$AB = \sqrt{p+q}$

20. $\frac{QT}{PR} = \frac{QR}{QS}$ (Given)

In ΔPQR, $\angle 1 = \angle 2$

$\therefore$ PQ = PR

[sides opposite to equal angles are equal]

So, $\frac{QT}{PR} = \frac{QR}{QS}$

Also, $\angle Q = \angle Q$ (common)

$\therefore$ $\Delta PQS \sim \Delta TQR$

[By SAS Similarity cretirion]

21. In ΔCBQ and ΔCAP,

$\angle BCQ = \angle ACP$ (common)

$\angle QBC = \angle PAC = 90°$

(PA and QB are perpendicular)

$\therefore$ $\Delta CBQ \sim \Delta CAP$ (AA Similarity creterion)

$$\frac{BC}{AC} = \frac{BQ}{AP} = \frac{CQ}{CP}$$

[Corresponding sides of similar triangles are proportional]

$$\frac{BC}{AC} = \frac{Z}{x} \quad \text{(i)}$$

In ΔABQ and ΔACR,

$\angle BAQ = \angle CAR$ (common)

$\angle ABQ = \angle ACR = 90°$

(BQ and RC are perpendicular)

$\therefore$ $\Delta ABQ \sim \Delta ACR$ (AA Similarity creterior)

$$\frac{AB}{AC} = \frac{BQ}{CR} = \frac{AQ}{AR}$$

$$\frac{AB}{AC} = \frac{Z}{Y} \quad \text{(ii)}$$

From (i),

$$1 - \frac{BC}{AC} = 1 - \frac{z}{x}$$

$$\frac{AC - BC}{AC} = \frac{x - z}{x}$$

$$\frac{AB}{AC} = \frac{x - z}{x} \quad \text{(iii)}$$

From (ii) and (iii)

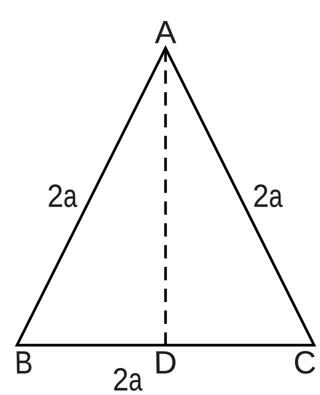
$$\frac{AB}{AC} = \frac{z}{y} = \frac{x - z}{x}$$

$$\frac{z}{y} = 1 - \frac{z}{x}$$

$$\frac{z}{x} + \frac{z}{y} = 1$$

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$$

22.



Draw $AD \perp BC$

In ΔADB and ΔADC

$AB = AC = 2a$ (Given)

$AD = AD$ (Common)

$\angle ADB = \angle ADC$

$= 90°$ (By Construction)

$\therefore \Delta ADB \cong \Delta ADC$ (RHS)

$\therefore BD = DC = \frac{1}{2}BC$

$= a$ (CPCT)

In ΔADC, right angled at D

$AC^2 = AD^2 + DC^2$

$(2a)^2 = AD^2 + a^2$

$AD^2 = 4a^2 - a^2 = 3a^2$

$AD = \sqrt{3}\,a$

So, length of the altitude of an equilateral triangle $= \sqrt{3}\,a$ cm

23. In ΔAOB, $XY \parallel AB$

$\therefore \frac{OX}{AX} = \frac{OY}{BY}$

(i) [Basic Proportionality theorem]

In ΔAOC, $XZ \parallel AC$

$\therefore \frac{OZ}{ZC} = \frac{OX}{AX}$

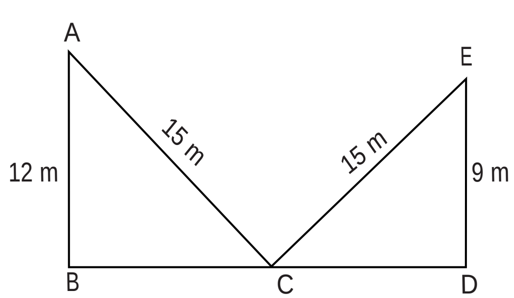
(ii) [Basis Proportionality theorem]

By (i) and (ii), $\frac{OY}{BY} = \frac{OZ}{ZC}$

$\therefore YZ \parallel BC$

[By Converse of Basic proportionality theorem]

24.



Let $AC = CE$ denotes the ladder

In ΔABC, $AC^2 = AB^2 + BC^2$

$15^2 = 12^2 + BC^2$

$225 - 144 = BC^2$

$BC^2 = 81$

$BC = 9$ m

In ΔCDE, $CE^2 = DE^2 + CD^2$

$15^2 = 9^2 + CD^2$

$225 - 81 = CD^2$

$144 = CD^2$

$12 = CD$

So, $BD = BC + CD$

$= 9 + 12 = 21$ m

Section D

25. In ΔXPQ and ΔXYZ,

$\frac{XP}{PY} = \frac{XQ}{XZ} = 3$ (Given)

$\angle X = \angle X$ (Common)

$\therefore \Delta XPQ \sim \Delta XYZ$

(SAS Similarity creterion)

So, $\frac{ar\,\Delta XPQ}{ar\,\Delta XYZ} = \left(\frac{XP}{XY}\right)^2 = \left(\frac{PQ}{YZ}\right)^2 = \left(\frac{XQ}{XZ}\right)^2$

[Ratio of area of two similar triangles is equal to square of their corresponding sides]

$\frac{ar\,\Delta XPQ}{32} = \left(\frac{XP}{XY}\right)^2 = \left(\frac{3}{4}\right)^2$

ar $\Delta XPQ = \frac{9}{16} \times 32$

$$\left[\begin{array}{l} \frac{XP}{Py} = 3 \\ \frac{Py}{XP} = \frac{1}{3} \\ \frac{Py}{XP} + 1 = \frac{1}{3} + 1 \\ \frac{XY}{XP} = \frac{4}{3} \end{array}\right]$$

= 18 cm²

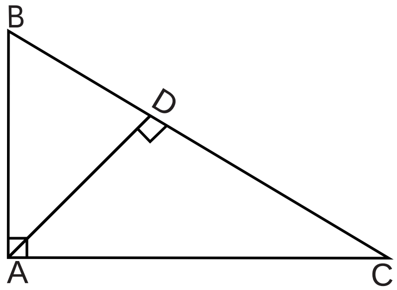
area of quadrilateral PYZQ

= ar ΔXYZ – ar ΔXPQ

= 32 – 18

= 14 cm²

26.



In ΔABC, right angled at B,

We need to prove $AC^2 = AB^2 + BC^2$

Draw BD⊥AC

We know that if a perpendicular drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.

So, ΔCBA and ΔCDB

[Corresponding sides of similar triangles are proportionals

$$\frac{CB}{CD} = \frac{CA}{CB}$$

$CB^2 = CA \times CD$ (i)

Also, ΔABC and ΔADB

$$\frac{AB}{AD} = \frac{BC}{BD} = \frac{AC}{AB}$$

$$\frac{AB}{AD} = \frac{AC}{AB}$$

$AB^2 = AC \times AD$ (ii)

From (i) and (ii),

$AB^2 + BC^2 = AC \times AD + AC \times CD$

$= AC (AD + CD)$

$= AC \times AC$

$= AC^2$

$\therefore AB^2 + BC^2 = AC^2$

27. As XY || AC

∠BXY = ∠A (Corresponding angles)

∠BYX = ∠C (Corresponding angles)

∴ ΔABC ~ ΔXBY (AA Similarity Creterion)

So, $$\frac{ar\,\triangle ABC}{ar\,\triangle XBY} = \left(\frac{AB}{XB}\right)^2 \quad \text{(i)}$$

[Ratio of areas of two similar triangles is equal to square of ratio of their corresponding sides]

Also, ar ΔABC = 2 ar (XBY)

i.e. $$\frac{ar\,\triangle ABC}{ar\,\triangle XBY} = \frac{2}{1} \quad \text{(ii)}$$

From (i) and (ii),

$$\left(\frac{AB}{XB}\right)^2 = \frac{2}{1}$$

$$\frac{AB}{XB} = \frac{\sqrt{2}}{1}$$

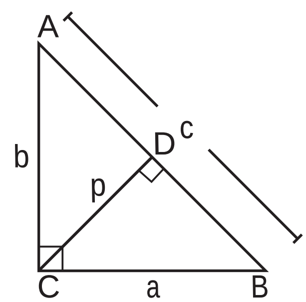
$$\frac{XB}{AB} = \frac{1}{\sqrt{2}}$$

$$\therefore 1 - \frac{XB}{AB} = 1 - \frac{1}{\sqrt{2}}$$

$$\frac{AB - XB}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}}$$

$$\frac{XB}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}} = \frac{2-\sqrt{2}}{2}$$

28.



In ΔACB, right angled at C such that CD ⊥ AB.

We know that if a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other

So, $\Delta BDC \sim \Delta BCA$

$$\therefore \frac{BD}{BC} = \frac{DC}{CA} = \frac{BC}{BA}$$

i.e. $\frac{p}{b} = \frac{a}{c}$

$pc = ab$

$\Rightarrow p = \frac{ab}{c}$

$\Rightarrow p^2 = \frac{a^2b^2}{c^2}$

$\Rightarrow \frac{1}{p^2} = \frac{c^2}{a^2b^2}$

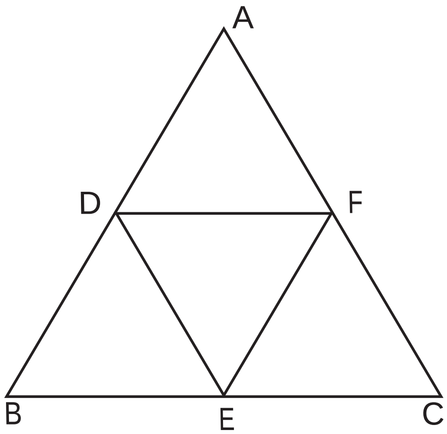
In ΔACB, $AC^2 + BC^2 = AB^2 b^2 + a^2 = c^2$

$$\frac{1}{p^2} = \frac{a^2+b^2}{a^2 b^2}$$

$$\frac{1}{p^2} = \frac{a^2}{a^2 b^2} + \frac{b^2}{a^2 b^2}$$

$$\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

29.



Given that:-

ΔABC in whichD, E, F are the mid points of sides AB, BC and CA respectively.

To prove:- each of the triangles are similar to the original triangle, i.e.,

$\Delta ADF \sim \Delta ABC$

$\Delta BDE \sim \Delta ABC$

$\Delta CEF \sim \Delta ABC$

Proof:-

Consider the ΔADF and ΔABC

Since D and F are the mid points of AB and AC respectively.

$\therefore$ DF // BC

$\Rightarrow \angle AFD = \angle B$ (Corresponding angles are equal)

Now, in ΔADF and ΔABC, we have

$\angle ADF = \angle B$(Corresponding angles)

$\angle A = \angle A$(Common)

By AA similar conditions,

$\Delta ADF \sim \Delta ABC$

Similarly, we have

$\Delta BDE \sim \Delta ABC$

$\Delta CEF \sim \Delta ABC$

$\therefore$ EF // AB

$\Rightarrow$ EF // AD..........(1)

And, DE // AC

$\Rightarrow$ DE // AF..........(2)

From eqn (1) and (2), we have

ADEF is a parallelogram.

Similarly, BDFE is a parallelogram.

Now, in ΔABC and ΔDEF

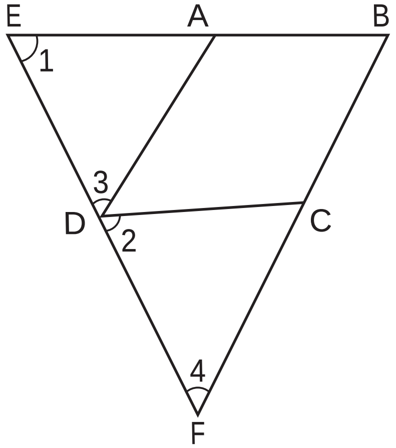
$\angle A = \angle FED$ ($\because$ Opposite angles of parallelogram)

$\angle B = \angle DFE$ ($\because$ Opposite angles of parallelogram)

Therefore, by AA similar condition

$\Delta ABC \sim \Delta DEF$

Hence proved that each of the triangles are similar to the original triangle.

30. 

Consider ΔEDA and ΔEFB

$\angle 1 = \angle 2$ (Common)

$\angle 3 = \angle 4$

[Corresponding angles as AD || BF]

$\therefore \quad \Delta EDA \sim \Delta EFB$

(AA Similarity Creterion)

$\therefore \quad \frac{DA}{FB} = \frac{EA}{EB}$

[Corresponding sides of similar triangles proportional]

$\Rightarrow \frac{DA}{AE} = \frac{FB}{BE}$ (i)

Consider ΔEDA and ΔDFC

$\angle 1 = \angle 2$ (Corresponding angles as BE || CD)

$\angle 3 = \angle 4$ (Corresponding angles as AD || BF)

$\therefore \quad \Delta EDA \sim \Delta DFC$ (AA Similarity Creterion)

$\therefore \quad \frac{ED}{DF} = \frac{DA}{FC} = \frac{EA}{DC}$

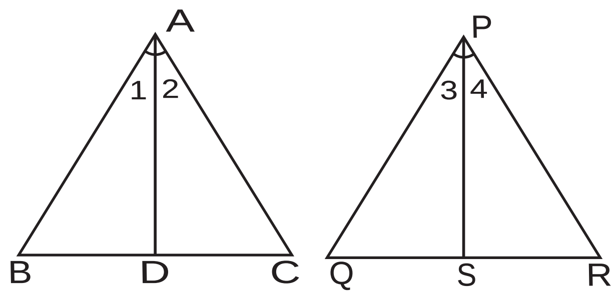
[Corresponding sides of similar triangles are proportional]

i.e. $\frac{DA}{FC} = \frac{EA}{DC}$

$\Rightarrow \frac{DA}{AE} = \frac{FC}{CD}$ (ii)

From (i) and (ii),

$\frac{DA}{AE} = \frac{FB}{BE} = \frac{FC}{CD}$

31. 

Given :AD and PS are bisectors of $\angle A$ and $\angle P$ respectively. Such that

$\frac{BD}{DC} = \frac{QS}{SR}$

To prove = $\Delta ABC \sim \Delta PQR$

ProofIn ΔABC, AD is bisector of $\angle A$

$\therefore \quad \frac{AB}{BD} = \frac{AC}{CD}$

i.e. $\frac{AB}{AC} = \frac{BD}{CD}$ (i)

In ΔPQR, PS is bisector of $\angle P$

$\therefore \quad \frac{PQ}{QS} = \frac{PR}{RS}$

i.e. $\frac{PQ}{PR} = \frac{QS}{RS}$ (ii)

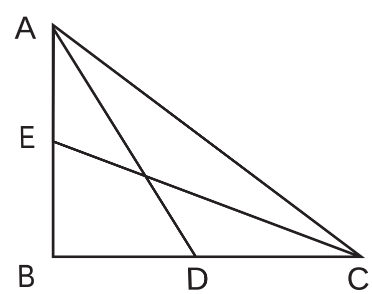
Also, $\frac{BD}{DC} = \frac{QS}{SR}$ (iii)

From (i), (ii), (iii), we get

$\frac{AB}{AC} = \frac{PQ}{PR} \Rightarrow \frac{AB}{PQ} = \frac{AC}{PR}$

Also, $\angle A = \angle P$ (Given)

$\therefore \Delta ABC \sim \Delta PQR$

32. 

ΔABC is a right triangle right-angled at B

$\therefore \quad AD^2 = AB^2 + BD^2$

(By Pythagoras tearoom)

$\Rightarrow \quad AD^2 = AB^2 + \left(\frac{BC}{2}\right)^2 \qquad [\because BD = DC]$

$\Rightarrow \quad AD^2 = AB^2 + \frac{1}{4} BC^2 \qquad$ (i)

Also, ΔBCE is a right triangle right angled at B

$\therefore \quad CE^2 = BC^2 + BE^2$

$\Rightarrow \quad CE^2 = BC^2 + \left(\frac{AB}{2}\right)^2 \qquad [\because BE = EA]$

$\Rightarrow mCE^2 = BC^2 + \frac{1}{4} AB^2 \qquad$ (ii)

On adding (i) and (ii), we get

$AD^2 + CE^2 = \frac{5}{4}(AB^2 + BC^2)$

$\Rightarrow \quad AD^2 + CE^2 = \frac{5}{4} AC^2$

[As ΔABC is right triangle
$\therefore AC^2 = AB^2 + BC^2$]

$\Rightarrow \quad \left(\frac{3\sqrt{5}}{2}\right)^2 + CE^2 = \frac{5}{4}(25)$

$\Rightarrow \quad CE^2 = \frac{125}{4} - \frac{45}{4} = 20$

$\therefore \quad CE = \sqrt{20}$ cm $= 2\sqrt{5}$ cm

WORKSHEET 2

Section A

1. $\Delta ABC \sim \Delta RPQ$

$\therefore \frac{AB}{RP} = \frac{BC}{PQ} = \frac{AC}{RQ}$

[Corresponding sides of similar triangles are proportional]

$\therefore \frac{3}{6} = \frac{5}{10} = \frac{6}{RQ}$

$RQ = \frac{6 \times 10}{5} = 12$ cm

2. DABC ~ DDEF

$\therefore \frac{ar\triangle ABC}{ar\triangle DEF} = \left(\frac{AB}{DE}\right)^2$

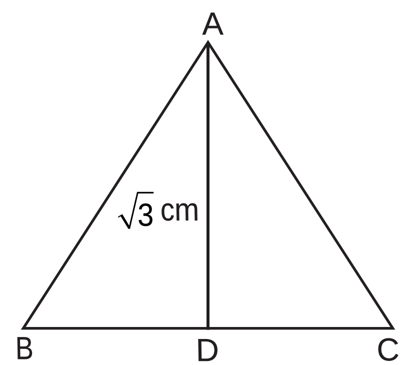
[Ratio of areas of similar triangles is Proportional to the square of ratio of ratio of their corresponding sides]

$\Rightarrow \quad \frac{169}{121} = \frac{(26)^2}{DE}$

$\Rightarrow \quad DE^2 = \frac{(26)^2 \times 121}{169}$

$\Rightarrow \quad DE = \frac{26 \times 11}{13} = 22$ cm

3.



ΔABC is equilateral and AD is the Median such that $AD = \sqrt{3}$ cm

In an equilateral triangle, median and altitude are same

$\therefore \quad AD \perp BC$

Also, $DC = \frac{1}{2} AC$

[As AD is the Median]

$AC^2 = AD^2 + CD^2$

$AC^2 = (\sqrt{3})^2 + \left(\frac{1}{2}AC\right)^2$

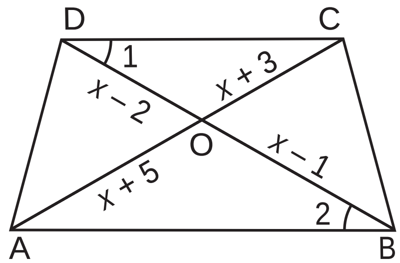
$AC^2 = 3 + \frac{1}{4} AC^2$

$\frac{3}{4} AC^2 = 3$

$AC^2 = 4$

$AC = 2$ cm

4.



In ΔCOD and ΔAOB,

$\angle 1 = \angle 2$

[Corresponding angles as AB || CD]

$\angle AOB = \angle COD$

[Vertically opposite angles]

$\therefore \quad \Delta COD \sim \Delta AOB$

$$\therefore \quad \frac{CO}{AO} = \frac{OD}{OB} = \frac{CD}{AB}$$

[Corresponding sides of similar triangles are proportional.]

$$\frac{x+3}{x+5} = \frac{x-2}{x-1}$$

$\Rightarrow$ (x + 3) (x – 1) = (x – 2) (x + 5)

$\Rightarrow x^2 + 2x - 3 = x^2 + 3x - 10$

$\Rightarrow 7 = x$

5. In ΔSPT and ΔQPR,

$\angle PST = \angle PQR$

[Corresponding angles as ST || QR]

$\angle PTS = \angle PRQ$

$\therefore \quad \Delta SPT \sim \Delta QPR$

[AA Similarity Creterion]

$$\therefore \quad \frac{ar_{\triangle}PST}{ar_{\triangle}PQR} = \left(\frac{PT}{PR}\right)^2$$

[Ratio of areas of two similar triangles is equal to square of ratio of their corresponding sides]

$$= \left(\frac{PT}{PT+TR}\right)^2$$

$$= \left(\frac{2}{2+4}\right)^2$$

$$= \left(\frac{2}{6}\right)^2$$

$$= \frac{1}{9}$$

6. DE || BC

$$\therefore \quad \frac{AD}{BD} = \frac{AC}{CE}$$

(Basic Proportionality theorem)

$$\Rightarrow \quad \frac{BD}{AD} = \frac{CE}{AE}$$

$$\Rightarrow \quad \frac{BD}{AD} + 1 = \frac{CE}{AE} + 1$$

$$\Rightarrow \quad \frac{BD+AD}{AD} = \frac{CE+AE}{AE}$$

$$\frac{AB}{AD} = \frac{AC}{AE}$$

Also, $\angle A = \angle A$ (Common)

$\therefore \Delta ADE \sim \Delta ABC$

$$\Rightarrow \quad \frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC}$$

[Corresponding sides of similar triangles are proportional]

$$\Rightarrow \quad \frac{AD}{AB} = \frac{DE}{BC}$$

$$\Rightarrow \quad \frac{1.5}{6} = \frac{DE}{8}$$

$$\Rightarrow \quad DE = \frac{1.5 \times 8}{6} = 2 \text{ cm}$$

7. As MN || AB,

$$\frac{CM}{AM} = \frac{CN}{BN}$$

[Basic proportionality theorem]

$$\frac{2}{4} = \frac{BC - BN}{BN}$$

$$\frac{1}{2} = \frac{7.5 - BN}{BN}$$

$\therefore$ BN = 15 – 2BN

$\Rightarrow$ 3BN = 15

BN = 5 cm

8. We know that ratio of area of two similar triangles is equal to square of ratio of their corresponding sides.

So, Ratio of corresponding sides

$$= \sqrt{\frac{25}{64}} = \frac{5}{8}$$

9. DE || BC

$$\Rightarrow \frac{AD}{DB} = \frac{AE}{CE}$$

[Basic proportionality theorem]

$$\Rightarrow \frac{DB}{AD} + 1 = \frac{CE}{AE} + 1$$

$$\Rightarrow \frac{AB}{AD} = \frac{AC}{AE}$$

Also, $\angle A = \angle A$ (Common)

$\therefore$ $\Delta ADE \sim \Delta ABC$

(SAS Similarity Creterion)

$$\therefore \frac{ar_{\triangle}ADE}{ar_{\triangle}ABC} = \left(\frac{DE}{BC}\right)^2$$

$$\frac{ar_{\triangle}ADE}{81} = \frac{\left(\frac{2}{3}BC\right)^2}{BC^2}$$

$$\frac{ar_{\triangle}ADE}{81} = \frac{4}{9}$$

$$ar\ \Delta ADE = \frac{4}{9} \times 81 = 36\ cm^2$$

10. Considers $AC^2 + BC^2$

$= AC^2 + AC^2$ ($\because$ AC = BC)

$= 2AC^2$

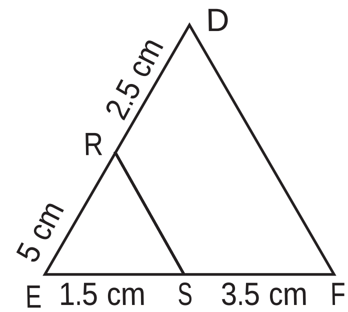
$= AB^2$

$\therefore$ ΔABC is right angled triangle

[As we know that in a triangle, if square of one side is equal to sum of the squares of other two sides then the angle opposite the first side is a right angle.]

Section B

11.



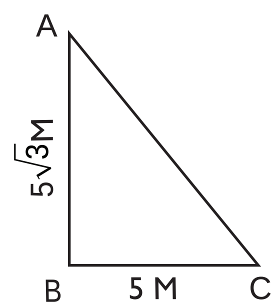
In a triangle ΔDEF, R and S are two points on the sides DE and EF respectively. ER=5cm, RD=2.5cm, FS=3.5 and SD=1.5cm.

$$\therefore \frac{ER}{RD} = \frac{5}{2.5} = \frac{2}{1} \text{ and } \frac{FS}{SD} = \frac{3.5}{1.5} = \frac{7}{3}$$

$$\because \frac{ER}{RD} \neq \frac{FS}{SD}$$

$\therefore$, RS is not parallel to DF.

12.



In ΔABC, right angled at B

$AC^2 = AB^2 + BC$

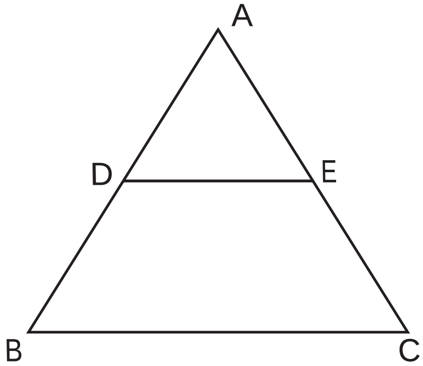
$= \left(5\sqrt{3}\right)^2 + \left(5\right)^2$

= 75 + 25

= 100

$\therefore$ AC = 10 M

13.



As DE || BC

$$\frac{AD}{DB} = \frac{AE}{CE}$$

$$\frac{BD}{AD} = \frac{CE}{AE}$$

$$\Rightarrow \frac{BD}{AD} + 1 = \frac{CE}{AE} + 1$$

$$\Rightarrow \frac{AB}{AD} = \frac{AC}{AE}$$

$$\Rightarrow \frac{AD}{AB} = \frac{AE}{AC} \qquad \text{(i)}$$

Also, $\angle A = \angle A$ (Common)

$\therefore$ $\Delta ADE \sim \Delta ABC$ (SAS Similarity Creterion)

$$\therefore \frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC}$$

[Corresponding sides of similar triangles are proportional]

$$\Rightarrow \frac{AD}{AD+BD} = \frac{AE}{AC}$$

$$\Rightarrow \frac{AD}{AD+3AD} = \frac{4.5}{AC}$$

$$\Rightarrow \frac{AD}{4AD} = \frac{4.5}{AC}$$

$\Rightarrow$ AC = 4.5 × 4 = 18 cm

Also, $\frac{AD}{AB} = \frac{AE}{AC}$ (From (i))

$$\frac{AD}{AD+BD} = \frac{AE}{18}$$

$$\frac{AD}{AD+3AD} = \frac{AE}{18}$$

$$\frac{1}{4} = \frac{AE}{18}$$

$$AE = \frac{18}{4} = \frac{9}{2} = 4.5 \text{ cm}$$

14. Consider ΔABC with sides as

AB = (a – 1) cm

BC = $\left(2\sqrt{a}\right)$ cm

AC = (a + 1) cm

Consider $AB^2 + BC^2$

$= (a-1)^2 + \left(2\sqrt{a}\right)^2$

$= a^2 + 1 - 2a + 4a$

$= a^2 + 2a + 1$

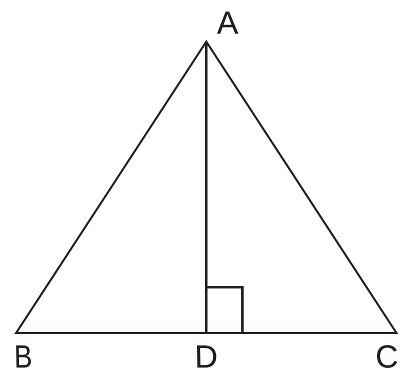
$= (a+1)^2$

$= AC^2$

$\therefore$ ΔABC is a right angled triangle

[As we know that in a triangle if square of one side is equal to the sum of squares of other two sides, then the angle opposite the first side is a right angle i.e. triangle is right angled]

15.



Draw AD ⊥ BC

In ΔADB and ΔADC

AD = AD (Common)

AB = AC (ΔABC is equilateral)

$\angle ADB = \angle ADC = 90°$ (By Construction)

$\therefore \quad \Delta ADB \cong \Delta ADC \quad$ (RHS)

$\Rightarrow \quad CD = \frac{1}{2} BC = \frac{1}{2} 3\sqrt{3}$ cm [CPCT]

In ΔADC,

$AC^2 = AD^2 + CD^2$

$\left(3\sqrt{3}\right)^2 = AD^2 + \left(\frac{3\sqrt{3}}{2}\right)^2$

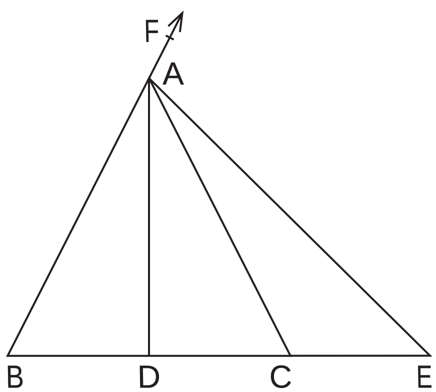
$AD^2 = 27 - \frac{27}{4}$

$= \frac{108 - 27}{4}$

$= \frac{81}{4}$

$\therefore AC = \frac{9}{2} = 4.5$ cm

16.



To prove $= \frac{BD}{BE} = \frac{CD}{CE}$

As AD bisects $\angle BAC$,

$\frac{AB}{BD} = \frac{AC}{CD}$ [Interior angle bisector theorem]

$\therefore \quad \frac{CD}{BD} = \frac{AC}{AB} \quad$ (i)

Also, AE bisects $\angle CAF$

$\therefore \quad \frac{BE}{AB} = \frac{CE}{AC}$

$\Rightarrow \quad \frac{BE}{CE} = \frac{AB}{AC}$

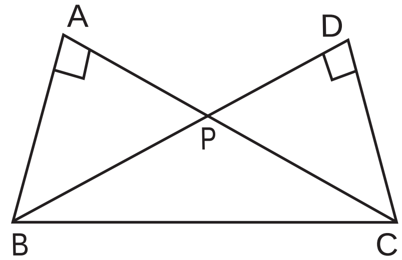
$\Rightarrow \quad \frac{CE}{BE} = \frac{AC}{AB} \quad$ (ii)

From (i) and (ii)

$\frac{CD}{BD} = \frac{CE}{BE}$

$\Rightarrow \quad \frac{BD}{BE} = \frac{CD}{CE}$

17.



To prove: $AP \times PC = BP \times PD$

Consider ΔAPB and ΔDPC

$\angle BAP = \angle CDP = 90°$ (Given)

$\angle APB = \angle DPC$

(Vertically opposite angles)

$\therefore \quad$ DAPB ~ ΔDPC (AA similarity crelerion)

$\therefore \quad \frac{AP}{DP} = \frac{PB}{PC} = \frac{AB}{DC}$

(Corresponding sides of similar triangles are proportional)

$\Rightarrow \quad \frac{AP}{DP} = \frac{PB}{PC}$

$\Rightarrow \quad AP \times PC = BP \times PD$

18. Consider ΔQPM and ΔRSM

$\angle QPM = \angle RSM = 90°$

$\angle QMP = \angle RMS$

(Vertically opposite angles)

$\therefore \quad \Delta QPM \sim \Delta RSM$ (AA similarity Creterion)

$\therefore \quad \frac{QP}{RS} = \frac{PM}{SM} = \frac{QM}{RM}$

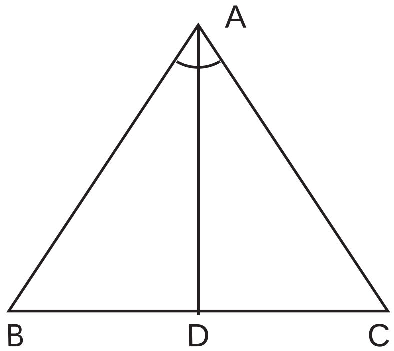
[Corresponding sides of similar triangles are proportional]

i.e. $\frac{PM}{SM} = \frac{QM}{RM}$

$$\frac{3}{4} = \frac{QM}{6}$$

$$QM = \frac{3\times6}{4} = \frac{3\times3}{2} = 4.5 \text{ cm}$$

19.



AD bisects $\angle A$ So, by Interior angle bisector theorem

$$\frac{AB}{AC} = \frac{BD}{DC}$$

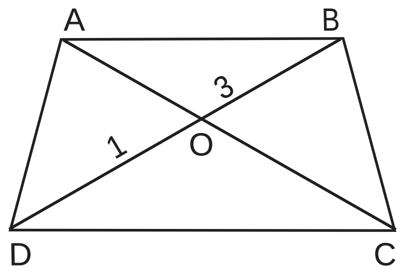
$$\frac{AB}{AC} = \frac{BD}{DC} = 1$$

[$\because$ BD = DC as D is a midpoint of BC]

AB = AC

$\therefore$ ΔABC is an isosceles.

20.



Here, AC divides the diagonal BD in the ratio 1 : 3

Consider ΔAOB and ΔCOD

$\angle BAO = \angle DCO$

(Alternate interior angles as AB || CD)

$\angle AOB = \angle COD$

(Vertically opposite angles)

$\therefore$ $\Delta AOB \sim \Delta COD$

(AA similarity creterion)

$$\therefore \frac{AO}{CO} = \frac{OB}{OD} = \frac{AB}{CD}$$

[Corresponding sides of similar triangles are proportional.]

$$\Rightarrow \frac{OB}{OD} = \frac{AB}{CD}$$

$$\Rightarrow \frac{3}{1} = \frac{AB}{CD}$$

$$\Rightarrow AB = 3CD$$

Section C

21. In ΔADE and ΔABC,

$\angle A = \angle A$ (Common)

$\angle ADE = \angle ABC$ (Given)

$\therefore$ $\Delta ADE \sim \Delta ABC$ (AA similarity createrion)

$$\Rightarrow \frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC}$$

[Corresponding sides of similar triangles are proportional]

$$\therefore \frac{AD}{AB} = \frac{DE}{BC}$$

$$\Rightarrow \frac{7.6}{AE+BE} = \frac{DE}{8.4}$$

$$\Rightarrow \frac{7.6}{7.2+4.2} = \frac{DE}{8.4}$$

$$\Rightarrow \frac{7.6}{11.4} = \frac{DE}{8.4}$$

$$\Rightarrow DE = \frac{7.6\times8.4}{11.4}$$

$= 5.6$ cm

22. In ΔABC, LM || BC

$$\therefore \frac{AM}{BM} = \frac{AL}{CL}$$

(i) [Basic proportionality theorem]

In ΔADC, LN || CD

$\therefore \quad \frac{AN}{DN} = \frac{AL}{CL}$

(ii) [Basic proportionality theorem]

From (i) and (ii), $\frac{AM}{BM} = \frac{AN}{DN}$

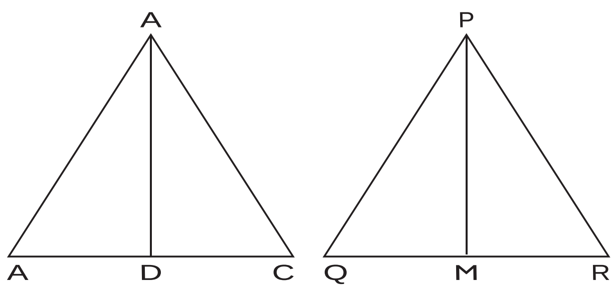
$\Rightarrow \quad \frac{BM}{AM} = \frac{DN}{AN}$

$\Rightarrow \quad \frac{BM}{AM} + 1 = \frac{DN}{AN} + 1$

$\Rightarrow \quad \frac{AB}{AM} = \frac{AD}{AN}$

$\Rightarrow \quad AM \times AD = AB \times AN$

23.



In ΔABD and ΔPQM

$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{BC}{QR}$ (Given)

$\Rightarrow \quad \frac{AB}{PQ} = \frac{AD}{PM} = \frac{BD}{QM}$

$\Rightarrow \quad \frac{AB}{PQ} = \frac{AD}{PM} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR}$

(As AD and PM are the medians)

$\therefore \quad \Delta ABD \sim \Delta PQM$

(SSS similarity creterion)

$\therefore \quad \angle B = \angle Q$

[Corresponding angles of similar triangles are equal]

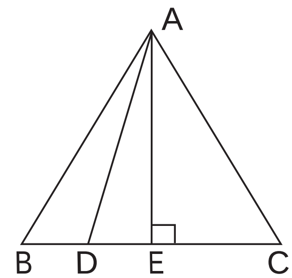
Now, In ΔABC and ΔPQR

$\frac{AB}{PQ} = \frac{BC}{QR}$ (Given)

$\angle B = \angle Q$ (Proved)

$\therefore \Delta ABC \sim \Delta PQR$ (SSS similarity creterion)

24.



Let AB = BC = AC = a

$\therefore$ BD = $\frac{BC}{4} = \frac{a}{4}$

Draw AE $\perp$ BC

$\therefore$ BE = EC = $\frac{a}{2}$

[In Equilateral triangle altitude is same as Median]

In right angled triangle ΔAED,

$AD^2 = DE^2 + AE^2$ (i)

Now, DE = BE – BD

$= \frac{a}{2} - \frac{a}{4}$ $\quad [\because BD = \frac{1}{4} = \frac{a}{4}]$

$= \frac{a}{4}$ (ii)

In ΔAEC,

$AC^2 = AE^2 + CE^2$

$a^2 = AE^2 + \left(\frac{a}{2}\right)^2$

$AE^2 = a^2 - \frac{a^2}{4} = \frac{3a^2}{4}$ (iii)

On putting (ii), (iii) in (i), we get

$AD^2 = \left(\frac{a}{4}\right)^2 + \left(\frac{3a}{4}\right)^2$

$= \frac{a^2}{16} + \frac{3a^2}{4}$

$= \frac{a^2 + 12a^2}{16}$

$$= \frac{13a^2}{16}$$

$16\,AD^2 = 13\,a^2$

$16AD^2 = 13\,BC^2$

25. As ΔABC is isosceles,

AB = AC

$\therefore\ \angle B = \angle C$

(Angles opposite to equal sides are equal)

In ΔADB and ΔEFC

$\angle ADB = \angle EFC$

(AS $EF \perp AC$ and $AD \perp CD$)

$\angle B = \angle C$ (Proved)

$\therefore\ \Delta ADB \sim \Delta EFC$ (AA similarity creterion)

$$\therefore\ \frac{AD}{EF} = \frac{BD}{FC} = \frac{AB}{EC}$$

i.e. $\frac{AD}{EF} = \frac{AB}{EC}$

$\Rightarrow\ AD \times EC = AB \times EF$

26. In ΔABC and ΔADE,

$\angle A = \angle A$ (Common)

$\angle ACB = \angle AED = 90°$

(As $DE \perp AB$ and ΔABC is right angled at C)

$\therefore\ \Delta ABC \sim \Delta ADE$

(By AA Similarity creterion)

$$\Rightarrow\ \frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE}$$

[Corresponding sides of similar triangles are proportional]

In ΔABC, $\angle C = 90°$

$\therefore\ AB^2 = AC^2 + BC^2$

[By Pythagoras theorem]

$= (3 + 2)^2 + 12^2$

$= 25 + 144$

$= 169$

$\therefore\ AB = 13$ cm

As $\frac{AB}{AD} = \frac{BC}{DE}$

$\therefore\ \frac{13}{3} = \frac{12}{DE}$

$\therefore\ DE = \frac{12 \times 3}{13} = \frac{36}{13}$ cm

Also, $\frac{BC}{DE} = \frac{AC}{AE}$

$$\Rightarrow\ \frac{12}{\frac{36}{13}} = \frac{5}{AE}$$

$$\Rightarrow\ \frac{12 \times 13}{36} = \frac{5}{AE}$$

$$\Rightarrow\ AE = \frac{5 \times 36}{12 \times 13}$$

$$= \frac{15}{13} \text{ cm}$$

27. As $\Delta NSQ \cong \Delta MTR$,

$\angle NQS = \angle MRT$ (CPCT)

$\Rightarrow\ PQ = PR$ (i)

(Sides opposite to equal angles are equal)

Also, as $\angle 1 = \angle 2$

$\therefore\ PS = PT$ (ii)

(Sides opposite to equal angles are equal.)

On Subtracting (ii) from (i), we get

$PQ - PS = PR - PT$

$QS = TR$ (iii)

From (ii) and (iii),

$$\frac{PS}{QS} = \frac{PT}{TR} \quad \Rightarrow \frac{PS}{PQ} = \frac{PT}{PR}$$

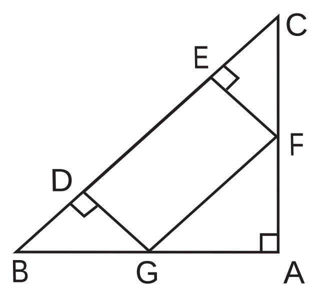
Also, $\angle P = \angle P$ (Common)

$\therefore \quad \Delta PST \sim \Delta PQR$

(SAS similarity creterion)

Section D

28.



In ΔAFG and ΔDBG,

$\angle AGF = \angle DBG$

(Corresponding angles as $GF \parallel BC$)

$\angle GAF = \angle BDG = 90°$ ($\because$ DE FG is a square)

$\therefore \Delta AFG \sim \Delta DBG$ (i)

(AA similarity creterion)

In ΔAGF and ΔEFC,

$\angle AFG = \angle CEF = 90°$

$\angle AFG = \angle ECF$

(Corresponding angles as $GF \parallel BC$)

$\therefore \quad \Delta AGF \sim \Delta EFC$ (ii)

(AA similarity creterion)

From (i), (ii), we get

$\Delta DBG \sim \Delta EFC$

$\Rightarrow \quad \frac{BD}{EF} = \frac{DG}{EC}$

[Corresponding sides of similar triangles are proportional]

$\Rightarrow \quad \frac{BD}{DE} = \frac{DE}{EC}$

[As DEFG is a square, $EF = DE$ and $DG = DE$]

$\Rightarrow \quad DE^2 = BD \times EC$

29. In ΔAOD, MO bisects $\angle AOD$,

So, by interior angle bisector theorem,

$$\frac{AO}{OD} = \frac{AM}{DM} \qquad \text{(i)}$$

In ΔBOC, NO bisects $\angle BOC$

So, by interior angle bisector theorem,

$$\frac{BO}{CO} = \frac{BN}{CN}$$

$$\Rightarrow \quad \frac{CO}{BO} = \frac{CN}{BN} \qquad \text{(ii)}$$

We know that $AO = OD \quad \Rightarrow \frac{AO}{OD} = 1$

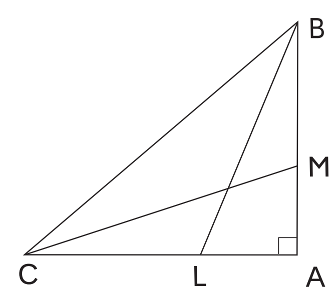
and $CO = BO \quad \Rightarrow \frac{CO}{BO} = 1$

(Radii of same circle)

So, From (i) and (ii), we get

$$\frac{AM}{DM} = \frac{CN}{BN}$$

30.



In ΔABC, $BC^2 = AB^2 + AC^2$

(By Pythagoras theorem)

In ΔABL, $BL^2 = AB^2 = AL^2$

$$= AB^2 + \left(\frac{1}{2}AC\right)^2$$

[As L is a midpoint of AC $\therefore AL = \frac{1}{2}AC$]

$$BL^2 = AB^2 + \frac{AC^2}{4}$$

$4BL^2 = 4AB^2 + AC^2$ (i)

In ΔCMA, $CM^2 = AC^2 + AM^2$

$$= AC^2 + \left(\frac{1}{2}AB\right)^2$$

$= AC + \frac{AB^2}{4}$

[As M is a midpoint of AB $\therefore$ AM = $\frac{1}{2}AB$]

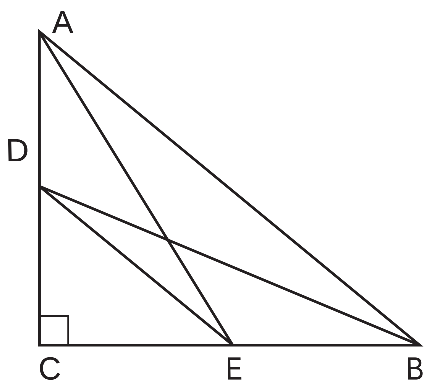
$\Rightarrow$ $4\ CM^2 = 4AC^2 + AB^2$ (ii)

From (i), (ii), we get

$4(BL^2 + CM^2) = 5\ AB^2 + 5\ AC^2$

$= 5\ BC^2$

31.



To prove : $AE^2 + BD^2 = AB^2 + DE^2$

Proof In $\Delta ACE, AE^2 = AC^2 + CE^2$ (i)

(By Pythagoras theorem)

In $\Delta DCB, BD^2 = DC^2 + BC^2$ (ii)

(By Pythagoras theorem)

In $\Delta ABC, AB^2 = AC^2 + BC^2$ (iii)

(By Pythagoras theorem)

In $\Delta DCE, DE^2 = DC^2 + CE^2$ (iv)

(By Pythagoras theorem)

Consider $AE^2 + BD^2$

$= AC^2 + CE^2 + DC^2 + BC^2$ (By ① and ②)

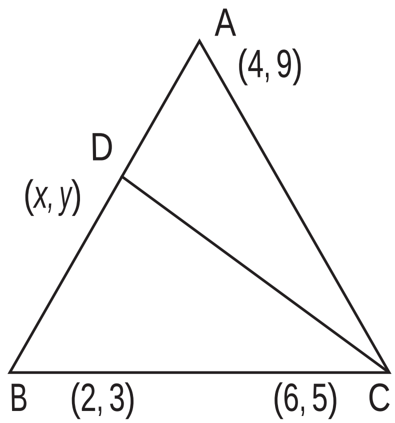
$= (AC^2 + BC^2) + (CE^2 + DC^2)$

$= AB^2 + DE^2$ (By ③,④)

Chapter 07 Coordinate Geometry

MULTIPLE CHOICE QUESTIONS

1.



$D(x, y) = \left(\frac{4+2}{2}, \frac{9+3}{2}\right) = (3, 6)$

So, $CD = \sqrt{(6-3)^2 + (5-6)^2}$

$= \sqrt{9+1}$

$= \sqrt{10}$ units

Option (b)

2. As A, B and C are collinear

$\therefore$ x (– 4 + 5) – 3 (– 5 – 2) + 7 (2 + 4) = 0

x + 21 + 42 = 0

x = – 63

Option (c)

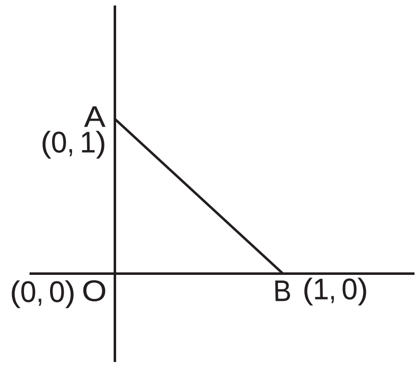
3. $(2, p) = \left(\frac{6-2}{2}, \frac{-5+11}{2}\right)$

$= (2, 3)$

$\Rightarrow$ p = 3

Option (b)

4.



In DAOB,

AB² = AO² + OB²

1² + 1²

2

AB = $\sqrt{2}$

Perimeter = AO + OB + AB

= 1 + 1 + $\sqrt{2}$

Option (d)

WORKSHEET 1

Section A

1. Centroid = $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

$(1, 4) = \left(\frac{4-9+x_3}{3}, \frac{-3+7+y_3}{3}\right)$

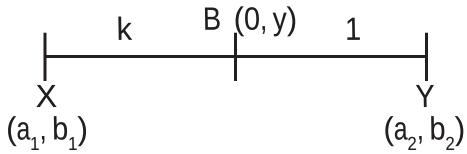
$(1, 4) = \left(\frac{-5+x_3}{3}, \frac{4+y_3}{3}\right)$

$\frac{-5+x_3}{3} = 1$ | $\frac{4+y_3}{3} = 4$

$x_3 - 5 = 3$ | $y_3 + 4 = 12$

$x_3 = 8$ | $y_3 = 8$

So, third vertex is (8, 8)

2. 

Let the ratio be k : 1

So, (0, y) $= \left(\dfrac{ka_2 + a_1}{k+1}, \dfrac{kb_2 + b_1}{k+1}\right)$

$\dfrac{ka_2 + a_1}{k+1} = 0$

$ka_2 + a_1 = 0$

$ka_2 = -a_1$

$k = \dfrac{-a_1}{a_2}$

3. Distance $= \sqrt{\left(\dfrac{2}{5}+\dfrac{8}{5}\right)^2 + (2-2)^2}$

$= \sqrt{\left(\dfrac{2}{5}+\dfrac{8}{5}\right)^2 + 0}$

$= \sqrt{\left(\dfrac{2+8}{5}\right)^2}$

= 2 sq. units

4. Let Point on y - axis be (0, y).

$\sqrt{(6-0)^2 + (5-y)^2} = \sqrt{(-4-0)^2 + (3-y)^2}$

$\sqrt{36 + 25 + y^2 - 10y} = \sqrt{16 + 9 + y^2 - 6y}$

$\sqrt{61 + y^2 - 10y} = \sqrt{25 + y^2 - 6y}$

$61 + y^2 - 10y = 25 + y^2 - 6y$

$36 = 4y$

$y = 9$

So, point on y - axis which is equidistant from point A (6, 5) and B (– 4, 3) is (0, 9)

5. As point A (0, 2) is equidistant from the points B (3, P) and C (P, 5), So,

$\sqrt{(3-0)^2 + (P-2)^2} = \sqrt{(P-0)^2 + (5-2)^2}$

$\sqrt{9 + (P-2)^2} = \sqrt{P^2 + 9}$

$(P-2)^2 = P^2$

$P^2 + 4 - 4P = P^2$

$4P = 4$

$P = 1$

6. $\sqrt{(4-1)^2 + (K-0)^2} = 5$

$\sqrt{3^2 + K^2} = 5$

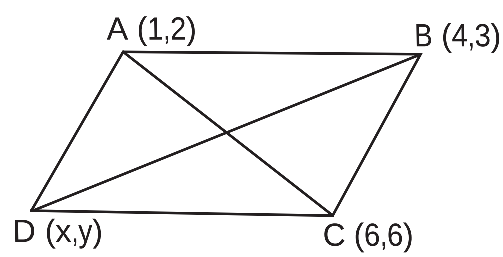
On Squarring both sides, we get

$9 + K^2 = 25$

$K^2 = 25 - 9 = 16$

$K^2 = 16$

$K = \pm 4$

7. 

We know that diagonals of a parallelogram bisect each other

$\therefore \left(\dfrac{1+6}{2}, \dfrac{2+6}{2}\right) = \left(\dfrac{4+x}{2}, \dfrac{3+y}{2}\right)$

$\left(\dfrac{7}{2}, 4\right) = \left(\dfrac{4+x}{2}, \dfrac{3+y}{2}\right)$

$\therefore \dfrac{7}{2} = \dfrac{4+x}{2}$ and $4 = \dfrac{3+y}{2}$

7 = 4 + x and 8 = 3 + y

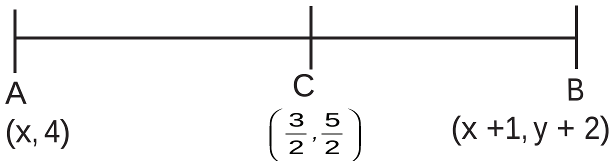
x = 3 and y = 5

So, coordinates of fourth uertex

= (x, y)

= (3, 5)

8.



As C is a midpoint of AB,

$$\left(\frac{x+x+1}{2}, \frac{4+y+2}{2}\right) = \left(\frac{3}{2}, \frac{5}{2}\right)$$

$$\left(\frac{2x+1}{2}, \frac{y+6}{2}\right) = \left(\frac{3}{2}, \frac{5}{2}\right)$$

$$\therefore \frac{2x+1}{2} = \frac{3}{2} \text{ and } \frac{y+6}{2}\ \frac{5}{2}$$

2x + 1 = 3 and y + 6 = 5

2x = 2 and y = 5 – 6

x = 1 and y – 1

Section B

9. Let y – coordinate by v

$\therefore$ x – coordinate = 2v

So, point P is (2v, v)

PQ = PR

$$\sqrt{(2-2v)^2+(-5-v)^2} = \sqrt{(3-2v)^2+(6-v)^2}$$

On squaring both sides, we get

$(2 - 2v)^2 + (-5 - v)^2 = (-3 - 2v) + (6 - v)^2$

$\therefore$ $4 + 4v^2 - 8v + 25 + v^2 + 10v$

$= 9 + 4v^2 = 12v + 36 + v^2 - 12v$

$\Rightarrow$ $5v^2 + 2v + 29 = 5v^2 + 45$

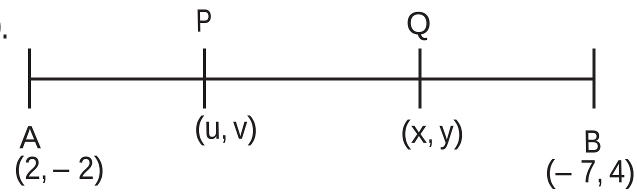
$\Rightarrow$ 2v = 45 –29

2v = 16

v = 8

So, Point P is (2v, v) i.e. (16, 8)

10.



Point P divides AB in ratio 1: 2

$$\text{So, P (u,v)} = \left(\frac{1(-7)+2(2)}{3}, \frac{1(4)+2(-2)}{3}\right)$$

$$= \left(\frac{-7+4}{3}, \frac{4-4}{3}\right)$$

= (– 1, 0)

Point Q divides AB in ratio 2 : 1

$$\text{So, Q (x, y)} = \left(\frac{2(-7)+1(2)}{3}, \frac{2(4)+11(-2)}{3}\right)$$

$$= \left(\frac{-14+2}{3}, \frac{8-2}{3}\right)$$

$$= \left(\frac{-12}{3}, \frac{6}{3}\right)$$

= (– 4, 2)

11. Let A(3, 0), B(6, 4) and C(–1, 3) be the given points

$AB = \sqrt{(6-3)^2+(4-0)^2}$

$\Rightarrow AB = \sqrt{(3)^2+(4)^2}$

$\Rightarrow AB = \sqrt{9+16}$

$\Rightarrow AB = \sqrt{25}$

$BC = \sqrt{(-1-6)^2+(3-4)^2}$

$\Rightarrow BC = \sqrt{(-7)^2+(-1)^2}$

$\Rightarrow BC = \sqrt{49+1}$

$\Rightarrow BC = \sqrt{50}$

$AC = \sqrt{(-1-3)^2+(3-0)^2}$

$\Rightarrow AC = \sqrt{(-4)^2 + (3)^2}$

$\Rightarrow AC = \sqrt{16+9}$

$\Rightarrow AC = \sqrt{25}$

$\Rightarrow AB^2 = \left(\sqrt{25}\right)^2$

$\Rightarrow AB^2 = 25$

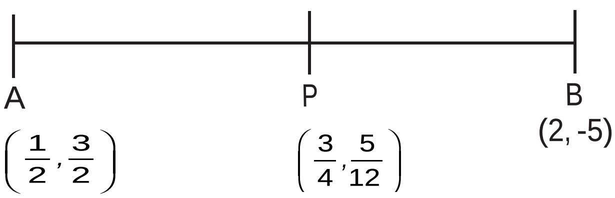
$\Rightarrow AC^2 = 25$

$\Rightarrow BC^2 = \left(\sqrt{50}\right)^2$

$\Rightarrow BC^2 = 50$

Since $AB^2 + AC^2 = BC^2$ and AB = AC

$\therefore$ ABC is a right angled isosceles triangle.

12. 

Let point P divides AB in ratio k : 1

So, $\left(\frac{3}{4}, \frac{5}{12}\right) = \left(\frac{2k+\frac{1}{2}}{k+1}, \frac{-5k+\frac{3}{2}}{k+1}\right)$

$\frac{3}{4} = \frac{2k+\frac{1}{2}}{k+1}$

$\Rightarrow 3k + 3 = 8k + 2$

$\Rightarrow 1 = 5k$

$k = \frac{1}{5}$

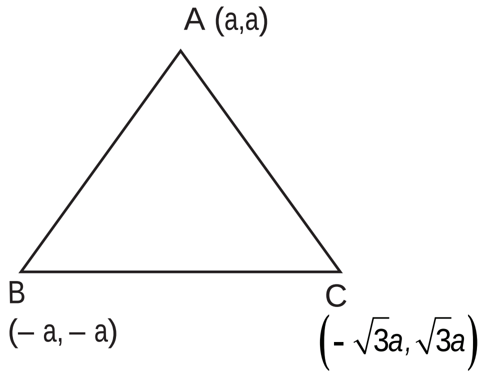
$\frac{5}{12} = \frac{-5k+\frac{3}{2}}{k+1}$

$5k + 5 = -60k + 18$

$65k = 13$

$k = \frac{1}{5}$

So, point P divides AB in ratio 1 : 5

13. 

$AB = \sqrt{(-a-a)^2 + (-a-a)^2}$

$= \sqrt{4a^2 + 4a^2}$

$= \sqrt{8a^2}$

$= 2\sqrt{2}a$ Units

$BC = \sqrt{(-\sqrt{3}a+a)^2 + (\sqrt{3}a+a)^2}$

$= \sqrt{3a^2 + a^2 - 2\sqrt{3}a^2 + 3a^2 + 2\sqrt{3}a^2}$

$= \sqrt{8a^2}$

$= 2\sqrt{2}a$

$AC = \sqrt{(-\sqrt{3}a-a)^2 + (\sqrt{3}a-a)^2}$

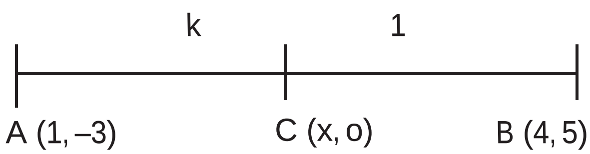
$= \sqrt{3a^2 + a^2 + 2\sqrt{3}a^2 + 3a^2 + a^2 - 2\sqrt{3}a^2}$

$= \sqrt{3a^2 + a^2 + 3a^2 + a^2}$

$= \sqrt{8a^2}$

$= 2\sqrt{2}a$

As AB = AC, ΔABC is an equilateral triangle.

14. 

Let point C (x, o) divides AB in ratio k : 1

So,

$(x, o) = \left(\frac{4k+1}{k+1}, \frac{5k-3}{k+1}\right)$

$\therefore \frac{5k-3}{k+1} = 0$

$5k - 3 = 0$

$k = \frac{3}{5}$

So, x – axis divides the line segment joining point (1, – 3) and (4, 5) in ratio 3 : 5

15. $\sqrt{(9-x)^2+(10-4)^2} = 10$

$81 + x^2 - 18x + 36 = 100$

$x^2 - 18x + 17 = 0$

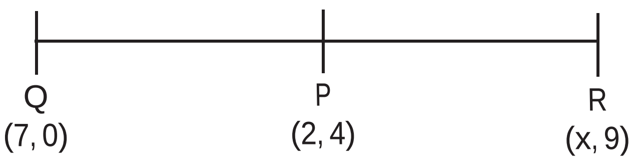
$x^2 - 17x - x + 17 = 0$

$x(x-17) - 1(x-17) = 0$

$(x-1)(x-17) = 0$

$x = 1, 17$

16.



PQ = PR

$\Rightarrow \sqrt{(2-7)^2+(4-2)^2} = \sqrt{(x-2)^2+(9-4)^2}$

$\Rightarrow \sqrt{25+16} = \sqrt{x^2+4-4x+25}$

$\Rightarrow \sqrt{41} = \sqrt{x^2-4x+29}$

On squarring both sides, we get

$41 = x^2 - 4x + 29$

$0 = x^2 - 4x - 12$

$0 = x^2 - 6x + 2x - 12$

$0 = x(x-6) + 2(x-6)$

$0 = (x+2)(x-6)$

$x = -2$ or 6

$PQ = \sqrt{(7-2)^2+(0-4)^2}$

$= \sqrt{5^2+(-4)^2}$

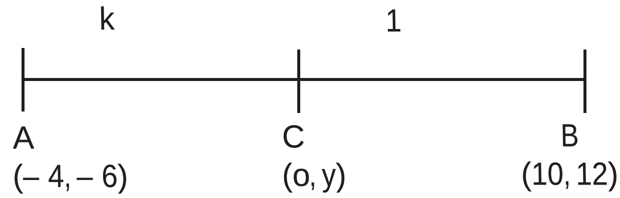
$= \sqrt{25+16}$

$= \sqrt{41}$

Section C

17. Let y – axis divides the line segment joining the points (– 4, – 6) and (10, 12) in ratio k : 1

Point on y – axis must be of form (o, y)



$(o, y) = \left(\frac{10k+(-4)}{k+1}, \frac{12k-6}{k+1}\right)$

$(o, y) = \left(\frac{10k-4}{k+1}, \frac{12k-6}{k+1}\right)$

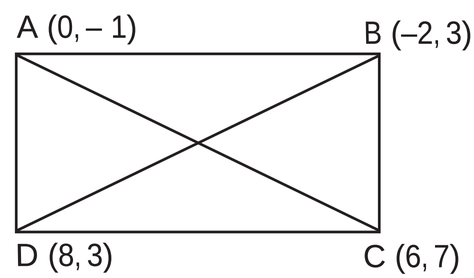
$\Rightarrow \frac{10k-4}{k+1} = 0$

$\Rightarrow 10k = 4$

$k = \frac{2}{5}$

So, ratio is 2 : 5

18.



$AB = \sqrt{(-2-0)^2+(3+1)^2}$

$= \sqrt{4+16}$

$= \sqrt{20}$

$= 2\sqrt{5}$ Unit

$CD = \sqrt{(-8-0)^2+(3+1)^2}$

$= \sqrt{4+16}$

$= \sqrt{20}$

$= 2\sqrt{5}$ Unit

$\therefore$ AB = CD

$AD = \sqrt{(-8-0)^2+(3+1)^2}$

$= \sqrt{64+16} = \sqrt{80} = 4\sqrt{5}$ Units

$BC = \sqrt{(6+2)^2 + (7-3)^2}$

$= \sqrt{64+16}$

$= \sqrt{80}$

$= 4\sqrt{5}$ Units

$\therefore$ AD = BC

As AB = CD and AD = BC,

So, ABCD is a parallelogram

$AC = \sqrt{(6-0)^2 + (7+1)^2}$

$= \sqrt{36+64}$

$= \sqrt{100}$

= 10 Units

$BD = \sqrt{(8+2)^2 + (3-3)^2}$

$= \sqrt{100}$

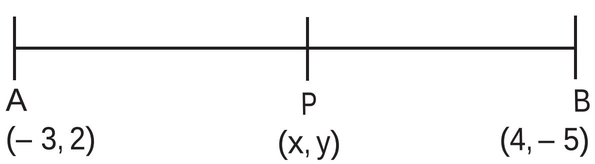
= 10 Units

So, AC = BD

$\therefore$ ABCD is a parallelogram in which both diagonals are equal.

So, ABCD is a rectangle.

19.



As point P is equidistant from A and B,

AP = BP

$\sqrt{(x+3)^2 + (y-2)^2} = \sqrt{(4-x)^2 + (-5-y)^2}$

On squarring both sides, we get

$(x + 3)^2 + (y - 2)^2 = (4 - x)^2 + (-5 - y)^2$

$\underline{x^2} + \underline{9} + \underline{6x} + \underline{y^2} + \underline{4} - \underline{4y}$

$= 16 + \underline{x^2} - \underline{8x} + 25 + \underline{y^2} + \underline{10}y$

$14x - 14y + 13 = 41$

$14x - 14y - 28 = 0$

$x - y = 2$

$\therefore \quad y = x - 2$

20.

Point P divides AB in ratio 1 : 2

So,

$p\,(p, -2) = \left(\dfrac{1(1)+2(3)}{3}, \dfrac{1(2)+2(-4)}{3}\right)$

$p\,(p, -2) = \left(\dfrac{7}{3}, -2\right)$

$\therefore\ p = \dfrac{7}{3}$

Point Q divides AB in ratio 2 : 1

So,

$Q\left(\dfrac{5}{3}, q\right) = \left(\dfrac{2(1)+1(3)}{3}, \dfrac{2(2)+1(-4)}{3}\right)$

$Q\left(\dfrac{5}{3}, q\right) = \left(\dfrac{5}{3}, 0\right)$

$\therefore \quad q = 0$

21. As the points A (3p + 1, p), B (p + 2, p – 5) and C (p + 1, – p) are collinear,

area of ΔABC = 0

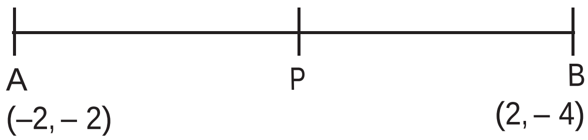
i. e. $\dfrac{1}{2}$ [(3p + 1) (p – 5 + p) + (p + 2) (– p – p) + (p + 1) (p – p + 5)] = 0

$\Rightarrow$ [(3p + 1) (2p – 5)–2p (p + 2) + 5 (p + 1)] = 0

$\Rightarrow$ $[6p^2 - 15p + 2p - 5 - 2p^2 - 4p + 5p + 5] = 0$

$\Rightarrow$ $[4p^2 - 12p] = 0$

p = 0, 3

22. 

$AP = \frac{3}{7} AB$

$\Rightarrow \quad AP = \frac{3}{7}(AP + BP)$

$\Rightarrow \quad 7 AP = 3 AP + 3 BP$

$\Rightarrow \quad 4 AP = 3 BP$

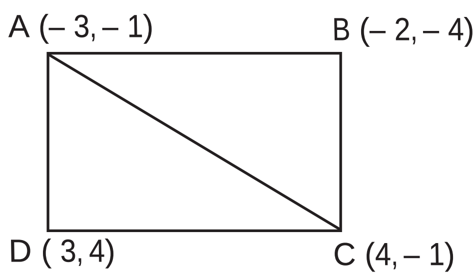
$\Rightarrow \quad \frac{AP}{PB} = \frac{3}{4}$

Let point P be (x, y), using section formula,

$$(x, y) = \left(\frac{3(2)+4(-2)}{7}, \frac{3(-4)+4(-2)}{7}\right)$$

$$= \left(\frac{6-8}{7}, \frac{-12-8}{7}\right)$$

$$= \left(\frac{-2}{7}, \frac{-20}{7}\right)$$

23. 

Join AC

Area of ΔACD

$= \frac{1}{2}$ [–3 (– 1 – 4) + 4 (4 + 1) + 3 (–1+1)]

$= \frac{1}{2}$ [–3 (– 5) + 20]

$= \frac{1}{2}$ [15 + 20]

$= \frac{35}{2}$ sq. Units

Area of ΔACD

$= \frac{1}{2}$ [(–3) (– 4 – 1) – 2 (– 1 + 1) + 4 (–1+4)]

$= \frac{1}{2}$ [–3 (–3) – 2 (0) + 4 (3)]

$= \frac{1}{2}$ [9 + 12] $= \frac{21}{2}$ sq. Units

So, area of quadrilateral ABCD

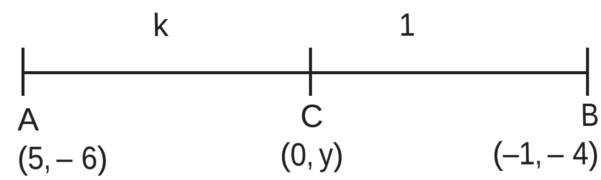
= area of ΔACD + area of ΔABC

$= \frac{35}{2} + \frac{21}{2}$

$= \frac{56}{2}$

= 28 sq. Units

24. Let y – axis divides the line segment joining points A (5, – 6), B (– 1, – 4) in ratio k : 1. Point on y – axis is of from (0, y).



By section formula,

$$(0, y) = \left(\frac{-k+5}{k+1}, \frac{-4k-6}{k+1}\right)$$

$$0 = \frac{-k+5}{k+1}$$

k = 5

So, y – axis divides AB in ratio 5 : 1

Also, $y = \frac{-4k-6}{k+1}$

$= \frac{-20-6}{5+1}$

$= \frac{-26}{6}$

$= \frac{-13}{3}$

So, C (0, y) = $(0, \frac{-13}{3})$

Section D

25. Consider points $(x_1, y_1) = (t, t-2)$

$= (x_2, y_2) = (t + 2, t - 2)$

$= (x_3, y_3) = (t + 3, t)$

Area of triangle

$= \frac{1}{2} [x_1 (y_2, y_3) + x_2 (y_3, y_1) + x_3 (y_1, - y_2)]$

$= \frac{1}{2} [t(t - 2 - t) + (t + 2)(t - t + 2) + (t + 3)(t - 2 - t + 2)]$

$= \frac{1}{2} [t(-2) + (t + 2)(2)]$

$= \frac{1}{2} [-2t + 2t + 4]$

$= \frac{1}{2} (4)$

= 2 sq. Units

So, area of triangle is independent of t.

26. $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{3}$

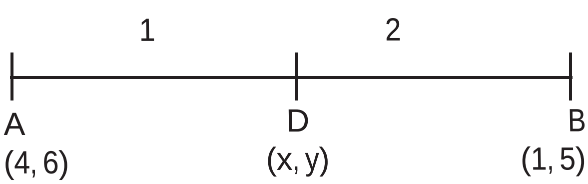
$\Rightarrow \frac{AD}{AB} = \frac{AC}{AE} = \frac{3}{1}$

$\Rightarrow \frac{AD}{AB} - 1 = \frac{AC}{AE} - 1 = 3 - 1$

$\Rightarrow \frac{BD}{AD} = \frac{CE}{AE} = 2$

$\therefore \frac{AD}{BD} = \frac{AE}{CE} = \frac{1}{2}$

For coordinates of D



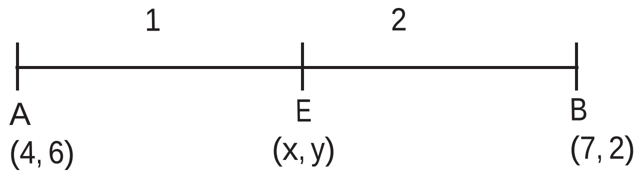
By section formula,

$(x, y) = \left(\frac{1(1) + 2(4)}{3}, \frac{1(5) + 2(6)}{3}\right)$

$(x, y) = \left(\frac{9}{3}, \frac{17}{3}\right)$

$= \left(3, \frac{17}{3}\right)$

For coordinates of E

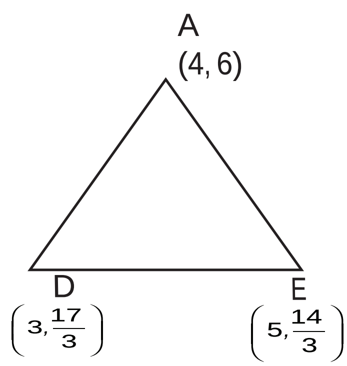


By section formula,

$(x, y) = \left(\frac{1(7) + 2(4)}{3}, \frac{1(2) + 2(6)}{3}\right)$

$= \left(\frac{7 + 8}{3}, \frac{2 + 12}{3}\right)$

$= \left(5, \frac{14}{3}\right)$



ar ΔADE

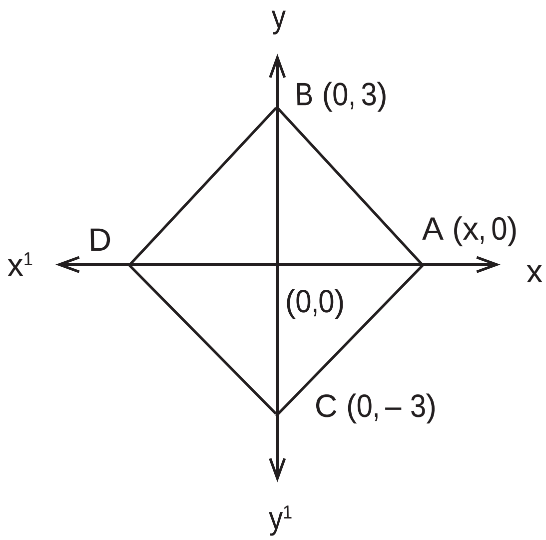
$= \frac{1}{2} [4\left(\frac{17}{3}, \frac{-14}{3}\right) + 3\left(\frac{14}{3}, -6\right) + 5\left(6 - \frac{17}{3}\right)]$

$= \frac{1}{2} [4\left(\frac{3}{3}\right) + 3\left(\frac{14 - 18}{3}\right) + 5\left(\frac{18 - 17}{3}\right)]$

$= \frac{1}{2} [4 + (-4) + \frac{5}{3}]$

$= \frac{5}{6}$ sq. Units

27.



Let coordinates of B be (0, y)

As (0, 0) is a Midpoint of BC

$\therefore$ (0, 0) = $\left(\frac{0+0}{2}, \frac{y-3}{2}\right)$

(0, 0) = $\left(\frac{0}{2}, \frac{y-3}{2}\right)$

(0, 0) = $\left(0, \frac{y-3}{2}\right)$

$\frac{y-3}{2} = 0$

y = 3

So, point B is (0, 3)

Let coordinates of point A be (x, 0)

Using distance formula,

AB = $\sqrt{(x-0)^2+(0-3)^2}$

= $\sqrt{x^2+9}$

BC = $\sqrt{(0-0)^2+(-3-3)^2}$

= $\sqrt{36}$

= 6

As ΔABC is equilateral,

AB = BC

i.e. $\sqrt{x^2+9}$ = 6

$x^2 + 9 = 36$

$x^2 = 27$

$x = \pm 3\sqrt{3}$

$\therefore$ Coordinates of point A are $(3\sqrt{3}, 0)$

As BACD is a rhombus and diagonals of rhombus bisect each other. So, 0D = 0A = 3 $\sqrt{3}$ units

$\therefore$ Point D is $(-3\sqrt{3}, 0)$

28. Area of triangle = 5 sq. units

As third vertex leis on y = x + 3,

So, it must be of form (x, x + 3)

Let $(x_1, y_1) = (2, 1)$

$(x_2, y_2) = (3, -2)$

$(x_3, y_3) = (x, x + 3)$

Area of triangle

$= \frac{1}{2}\ [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$5 = \frac{1}{2}[2(-2 - x - 3) + (x + 3 - 1) + x(1 + 2)]$

10 = [2 (– 5 – x) + 3 (x + 2) + 3x]

10 = [– 10 2x + 3x + 6 + 3x]

10 = [4x – 4]

$\therefore$ ± 10 = 4x – 4

4x – 4 = 10	4x – 4 = – 10
4x = 10	4x = – 6
$x = \frac{7}{2}$	$x = \frac{-3}{2}$
So, third vertex is	So, third vertex is
(x, x + 3)	(x, x + 3)
$= \left(\frac{7}{2}, \frac{7}{2}+3\right)$	$= \left(\frac{-3}{2}, \frac{-3}{2}+3\right)$
$= \left(\frac{7}{2}, \frac{13}{2}\right)$	$= \left(\frac{-3}{2}, \frac{3}{2}\right)$

29. Let $(x_1, y_1) = (a, a^2)$

$(x_2, y_2) = (b, b^2)$

$(x_3, y_3) = (c, c^2)$

Consider Area of triangle

$= \frac{1}{2}$ [x_1 (y_2, – y_3) + x_2 (y_3, y_1) + x_3 (y_1, – y_2)]

$= \frac{1}{2}$ [a (b² – c²) + b (c² – a²) + c (a² – b²)]

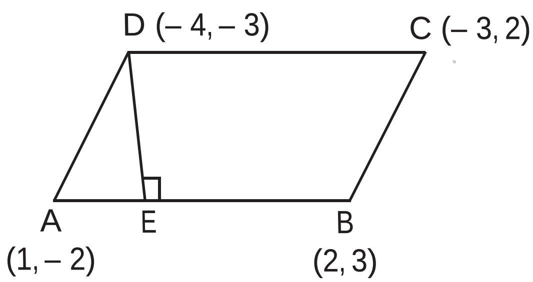
$= \frac{1}{2}$ [ab² – ac² + bc² – a²b + a²c – b²c]

$= \frac{1}{2}$ [ab (b – a) + ac (a – c) + bc (c – b)]

Here, it is clear that area of triangle is 0 if a = b = c

but it is given that a ≠ b ≠ c

30.



Let be the height of parallelogram ABCD.

For ΔABD,

Let (x_1, y_1) = (1, – 2)

(x_2, y_2) = (2, 3)

(x_3, y_3) = (– 4, – 3)

area of ΔABD

$= \frac{1}{2}$ [x_1 (y_2, – y_3) + x_2 (y_3, y_1) + x_3 (y_1, – y_2)]

$= \frac{1}{2}$ [1 (3 + 3) + 2 (– 3 + 2) – 4 (– 2 – 3)]

$= \frac{1}{2}$ [6 – 2 + 20]

= 12 sq. units

For ΔBCD,

Let (x_1, y_1) = (2, 3)

(x_2, y_2) = (– 3, 2)

(x_3, y_3) = (– 4, – 3)

area of ΔBCD

$= \frac{1}{2}$ [x_1 (y_2, – y_3) + x_2 (y_3, y_1) + x_3 (y_1, – y_2)]

$= \frac{1}{2}$ [2 (2 + 3) + – 3 (– 3 – 3) – 4 (3 – 2)]

$= \frac{1}{2}$ [10 + 18 – 4]

= 12 sq. units

Area of parallelogram ABCD

= area of ΔABC + area of ΔBCD

= 12 + 12

= 24 sq. units

We know that area of parallelogram

= base x height

24 = AB x height

By Distance formula,

AB = $\sqrt{(2-1)^2 + (3+2)^2}$

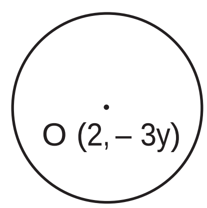
= $\sqrt{1+25}$

= $\sqrt{26}$ units

∴ 24 = $\sqrt{26}$ x height

height = $\frac{24}{\sqrt{26}}$ units = $\frac{12}{13}$ $\sqrt{26}$ units

31.



Let the center be 0 (2, – 3y)

As points A and B lie on a circle,

AO = BO

$\sqrt{(2+1)^2 + (-3y - y)^2} = \sqrt{(2-5)^2 + (-3y-7)^2}$

$\sqrt{9 + 16y^2} = \sqrt{9 + 9y^2 + 49 + 42y}$

On squarraig both sides, we get

9 + 16y² = 9y² + 42y + 58

$7y^2 - 42y - 49 = 0$

$y^2 - 6y - 7 = 0$

$y^2 - 7y + y - 7 = 0$

$y(y - 7) + (y - 7) = 0$

$(y + 1)(y - 7) = 0$

$y = -1, 7$

When y = 1

A = (– 1, y) = (– 1, – 1)

O = (2, 3)

So,

radius = AO

$= \sqrt{(2+1)^2 + (3+1)^2}$

$= \sqrt{9+16}$

$= \sqrt{25}$

= 5 units

When y = 7

A = (–1, y)

= (–1, 7)

O = (2, – 3y)

= (2, – 21)

So,

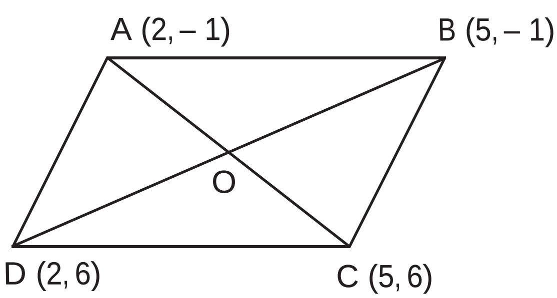
ratio = AO

$= \sqrt{(2+1)^2 + (-2-7)^2}$

$= \sqrt{9+784}$

$= \sqrt{793}$ units

32.



By distance formula,

$AC = \sqrt{(5-2)^2 + (6+1)^2}$

$= \sqrt{9+49} = \sqrt{58}$ units

$BD = \sqrt{(2-5)^2 + (6+1)^2}$

$= \sqrt{9+49}$

$= \sqrt{58}$

So, AC = BD

Also, By Midpoint formula,

Midpoint of AC $= \left(\frac{2+5}{2}, \frac{-1+6}{2}\right)$

$= \left(\frac{7}{2}, \frac{5}{2}\right)$

Midpoint of BD $= \left(\frac{5+2}{2}, \frac{-1+6}{2}\right)$

$= \left(\frac{7}{2}, \frac{5}{2}\right)$

So, Midpoint of AC = Midpoint of BD.

So, AC and BD bisects each other.

WORKSHEET 2

Section A

1. Let P (x, y) be the point equidistant from the point A (5, 1), B (–3, – 7) and C (7, – 1)

$\therefore$ PA = PB = PC

PA = PB

$\Rightarrow \sqrt{(5-x)^2 + (1-y)^2}$

$= \sqrt{(-3-x)^2 + (-7-y)^2}$

$\Rightarrow \sqrt{25 + x^2 - 10x + 1 + y^2 - 2y}$

$= \sqrt{9 + x^2 + 6x + 49 + y^2 + 14y}$

On squarring both sides we get

$x^2 + y^2 - 10x - 2y + 26$

$= x^2 + y^2 + 6x + 14y + 58$

$0 = 16x + 16y + 32$

$x + y = -2$ ①

PB = PC

$\Rightarrow \sqrt{(-3-x)^2 + (-7-y)^2}$

$= \sqrt{(7-x)^2 + (-1-y)^2}$

$\Rightarrow \sqrt{9 + x^2 + 6x + 49 + y^2 + 14y}$

$= \sqrt{49 + x^2 + 14x + 1 + y^2 + 2y}$

On squarring both sides, we get

$x^2 + y^2 + 6x + 14y + 58$

$= x^2 + y^2 - 14x + 2y + 50$

$20x + 12y + 8 = 0$

$5x + 3y = -2$ ②

From ①, we get

$x = -2 - y$

On potting in ②, we get

$5(-2 - y) + 3y = -2$

$-10 - 5y + 3y = -2$

$-2 = 8$

$sy = -4$

So, $x = -2 - y$

$= -2 + 4$

$= 2$

So, point (2, – 4) is equidistant

From point A (5, 1), B (–3, – 7) and C (7, – 1)

2. Reflexion of (–3, 4) in X – axis (Q) = (–3, –4)

Reflexion of (– 3, 4) in Y – axis (R) = (3, 4)

So, by distance formula,

$QR = \sqrt{(3+3)^2 + (4+4)^2}$

$= \sqrt{36 + 64}$

$= \sqrt{100}$

= 10 units

3. As point (3, a) lies on line

$2x - 3y + 5 = 0$

$\therefore \quad 6 - 3a + 5 = 0$

$3a = 11$

$a = \frac{11}{3}$

4. By Distance formula,

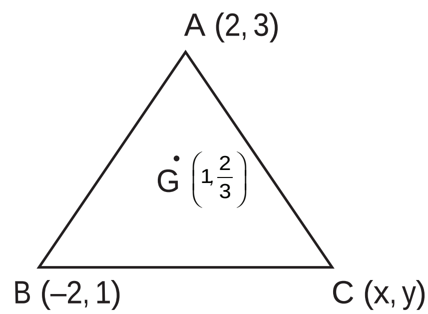
Distance $= \sqrt{(0+6)^2 + (0-8)^2}$

$= \sqrt{(36 + 64}$

$= \sqrt{100}$

= 10 units

5.



Let $(x_1, y_1) = (2, 3)$

$(x_2, y_2) = (-2, 1)$

$(x_3, y_3) = (x, y)$

Centroid (G) = $\left(1, \frac{2}{3}\right)$

We know that

Centroid = $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

$\left(1, \frac{2}{3}\right) = \left(\frac{2 - 2 + x}{3}, \frac{3 + 1 + y}{3}\right)$

$\left(1, \frac{2}{3}\right) = \left(\frac{x}{3}, \frac{4 + y}{3}\right)$

$\Rightarrow \quad 1 = \frac{x}{3}$ and $\frac{2}{3} = \frac{4 + y}{3}$

$\Rightarrow \quad x = 3$ and $y = -2$

6. Let $(x_1, y_1) = (k, 2k)$

$(x_2, y_2) = (3k, 3k)$

$(x_3, y_3) = (3, 1)$

Since the points are collinear, area of triangle is zero

i.e.

$\frac{1}{2}$ $[\, x_1 (y_2, - y_3) + x_2 (y_3, y_1) + x_3 (y_1, - y_2) \,] = 0$

$[\, k\ (3k - 1) + 3k\ (1 - 2k) + 3\ (2k - 3k) \,] = 0$

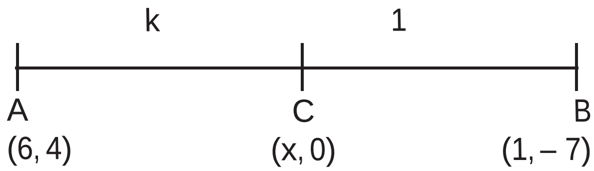
$[\, 3k^2 - k + 3k - 6k^2 - 3k \,] = 0$

$[-3k^2 - k] = 0$

$k(3k+1) = 0$

$k = \frac{-1}{3}$

7.



Let the x – axis divides AB in ratio k : 1

Point on x – axis must be of form (x, 0), so, by section formula

$$(x, 0) = \left(\frac{k+6}{k+1}, \frac{7k+4}{k+1}\right)$$

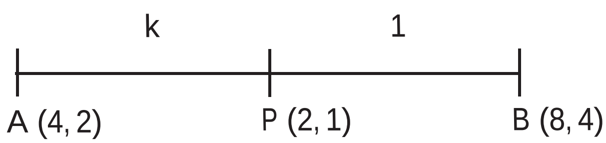
$$(x, 0) = \left(\frac{6+k}{k+1}, \frac{-7k+4}{k+1}\right)$$

$$\therefore \quad \frac{-7k+4}{k+1} = 0$$

$$k = \frac{4}{7}$$

So, x – axis divides line AB in ratio 4 : 7.

8.



Let AP : PB = K : 1

By section formula,

$$P(2, 1) = \left(\frac{8k+4}{k+1}, \frac{4k+2}{k+1}\right)$$

$$\therefore \quad 2 = \frac{8k+4}{k+1}, \; 1 = \frac{4k+2}{k+1}$$

$8k + 4 = 2k + 2$

$6k = -2$

$$k = \frac{-2}{6} = \frac{-1}{3}$$

$$\therefore \quad \frac{AB}{PB} = \frac{-1}{3}$$

$$\frac{PB}{AP} = -3$$

$$\frac{PB}{AP} + 1 = -3 + 1$$

$$\frac{AP+PB}{AP} = -2$$

$$\frac{AB}{PB} = -2$$

$$\frac{AP}{AB} = \frac{-1}{2}$$

$$AP = \frac{-1}{2} AB$$

9. Consider the two points P(a sin α, –b cos α) and Q (–a cos α, b sin α).

We need to find the distance between P and Q

Let d be the distance PQ.

Thus, by distance formula

$$d = \sqrt{(a \sin\alpha + a\cos\alpha)^2 + (-b\cos\alpha - b\sin\alpha)^2}$$

$$= \sqrt{a^2(\sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha) + b^2(\sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha)}$$

$$= \sqrt{a^2(1 + \sin 2\alpha) + b^2(1 + \sin 2\alpha)}$$

$[\because \sin^2\alpha + \cos^2\alpha = 1 \text{ and } \sin 2\alpha = 2\sin\alpha\cos\alpha]$

$$\therefore d = \sqrt{(a^2 + b^2)(1 + \sin 2\alpha)}$$

10. We have to write the condition of three points.

If three points (x_1, y_1), (x_2, y_2) and (x_3, y_3) are collinear, then they will not form a triangle.

In other words, the triangle having vertices (x_1, y_1), (x_2, y_2) and C (x_3, y_3) will have area 0.

The formula to calculate the area of a triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is,

$$A = \frac{1}{2}\begin{vmatrix} x_1 - x_2 & y_1 - y_2 \\ x_2 - x_3 & y_2 - y_3 \end{vmatrix}$$

$$A = \frac{1}{2}|(x_1 - x_2)(y_2 - y_3) - (x_2 - x_3)(y_1 - y_2)$$

Therefore,

$$\frac{1}{2}|\, x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| = 0$$

$$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$$

Therefore, the condition of collinearity of (x_1, y_1), (x_2, y_2) and C (x_3, y_3) is

$$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$$

Section B

11. Let the vertices of triangle be $(x_1, y_1) = (-3, 1)$, $(x_2, y_2) = (0, -2)$ and (x_3, y_3)

Centroid of triangle (x, y) = (0, 0)

We Know that

Centroid of triangle

$$(x, y) = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

i.e. $$(0, 0) = \left(\frac{-3 + 0 + x_3}{3}, \frac{1 - 2 + y_3}{3}\right)$$

$$\Rightarrow \frac{-3 + x_3}{3} = 0, \frac{-1 + y_3}{3} = 0$$

$$\Rightarrow x_3 = 3, \quad y_3 = 1$$

So, third vertex is $(x_3, y_3) = (3, 1)$

12. The required ratio would be k : 1.

The coordinates of the point of divisions will be

$$\left(\frac{3k - 4}{k + 1}, \frac{7 + 5}{k + 1}\right)$$

The point which we have identified is on y axis and there the point is zero on x coordinate.

So, $\frac{3k - 4}{k + 1} = 0$

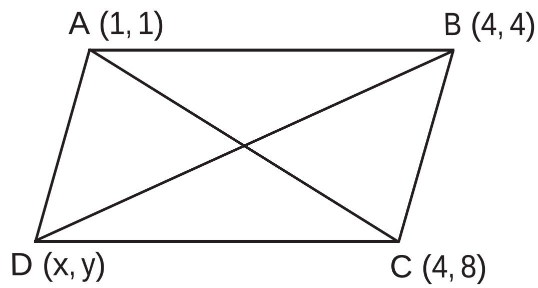
$$3k - 4 = 0$$

$$k = \frac{4}{3}$$

So the required ratio is

$k = \frac{4}{3}$ or 4 : 3

13.



We know that diagonals of parallelogram bisect each other

$$\therefore \left(\frac{1+4}{2}, \frac{1+8}{2}\right) = \left(\frac{x+4}{2}, \frac{y+8}{2}\right)$$

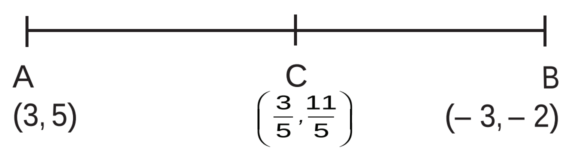
$$(5, 9) = (x + 4, y + 4)$$

$$x + 4 = 5, \quad y + 4 = 9$$

$$x = 1, \qquad y = 5$$

So, fourth vertex is (1, 5).

14. Let the point C $\left(\frac{3}{5}, \frac{11}{5}\right)$ divide the line segment joining point A (3, 5) and B (– 3, – 2) in ratio k : 1.



By section formula,

$$\left(\frac{3}{5},\frac{11}{5}\right) = \left(\frac{-3k+3}{k+1},\frac{-2k+5}{k+1}\right)$$

$$\therefore \quad \frac{-3k+3}{k+1} = \frac{3}{5}$$

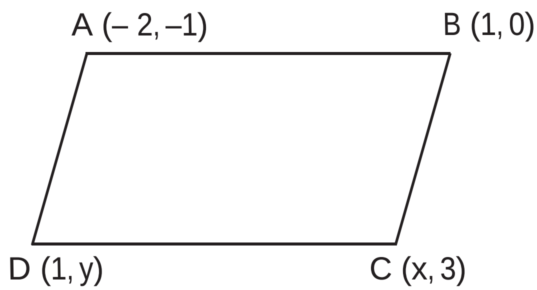
$$5(-3k+3) = 3(k+1)$$

$$-15k+15 = 3k+3$$

$$12 = 18k$$

$$k = \frac{2}{3}$$

15.



We know that diagonals of parallelogram bisect each other

$$\therefore \quad \left(\frac{-2k+x}{2},\frac{-1+3}{2}\right) = \left(\frac{1+1}{2},\frac{y+0}{2}\right)$$

$$\Rightarrow \quad \left(\frac{-2+x}{2},1\right) = \left(1,\frac{y}{2}\right)$$

$$\therefore \quad \frac{-2+x}{2} = 1, \; 1 = \frac{y}{2}$$

$$x = 4, \; y = 2$$

16. Given the vertices of a ΔABC are right - angled at A.

$\therefore AB^2 + AC^2 = BC^2$

$AB^2 = (-2 - 0)^2 + (a - 3)^2 = 4 + (a - 3)^2$

$BC^2 = (-1 + 2)^2 + (4 - a)^2 = 1 + (4 - a)^2$

$AC^2 = (-1 - 0)^2 + (4 - 3)^2 = 1 + 1 = 2$

Since $AB^2 + AC^2 = BC^2$

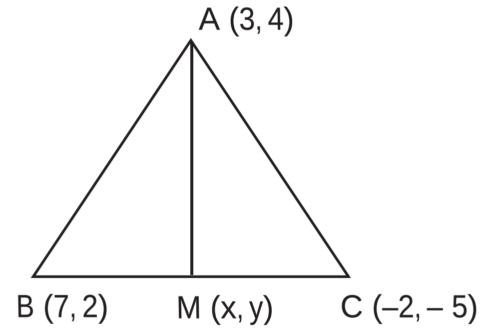
$4 + (a - 3)^2 + 2 = 1 + (4 - a)^2$

$4 + a^2 + 9 - 6a + 2 = 1 + 16 + a2 - 8a$

$2a = 2$

$\therefore a = 1.$

17.



By midpoint formula,

$$M(x, y) = \left(\frac{7-2}{2},\frac{2-5}{2}\right)$$

$$= \left(\frac{5}{2},\frac{-3}{2}\right)$$

By Distance formula,

$$AM = \sqrt{\left(\frac{5}{2}-3\right)^2 + \left(\frac{-3}{2}-4\right)^2}$$

$$= \sqrt{\left(\frac{-1}{2}\right)^2 + \left(\frac{-3-8}{2}\right)^2}$$

$$= \sqrt{\frac{1}{4}+\frac{121}{4}}$$

$$= \sqrt{\frac{121}{4}} = \sqrt{\frac{61}{2}}$$

18. As point A (x, y) is equidistant from B (6, – 1) and C (2, 3)

$\therefore$ AB = AC

$$\sqrt{(6-x)^2 + (-1-y)^2} = \sqrt{(2-x)^2 + (3-y)^2}$$

On squarring both sides, we get

$(6 - x)^2 + (-1 - y)^2 = (2 - x)^2 + (3 - y)^2$

$36 + \underline{x^2} - 12x + 1 + \underline{y^2} + 2y = 4 + \underline{x^2} - 4x + 9 + \underline{y^2} - 6y$

$\therefore -12x + 2y + 37 = -4x - 6y + 13$

$\Rightarrow \quad 0 = 8x - 8y - 24$

$\Rightarrow \quad 8x - 8y = 24$

$\Rightarrow \quad x - y \quad = 3$

$\Rightarrow \quad x \quad = y + 3$

19. As the points A (2, 1) and B (1, 2) are equidistant from the point C (x, y),

BC = AC

$$\sqrt{(x-1)^2 + (y+2)^2} = \sqrt{(x-2)^2 + (y-1)^2}$$

On squarring both sides, we get

$(x - 1)^2 + (y + 2)^2 = (x - 2)^2 + (y - 1)^2$

$\underline{x^2} + 1 - 2x + \underline{y^2} + 4 + 4y = \underline{x^2} + 4 - 4x + \underline{y^2} + 1 - 2y$

$-2x + 4y + 5 = -4x - 2y + 5$

$2x + 6y = 0$

$x + 3y = 0$

20. Let the vertices of triangle be

$(x_1, y_1) = (k, 2k)$

$(x_2, y_2) = (3k, 3k)$

$(x_3, y_3) = (3, 1)$

Area of triangle

$= \frac{1}{2}[x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)]$

$= \frac{1}{2}[5 (7 + 4) + 4 (-4 - 2) + 7 (2 - 7)]$

$= \frac{1}{2}[5 (11) + 4 (-6) + 7 (-5)]$

$= \frac{1}{2}[55 - 24 - 35]$

$= \frac{1}{2}[55 - 59]$

$= \frac{4}{2} = 2$ sq. units

Section C

21. PA=PB

Take square both side

$PA^2 = PB^2$

Now use distance formula,

$\{x - (a + b)\}^2 + \{y - (b - a)\}^2 = \{x - (a - b)\}^2 + \{y - (a + b)\}^2$

$\Rightarrow \quad x^2 + (a + b)^2 - 2x(a + b) + y^2 + (b - a)^2 - 2y(b - a)y = x^2 + (a - b)^2 - 2x(a - b) + y^2 + (a + b)^2 - 2y(a + b)$

$\Rightarrow \quad 2x(a - b) - 2x(a + b) = 2y(b - a) - 2y(a + b)$

$\Rightarrow \quad 2x\{a - b - a - b\} = 2y\{b - a - a - b\}$

$\Rightarrow \quad 2x(-2b) = 2y(-2a)$

$\Rightarrow \quad bx = ay$

Hence proved.

22. Any point on the x -axis will be the form A(x,0).Let this point divides the line segment

Joining (3, –2) and (–7, –1) in the ratio m:n internally.

Thus the coordinate of A is

$$\left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}\right)$$

where (3, –2) and (–7, –1) are (x_1, y_1) and (x_2, y_2) respectively.

Thus the coordinates of

$$A = \frac{(m(-7) + n(3))}{m + n}, \frac{m(-1) + n(-2)}{m + n} = (x,0)$$

Here the y coordinates of A is Zero.

Thus $\left(\frac{-m + (-2n)}{m + n}\right) = 0.$

Hence $-m - 2n = 0.$

$\Rightarrow -m = 2n$

$\Rightarrow \frac{m}{n} = \frac{2}{-1}.$

$\Rightarrow m : n = 2 : -1$ internally

Thus $m : n = 2 : 1$ externally.

23. P (8, 4) — R (5, 1) — Q (x, y)

By mid-point formula,

$$(5, 1) = \left(\frac{8 + x}{2}, \frac{4 + y}{2}\right)$$

$$5 = \frac{8+x}{2}, 1 = \frac{4+y}{2}$$

x + 8 = 10, y + 4 = 2

x = 2, y = – 2

So, Coordinates of Q = (x, y)

= (2, –2)

24. Let points be

$(x_1, y_1) = (c, a+b)$

$(x_2, y_2) = (b, b+c)$

$(x_3, y_3) = (a, a+c)$

Area of triangle

$$= \frac{1}{2}[x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)]$$

$$= \frac{1}{2} [c (b+c-a-c) + a (a+c-a-b) + b (a+b-b-c)]$$

$$= \frac{1}{2} [c (b-a) + a (c-6) + b (a-c)]$$

$$= \frac{1}{2} [bc - ac + ac - ab + ab - bc]$$

= 0

As area of triangle = 0

So, points A, B and C are collinear.

25. Let the point be

$(x_1, y_1) = (a, 0)$

$(x_2, y_2) = (0, b)$

$(x_3, y_3) = (1, 1)$

Points are collinear, if area of triangle = 0

i.e. $\frac{1}{2}[x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)] = 0$

$\Rightarrow \frac{1}{2} [a (b-1) + 0 (1+0) + 1 (0-b)] = 0$

$\Rightarrow \frac{1}{2} [ab - a - b] = 0$

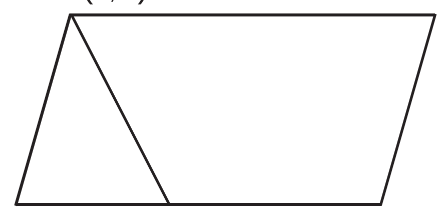
$\Rightarrow$ ab – a – b = 0

$\Rightarrow$ ab = a + b

$\Rightarrow 1 = \frac{a}{ab} + \frac{b}{ab}$

$\Rightarrow 1 = \frac{1}{a} + \frac{1}{b}$

26.



We know that diagonals of parallelogram bisect each other.

$\therefore$ Midpoint of AC = midpoint of BD

So, by midpoint formula,

$$\left(\frac{6+9}{2}, \frac{1+4}{2}\right) = \left(\frac{x+8}{2}, \frac{y+2}{2}\right)$$

i.e. $\left(\frac{15}{2}, \frac{5}{2}\right) = \left(\frac{x+8}{2}, \frac{y+2}{2}\right)$

i.e. x + 8 = 15, y + 2 = 5

x = 7, y = 3

So, point D = (7, 3)

Again, by midpoint formula,

$$E (u, v) = \left(\frac{6+9}{2}, \frac{1+4}{2}\right)$$

$$= \left(\frac{7+9}{2}, \frac{3+4}{2}\right) = \left(\frac{16}{2}, \frac{7}{2}\right)$$

$$= \left(8, \frac{7}{2}\right)$$

For area of ΔADE

Let $(x_1, y_1) = (c, a+b)$

$(x_2, y_2) = (b, b+c)$

$(x_3, y_3) = (a, a+c)$

area of ΔADE

$$= \frac{1}{2} [x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)]$$

$$= \frac{1}{2}\left[6\left(3-\frac{7}{2}\right)+7\left(\frac{7}{2}-1\right)+8(1-3)\right]$$

$$= \frac{1}{2}\left[6\left(\frac{-1}{2}\right)+7\left(\frac{5}{2}\right)+8(-2)\right]$$

$$= \frac{1}{2}\left[-3+\frac{35}{2}-16\right]$$

$$= \frac{1}{2}\left[\frac{35}{2}-19\right]$$

$$= \frac{1}{2}\left[\frac{35-38}{2}\right]$$

$$= \frac{3}{4}$$

27. Given ,

Points = (p + 1, 2p – 2), (p – 1, p) and (p – 3, 2p – 6)

For the given points (x_1, y_1), (x_2, y_2) and (x_3, y_3) to be collinear then

$[x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_3)] = 0$

Here,

$x_1 = p + 1$ $\quad$ $y_1 = 2p - 2$

$x_2 = p - 1$ $\quad$ $y_2 = p$

$x_3 = p - 3$ $\quad$ $y_3 = 2p - 6$

Substituting the values in the formula ,

(p + 1) (p – (2p – 6)) + (p – 1) (2p – 6 – (2p – 2)) + (p – 3) (2p – 2 – (p)) = 0

(p + 1) (p – 2p + 6)) + (p – 1) (2p – 6 – 2p + 2)) + (p – 3) (2p – 2 – p) = 0

(p + 1) (–p + 6) + (p – 1) (–4) + (p – 3) (p – 2) = 0

$-p^2 - p + 6p + 6 - 4p + 4 + p^2 - 3p - 2p + 6 = 0$

– 4p + 16 = 0

4p = 16

Dividing both the sides by 4

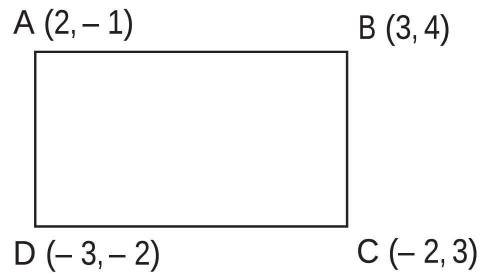
$$\frac{4p}{4} = \frac{16}{4}$$

p = 4

Hence,

For the points to be collinear, p = 4

28.



By Distance formula,

$AB = \sqrt{(3-2)^2+(4+1)^2}$

$= \sqrt{1+25} = \sqrt{26}$

$BC = \sqrt{(-2-3)^2+(3-4)^2}$

$= \sqrt{25+1} = \sqrt{26}$

$CD = \sqrt{(-3+2)^2+(-2-3)^2}$

$= \sqrt{1+25}$

$= \sqrt{26}$

$AD = \sqrt{(-3-2)^2+(-2+1)^2}$

$= \sqrt{26}$

As AB = BC = CD = AD,

ABCD is a rhombus

Again, by distance formula,

$AC = \sqrt{(-2-2)^2+(3+1)^2}$

$= \sqrt{16+16}$

$= \sqrt{32}$

$= 4\sqrt{2}$ units

$BD = \sqrt{(-3-3)^2+(-2-4)^2}$

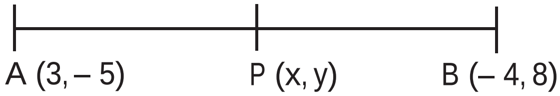
$= \sqrt{(-6)^2+(-6)^2}$

$= \sqrt{72}$

$= 6\sqrt{2}$ units

$\therefore$ $AC \neq BD$

As diagonals are not equal, ABCD is a rhombus but not a square.

29. 

$\frac{AP}{PB} = \frac{k}{1}$

Let point P be (x, y).

By section formula,

$(x, y) \quad = \left(\frac{-4k+3}{k+1}, \frac{8k-5}{k+1}\right)$

$(x, + y) \; = \left(\frac{-4k+3}{k+1}, \frac{8k-5}{k+1}\right)$

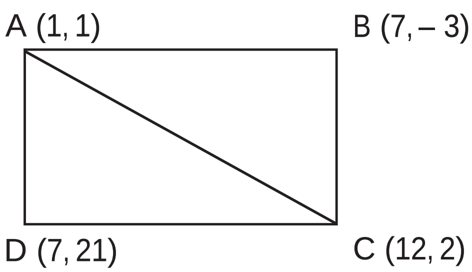
$\therefore \quad x = \frac{-4k+3}{k+1}, \quad y = \frac{8k-5}{k+1}$

As point P lies on line x + y = 0

$\therefore \quad \left(\frac{-4k+3}{k+1}\right) + \left(\frac{8k-5}{k+1}\right) = 0$

$\Rightarrow \quad 4k - 2 = 0$

$\Rightarrow \quad k = \frac{1}{2}$

30. 

Area of ΔABC

$= \frac{1}{2}$ [1 (– 3 – 2) + 7 (2 – 1) + 12 (1 + 3)]

$= \frac{1}{2}$ [– 5 + 7 + 48]

$= \frac{1}{2}$ [50]

= 25 sq. units

Area of ACD

$= \frac{1}{2}$ [1 (2 – 21) + 12 (21 – 1) + 7 (1 – 2)]

$= \frac{1}{2}$ [– 19 + 12 (20) – 7]

$= \frac{1}{2}$ [– 26 + 240]

$= \frac{1}{2}$ [214]

= 107 sq. units

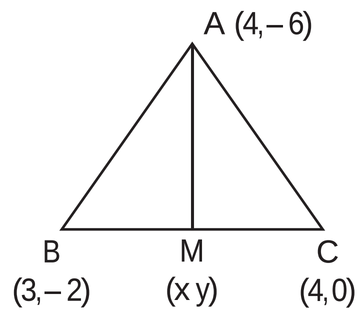
So, area of quadrilateral ABCD

= area of ΔABC + area of ΔACD

= 25 + 107

= 132 sq. units

Section D

31. 

Let AM be the median such that point M is (x, y)

$(x, y) \quad = \left(\frac{3+5}{3}, \frac{-2+2}{3}\right)$

(x, y) = (4, 0)

So, point M (x, y) = (4, 0)

Area of ΔAMB

$= \frac{1}{2}$ [4 (y + 2)+ x (– 2 + 6) + 3 (– 6 – y)]

$= \frac{1}{2}$ [4y + 8 – 2x + 6 – 18 – 3y]

$= \frac{1}{2}$ [4x + y – 10]

$= \frac{1}{2}$ [4 (4) + 0 – 10]

= 3 sq. units

Area of ΔAMC

$= \frac{1}{2}$ [4 (y – 2)+ x (2 + 6) + 5 (– 6 – y)]

$= \frac{1}{2}$ [4y – 8 + 8x – 30 – 5y]

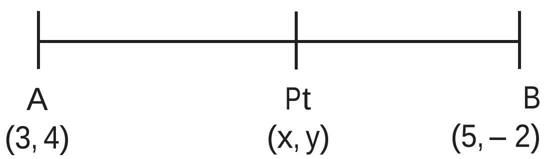
$= \frac{1}{2}$ [8x – y – 38]

$= \frac{1}{2}$ [8 (4) – 0 – 38]

= 3 sq. units

So, median divides the triangle into two triangle of equal area.

32.



PA = PB

$$\sqrt{(-3-x)^2+(4-y)^2} = \sqrt{(5-x)^2+(-2-y)^2}$$

On squarring both sides, we get

$(3 – x)^2 + (4 – y)^2 = (5 – x)^2 + (– 2 – y)^2$

⇒ $9 + \underline{x^2} – 6x + 16 + \underline{y^2} – 8y = 25 + \underline{x^2} – 10 + 4 + \underline{y^2} + 4y$

⇒ – 6x – 8y + 25 = – 10x + 4y + 29

⇒ 4x – 12y – 4 = 0

⇒ x – 3y = 1 ①

Also, area of ΔPAB = 10

∴ $\frac{1}{2}$ [x (4 + 2) + 3 (– 2 – y) + 5 (y – 4)] = 10

⇒ [6x – 6 – 3y + 5y – 20] = 20

⇒ [6x + 2y – 26] = 20

⇒ [3x + y – 13] = 10

⇒ 3x + y – 13 = ± 10

⇒ 3x + y = 23 ② or 3x + y = 3 ③

From ①, x = 1 + 3y

So, eq. ② becomes 3 + 9y + y = 23

10y = 20

y = 2

So, x = 1 + 3y

= 1 + 6

= 7

So, P (x, y) = (7, 2)

On putting x = 1 + 3y in ③, we get

3 (1 + 3y) + y = 3

3 + 9y + y = 3

10y = 0

y = 0

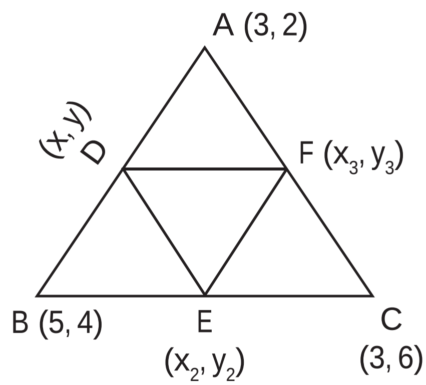
So, x = 1 + 3y

= 1 + 0

= 1

So, P (x, y) = (1, 0)

33.



By midpoint formula,

D $(x_2, y_2) = \left(\frac{3+5}{2}, \frac{2+4}{2}\right)$

D (x_2, y_2) = (<u>4, 3</u>)

Again, E $(x_2, y_2) = \left(\frac{5+3}{2}, \frac{4+6}{2}\right)$

= (<u>4, 5</u>)

F $(x_3, y_3) = \left(\frac{3+3}{2}, \frac{2+6}{2}\right)$

= (<u>3, 4</u>)

Area of ΔDEF

$= \frac{1}{2}$ [$x_1 (y_2, – y_3) + x_2 (y_3, – y_1) + x_3 (y_1, – y_2)$]

$= \frac{1}{2}$ [4 (5 – 4) + 4 (4 – 3) + 3 (3 – 5)]

$= \frac{1}{2}$ [4 + 4 – 6] = 1 sq. unit

34. Let $A(x_1 = -2, y_1 = 5)$, $B(x_2 = k, y_2 = -4)$ and $C(x_3 = 2k + 1, y_3 = 10)$ be the vertices of the triangle, so

Area of (ΔABC)

$= \frac{1}{2} [x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)]$

$53 = \frac{1}{2} [(-2)(-4-10) + k(10-5) + (2k+1)(5+4)]$

$\Rightarrow \quad 53 = \frac{1}{2} [28 + 5k + 9(2k+1)]$

$\Rightarrow \quad 28 + 5k + 18k + 9 = 106$

$\Rightarrow \quad 37 + 23k = 106$

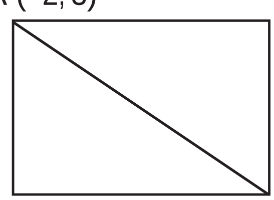
$\Rightarrow \quad 23k = 106 - 37 = 69$

$\Rightarrow \quad k = \frac{69}{23} = 3$

Hence, k = 3

The value of k is 3.

35.



Area of quadrilateral ABCD = 80 sq. units

i.e. area of ΔABC + area of ΔACD = 80

i.e. $\frac{1}{2} [-2(5+5) + 6(-5-3)] +$

$\frac{1}{2} [-2(-5+3) + x(-3-3)] = 80$

$\Rightarrow \frac{1}{2} [4 - 6x - 32] = 160$

$\Rightarrow [-6x - 28] = 160$

$\Rightarrow \quad -6x - 28 = \pm 160$

$-6x - 28 = 160$	$-6x - 28 = -160$
$6x = -188$	$-6x = -132$
$x = \frac{-94}{3}$	$x = 22$

36. Let D (x, y) be the Circumcentre.

We know that Circumcentre of a triangle is equidistant from each of the vertices.

Let the vertices be $A(x_1, y_1) = (8, 6)$, $B(x_2, y_2) = (8, -2)$ and $C(x_3, y_3) = (2, -2)$.

So, AD = BD

$$\sqrt{(8-x)^2 + (6-y)^2} = \sqrt{(8-x)^2 + (-2-y)^2}$$

On squarring both sides, we get

$(8-x)^2 + (6-y)^2 = (8-x)^2 + (-2-y)^2$

$(6-y)^2 = (-2-y)^2$

$36 + y^2 - 12y = 4 + y^2 + 4y$

$32 = 16y$

$y = 2$

Also, BD = CD

$$\sqrt{(8-x)^2 + (-2-y)^2} = \sqrt{(2-x)^2 + (-2-y)^2}$$

$(8-x)^2 + (-2-y)^2 = (2-x)^2 + (-2-y)^2$

$64 + \underline{x^2} - 16x + 4 \underline{y^2} + 4y = 4 + \underline{x^2} - 4x + 4 + \underline{y^2} + 4y$

$\Rightarrow -16x + 4y + 68 = -4x + 4y + 8$

$\Rightarrow 12x = 60$

$\Rightarrow \quad x = 5$

So, Circumcentre is (x, y) = (5, 2)

Circumradius

= AD

$= \sqrt{(8-x)^2 + (6-y)^2}$

$= \sqrt{(8-5)^2 + (6-2)^2}$

$= \sqrt{9+16}$

$= \sqrt{25}$

$= 5$

37. By ,midpoint formula,

$$C\ (x, y) = \left(\frac{O+2a}{3}, \frac{26+O}{3}\right)$$

$$= (a, b)$$

Using distance formula, we have

$$BC = \sqrt{(a-o)^2+(b-2b)^2}$$

$$= \sqrt{a^2+b^2}$$

$$OC = \sqrt{(a-o)^2+(b-o)^2}$$

$$= \sqrt{a^2+b^2}$$

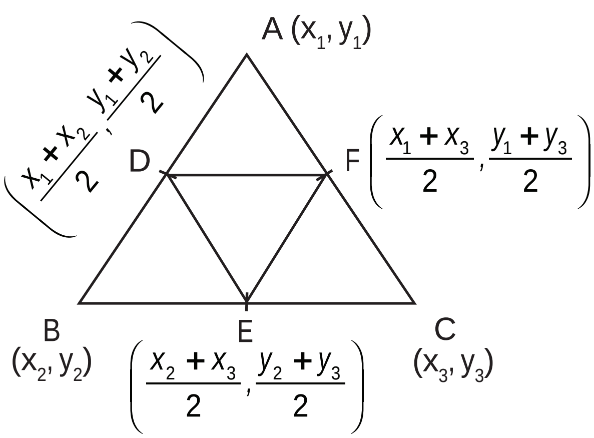
$$AC = \sqrt{(a-2a)^2+(b-o)^2}$$

$$= \sqrt{a^2+b^2}$$

So, BC = CO = AC

$\therefore$ Point C is equidistant from the vertices O, and B.

38.



By midpoint formula,

D is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

E is $\left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right)$

F is $\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$

Area of ΔABC

$$\frac{1}{2}\ [\ x_1\ (y_2 - y_3) + x_2\ (y_3 - y_1) + x_3\ (y_1 - y_2)\ ]$$

Consider ar ΔDEF

$$= \frac{1}{2}\left[\begin{array}{l}\left(\frac{x_1+x_2}{2}\right)\left[\left(\frac{y_2+y_3}{2}\right)-\left(\frac{y_1+y_3}{2}\right)\right] \\ +\left(\frac{x_2+x_3}{2}\right)\left[\left(\frac{y_1+y_3}{2}\right)-\left(\frac{y_1+y_3}{2}\right)\right] \\ +\left(\frac{x_1+x_3}{2}\right)\left[\left(\frac{y_1+y_2}{2}\right)-\left(\frac{y_2+y_3}{2}\right)\right]\end{array}\right]$$

$$= \frac{1}{8}\left[\begin{array}{l}(x_1+x_2)(y_2-y_1) \\ +(x_2+x_3)(y_3-y_2) \\ +(x_1+x_3)(y_1-y_3)\end{array}\right]$$

$$= \frac{1}{8}\left[\begin{array}{l}x_1[(y_2-y_1)+(y_1-y_3)] \\ +x_2[(y_2-y_1)+(y_3-y_2)] \\ +x_3[(y_3-y_2)+(y_1-y_3)]\end{array}\right]$$

$$= \frac{1}{8}\left[\begin{array}{l}x_1(y_2-y_3)+x_2\ (y_3-y_1) \\ +x_3(y_1-y_2)\end{array}\right]$$

$$= \left(\frac{1}{4}\right)\left(\frac{1}{2}\right)\ [x_1\ (y_2-y_3) + x_2\ (y_3-y_1) + x_3\ (y_1-y_2)\ ]$$

$$= \frac{1}{4}\ \text{area of}\ \Delta ABC$$

39. Using formula for area of triangle,

ar $\Delta DBC = \frac{1}{2}$ [x (5 + 2) – 3 (– 2 – 3x) + 4 (3x – 5)]

$$= \frac{1}{2}\ [\ 7x + 6 + 9x + 12x - 20\ ]$$

$$= \frac{1}{2}\ [\ 28x - 14\ ]$$

$= [\,14x - 7\,]$ ①

Using formula for area of triangle,

$$= \frac{1}{2}\ [\,6\,(5+2) - 3\,(-2-3) + 4\,(3-5)\,]$$

$$= \frac{1}{2}\ [\,42 + 15 - 8\,]$$

$$= \frac{1}{2}\ [\,49\,] \text{ sq. units}$$

As $\dfrac{ar\Delta DBC}{ar\Delta ABC} = \dfrac{1}{2}$

$$\Rightarrow \frac{|14x-7|}{\frac{|49|}{2}} = \frac{1}{2}$$

$$\Rightarrow \frac{2|14x-7|}{49} = \frac{1}{2}$$

$$\Rightarrow |14x-7| = \frac{49}{4}$$

$$\Rightarrow 14x - 7 = \pm\frac{49}{4}$$

If $14x - 7 = \dfrac{49}{4}$

$$14x = \frac{49}{4} + 7 = \frac{49+28}{4} = \frac{77}{4}$$

$$\Rightarrow x = \frac{11}{8}$$

If $14x - 7 = \dfrac{-49}{4}$

$$14x = \frac{-49}{4} + 7 = \frac{-49+28}{4} = \frac{-21}{4}$$

$$\Rightarrow x = \frac{-3}{8}$$

40. As the point (x_1, y_1), (x_2, y_2) and (x_3, y_3) lie on the same line, area of triangle formed by these points is 0.

i.e. $\frac{1}{2}\ [\,x_1\,(y_2, - y_3) + x_2\,(y_3, - y_1) + x_3\,(y_1, - y_2)\,] = 0$

$[\,x_1\,(y_2, - y_3) + x_2\,(y_3, - y_1) + x_3\,(y_1, - y_2)\,] = 0$

On dividing by $x_1\,x_2\,x_3$, we get

$$\left[\frac{x_1\,(y_2 - y_3) + x_2\,(y_3 - y_1) + x_3\,(y_1 - y_2)}{x_1\,x_2\,x_3}\right] = 0$$

$$\left[\frac{y_2 - y_3}{x_2\,x_3} + \frac{y_3 - y_1}{x_1\,x_3} + \frac{y_1 - y_2}{x_1\,x_2}\right] = 0$$

$$\therefore\ \frac{y_2 - y_3}{x_2\,x_3} + \frac{y_3 - y_1}{x_1\,x_3} + \frac{y_1 - y_2}{x_1\,x_2} = 0$$

Chapter 08

Introduction to Trigonometry

MULTIPLE CHOICE QUESTIONS

1. $\text{Cot } x = \frac{12}{16} = \frac{3}{4}$

$$\frac{\sin x - \cos x}{\sin x + \cos x} = \frac{1 - \cot x}{1 + \cot x}$$

$$= \frac{1 - \frac{3}{4}}{1 + \frac{3}{4}}$$

$$= \frac{1}{4} \times \frac{4}{7}$$

$$= \frac{1}{7}$$

Option (a)

2. $$\frac{x(2)^2(\sqrt{2})^2}{8\left(\frac{1}{\sqrt{2}}\right)^2\left(\frac{\sqrt{3}}{2}\right)^2} = (\sqrt{3})^2 - \left(\frac{1}{\sqrt{3}}\right)^2$$

$$\frac{8x}{3} = 3 - \frac{1}{3} = \frac{8}{3}$$

$$x = 1$$

Option (a)

3. A + B + C = 180°

$\Rightarrow$ B + C = 180° – A

$$\therefore \quad = \sin\left(\frac{B + C}{2}\right) = \sin\left(\frac{180° - A}{2}\right)$$

$$= \sin\left(90° - \frac{A}{2}\right)$$

$$= \cos\frac{A}{2}$$

Option (b)

4. $$\frac{\tan 30°}{\tan 0° - \cot 30°}$$

$$= \frac{\frac{1}{\sqrt{3}}}{0 - \sqrt{3}}$$

$$= \frac{-1}{3}$$

Option (b)

5. Consider

$(a \sin\theta + b \cos\theta)^2$

$= a^2 \sin^2\theta + b^2 \cos^2\theta + 2ab \sin\theta \cos\theta$

$= a^2 (1 - \cos^2\theta) + b^2 (1 - \sin^2\theta) + 2ab \sin\theta \cos\theta$

$= a^2 + b^2 - a^2 \cos^2\theta - b^2\sin^2\theta + 2ab \sin\theta \cos\theta$

$= a^2 + b^2 - (a^2 \cos^2\theta + b^2 \sin^2\theta) + 2ab \sin\theta \cos\theta$ (i)

Also, $a \cos\theta - b \sin\theta = c$

$\Rightarrow (a \cos\theta - b \sin\theta)^2 = c^2$

$\Rightarrow a^2 \cos^2\theta + b^2 \sin^2\theta - 2ab \sin\theta \cos\theta = c^2$

$\Rightarrow a^2 \cos^2\theta + b^2 \sin^2\theta = c^2 + 2ab \sin\theta \cos\theta$ (ii)

So, $(a \sin\theta + b \cos\theta)^2$

$= a^2 + b^2 + 2ab \sin\theta \cos\theta - c^2 - 2ab \sin\theta \cos\theta$

[From (i) and (ii)]

$= a^2 + b^2 - c^2$

$\therefore$ a sin + b cos $= \pm\sqrt{a^2 + b^2 - c^2}$

Option (b)

WORKSHEET 1

Section A

1. $$\frac{\cos(90° - \theta)\sec(90° - \theta)\tan\theta}{\cos(90° - \theta)\sin(90° - \theta)\cot(90° - \theta)} + \frac{\tan(90° - \theta)}{\cot\theta}$$

$$= \frac{\sin\theta\ \mathrm{cosec}\theta\ \tan\theta}{\sec\theta\cos\theta\tan\theta}$$

$$= \frac{\cot\theta}{\cot\theta}$$

$$= \frac{1}{1} + \frac{1}{1}$$

$= 2$

2. Consider

$$\frac{\tan A\tan B + \tan A\cot B}{\sin A\sec B} - \frac{\sin^2 B}{\cos^2 A}$$

$$= \frac{\tan(90° - B)\tan B + \tan A\cot(90° - A)}{\sin A\sec(90° - A)} - \frac{\sin^2 B}{\cos^2(90° - B)}$$

$$= \frac{\cot B\tan B + \tan^2 A}{\sin A\ \mathrm{cosec} A} - \frac{\sin^2 B}{\sin^2 B}$$

$$= \frac{1 + \tan^2 A}{1} - 1$$

$= \tan^2 A$

3. $$\sqrt{\frac{1 + \sin\theta}{1 - \sin\theta}}$$

$$= \sqrt{\frac{1 + \sin\theta}{1 - \sin\theta} \times \frac{1 + \sin\theta}{1 - \sin\theta}}$$

$$= \sqrt{\frac{(1 + \sin\theta)^2}{1 - \sin^2\theta}}$$

$$= \sqrt{\frac{(1 + \sin\theta)^2}{\cos^2\theta}}$$

$$= \frac{1 + \sin\theta}{\cos\theta}$$

4. $\cos(a + b) = 0$

$a + b = 90°$

$\therefore\ a = 90° - 6$

Consider $\sin(a - b) = \sin[90° - 26]$

$= \cos 26°$

5. $$\frac{\tan^2 - \sec^2\theta}{\cot^2 - \mathrm{cosec}^2\theta}$$

$$= \frac{\sec^2\theta - \tan^2\theta}{\mathrm{cosec}^2\theta - \cot^2\theta}$$

$$= \frac{1}{1}$$

$= 1$

6. $\mathrm{cosec}\theta = 3x \Rightarrow\ x = \frac{1}{3}\ \mathrm{cosec}\theta$

$\cot\theta = \frac{3}{x}\ \Rightarrow \frac{1}{x} = \frac{1}{3}\ \cot\theta$

consider $x^2 - \frac{1}{x^2} = \frac{1}{9}\ \ \mathrm{cosec}^2\theta - \frac{1}{9}\cot^2\theta = \frac{1}{9}$

7. $\tan A = \frac{5}{12}$

Consider $(\sin A + \cos A)\sec A$

$= (\sin A + \cos A)\ \frac{1}{\cos A}$

$= \tan A + 1$

$= \frac{5}{12} + 1$

$= \frac{17}{12}$

8. Consider $6\tan^2\theta - \frac{6}{\cos^2\theta}$

$= 6(\tan^2\theta - \sec^2\theta)$

$= 6\ \ (1)$

$= 6$

Section B

9. $2\sin^2 30° - 3\cos^2 45° + \tan^2 60°$

$= 2\left(\frac{1}{2}\right)^2 - 3\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\sqrt{3}\right)^2$

$= \frac{1}{2} - \frac{3}{2} + 3$

$= 2$

10. (i) We know that $-1 \le \sin\theta \le 1$

$\therefore \quad 0 \le \sin^2\theta \le 1$

If $\sin\theta = x + \frac{1}{x}$,

On squaring both sides, we get

$\sin^2\theta = x^2 + \frac{1}{x^2} + 2$

Here, R H S $= x^2 + \frac{1}{x^2} + 2 > 2$

but Maximum value of $\sin^2\theta$ is

$\therefore \quad \sin^2\theta$ is $\neq x + \frac{1}{x}$

(ii) As $(a - b)^2 \ge 0$

$\Rightarrow \quad a^2 + b^2 - 2ab \ge 0$

$\Rightarrow \quad a^2 + b^2 \ge 2ab$

$\therefore \quad \cos\theta = \frac{a^2 + b^2}{2ab} \ge \frac{2ab}{2ab} = 1$

$\Rightarrow \quad \cos\theta \ge 1$

if $\cos = 1$

$\frac{a^2 + b^2}{2ab} = 1$

$a^2 + b^2 = 2ab$

$(a - b)^2 = 0$

$a = b$

but a and b are distvied

$\therefore \quad \cos\theta > 1$

but $-1 \le \cos\theta \le 1$

So, $\cos\theta \neq \frac{a^2 + b^2}{2ab}$

11. (i) $2\sin 3x = \sqrt{3}$

$\sin 3x = \frac{\sqrt{3}}{2}$

$3x = 60°$

$x = 20°$

(ii) $2\sin\frac{x}{2} = 1$

$\sin\frac{x}{2} = \frac{1}{2} = \sin 30°$

$\frac{x}{2} = 30°$

$x = 60°$

12. $\text{Sin}\theta + \sin^2\theta = 1$

$\Rightarrow \quad \sin\theta = 1 - \sin^2\theta$

$\Rightarrow \quad \sin\theta = \cos^2\theta \qquad$ (i)

$\Rightarrow \quad \tan\theta = \cos\theta$

Consider $\cos^2\theta + \cos^4\theta$

$= \tan^2\theta + \tan 4\theta$

$= \tan^2\theta\,(1 + \tan^2\theta)$

$= \tan^2\theta \sec^2\theta$

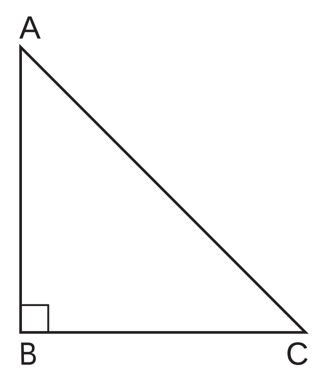
$= \tan^2\theta \frac{1}{\sin\theta} \qquad$ By (i)

$= \frac{\sin\theta}{\cos^2\theta}$

$= \frac{\sin\theta}{\sin\theta} \qquad$ By (i)

$= 1$

13.



$\tan A = \frac{1}{\sqrt{3}}$

$$\frac{BC}{AB} = \frac{1}{\sqrt{3}}$$

Let BC = k, AB = $\frac{A + B = 90°}{A - B = 30°}$ k

$\therefore$ $AC^2 = BC^2 + AB^2 = 4k^2$

AC = 2k

Consider

sinA cosC + cosA sinC

$$= \left(\frac{BC}{AC}\right)\left(\frac{BC}{AC}\right) + \left(\frac{AB}{AC}\right)\left(\frac{AB}{AC}\right)$$

$$= \frac{BC^2}{AC^2} + \frac{AB^2}{AC^2}$$

$$= \frac{BC^2 + AB^2}{AC^2}$$

$$= \frac{AC^2}{AC^2}$$

= 1

14. Consider

$4\cot^2 45° - \mathrm{Sec}^2 60° + \sin^2 60° - \cos^2 90°$

$$= 4(1)^2 - (2)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - (0)^2$$

$$= 4 - 4 + \frac{3}{4}$$

$$= \frac{3}{4}$$

15. Consider

$$(\mathrm{cosec}\theta - \cot\theta)^2 = \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$$

$$= \left(\frac{1 - \cos\theta}{\sin\theta}\right)^2$$

$$= \frac{(1 - \cos\theta)^2}{\sin\theta}$$

$$= \frac{(1 - \cos\theta)^2}{1 - \cos^2\theta}$$

$$= \frac{1 - \cos\theta}{1 + \cos\theta}$$

16. Consider

$3\cos^2 30° + \sec^2 30° + 2\cos^2 0° + 3\sin^2 90° - \tan^2 60°$

$$= 3\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{2}{\sqrt{3}}\right)^2 + 2(1)^2 + 3(1)^2 - \left(\sqrt{3}\right)^2$$

$$= \frac{9}{4} + \frac{4}{3} + 2 + 3 - 3$$

$$= \frac{9}{4} + \frac{4}{3} + 2$$

$$= \frac{27 + 16 + 24}{12}$$

$$= \frac{67}{12}$$

Section C

17. $\tan\theta + \cot\theta = 2$

$$\tan\theta + \frac{1}{\tan\theta} = 2$$

$\tan^2\theta - 2\tan\theta + 1 = 0$

$(\tan\theta - 1)^2 = 0$

$\tan\theta - 1 = 0$

$\tan\theta = 1 = \tan 45°$

$\theta = 45°$

Consider $\tan^7\theta + \cot^7\theta$

$= \tan^7(45°) + \cot^7(45°)$

$= 1^7 + 1^7$

$= 1 + 1$

$= 2$

18. Consider

$$\frac{\sin\theta}{1 + \cos\theta} + \frac{1 + \cos\theta}{\sin\theta}$$

$$= \frac{\sin\theta(\sin\theta) + (1 + \cos\theta)(1 + \cos\theta)}{\sin\theta(1 + \cos\theta)}$$

$$= \frac{\sin^2\theta + 1 + \cos^2\theta + 2\cos\theta}{\sin\theta(1 + \cos\theta)}$$

$$= \frac{2 + 2\cos\theta}{\sin\theta\,(1 + \cos\theta)}$$

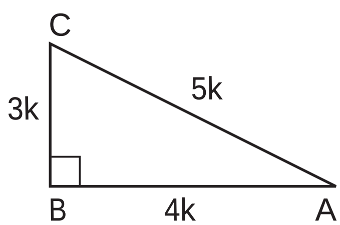
$$= \frac{2 + (1 + \cos\theta)}{\sin\theta\,(1 + \cos\theta)}$$

$$= \frac{2}{\sin\theta} = 2\,\text{cosec}\,\theta$$

19. $\sec A = \frac{5}{4} = \frac{AC}{AB}$

L H S

$$\frac{3\sin A - 4\sin^3 A}{4\cos^3 A - 3\cos A}$$



$BC^2 = AC^2 - AB^2$

$= 25k^2 - 16k^2$

$= 9k^2$

$\therefore\ BC = 3k$

So, $\frac{3\sin A - 4\sin^2 A}{4\cos^3 A - 3\cos A}$

$$= \frac{3\left(\frac{3}{5}\right) - 4\left(\frac{3}{5}\right)^3}{4\left(\frac{4}{5}\right)^3 - 3\left(\frac{4}{5}\right)}$$

$$= \frac{\frac{9}{5} - \frac{108}{125}}{\frac{256}{125} - \frac{12}{5}}$$

$$= \frac{\frac{9}{5} - \frac{108}{125}}{\frac{256}{125} - \frac{12}{5}}$$

$$= \frac{\frac{225 - 108}{125}}{\frac{256 - 300}{125}}$$

$$= \frac{117}{-44}$$

R H S

$$\frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$$

$$= \frac{3\left(\frac{3}{4}\right) - \left(\frac{3}{4}\right)^3}{1 - 3\left(\frac{3}{4}\right)^2}$$

$$= \frac{\frac{9}{4} - \frac{27}{64}}{1 - \frac{27}{16}}$$

$$= \frac{\frac{144 - 27}{64}}{\frac{16 - 24}{16}}$$

$$= \frac{117}{-44}$$

$$= \frac{-117}{44}$$

So, L H S = R H S

20. $a\cos\theta + b\sin\theta = m$

$a\sin\theta - b\cos\theta = n$

To prove : $a^2 + b^2 = m^2 + n^2$

Proof $a\cos\theta + b\sin\theta = m$

On squarring both sides, we get

$a^2\cos^2\theta + b^2\sin^2\theta + 2ab\sin\theta\cos\theta = m^2$ (1)

$a\sin\theta - b\cos\theta = n$

On squarring both sides, we get

$a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta = n^2$ (2)

On adding (1) and (2), we get

$a^2(\sin^2\theta + \cos^2\theta) + b^2(\sin^2\theta + \cos^2\theta) = m^2 + n^2$

$\Rightarrow\ a^2 + b^2 = m^2 + n^2$

21. $x = a\cos^3\theta$

$y = b\sin^3\theta$

Consider $\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}}$

$$= \left(\frac{a\cos^3\theta}{a}\right)^{\frac{2}{3}} + \left(\frac{b\cos^3\theta}{b}\right)^{\frac{2}{3}}$$

$= \cos^2\theta + \sin^2\theta$

$= 1$

22. $\sin(A + B) = 1 = \sin 90°$

$A + B = 90°$ (1)

$\cos(A - B) = \frac{\sqrt{3}}{2} = \cos 30°$

$A - B = 30°$ (2)

On solving (1) and (2), we get

$$\begin{aligned} A + B &= 90° \\ A - B &= 90° \\ \hline 2A &= 120° \\ A &= 60° \end{aligned}$$

From (1), $B = 90° - A$

$= 90° - 60°$

$= 30°$

23. Consider

$(1 - \sin\theta + \cos\theta)^2$

$= [(1 - \sin\theta) + \cos\theta]^2$

$= (1 - \sin\theta)^2 + \cos^2\theta + 2\cos\theta\,(1 - \sin\theta)$

$= (1 - \sin\theta)^2 + (1 - \sin^2\theta) + 2\cos\theta(1 - \sin\theta)$

$= (1 - \sin\theta)\,[1 - \sin\theta + 1 + \sin\theta + 2\cos\theta]$

$= (1 - \sin\theta)\,(2\cos\theta + 2)$

$= 2\,(1 + \cos\theta)\,(1 - \sin\theta)$

= R H S

24. LHS $= \dfrac{\tan A + \sin A}{\tan A - \sin A}$

$$= \frac{\frac{\sin A}{\cos A} + \sin A}{\frac{\sin A}{\cos A} - \sin A}$$

$$= \frac{\frac{\sin A + \sin A\cos A}{\cos A}}{\frac{\sin A - \sin A\cos A}{\cos A}}$$

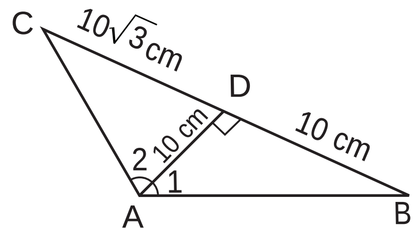
$$= \frac{\sin A + \sin A\cos A}{\sin A - \sin A\cos A}$$

$$= \frac{\sin A(1 + A\cos A)}{\sin A(1 - A\cos A)} = \frac{1 + \frac{1}{\sec A}}{1 - \frac{1}{\sec A}}$$

$$= \frac{\frac{\sec A + 1}{\sec A}}{\frac{\sec A - 1}{\sec A}} = \frac{\sec A + 1}{\sec A - 1} = \text{RHS}$$

Section D

25.



In ΔADB,

$\tan(\angle 1) = \dfrac{BD}{AD} = \dfrac{10}{10} = 1$

$\therefore \quad \angle 1 = 45°$

In ΔADC,

$\tan(\angle 2) = \dfrac{10\sqrt{3}}{10} = \sqrt{3}$

$\angle 2 = 60°$

So, $\angle A = \angle 1 + \angle 2$

$= 45° + 60°$

$= 105°$

26. Consider

L H S $= \dfrac{\sin\theta}{1 - \cos\theta}$

$$= \frac{\sin\theta\,(1 + \cos\theta)}{(1 - \cos\theta)\,(1 + \cos\theta)}$$

$$= \frac{\sin\theta\,(1 + \cos\theta)}{1 - \cos^2\theta}$$

$$= \frac{\sin\theta\,(1 + \cos\theta)}{\sin^2\theta}$$

$$= \frac{1 + \cos\theta}{\sin\theta}$$

$$= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$$

$= \text{cosec}\,\theta + \cot\theta$

$=$ R H S

27. Given : $(2\theta + 45°)$ & $(30° + \theta)$ and $\sin(2\theta + 45°) = \cos(30° + \theta)$

$\sin(2\theta + 45°) = \cos(30° + \theta)$

$\sin(2\theta + 45°) = \sin(90°\ \theta(30°– \theta))$

$[\sin(90° – \theta) = \cos\theta]$

$\sin(2\theta + 45°) = \sin(90° – 30° + \theta)$

On equating both sides,

$(2\theta + 45°) = (60° + \theta)$

$2\theta – \theta = 60° - 45°$

$\theta = 15°$

28. Consider

L H S $= \dfrac{\tan A}{1 – \cot A} + \dfrac{\cot A}{1 – \tan A}$

$$= \frac{\frac{\sin A}{\cos A}}{1 – \frac{\cos A}{\sin A}} + \frac{\frac{\cos A}{\sin A}}{1 – \frac{\sin A}{\cos A}}$$

$$= \frac{\sin^2 A}{\cos A\,(\sin A – \cos A)} + \frac{\cos^2 A}{\sin A\,(\cos A – \sin A)}$$

$$= \frac{\sin^2 A}{\cos A\,(\sin A – \cos A)} – \frac{\cos^2 A}{\sin A\,(\cos A – \sin A)}$$

$$= \frac{\sin^3 A – \cos^3 A}{\sin A \cos A\,(\sin A – \cos A)}$$

$$= \frac{(\sin A – \cos A)\,(\sin^2 A + \cos^2 A + \sin A \cos A}{\sin A \cos A\,(\sin A – \cos A)}$$

$$= \frac{1 + \sin A \cos A}{\sin A \cos A}$$

$= 1 + \text{cosec}\,A \sec A$

$=$ R H S

29. To prove :

$$\frac{1}{\text{cosec}A – \cot A} – \frac{1}{\sin A}$$

$$= \frac{1}{\sin A} – \frac{1}{\text{cosec}A + \cot A}$$

i.e. To prove

$$\frac{1}{\text{cosec}A – \cot A} + \frac{1}{\text{cosec}A + \cot A} = \frac{2}{\sin A}$$

Consider

$$\frac{1}{\text{cosec}A – \cot A} + \frac{1}{\text{cosec}A + \cot A}$$

$$= \frac{1}{\frac{1}{\sin A} – \frac{\cos A}{\sin A}} + \frac{1}{\frac{1}{\sin A} + \frac{\cos A}{\sin A}}$$

$$= \frac{\sin A}{1 – \cos A} + \frac{\sin A}{1 + \cos A}$$

$$= \left(\frac{1 + \cos A + 1 – \cos A}{(1 – \cos A)\,(1 + \cos A)}\right) \sin A$$

$$= \left(\frac{2}{1 – \cos^2 A}\right) \sin A$$

$$= \frac{2}{\sin^2 A} \sin A$$

$$= \frac{2}{\sin A}$$

30. $\sin\theta + \cos\theta = p$, $\sec\theta + \text{cosec}\theta = q$

Consider

$q\,(p^2 – 1)$

$= (\sec\theta + \text{cosec}\theta)\ [\ \sin^2\theta + \cos\theta^2) – 2\ ]$

$= (\sec\theta + \text{cosec}\theta)\ [\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta -1]$

$$= \left(\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right) (2\sin\theta\cos\theta)$$

$$= \left(\frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}\right) (2\sin\theta\cos\theta)$$

$= 2\,(\sin\theta + \cos\theta)$

$= 2p$

31. $\text{Sec}\theta + \tan\theta = p$ (i)

We know that $\text{Sec}^2\theta – \tan^2\theta = 1$

$\Rightarrow \quad (\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$

$\Rightarrow \quad (\sec\theta - \tan\theta)\, p = 1$

$\Rightarrow \quad \sec\theta - \tan\theta = \frac{1}{p} \quad \text{(ii)}$

On adding (i) and (ii), we get

$2\sec\theta = p + \frac{1}{p}$

$\sec\theta = \frac{1}{2}\left(p + \frac{1}{p}\right)$

On subtracting (i) from (ii), we get

$-2\tan\theta = \frac{1}{p} - p$

$\tan\theta = \frac{1}{2}\left(p - \frac{1}{p}\right)$

Also,

$$\sin\theta = \frac{\tan\theta}{\sec\theta} = \frac{\frac{1}{2}\left(p - \frac{1}{p}\right)}{\frac{1}{2}\left(p + \frac{1}{p}\right)} = \frac{p^2 - 1}{p^2 + 1}$$

32. $\sin\theta + \cos\theta = \sqrt{2}$

$\Rightarrow \quad \frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta = 1$

$\Rightarrow \quad \cos\frac{\pi}{4}\sin\theta + \sin\frac{\pi}{4}\cos\theta = 1$

$\Rightarrow \quad \sin\left(\frac{\pi}{4} + \theta\right) = 1 = \sin\frac{\pi}{2}$

$\Rightarrow \quad \frac{\pi}{4} + \theta = \frac{\pi}{2}$

$\Rightarrow \quad \theta = \frac{\pi}{2} - \frac{\pi}{4}$

$\frac{\pi}{4}$

Consider

$\tan\theta + \cot\theta$

$= \tan\frac{\pi}{4} + \cot\frac{\pi}{4}$

$= 1 + 1$

$= 2$

WORKSHEET 2

Section A

1. Consider

$(1 + \cot^2\theta)\sin^2\theta$

$= \left(1 + \frac{\cos^2\theta}{\sin^2\theta}\right)\sin^2\theta$

$= \sin^2\theta + \cos^2\theta$

$= 1$

2. Consider

$\text{cosec}^2\theta\,(1 + \cos\theta)(1 - \cos\theta) = x$

$\Rightarrow \quad \text{cosec}^2\theta\,(1 - \cos^2\theta) = x$

$\Rightarrow \quad \text{cosec}^2\theta \sin^2\theta = x$

$\Rightarrow \quad \frac{1}{\sin^2\theta}\sin^2\theta = x$

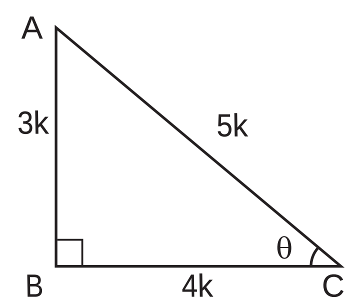
$\Rightarrow \quad 1 = x$

3. $\cos1^\circ \cos2^\circ \cos3^\circ \ldots \cos179^\circ \cos188^\circ$

$= \cos1^\circ \cos2^\circ \cos3^\circ \ldots \cos90^\circ \ldots \cos179^\circ \cos188^\circ$

$= 0$

4.



$\cot\theta = 4$

$\cot\theta = \frac{4}{3}$

$= \frac{BC}{AB}$

Let BC = 4k, AB = 3k

By Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$= (3k)^2 + (4k)^2$$

$$= 9k^2 + 16k^2$$

$$= 25k^2$$

$$\therefore\ AC = 5k$$

Consider $\dfrac{4\cos\theta - \sin\theta}{2\cos\theta + \sin\theta}$

$$= \frac{4\left(\frac{4}{5}\right) - \left(\frac{3}{5}\right)}{2\left(\frac{4}{5}\right) - \frac{3}{5}}$$

$$= \frac{\frac{16}{5} - \frac{3}{5}}{\frac{8}{5} + \frac{3}{5}}$$

$$= \frac{13}{5} \times \frac{5}{11}$$

$$= \frac{13}{11}$$

5. Consider (secA + tanA) (1 – sinA)

$$= \left(\frac{1}{\cos A}\ \frac{\sin}{\cos A}\right)(1 - \sin A)$$

$$= \frac{(1 + \sin A)(1 - \sin A)}{\cos A}$$

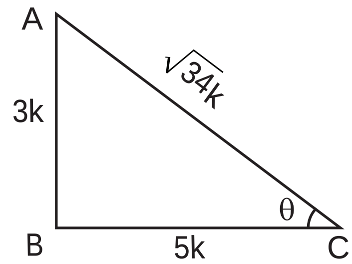
$$= \frac{1 - \sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A}$$

$$= \cos A$$

6. $3\cos\theta = 5\sin\theta$

$$\frac{3}{5} = \frac{\sin\theta}{\cos\theta}$$

$$\tan\theta = \frac{3}{5} = \frac{AB}{BC}$$



Let AB = 3k

BC = 5k

By Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$= 9k^2 + 25k^2$$

$$= 34k^2$$

$$\Rightarrow\ AC = \sqrt{34}\ k$$

Consider

$$\frac{5\sin\theta - 2\sec^3\theta + 2\cos\theta}{5\sin\theta + 2\sec^3\theta + 2\cos\theta}$$

$$= \frac{5\left(\frac{3}{\sqrt{34}}\right) - 2\left(\frac{\sqrt{34}}{5}\right)^3 + 2\left(\frac{5}{\sqrt{34}}\right)}{5\left(\frac{3}{\sqrt{34}}\right) + 2\left(\frac{\sqrt{34}}{5}\right)^3 - 2\left(\frac{5}{\sqrt{34}}\right)}$$

$$= \frac{\frac{15}{\sqrt{34}} + \frac{10}{\sqrt{34}} - \frac{68}{125}\sqrt{34}}{\frac{15}{\sqrt{34}} + \frac{68}{125}\sqrt{34} - \frac{10}{\sqrt{34}}}$$

$$= \frac{\frac{25\sqrt{34}}{34} - \frac{68}{125}\sqrt{34}}{\frac{5\sqrt{34}}{34} + \frac{68}{125}\sqrt{34}}$$

$$= \frac{\frac{3125\sqrt{34} - 2312\sqrt{34}}{4250}}{\frac{625\sqrt{34} + 2312\sqrt{34}}{4250}}$$

$$= \frac{813\sqrt{34}}{2937\sqrt{34}}$$

$= \dfrac{271}{979}$

$$\frac{\cos(90° - \theta)\sec(90° - \theta)\tan\theta}{\text{cosec}(90° - \theta)\sin(90° - \theta)\cot(90° - \theta)} + \frac{\tan(90° - \theta)}{\cot\theta}$$

$$= \frac{\sin\theta\ \text{cosec}\theta\ \tan\theta}{\sec\theta\cos\theta\tan\theta} + \frac{\cot\theta}{\cot\theta}$$

$$= \frac{1}{1} + 1$$

$= 2$

7. $\cos\phi$

$= \cos(180° - (90° + \theta))$

$= \cos(90° - \theta)$

$= \sin\theta$

$= \dfrac{4}{5}$

8. $\cos^2 17° - \sin^2 73°$

$= \cos^2(90° - 73°) - \sin^2 73°$

$= \sin^2 73° - \sin^2 73°$

$= 0$

9. $\dfrac{2\tan 30°}{1 + \tan^2 30°}$

$$= \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1 + \left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{2}{\sqrt{3}} \times \frac{3}{4}$$

$$= \frac{\sqrt{3}}{2}$$

Section B

10. $\tan 2\theta = \cot(\theta + 6°)$

$\Rightarrow \cot(90° - 2\theta) = \cot(\theta + 6°)$

$\Rightarrow (90° - 2\theta) = (\theta + 6°)$

$\Rightarrow 90° - 6° = 3\theta$

$\Rightarrow \dfrac{84}{3} = \theta$

$\Rightarrow 28° = \theta$

11. $\dfrac{2\cos 67°}{\sin 23°} - \dfrac{\tan 40°}{\cot 50°} - \cos 0°$

$$= \frac{2\cos(90° - 23°)}{\sin 23°} - \frac{\tan(90° - 50°)}{\cot 50°} - \cos 0°$$

$$= \frac{2\sin 23°}{\sin 23°} - \frac{\cot 50°}{\cot 50°} - \cos 0°$$

$= 2 - 1 - 1$

$= 0$

12. $\sec 4A = \text{cosec}(A - 20°)$

$\text{cosec}(90° - 4A) = \text{cosec}(A - 20°)$
$(\sec\theta = \text{cosec}(90 - \theta)$

Comparing angles

$90° - 4A = A - 20°$

$-4A - A = -20° - 90°$

$-5A = -110°$

$A = \dfrac{-110°}{-5}$

$A = 22°$

13. $\dfrac{\cos 70°}{\sin 20°} + \dfrac{\cos 59°}{\sin 31°} - 8\sin^2 30°$

$$= \frac{\cos(90° - 20°)}{\sin 20°} + \frac{\cos(90° - 31°)}{\sin 31°} - 8\left(\frac{1}{2}\right)^2$$

$$= \frac{\sin 20°}{\sin 20°} + \frac{\sin 31°}{\sin 31°} - \frac{8}{4}$$

$= 1 + 1 - 2$

$= 0$

14. Sin 75 = Sin (30 + 45)

= Sin 30 Cos 45 + Sin 45 Cos 30

$= \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \left(\frac{1}{2\sqrt{2}}\right) + \left(\frac{\sqrt{3}}{2\sqrt{2}}\right)$

$= \frac{(1+\sqrt{3})}{2\sqrt{2}}$

On rationalising :-

$\frac{(1+\sqrt{3})(2\sqrt{2})}{(2\sqrt{2})(2\sqrt{2})}$

$\frac{(2\sqrt{2} + 2\sqrt{6})}{8}$

$\frac{2(\sqrt{2} + \sqrt{6})}{8}$

$\frac{(\sqrt{2} + \sqrt{6})}{4}$

Therefore Sin75 = $\frac{(\sqrt{2} + \sqrt{6})}{4}$

15. $\sin\theta = \cos\theta$

$\frac{\sin\theta}{\cos\theta} = 1$

$\tan\theta = 1$

$\therefore \quad \theta = \frac{\pi}{4}$

Consider $2\tan^2\theta + \sin^2\theta - 1$

$= 2\tan^2\frac{\pi}{4} + \sin^2\frac{\pi}{4} - 1$

$= 2(1)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 - 1$

$= 2 - \frac{1}{2} - 1$

$= 1 - \frac{1}{2}$

$= \frac{1}{2}$

16. $\alpha + \beta = 90°$

To prove: $= \sqrt{\cos\alpha \operatorname{cosec}\beta - \cos\alpha \sin\beta}$

$= \sin\alpha$

Consider $\sqrt{\cos\alpha \sec\alpha - \cos\alpha \cos\alpha}$

$= \sqrt{\cos\alpha \operatorname{cosec}(90° - \alpha) - \cos\alpha \sin(90° - \alpha)}$

$= \sqrt{\cos\alpha \sec\alpha - \cos\alpha \cos\alpha}$

$= \sqrt{1 - \cos^2\alpha}$

$= \sqrt{\sin^2\alpha}$

$= \sin\alpha$

17. Consider

$2\left(\frac{\cos 58°}{\sin 32°}\right) - \sqrt{3}\left(\frac{\cos 38° \operatorname{cosec} 52°}{\tan 15° \tan 60° \tan 75°}\right)$

$= 2\left(\frac{\cos(90° - 32°)}{\sin 32°}\right)$

$- \sqrt{3}\left(\frac{\cos(90° - 52°) \operatorname{cosec} 52°}{\tan(90° - 75°) \tan 60° \tan 75°}\right)$

$= 2\left(\frac{\sin 32°}{\sin 32°}\right) - \sqrt{3}\left(\frac{\sin 52° \operatorname{cosec} 52°}{\cot 75° \tan 60° \tan 75°}\right)$

$= 2 - \sqrt{3}\left(\frac{1}{\sqrt{3}}\right)$

$= 2 - 1$

$= 1$

18. $\tan\theta + \frac{1}{\tan\theta} = 2$

On squarring both sides, we get

$\tan^2\theta + \frac{1}{\tan^2\theta} + 2\tan\theta \frac{1}{\tan\theta} = 4$

$\tan^2\theta + \frac{1}{\tan^2\theta} + 2 = 4$

$\tan^2\theta + \frac{1}{\tan^2\theta} = 4 - 2$

$\frac{1}{\tan^2\theta} = 2$

19. $\frac{2\tan 67°}{\cot 23°} - \frac{\sin 40°}{\cos 50°} - \tan 0°$

$= \frac{2\tan (90° - 23°)}{\cot 23°} - \frac{\sin (90° - 50°)}{\cos 50°} - \tan 0°$

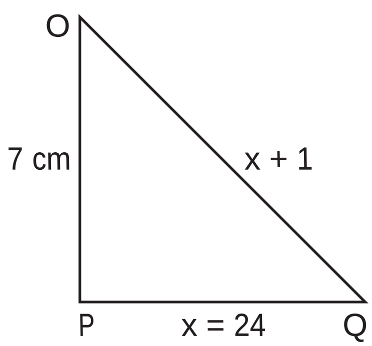
$= \frac{2\cot 23°}{\cot 23°} - \frac{\cos 50°}{\cos 50°} - \tan 0°$

$= 2 - 1 = 0$

$= 0$

Section C

20.



$OQ - PQ = 1$

Let $PQ = x$

$\therefore OQ = x + 1$

By Pythagoras theorem,

$OQ = OP + PQ$

$(x + 1)^2 = 7^2 + x^2$

$x^2 + 1 + 2x = 49 + x^2$

$2x = 48$

$x = 24$

$\therefore PQ = 24$ cm and $OQ = 25$ cm

$\sin Q = \frac{OP}{OQ}$

$= \frac{7}{25}$

$\cos Q = \frac{PQ}{OQ}$

$= \frac{24}{25}$

21. Consider

$(\sec\theta - \tan\theta)^2$

$= \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2$

$= \left(\frac{1 - \sin\theta}{\cos\theta}\right)^2$

$= \frac{(1 - \sin\theta)^2}{\cos^2\theta}$

$= \frac{(1 - \sin\theta)^2}{1 - \sin^2\theta}$

$= \frac{(1 - \sin\theta)(1 - \sin\theta)}{(1 - \sin\theta)(1 + \sin\theta)}$

$= \frac{1 - \sin\theta}{1 + \sin\theta}$

22. Consider

$\frac{\sec\theta - \tan\theta}{\sec\theta + \tan\theta}$

$= \frac{(\sec\theta - \tan\theta)(\sec\theta - \tan\theta)}{(\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}$

$= \frac{(\sec\theta - \tan\theta)^2}{\sec^2\theta - \tan^2\theta}$

$= \frac{\sec^2\theta + \tan^2\theta - 2\sec\theta\tan\theta}{1}$

$= 1 + \tan^2\theta + \tan^2\theta - 2\sec\theta\tan\theta$

$= 1 - 2\sec\theta\tan\theta + 2\tan^2\theta$

23. $\frac{\cos^2 20° + \cos^2 70°}{\sec^2 50° - \cot^2 40°} + 2\operatorname{cosec} 58°$

$- 2\cot 58° \tan 32° - (4\tan 13° \tan 37° \tan 45° \tan 53° \tan 77°)$

$= \frac{\cos^2(90° - 70°) + \cos^2 70°}{\sec^2(90° - 40°) - \cot^2 40°} + 2\operatorname{cosec} 58°$

$- 2\tan 32° \cot(90° - 32°) - 4\tan(90° - 77°)\tan(90° - 53°)(1)\tan 53° \tan 77°$

$= \frac{\sin^2 70° + \cos^2 70°}{\operatorname{cosec}^2 40° - \cot^2 40°} + 2\operatorname{cosec} 58°$

$- 2\tan 32° \operatorname{cosec} 32° - 4\cot 77° \cot 53° \tan 53° \tan 77°$

$= 1 + 2\operatorname{cosec} 58° - 2\sec 32° - 4$

$= 1 + 2\operatorname{cosec}(90° - 32°) - 2\sec 32° - 4$

$= 1 + 2\sec 32° - 2\sec 32° - 4$

$= -3$

24. $\operatorname{cosec}\theta + \cot\theta = p$ (i)

Consider

$\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$(\operatorname{cosec}\theta - \cot\theta)(\operatorname{cosec}\theta + \cot\theta) = 1$

$(\operatorname{cosec}\theta - \cot\theta) = \frac{1}{p}$ (ii)

On adding (i) and (ii), we get

$2\operatorname{cosec}\theta = p + \frac{1}{p}$

$\operatorname{cosec}\theta = \frac{1}{2}\left(p + \frac{1}{p}\right)$

On subtracting (i) and (ii), we get

$-2\cot\theta = \frac{1}{p} - p$

$\cot\theta = \frac{1}{2}\left(p - \frac{1}{p}\right)$

$\therefore \cos\theta = \frac{\cot\theta}{\operatorname{cosec}\theta}$

$= \frac{\frac{1}{2}\left(p - \frac{1}{p}\right)}{\frac{1}{2}\left(p + \frac{1}{p}\right)}$

$= \frac{p^2 - 1}{p^2 + 1}$

25. $\tan\theta = \frac{1}{\sqrt{7}}$

$\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta}$

$= \frac{(1 + \cot^2\theta) - (1 + \tan^2\theta)}{(1 + \cot^2\theta) + (1 + \tan^2\theta)}$

$= \frac{\cot^2\theta - \tan^2\theta}{2 + \cot^2\theta - \tan^2\theta}$

$= \frac{\left(\sqrt{7}\right)^2 - \left(\frac{1}{\sqrt{7}}\right)^2}{2 + \left(\sqrt{7}\right)^2 + \left(\frac{1}{\sqrt{7}}\right)^2}$

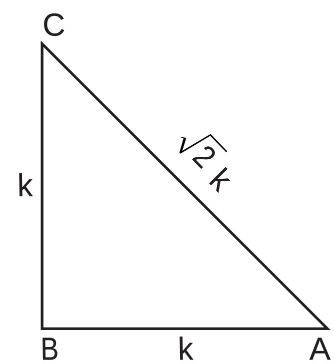
$= \frac{7 - \frac{1}{7}}{2 + 7 + \frac{1}{7}}$

$= \frac{48}{7} \times \frac{7}{64} = \frac{48}{64}$

$= \frac{12}{16}$

$= \frac{3}{4}$

26.



$\operatorname{cosec} A = \frac{\sqrt{2}}{1} = \frac{AC}{BC}$

Let $AC = \sqrt{2}k$

$BC = k$

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$2k^2 = AB^2 + k^2$

$AB^2 = 2k^2 - k^2 = k^2$

$AB = k$

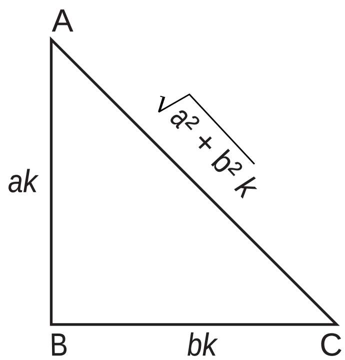
So, $\dfrac{2\sin^2 A + 3\cot^2 A}{4\tan^2 A - \cos^2 A}$

$= \dfrac{2\left(\frac{1}{\sqrt{2}}\right)^2 + 3(1)^2}{4(1)^2 - \left(\sqrt{2}\right)^2}$

$= \dfrac{1 + 3}{4 - 2}$

$= \ 2$

27.



$\sin\theta \ = \dfrac{a}{\sqrt{a^2 + b^2}}$

$= \dfrac{AB}{AC}$

Let AB = ak, AC = $\sqrt{a^2 + b^2}\,k$

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$(a^2 + b^2)\,k^2 = a^2k^2 + BC^2$

$BC^2 = b^2k^2$

$BC = bk$

$\therefore \quad \cos\theta = \dfrac{BC}{AC}$

$= \dfrac{b}{\sqrt{a^2 + b^2}}$

$\tan\theta = \dfrac{AB}{BC}$

$= \dfrac{a}{b}$

28. To prove

$\cos^6 A + \sin^6 A = 1 - 3\sin^2 A \cos^2 A$

LHS

$\cos^6 A + \sin^6 A$

$= (\cos^6 A)^3 + (\sin^2 A)^3$

$= (\cos^2 A + \sin^2 A)\ [(\cos^2 A)^2 - \cos^2 A \times \sin^2 A + (\sin^2 A)^2]$

(Because, $a^3 + b^3 = (a + b)\ (a^2 - ab + b^2)$)

$= (\cos^2 A)^2 - \cos^2 A \times \sin^2 A + (\sin^2 A)^2$

(Because, $\sin^2 x + \cos^2 x = 1$)

$= (\cos^2 A)^2 - \cos^2 A \times \sin^2 A + (\sin^2 A)^2 + 2\cos^2 A \sin^2 A - 2\cos^2 A \sin^2 A$

$= (\cos^2 A + \sin^2 A)^2 - 3\cos^2 A \sin^2 A$

(Because, $a^2 + 2ab + b^2 = (a + b)^2$)

$= \ 1 - 3\cos^2 A\ \sin^2 A$

= RHS

Hemce, proved.

29. $5\ \tan x \ = 4$

$\tan x \ = \dfrac{4}{5}$

Consider $\dfrac{5\sin x - 3\cos x}{5\sin x + 2\cos x}$

$= \dfrac{\frac{5\sin x - 3\cos x}{\cos x}}{\frac{5\sin x + 2\cos x}{\cos x}}$

$= \dfrac{5\tan x - 3}{5\tan x + 2}$

$= \dfrac{5\left(\frac{4}{5}\right) - 3}{5\left(\frac{4}{5}\right) + 2}$

$= \dfrac{4 - 3}{4 + 2}$

$$= \frac{1}{6}$$

Section D

30. $\sec\theta = x + \frac{1}{4x}$

We know that $\sec^2\theta - \tan^2\theta = 1$

$\tan^2\theta = \sec^2\theta - 1$

$$= \left(x + \frac{1}{4x}\right)^2 - 1$$

$$= x^2 + \left(\frac{1}{4x}\right)^2 + 2x\left(\frac{1}{4x}\right) - 1$$

$$= x^2 + \left(\frac{1}{4x}\right)^2 - \frac{1}{2}$$

$$= \left(x - \frac{1}{4x}\right)^2$$

So, $\tan^2\theta = \left(x - \frac{1}{4x}\right)$

$\therefore \quad \tan\theta = \pm\left(x - \frac{1}{4x}\right)$

Case 1

$\sec\theta = x + \frac{1}{4x}$

$\tan\theta = x - \frac{1}{4x}$

So,

$\sec\theta + \tan\theta$

$= 2x$

Case 2

$\sec\theta = x + \frac{1}{4x}$

$\tan\theta = -\left(x - \frac{1}{4x}\right)$

So,

$\sec\theta + \tan\theta$

$= x + \frac{1}{4x} - x + \frac{1}{4x}$

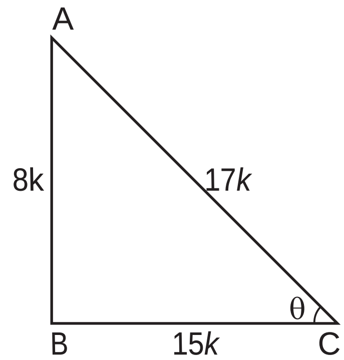
$= \frac{2}{4x}$

$= \frac{1}{2x}$

$\therefore \quad \sec\theta + \tan\theta = 2x + \frac{1}{2x}$

31. $\cot\theta = \frac{15}{8}$

$$= \frac{BC}{AB}$$



Let BC = 15k

AB = 8k

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$= 64k^2 + 225k^2$

$= 289k^2$

$\therefore \quad AC = 17k^2$

(i) Consider $\frac{(2 + 2\sin\theta)(1 - \sin\theta)}{(1 + \cos\theta)(2 - 2\cos\theta)}$

$$= \frac{2(1 + \sin\theta)(1 - \sin\theta)}{2(1 + \cos\theta)(1 - \cos\theta)}$$

$$= \frac{1 - \sin^2\theta}{1 - \cos^2\theta}$$

$$= \frac{\cos^2\theta}{\sin^2\theta}$$

$$= \cot^2\theta$$

$$= \frac{225}{64}$$

(ii) $\frac{\text{cosec}^2\theta - \cot^2\theta}{\text{cosec}^2\theta + \cot^2\theta}$

$$= \frac{1}{\text{cosec}^2\theta + \cot^2\theta}$$

$$= \frac{1}{\left(\frac{17}{8}\right)^2 + \left(\frac{15}{8}\right)^2}$$

$$= \frac{64}{289 + 225}$$

$$= \frac{64}{514}$$

$$= \frac{32}{257}$$

(iii) $\sec^2\theta + \tan^2\theta$

$$= \left(\frac{17}{15}\right)^2 + \left(\frac{8}{15}\right)^2$$

$$= \frac{289}{225} + \frac{64}{225}$$

$$= \frac{353}{225}$$

32. $\tan A = n \tan B$

$$\Rightarrow \frac{\sin A}{\cos A} = n \frac{\sin B}{\cos B} \quad \text{(i)}$$

Also, $\sin A = m \sin B$

$$\Rightarrow \frac{\sin A}{\sin B} = m \quad \text{(ii)}$$

From (i), (ii), we get

$$m = n\frac{\cos A}{\cos B}$$

$$\Rightarrow \cos B = \frac{n}{m}\cos A \quad \text{(iii)}$$

On putting value of sin B and cos B from (ii) and (iii) in $\cos^2 B + \sin^2 B = 1$, we get

$$\frac{n^2}{m^2}\cos^2 A + \frac{1}{m^2}\sin^2 A = 1$$

$$n^2\cos^2 A + \sin^2 A = m^2$$

$$n^2\cos^2 A + 1 - \cos^2 A = m^2$$

$$n^2\cos^2 A - \cos^2 A = m^2 - 1$$

$$(n^2 - 1)\cos^2 A = m^2 - 1$$

$$\therefore \quad \cos^2 A = \frac{m^2 - 1}{n^2 - 1}$$

33. $\text{cose c } \theta - \sin\theta = l$

$\sec\theta - \cos\theta = m$

Consider

$l^2 - m^2 \ (l^2 + m^2 + 3)$

$= (\text{cosec}\theta - \sin\theta)^2 (\sec\theta - \cos\theta)^2[(\text{cosec}\theta - \sin\theta)^2 (\sec\theta - \cos\theta)^2] + 3$

$$= \left(\frac{1}{\sin\theta} - \sin\theta\right)^2 \left(\frac{1}{\cos\theta} - \cos\theta\right)^2 \left[\left(\frac{1}{\sin\theta} - \sin\theta\right)^2 + \left(\frac{1}{\cos\theta} - \cos\theta\right)^2\right] + 3$$

$$= \left(\frac{1 - \sin^2\theta}{\sin\theta}\right)^2 = \left(\frac{1 - \cos^2\theta}{\cos\theta}\right)^2 \left[\left(\frac{1 - \sin^2\theta}{\sin\theta}\right)^2 + \left(\frac{1 - \cos^2\theta}{\cos\theta}\right)^2 + 3\right]$$

$$= \left(\frac{\cos^2\theta}{\sin\theta}\right)^2 \left(\frac{\sin^2\theta}{\cos\theta}\right)^2 \left[\left(\frac{\cos^2\theta}{\sin\theta}\right)^2 + \left(\frac{\sin^2\theta}{\cos\theta}\right)^2 + 3\right]$$

$$= \frac{\cos^4\theta}{\sin^2\theta}\frac{\sin^4\theta}{\cos^2\theta}\left[\frac{\cos^4\theta}{\sin^2\theta} + \frac{\sin^4\theta}{\cos^2\theta} + 3\right]$$

$$= \sin^2\theta\cos^2\theta\left[\frac{\cos^6\theta + \sin^6\theta + 3\sin^2\theta\cos^2\theta}{\sin^2\theta\cos^2\theta}\right]$$

$$= (\sin^2\theta)^3 + (\cos^2\theta)^3 + 3\sin^2\theta\cos^2\theta$$

$$= (\sin^2\theta + \cos^2\theta)(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta) + 3\sin^2\theta\cos^2\theta$$

$$= \sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta$$

$$= (\sin^2\theta + \cos^2\theta)^2$$

$$= 1^2$$

$$= 1$$

34. $\frac{\cos\alpha}{\cos\beta}$ m and $\frac{\cos\alpha}{\sin\beta} = n$

Consider

$(m^2 + n^2)\ \cos^2\beta$

$$= \left(\frac{\cos^2\alpha}{\cos^2\beta} + \frac{\cos^2\alpha}{\sin^2\beta}\right)\cos^2\beta$$

$$= \left(\frac{\cos^2\alpha\sin^2\beta + \cos^2\beta\cos^2\alpha}{\cos^2\beta\sin^2\beta}\right)\cos^2\beta$$

$$= \frac{\cos^2\alpha\sin^2\beta + \cos^2\beta\cos^2\alpha}{\sin^2\beta}$$

$= \left(\frac{\cos\alpha}{\sin\beta}\right)^2$

$= n^2$

35. $(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C)$

$= (\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$ (i)

On multiplying both side of (i) by $(\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$, we get

$(\sec^2 A - \tan^2 A)(\sec^2 B - \tan^2 B)(\sec^2 C - \tan^2 C)$

$= (\sec A - \tan A)^2 (\sec B - \tan B)^2 (\sec C - \tan C)^2$

$\Rightarrow 1 = (\sec A - \tan A)^2 (\sec B - \tan B)^2 (\sec C - \tan C)^2$

$\Rightarrow (\sec A - \tan A)^2 (\sec B - \tan B)^2 (\sec C - \tan C)^2 = \pm 1$

Again, Multiplying both sides of (i) by

$(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C)$

we get

$(\sec A + \tan A)^2 (\sec B + \tan B)^2 (\sec C + \tan C)^2$

$= (\sec^2 A - \tan^2 A)(\sec^2 B - \tan^2 B)(\sec^2 C - \tan^2 C)$

$= 1$

$\therefore$ $(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C) = \pm 1$

36. $x \sin^3\theta + y \cos^3\theta = \sin\theta \cos\theta$

$\Rightarrow x \sin^3\theta + \cos^2\theta (y \cos\theta) = \sin\theta \cos\theta$

$\Rightarrow x \sin^3\theta + \cos^2\theta\, x \sin\theta = \sin\theta \cos\theta$

[$\because$ $y \cos\theta = x \sin\theta$]

$\Rightarrow x \sin\theta + (\sin^2\theta + \cos^2\theta) = \sin\theta \cos\theta$

$\Rightarrow x \sin\theta = \sin\theta \cos\theta$

$\Rightarrow x = \cos\theta$

$\therefore \quad y = \frac{x \sin\theta}{\cos\theta} = \frac{\cos\theta \sin\theta}{\cos\theta} = \sin\theta$

Also, we know that

$\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \quad y^2 + x^2 = 1$

$\Rightarrow \quad x^2 + y^2 = 1$

37. $\operatorname{cosec}\theta - \sin\theta = m$

$\Rightarrow \frac{1}{\sin\theta} - \sin\theta = m$

$\Rightarrow \frac{1 - \sin^2\theta}{\sin\theta} = m$

$\Rightarrow \frac{\cos^2\theta}{\sin\theta} = m$

Also, $\sec\theta - \ = n$

$\frac{1}{\cos\theta} - \cos\theta = n$

$\frac{1 - \cos^2\theta}{\cos\theta} = n$

$\frac{\sin^2\theta}{\cos\theta} = n$

So,

L H S $= (m^2 n)^{\frac{2}{3}} + (mn^2)^{\frac{2}{3}}$

$= \left(\frac{\cos^4\theta}{\sin^2\theta} \frac{\sin^2\theta}{\cos\theta}\right)^{\frac{2}{3}} + \left(\frac{\cos^2\theta}{\sin\theta} \frac{\sin^4\theta}{\cos^4\theta}\right)^{\frac{2}{3}}$

$= (\cos^3\theta)^{\frac{2}{3}} + (\sin^3\theta)^{\frac{2}{3}}$

$= \cos^2\theta + \sin^2\theta$

$= 1$

38. $a \sec\theta + b \tan\theta + c = 0$

$p \sec\theta + q \tan\theta + r = 0$

To prove:

$(br - qc)^2 - (pc - ar)^2 = (aq - pb)^2$

Consider

$(br - qc)^2 - (pc - ar)^2$

$= [\, b(-p \sec\theta - q \tan\theta) + q(a \sec\theta + b \tan\theta)]^2 - [p(-a \sec\theta - b \tan\theta) + a(p \sec\theta + q \tan\theta)]^2$

$= [-bp\sec\theta - bq\tan\theta + aq\sec\theta + bq\tan\theta]^2 - [-ap\sec\theta - bp\tan\theta + ap\sec\theta + aq\tan\theta]^2$

$= [\sec\theta(aq - bp)]^2 - [(aq - bp)\tan\theta]^2$

$= (aq - bp)^2(\sec^2\theta - \tan^2\theta)$

$= (aq - bp)^2$

39. $\tan^2\theta = 1 - a^2$

Consider

$(\sec\theta + \tan^3\theta \operatorname{cosec}\theta)$

$= \sqrt{1 + \tan^2\theta} + \tan^2\theta \ \tan\theta \ \operatorname{cosec}\theta$

$= \sqrt{1 + \tan^2\theta} + \tan^2\theta \ \tan\theta \sqrt{1 + \cot^2\theta}$

$= \sqrt{1 + 1 - a^2} + (1 - a^2)\sqrt{1 - a^2}\sqrt{1 + \frac{1}{\tan^2\theta}}$

$= \sqrt{2 - a^2} + (1 - a^2)(\sqrt{1 - a^2})\sqrt{1 + \frac{1}{1 - a^2}}$

$= \sqrt{2 - a^2} + (1 - a^2)\frac{\sqrt{1 - a^2}}{\sqrt{1 - a^2}}\sqrt{2 - a^2}$

$= \sqrt{2 - a^2} + (1 - a^2)\sqrt{2 - a^2}$

$= \sqrt{2 - a^2} + (1 + 1 - a^2)$

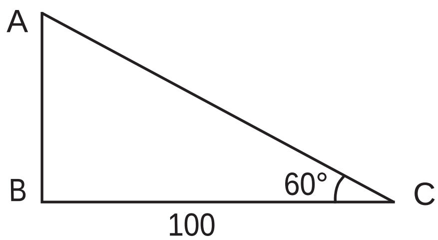
$= \sqrt{2 - a^2} + (2 - a^2)$

$= (2 - a^2)^{\frac{2}{3}}$

Chapter 09 Some Applications of Trigonometry

MULTIPLE CHOICE QUESTIONS

1.



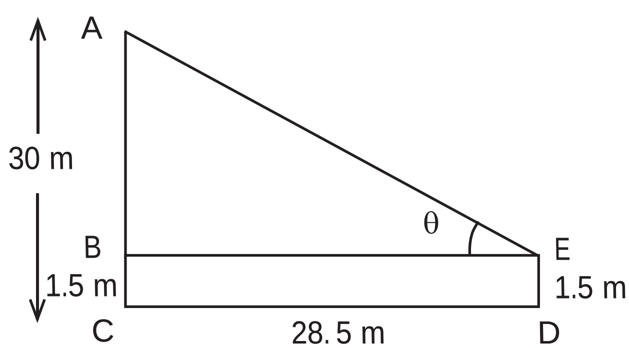
$\tan 60° = \frac{AB}{BC}$

$\sqrt{3} = \frac{AB}{100}$

$\therefore \quad AB = 100\sqrt{3}$ m

Option (d)

2.



$AB = AC - BC$

$= 30 - 1.5$

$= 28.5$ m

$BE = CD = 28.5$ m

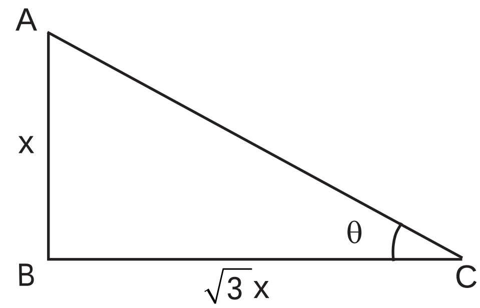
In ΔABE,

$\tan\theta = \frac{AB}{BE} = \frac{28.5}{28.5} = 1$

$\Rightarrow \theta = 45°$

Option (c)

3.

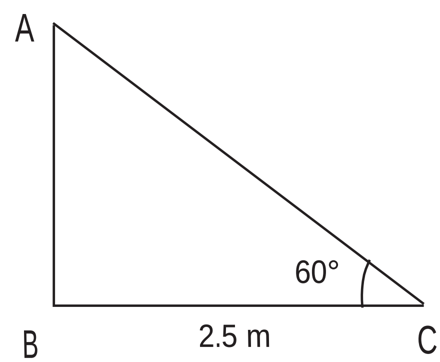


$\tan\theta = \frac{AB}{BC} = \frac{x}{\sqrt{3}x} = \frac{1}{\sqrt{3}}$

$\theta = 30°$

Option (d)

4.



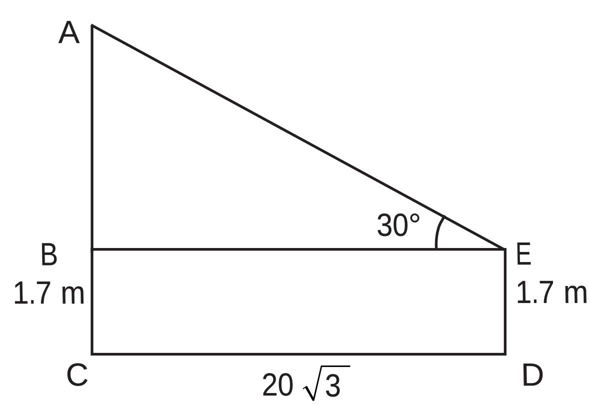
$\text{Cos } 60° = \frac{BC}{AC}$

$\frac{1}{2} = \frac{2.5}{AC}$

$AC = 5$ m

Option (b)

5.



In ΔABE, $BE = CD = 20\sqrt{3}$ m

$$\tan 30° = \frac{AB}{BE}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{20\sqrt{3}}$$

$$\therefore \quad AB = 20 \text{ m}$$

So, $AC = AB + BC = 20 + 1.7$

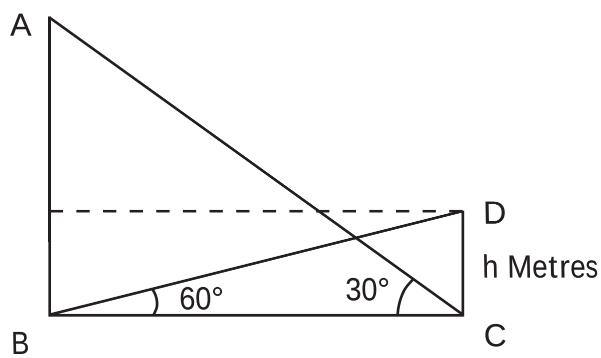
$= 21.7$ m

Option (a)

WORKSHEET 1

Section A

1.



Let AB denotes the tower.

In ΔBCD,

$$\tan 60° = \frac{CD}{BC}$$

$$\sqrt{3} = \frac{h}{BC}$$

$$BC = \frac{h}{\sqrt{3}} \text{ Metre}$$

In ΔABC,

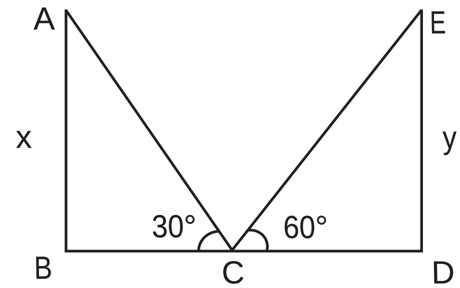
$$\tan 30° = \frac{AB}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{h}$$

$$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{h}$$

$$AB = \frac{h}{3} \text{ Metre}$$

2.



Let AB and DE denote two towers

In ΔABC,

$$\tan 30° = \frac{AB}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{x}{BC}$$

$$\Rightarrow \quad BC = \sqrt{3}$$

In ΔCDE,

$$\tan 60° = \frac{DE}{CD}$$

$$\sqrt{3} = \frac{y}{CD}$$

$$CD = \frac{y}{\sqrt{3}}$$

As $BC = CD$

$$\therefore \quad \sqrt{3}x = \frac{y}{\sqrt{3}}$$

$$3x = y$$

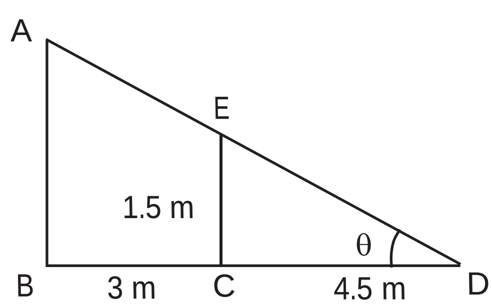
$$\Rightarrow \quad x : y = 1 : 3$$

3. In ΔABC,

$$\tan C = \frac{AB}{BC} = \frac{20}{20\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\therefore \quad C = 30°$$

4.



Let CE denotes the boy and AB denotes a lamp-post.

In ΔDCE,

$$\tan\theta = \frac{CE}{CD} = \frac{1.5}{45} = \frac{1}{3}$$

In ΔABC,

$$\tan\theta = \frac{AB}{BD}$$

$$\frac{1}{3} = \frac{AB}{7.5}$$

$$\therefore \quad AB = 2.5 \text{ m}$$

5. In ΔABC,

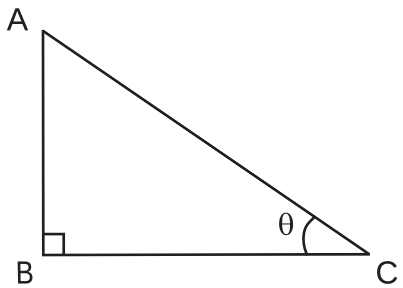
$$\sin 45° = \frac{BC}{AC}$$

$$\frac{1}{\sqrt{2}} = \frac{BC}{150}$$

$$BC = \frac{150}{\sqrt{2}}$$

$$= 75\sqrt{2} \text{ m}$$

6.



Let AB denotes the vertical role and BC denotes the shadow of the role.

Let $AB = BC = x$

In ΔABC,

$$\tan\theta = \frac{AB}{BC} = \frac{x}{x} = 1$$

$$\therefore \quad \theta = 45°$$

7. In ΔBAC,

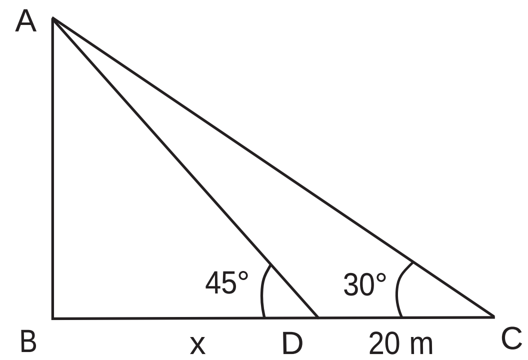
$$\tan\theta = \frac{AB}{AC}$$

$$= \frac{5a\sqrt{3}}{5a}$$

$$= \sqrt{3}$$

$$\theta = 60°$$

8.



Let AB denotes the chimney.

Let BD = x metre

In ΔABC,

$$\tan 30° = \frac{AB}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{x+20}$$

$$AB = \frac{1}{\sqrt{3}}(x+20) \qquad \text{(i)}$$

In ΔABC,

$$\tan 45° = \frac{AB}{BD}$$

$$1 = \frac{AB}{x}$$

$$AB = x \qquad \text{(ii)}$$

From (i) and (ii),

$$AB = x = \frac{1}{\sqrt{3}}(x+20)$$

$$\sqrt{3}x - x = 20$$

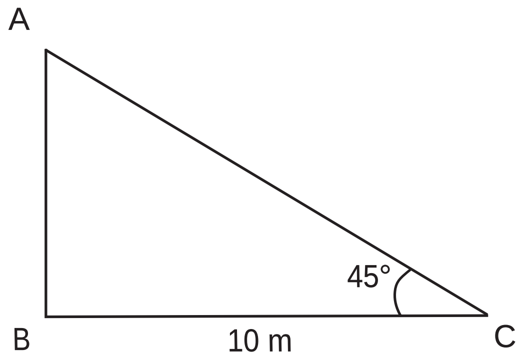
$$x = \frac{20}{\sqrt{3}-1}$$

$$= \frac{20}{2}(\sqrt{3}+1)$$

$$x = 10(\sqrt{3}+1)$$

$$\therefore \quad AB = x = 10(\sqrt{3}+1)$$

9.



Let AB denotes the tower and BC denotes the shadow

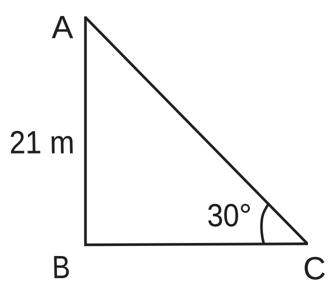
In ΔABC,

$\tan 45° = \frac{AB}{BC}$

$1 = \frac{AB}{10}$

$AB = 10$ m

10.



Let AC denotes the string of kite.

In ΔABC,

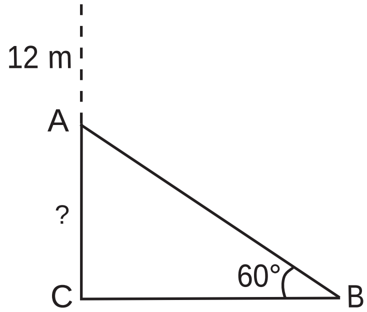
$\sin 30° = \frac{AB}{AC}$

$\frac{1}{2} = \frac{21}{AC}$

$AC = 42$ m

Section B

11.



Let AC be x and AB be 12 – x

$\therefore \sin 60° = \frac{AC}{AB}$

$\frac{\sqrt{3}}{2} = \frac{x}{12 - x}$

$12\sqrt{3} - \sqrt{3}x = 2x$

$12 \times 1.73 = 2x + 1.73x$

$20.76 = 3.73x$

$\frac{2076 \times \cancel{100}}{373 \times \cancel{100}} = x$

$5.6m = x = AC$

12. Let the aeroplane be at B and let the two ships be at C and D such that their angles of depression from B are 60° and 30° respectively.

In ΔCAB, we have,

$\tan 60° = \frac{AB}{CA}$

$\Rightarrow \sqrt{3} = \frac{1200}{x}$

$\Rightarrow x = \frac{1200}{\sqrt{3}} = 400\sqrt{3}$

In ΔBAD, we have

$\tan 30° = \frac{AB}{AD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{1200}{x + y}$

$\Rightarrow x + y = 1200\sqrt{3}$

$\Rightarrow y = 1200\sqrt{3} - x$

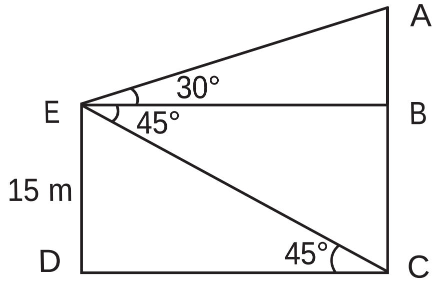
$\Rightarrow y = 1200\sqrt{3} - 400\sqrt{3}$

$= 800\sqrt{3} = 800 \times 1.732$

$= 1385.6$

Hence, the distance between the two ships is 1385.6 metres.

13.



Let the window be at point E and AC be the house.

Let

To find : AC

In ΔCDE,

$$\tan 45° = \frac{DE}{CD}$$

$$1 = \frac{15}{CD}$$

$\therefore$ CD = 15 m

$\therefore$ BE = CD = 15 m

In ΔABE,

$$\tan 30° = \frac{AB}{BE}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{15}$$

$$\therefore \quad AB = \frac{15}{\sqrt{3}} = 5\sqrt{3} \text{ m}$$

Also, BC = DE =15 m

$\therefore$ AC = AB = BC

$$AC = 5\sqrt{3} + 15$$

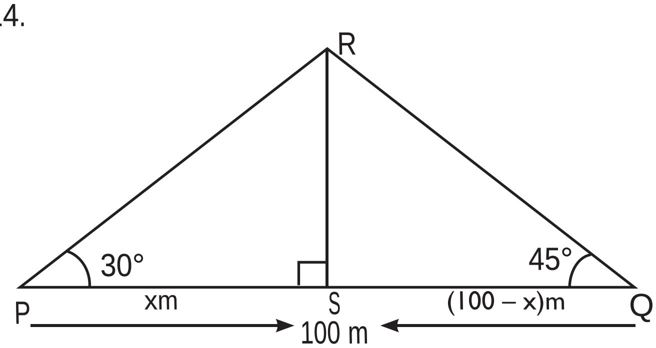
$$= 5(\sqrt{3} + 3) \text{ m}$$

$$= 5\,(1.732 + 3) \text{ m}$$

$$= 5 \times 4.732$$

$$= 23.66 \text{ m}$$

14.



Let AB denotes the three .

Step 1:

Given Data:

In rt. ΔPRS,

$x = RS \cot 30°$

$x = RS\sqrt{3}$

Step 2:

$x = \sqrt{3}\ RS$........ (i)

In rt. ΔRSQ,

$SQ = RS \cot 45°$

Step 3:

$(100 - x) = RS$

$x = 100 - RS$........ (ii)

Step 4:

Equating (i) and Equation(ii) we have:

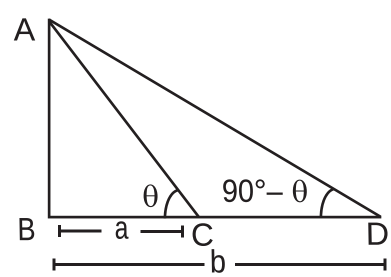
$RS = 100 - RS$

Step 5:

$2.73\ RS = 100$

$RS = 36.63$ m

15.



Let AB denotes the tree.

To prove : $AB = \sqrt{ab}$ metres

In ΔABC,

$$\tan\theta = \frac{AB}{BC}$$

$$\tan\theta = \frac{AB}{a}$$

$\therefore$ AB $= a \tan\theta$ (i)

In ΔABD,

$\tan(90 - \theta) = \frac{AB}{BD}$

$\cot\theta = \frac{AB}{b}$

$\therefore$ AB $= b \cot\theta$ (ii)

From (i) and (ii)

AB $= a \tan\theta = b \cot\theta$

$\tan^2\theta = \frac{b}{a}$

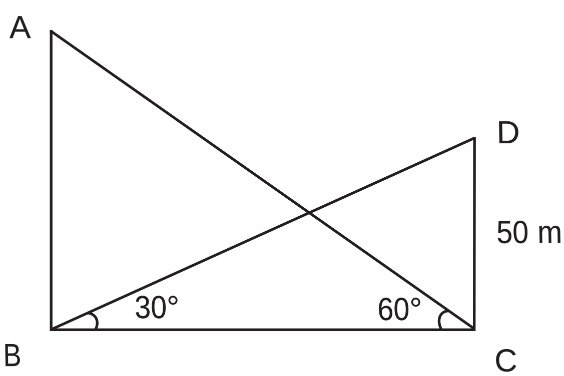
$\therefore \tan\theta = \sqrt{\frac{b}{a}}$

So, AB $= a \tan\theta$

$= a\sqrt{\frac{b}{a}}$

$= \sqrt{ab}$ metres

16.



Let AB denotes the hill and CD denotes the tower such that CD = 50 m

In ΔBCD,

$\tan 30° = \frac{CD}{BC}$

$\frac{1}{\sqrt{3}} = \frac{50}{BC}$

$\therefore$ BC $= 50\sqrt{3}$ m

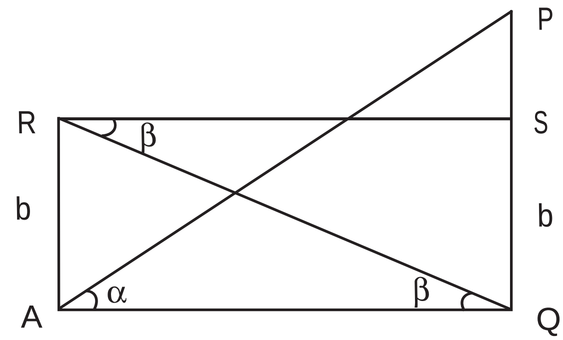
In ΔABC,

$\tan 60° = \frac{AB}{BC}$

$\sqrt{3} = \frac{AB}{50\sqrt{3}}$

$\therefore$ AB $= 150$ m

17.



Let PQ denotes the tower

To prove : $PQ = b \tan\alpha \cot\beta$

QS = AR = b ft

In ΔRAQ,

$\tan\beta = \frac{AR}{AQ}$

$\tan\beta = \frac{b}{AQ}$

AQ $= b \cot\beta$

In ΔPQA,

$\tan\alpha = \frac{PR}{AQ}$

$\tan\alpha = \frac{PS + QS}{b\cos\beta}$

$\tan\alpha = \frac{PS + b}{b\cos\beta}$

$\Rightarrow$ PS + b $= b \tan\alpha \cot\beta$

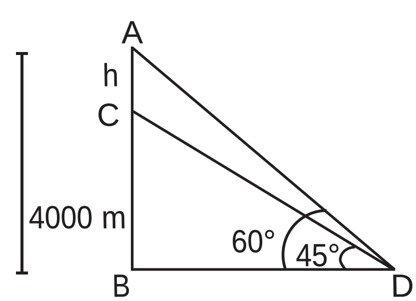
$\Rightarrow$ PS $= b \tan\alpha \cot\beta - b$

So, PQ = PS + SQ

$= b \tan\alpha \cot\beta - b + b$

$= b \tan\alpha \cot\beta$

18.



Height of first Airplane = AB = 4000m

Height of another lane = BC

The angles of elevation of two planes from the same point on the ground are 60° and 45°. i.e. $\angle ADB = 60°$ and $\angle CDB = 45°$

Let AC be h

CB = AB – AC = 4000 – h

In ΔABD

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\tan 60° = \frac{AB}{BD}$$

$$\sqrt{3} = \frac{4000}{BD}$$

$$BD = \frac{4000}{\sqrt{3}}$$

BD = 2309.401

In ΔCBD

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\tan 45° = \frac{CB}{BD}$$

$$1 = \frac{4000 - h}{2309.401}$$

2309.401 = 4000 – h

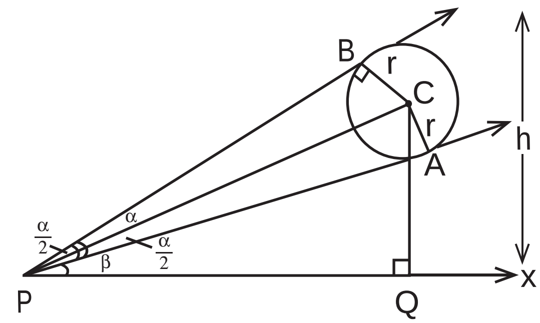
h = 4000 – 2309.401

h = 1690.599

Hence, the vertical distance between the aeroplane at that instant is 1690.599 m.

Section C

19. Let P be the eye of observer. Let PA and PB are tangents to the round balloon.



PX is the horizontal line and $CQ \perp PQ$. It is given that $\angle APB = \alpha$

$$\therefore \angle CPA = \angle CPB = \frac{\alpha}{2}$$

and $\angle CPA = \beta$

Let height of the centre C be h m and CA = CB = r

In right triangle CBP, we have

$$\sin\left(\frac{\alpha}{2}\right) = \frac{BC}{CP}$$

$$\Rightarrow \sin\left(\frac{\alpha}{2}\right) = \frac{r}{CP}$$

$$\Rightarrow CP = \frac{r}{\sin\left(\frac{\alpha}{2}\right)}$$

$$\Rightarrow CP = r \operatorname{cosec}\frac{\alpha}{2}$$

In right triangle CPQ, we have

$$\sin\beta = \frac{CQ}{CP}$$

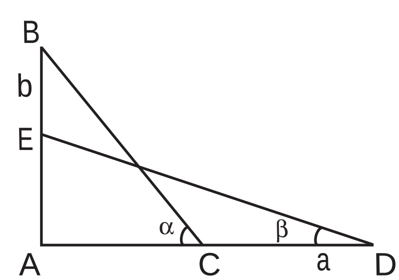
$$\Rightarrow CQ = CP \sin\beta$$

$$\Rightarrow CQ = r \operatorname{cosec}\frac{\alpha}{2}\sin\beta$$

Hence, the height of the centre

$$= r \sin\beta \operatorname{cosec}\frac{\alpha}{2}$$

20.



Let the kite be at point A such that AC is the string of kite.

Let BC be the ladder which slides down a distance b on the wall.

In right triangle ABC, we have

$$\sin\alpha = \frac{AB}{BC} = \frac{AE + EB}{BC}$$

$$\sin\alpha = \frac{AE + b}{BC}$$

But, AE = sin β × ED (In ΔAED)

So, replacing AE by ED sin β, we get

$$\sin\alpha = \frac{ED\sin\beta + b}{BC}$$

$$\Rightarrow b = BC\sin\alpha - ED\sin\beta$$

As, BC and ED both represent the same ladder.

BC = ED. (length of ladder does not change)

$$\Rightarrow BC\sin\alpha - BC\sin\beta = b$$

$$\Rightarrow BC(\sin\alpha - \sin\beta) = b \text{ ...(i)}$$

Similarly, in right triangle AED, we have

$$\cos\beta = \frac{AD}{ED} = \frac{AC + CD}{ED}$$

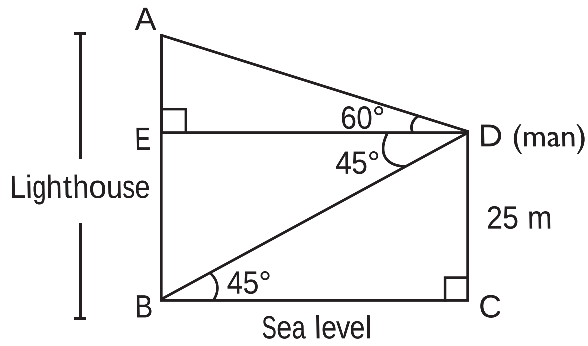
$$\cos\beta = \frac{AC + a}{ED}$$

But, AC = BC cos α (In ΔABC)

So, by replacing AC by BC cos α, we get ED cos β = BC cos α + a BC (cos, –cos α) = a [∴ ED = BC] ...(ii) Dividing (ii) by (i), we get

$$\frac{a}{b} = \frac{\cos\beta - \cos\alpha}{\sin\alpha - \sin\beta}$$

$$\frac{a}{b} = \frac{\cos\alpha - \cos\beta}{\sin\beta - \sin\alpha}$$

21. 

Let AB represents the light house (A shows its top), BC represents the sea level and D represents the position of the man,

By the below diagram,

In triangle DCB,

$$\tan 45° = \frac{CD}{BC}$$

$$1 = \frac{25}{BC} \Rightarrow BC = 25 \text{ m}$$

⇒ ED = 25 meters,

Now, in triangle DAE,

$$\tan 60° = \frac{AE}{ED}$$

$$\sqrt{3} = \frac{AE}{25}$$

$$\Rightarrow AE = 25\sqrt{3}$$

Hence, the height of lighthouse,

AB = AE + EB

$$= 25\sqrt{3} + 25$$

$$= 25(\sqrt{3} + 1) \text{ meters.}$$

22. Let the distance between the nearer kilometre stone and the hill be 'a' km. So, the distance between the farther kilometre stone and the hill is '1 + x' km since both are on the same side of the hill.

In triangle APB,

$$\tan 45° = \frac{h}{x}$$

$$\Rightarrow 1 = \frac{h}{x}$$

$\Rightarrow h = x$

In triangle AQB,

$\tan 30° = \frac{h}{1 + x}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{1 + x}$

$\Rightarrow 1 + x = \sqrt{3}h$

From equation 1,

$1 + h = \sqrt{3}h \Rightarrow 1 = \sqrt{3}h - h$

$\Rightarrow h = \frac{1}{\sqrt{3} - 1}$

$\Rightarrow h = 1.365\text{km}$

Hence, option A is correct.

Section D

23. Let A and B be the two positions of the ship. Let d be the distance travelled by the ship during the period of observation i.e. AB = d metres.

Let the observer be at O, the top of the lighthouse PO.

It is given that PO = 100 m and the angle of depression from O of A and B are 30° and 45° respectively.

$\therefore \angle OAP = 30°$ and $\angle OBP = 45°$

In ΔOPB, we have

$\tan 45° = \frac{OP}{BP}$

$\Rightarrow 1 = \frac{100}{BP}$

$\Rightarrow BP = 100$ m

In ΔOPA, we have

$\Rightarrow \tan 30° = \frac{OP}{AP}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{100}{d + BP}$

$\Rightarrow d + BP = 100\sqrt{3}$

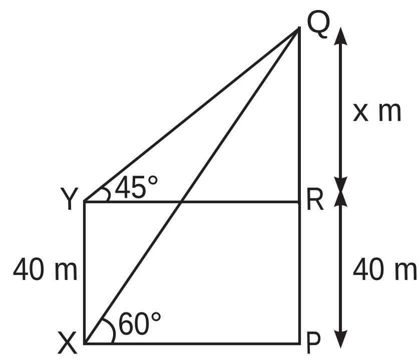
$\Rightarrow d + BP = 100\sqrt{3}$

$\Rightarrow d = 100\sqrt{3} - 100$

$\Rightarrow d = 100(\sqrt{3} - 1) = 100(1.732 - 1) = 73.2$ m.

Hence, the distance travelled by the ship from A to B is 73.2 m.

24.



In YRQ, we have

$\tan 45° = \frac{QR}{YR}$

$\Rightarrow 1 = \frac{x}{YR}$

$\Rightarrow YR = x$

or XP = x   [As YR = XP] ...(1)

Now, In ΔXPQ, we have

$\tan 60° = \frac{PQ}{PX}$

$\Rightarrow \sqrt{3} = \frac{x + 40}{x}$ [Using (1)]

$\Rightarrow \sqrt{3}x = x + 40$

$\Rightarrow x(\sqrt{3} - 1) = 40$

$\Rightarrow x = \frac{40}{\sqrt{3} - 1}$

On rationalising the denominator, we get

$x = \frac{40}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1} = \frac{40(\sqrt{3} + 1)}{3 - 1}$

$= 20(\sqrt{3} + 1) = 54.64$ m

So, height of the tower,

$PQ = x + 40 = 54.64 + 40 = 94.64$ metres

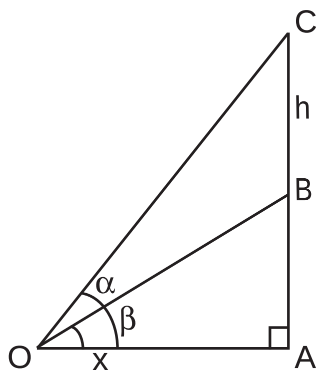
Now, In ΔXPQ, we have

$$\sin 60° = \frac{PQ}{PX}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{94.64}{XQ}$$

$$XQ = \frac{94.64 \times 2}{\sqrt{3}} = \frac{94.64 \times 2 \times \sqrt{3}}{3} = 109.3 \text{ m}$$

25.



Let AB be the tower and BC be the flagstaff. Let OA = x metres, AB = y metres and BC = h metres.

In right ΔOAB,

$$\tan \alpha = \frac{AB}{OA}$$

$$\Rightarrow y = x \tan \alpha \text{ or } x = \frac{y}{\tan \alpha} \quad ...(i)$$

In right ΔOAC,

$$\tan \beta = \frac{y + h}{x}$$

$$\Rightarrow x = \frac{(y + h)}{\tan \beta} \quad ...(ii)$$

From (i) and (ii),

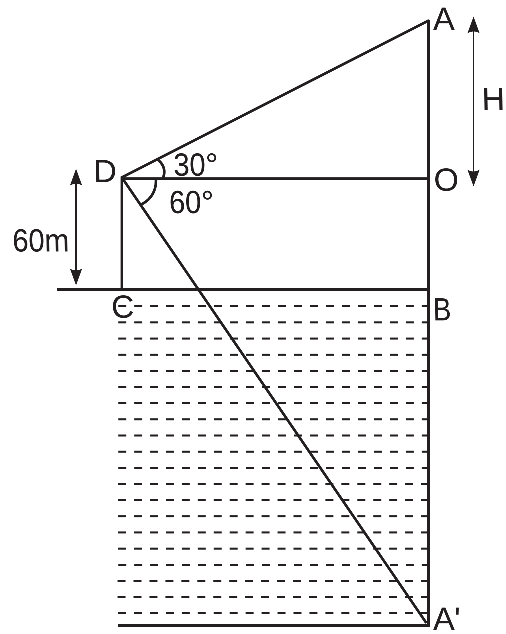
$$\frac{y}{\tan \alpha} = \frac{(y + h)}{\tan \beta}$$

$$y(\tan \beta - \tan \alpha) = h \tan \alpha$$

$$y = \frac{h \tan \alpha}{(\tan \beta - \tan \alpha)}$$

Thus, the height of the tower is $\frac{h \tan \alpha}{(\tan \beta - \tan \alpha)}$

26.



Let AO = H

CD = OB = 60 m

A'B = AB = 60 + H

In ΔAOD,

$$\tan 30° = \frac{AO}{OD} = \frac{H}{OD}$$

$$H = \frac{OD}{\sqrt{3}}$$

$$OD = \sqrt{3} H$$

Now, In $\Delta A'OD$,

$$\tan 60° = \frac{OA'}{OD} = \frac{OB + BA'}{OD}$$

$$\sqrt{3} = \frac{60 + 60 + H}{\sqrt{3} H} = \frac{120 + H}{\sqrt{3} H}$$

$$\Rightarrow 120 + H = 3H$$

$$\Rightarrow 2H = 120$$

$$\Rightarrow H = 60\text{m}$$

Thus, height of the cloud from the surface of the lake = AB + A'B = 60 + 60 = 120 m.

27. $\frac{AB}{AD} = \tan 30° = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{h}{x+y} = \frac{1}{\sqrt{3}}$

$\Rightarrow x + y = \sqrt{3}h$

$\therefore x = (x + y - y)$

$= \sqrt{3}h - h$

$= h(\sqrt{3} - 1)$

Now, $h(\sqrt{3} - 1)$ is covered in 12 min.

So, h will be covered in $\rightarrow$

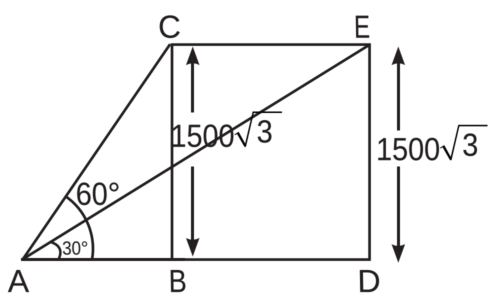
$\left[\frac{12}{h(\sqrt{3}-1)} \times h\right]$

$= \frac{12}{(\sqrt{3}-1)}$ min

$= \left(\frac{1200}{73}\right)$ min

= 16 min, 23 sec

28. Let A be the point of observation, C and E be the two points of the plane. It is given that after 15 seconds angle of changes from 60° to 30°.



i.e. $\angle BAC = 60°$ and $\angle DAE = 30°$. It is also given that height of the jet plane is $1500\sqrt{3}$ m.

i .e. $CB = 1500\sqrt{3}$

[Since jet plane is flying at constant height, therefore, $CB = ED = 1500\sqrt{3}$ m]

$\tan 60° = \frac{BC}{AB}$

$\Rightarrow \sqrt{3} = \frac{1500\sqrt{3}}{AB}$

$\Rightarrow AB = \frac{1500\sqrt{3}}{\sqrt{3}}$

$\Rightarrow AB = 1500\sqrt{3}$ m ...(i)

In right triangle ADE, we have

$\tan 30° = \frac{DE}{AD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{DE}{AB + BD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{1500\sqrt{3}}{AB + BD}$

$\Rightarrow AB + BD = 1500\sqrt{3} \times \sqrt{3}$

$\Rightarrow AB + BD = 4500$ ...(ii)

Putting the value of (i) in (ii), we get

1500 + BD = 4500

$\Rightarrow$ BD = 3000

$\because$ Distance travelled in 15 sec

= CE = BD = 3000 metres,

Now, speed of plane (m/s) $= \frac{3000}{15} = 200$ m/s

Now, speed of plane (km/s) $= \frac{200}{1000} \times 3600$

= 720 km/hr

WORKSHEET 2

Section A

1. Let AB be the height of the tower and C be the point.

In right ΔABC,

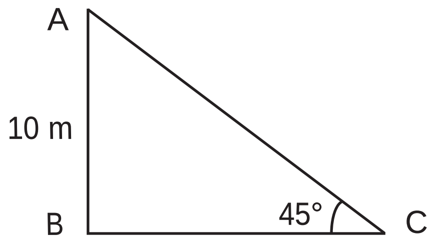
$\tan 30° = \frac{AB}{BC}$

$AB = BC \tan 30°$

$AB = \frac{20}{\sqrt{3}} = 11.56$m

Therefore, the height of the tower is 11.56m.

2.



Let AB and AC denotes the vertical role and wire respectively.

In ΔABC,

$$\sin 45° = \frac{AB}{AC}$$

$$\frac{1}{\sqrt{2}} = \frac{10}{AC}$$

$$\therefore \quad AC = 10\sqrt{2} \text{ m}$$

3. $BD = AB - AD$

$= 6 - 2.54$

$= 3.46$ m

In ΔCBD,

$$\sin 60° = \frac{BD}{CD}$$

$$\frac{\sqrt{3}}{2} = \frac{3.46}{CD}$$

$$\therefore \quad CD = \frac{3.46 \times 2}{\sqrt{3}}$$

$$= \frac{3.46 \times 2}{1.73}$$

$$= \frac{6.92}{1.732}$$

$= 4$ m

4. Let AB = 10 = Height of Pole

And AD be the length of the wire

From ΔABD,

$$\sin 45° = \frac{AB}{AD} \Rightarrow \frac{1}{\sqrt{2}} = \frac{10}{AD} \Rightarrow AD = 10\sqrt{2}$$

$\Rightarrow 10 \times 1.414$ (Take $\sqrt{2} = 1.414$)

$= 14.14$m

5. Distance from the foot of ladder to wall = 1.5cm

Angle made by ladder is 60°

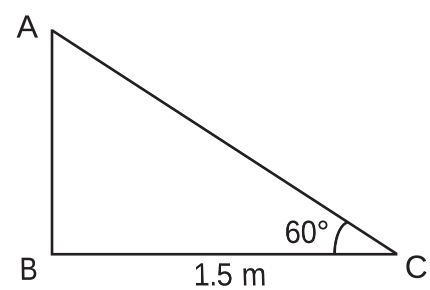
Consider tan 60° =

$$\frac{\text{Height of wall}}{\text{Distance from foot of ladder to wall}}$$

$$\sqrt{3} = \frac{\text{Height of wall}}{1.5}$$

Height of wall = $1.5\sqrt{3}$

6.



Let AB denotes the wall and AC denotes the ladder.

In ΔABC,

$$\tan 60° = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{AB}{1.5}$$

$$\therefore \quad AB = 1.5\sqrt{3} \text{ m}$$

7. Here is the position of balloon

Now, in ΔABC,

$$\sin 60° = \frac{AB}{AC}$$

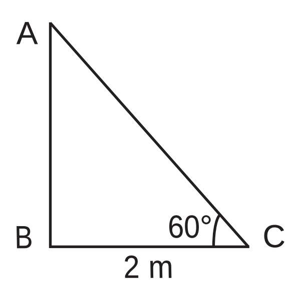
$AB = AC \sin 60°$

$$= 215 \times \frac{\sqrt{3}}{2}$$

$= 186$ m

$\Rightarrow$ Height of the balloon from the ground is 186 m.

8.



Let AB be the wall and AC be the ladder.

We have,

BC = 2 m and $\angle ACB = 60°$

In ΔABC,

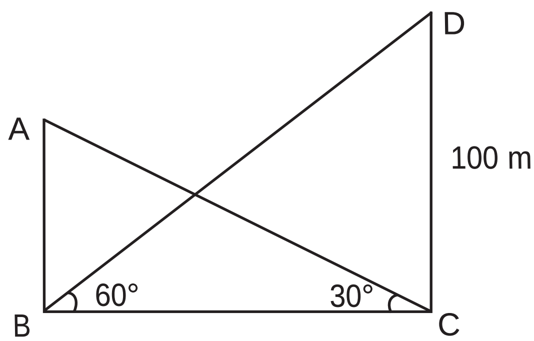
$$\cos 60° = \frac{BC}{AC}$$

$$\Rightarrow \frac{1}{2} = \frac{2}{AC}$$

$$AC = 4 \text{ m}$$

Section B

9.



let AB and CD denotes building and tower respectively.

In ΔBCD,

$$\tan 60° = \frac{CD}{BC}$$

$$\sqrt{3} = \frac{100}{BC}$$

$$BC = \frac{100}{\sqrt{3}}$$

$$= \frac{100}{3}\sqrt{3} \text{ m}$$

In ΔCBA,

$$\tan 30° = \frac{AB}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{\frac{100\sqrt{3}}{3}}$$

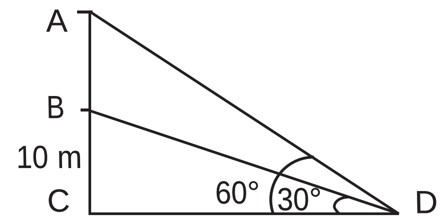
$$\frac{1}{\sqrt{3}} = \frac{3\,AB}{100\sqrt{3}}$$

$$3\,AB = 100$$

$$AB = \frac{100}{3} \text{ m}$$

So, height of building $= \dfrac{100}{3}$ m

10.



Let BC and AB denotes the building and tower respectively.

In ΔBCD

$$\tan 30° = \frac{BC}{CD}$$

$$\frac{1}{\sqrt{3}} = \frac{10}{CD}$$

$$\therefore \quad CD = 10\sqrt{3} \text{ m}$$

In ΔACD,

$$\tan 60° = \frac{AC}{CD}$$

$$\sqrt{3} = \frac{10 + AB}{10\sqrt{3}}$$

$$30 = AB + 10$$

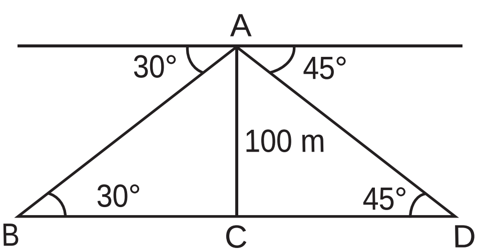
$$AB = 30 - 10$$

$$= 20 \text{ m}$$

So, height of tower = AB

= 20 m

11.



Let AC denotes the tower and the two buses be at points B and D respectively.

To find : BD

In ΔABC,

$$\tan 30° = \frac{AC}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{100}{BC}$$

$$\therefore \quad BC = 100\sqrt{3}$$

In ΔACD,

$$\tan 45° = \frac{AC}{CD}$$

$$1 = \frac{100}{CD}$$

$$CD = 100 \text{ m}$$

So, $BD = BC + CD$

$$= 100\sqrt{3} + 100$$

$$= 100(\sqrt{3} + 1) \text{ m}$$

12. In the first figure and from triangle BCD

$$\sin 30° = \frac{BC}{BD}$$

$$\Rightarrow \frac{1}{2} = \frac{1.5}{BD}$$

$$\Rightarrow BD = 1.5 \times 2$$

$$\Rightarrow BD = 3$$

So length of slide for child below 5 years = 3m

Again in the second figure and from triangle BCD

$$\sin 60° = \frac{BC}{BD}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{3}{BD}$$

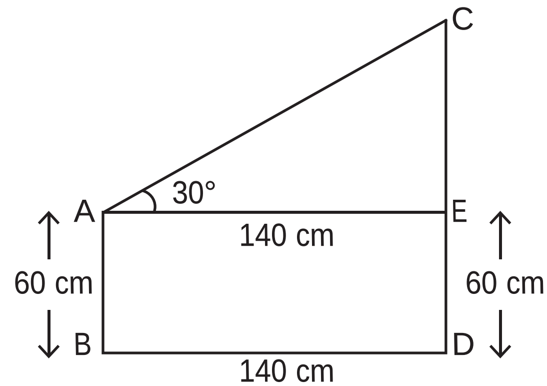
$$\Rightarrow BD = \frac{(3 \times 2)}{\sqrt{3}}$$

$$\Rightarrow BD = \frac{(\sqrt{3} \times \sqrt{3} \times 2)}{\sqrt{3}}$$

$$\Rightarrow BD = 2\sqrt{3}$$

So length of slide for child greater than 5 years $= 2\sqrt{3}$ m

13.



Let AB be the height of second tower and CD be the height of first tower.

Given, BD = AE = 140 m

And AB = DE = 60 m

In ΔAEC,

$$\tan 30° = \frac{\text{Perpendicular}}{\text{Base}} = \frac{CE}{AE}$$

$$\tan 30° = \frac{CE}{140}$$

$$\frac{1}{\sqrt{3}} = \frac{CE}{140}$$

$$CE = \frac{140}{\sqrt{3}}$$

$$CE = \frac{140 \times \sqrt{3}}{(\sqrt{3} \times \sqrt{3})}$$

[Rationalising the denominator]

$$CE = \frac{140\sqrt{3}}{3}$$

Height of the first tower CD = CE + DE

$$= \frac{140\sqrt{3}}{3} + 60$$

$$= \frac{(140 \times 1.73)}{3} + 60 \qquad [\sqrt{3} = 1.73]$$

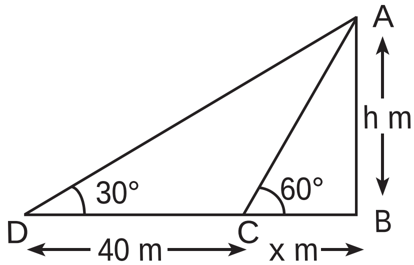
$$= \frac{242.2}{3} + 60$$

$$= 80.73 + 60$$

$$= 140.73 \text{ m}$$

Height of the first tower (CD)=140.73 m

14. AB is the tower and BC is the length of the shadow when the Sun's altitude is 60°, i.e., the angle of elevation of the top of the tower from the tip of the shadow is 60° and DB is the length of the shadow when the angle of elevation is 30°.



Now, let AB be h m and BC be x m. According to the question, DB is 40 m longer than BC.

So, BD = (40 + x) m

Now, we have two triangles ABC and ABD.

In ΔABC,

$\tan 60° = \frac{AB}{BC}$ or $\sqrt{3} = \frac{h}{x}$

$\Rightarrow x\sqrt{3} = h$ ...(i)

In ΔABD,

$\tan 30° = \frac{AB}{BD}$

i.e., $\frac{1}{\sqrt{3}} = \frac{h}{x + 40}$ ...(ii)

Using (i) in (ii), we get $(x\sqrt{3})\sqrt{3} = x + 40$,

i.e., $3x = x + 40$

i.e., $x = 20$

So, $h = 20\sqrt{3}$ [From (i)]

Therefore, the height of the tower is $20\sqrt{3}$ m.

Section C

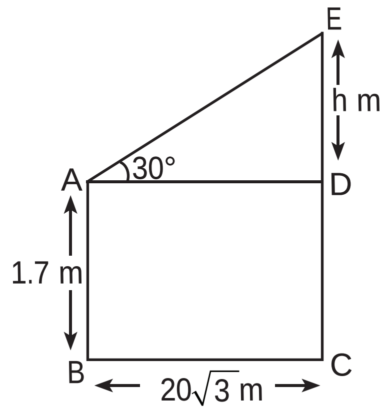
15. Let AB be the height of the observer and EC be the height of the tower.

Given:

AB = 1.7 m ⇒ CD = 1.7 m

BC = $20\sqrt{3}$ m

Let ED be h m.



In ΔADE,

$\tan 30° = \frac{ED}{AD}$

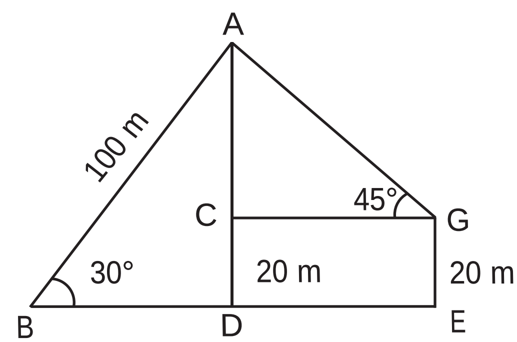
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{20\sqrt{3}}$

$\Rightarrow h = 20$ m

$\therefore$ EC = ED + DC = (h + 1.7)m = 21.7 m

Hence, the height of the tower is 21.7 m.

16.



Distance between girl and bird is $30\sqrt{2}$ m = 42.43 m.

Given: Distance between boy and bird =100 m

Height of building = 20 m

Angle of elevation boy = 30°

Angle of elevation girl = 45°

To find: Distance between girl and bird

In ΔABC

Using trigonometric ratio, we get

$\sin 30° = \frac{AC}{AB}$

$\frac{1}{2} = \frac{AC}{100}$

$AC = \frac{100}{2}$

AC = 50 m

$\Rightarrow$ AC = FA + CF (from figure)

50 = FA + 20 ($\because$ CF = ED = 20 m)

FA = 50 – 20

FA = 30 m

Now, In ΔAEF

Using trigonometric ratio, we get

$\sin 45° = \frac{FA}{AE}$

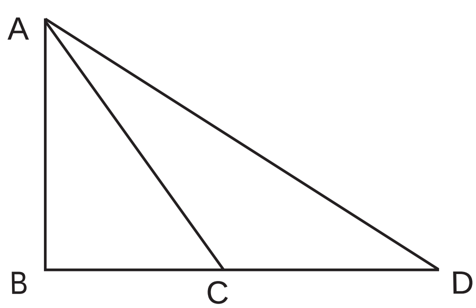
$\frac{1}{\sqrt{2}} = \frac{30}{AE}$

$AE = 30\sqrt{2} = 42.43$m

Therefore, distance between girl and bird is

$30\sqrt{2} = 42.43$m

17.



We are given that from the top of tower 96 m high, the angles of depression of two cars on a road at the same level as the base of the tower and on the same side of it are theta and phi, where tan theta = $\frac{3}{4}$ and tan phi = $\frac{1}{3}$

In the figure drawn above, let $\angle ACB = \tan\theta = \frac{3}{4}$ and $\angle ADB = \tan\phi = \frac{1}{3}$ and also the height of the tower = AB = 96 m.

Now, as we know that $\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$

So, in ΔABC,

$\tan\theta = \frac{AB}{BC}$

$\frac{3}{4} = \frac{96}{BC}$

$BC = \frac{96 \times 4}{3}$

BC = 32 $\times$ 4 = 128 m

Now, in ΔABD,

$\tan\phi = \frac{AB}{BD}$

$\frac{1}{3} = \frac{96}{BD}$

$BD = \frac{96 \times 3}{1}$

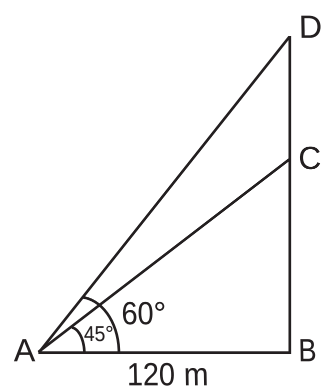
BD = 288 m

So, the distance between two cars =

CD = BD – BC

= 288 m – 128 m = 160 m

18.



Height of the flagstaff = CD

According to the figure,

$\tan 45° = \frac{CB}{120}$

$\Rightarrow 1 = \frac{CB}{120}$

$\therefore$ CB = 120 m

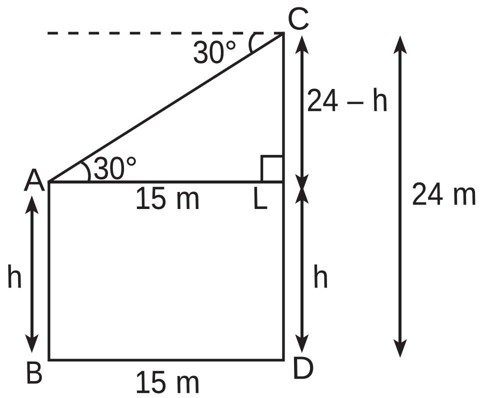
$\tan 60° = \frac{BD}{120}$

$\Rightarrow \sqrt{3} = \frac{BD}{120}$

$\therefore$ BD = 120 $\times$ 1.73 = 207.6 m

$\therefore$ Height of the flagstaff = CD = 207.6 – 120 = 87.6 m

19.



Let AB and CD be two poles, where CD = 24 m.

It is given that angle of depression of the top of the pole AB as seen from the top of the pole CD is 30° and horizontal distance between the two poles is 15 m.

$\therefore \angle CAL = 30°$ and BD = 15 m.

To find: Height of pole AB

Let the height of pole AB be h m.

AL = BD = 15 m and AB = LD = h

Therefore, CL = CD – LD = 24 – h

Consider right ΔACL:

$$\tan \angle CAL = \frac{\text{Perpendicular}}{\text{Base}} = \frac{CL}{AL}$$

$$\Rightarrow \tan 30° = \frac{24 - h}{15}$$

$$\Rightarrow 24 - h = \frac{15}{\sqrt{3}}$$

$$\Rightarrow 24 - h = 5\sqrt{3}$$

$$\Rightarrow h = 24 - 5\sqrt{3}$$

$$\Rightarrow h = 24 - 5 \times 1.732 \text{ [Taking } \sqrt{3} = 1.732]$$

$$\Rightarrow h = 15.34$$

Therefore, height of the pole AB = h m = 15.34 m.

20. Given AB is the tower.

P and Q are the points at distance of 4m and 9m respectively.

From fig, PB = 4m, QB = 9m.

Let angle of elevation from P be α and angle of elevation from Q be β.

Given that α and β are supplementary. Thus, $\alpha + \beta = 90$

In ΔABP,

$$\tan \alpha = \frac{AB}{BP} \quad ...(i)$$

In ΔABQ,

$$\tan \beta = \frac{AB}{BQ}$$

$\tan (90 - \alpha) = AB/BQ$ (Since, $\alpha + \beta = 90$)

$$\cot \alpha = \frac{AB}{BQ}$$

$$I/\tan \alpha = \frac{AB}{BQ}$$

$$\text{So, } \tan \alpha = \frac{BQ}{AB} \quad ...(ii)$$

From (i) and (ii)

$$\frac{AB}{BP} = \frac{BQ}{AB}$$

$AB^2 = BQ \times BP$

$AB^2 = 4 \times 9$

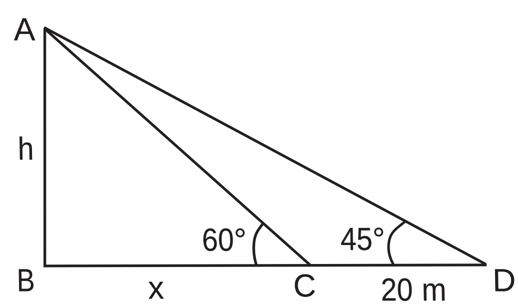
$AB^2 = 36$

Therefore, AB = 6.

Hence, height of tower is 6m.

Section D

21.



From the figure h = length of the tower

From the ΔABC

$\Rightarrow h = 1.732\,x$ ------- (1)

From the ΔABD

$\Rightarrow h = 20 + x$ ------ (2)

Equating equation (1) = equation (2)

$1.732\,x = x + 20$

$\Rightarrow 0.732\,x = 20$

$\Rightarrow x = 27.32$ m

Thus the height of the tower is given by

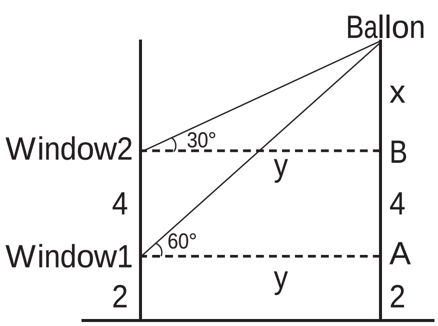
$h = 1.732\,x$

$\Rightarrow h = 1.732 \times 27.32$

$\Rightarrow h = 47.32$ m

This is the value of height of the tower.

22.



$\tan 30° = \frac{x}{y}$

$y = x\sqrt{3}$ ...(i)

$\tan 60° = \frac{x+4}{y}$

$y = \frac{x+4}{\sqrt{3}}$ ...(ii)

Equating (i) and (ii)

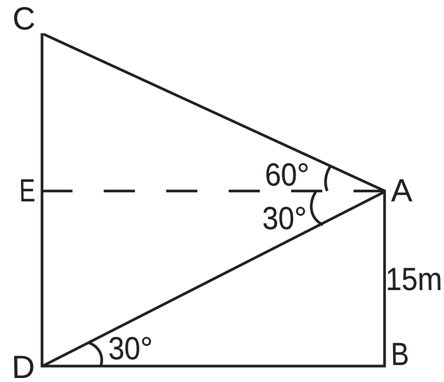
$x\sqrt{3} = \frac{x+4}{\sqrt{3}}$

$3x = 3 + 4$

$x = 2$

Height of ballon from the ground = 2 + 4 + 2 = 8 m

23.



AB = ED = 15 m

AE = BD

In ΔADB,

$\tan 30° = \frac{15}{DB}$

$DB = 15\sqrt{3}$ m

Or, $AE = 15\sqrt{3}$ m

In ΔACE,

$\tan 60° = \frac{CE}{AE}$

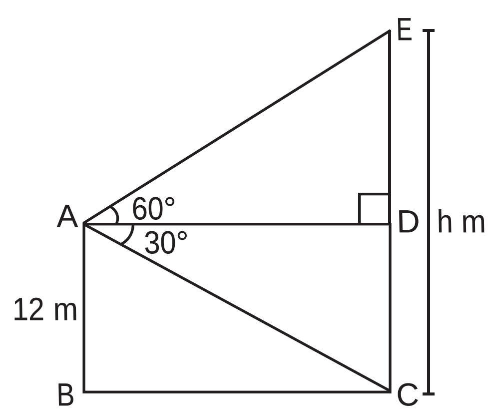
$CE = 15\sqrt{3}$ m = 45 m

So, CD = CE + ED = 60 m

AB is the building.

CD is the tower.

24.



Let AB be the deck of the ship. AB = 10 cm

Suppose CE be the cliff. C and E are the top and bottom of the cliff

Let CE = h m

Given, $\angle EAD = 60°$ and $\angle DAC = 30°$

CD = AB = 12 m

$\therefore$ DE = CE – CD = (h – 12)m

In ΔADE,

$$\tan 60° = \frac{DE}{AD} \qquad \left(\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}\right)$$

$$\therefore \sqrt{3} = \frac{(h-12)m}{AD}$$

$$\Rightarrow AD = \frac{(h-12)}{\sqrt{3}}m = \frac{(h-12)\sqrt{3}}{3}m \qquad ...(i)$$

In ΔADC,

$$\tan 30° = \frac{CD}{AD}$$

$$\therefore \frac{1}{\sqrt{3}} = \frac{12m}{AD}$$

$$\Rightarrow AD = 12\sqrt{3}\ m$$

From (i) and (ii), we have

$$\frac{(h-12)\sqrt{3}}{3}m = 12\sqrt{3}\ m$$

$$\Rightarrow h = 36 + 12 = 48$$

Height of the cliff = 48 m

Distance of cliff from the ship = BC = AD = $12\sqrt{3}$ m

Chapter 10

Circles

MULTIPLE CHOICE QUESTIONS

1. $C - r = 37$

$C = 37 - r$

$C = 2\prod r$

$37 + r = \frac{2 \times 22}{7 \times r}$

$37 + r = \frac{44r}{7}$

$37 = \frac{44r}{7 - r} = \frac{37r}{7}$

$r = \frac{37 \times 7}{37}$

$r = 7$ cm

$C = \frac{2 \times 22}{7 \times 7} = 44$ cm

Option (b)

2. $\pi r_1^2 + \pi r_2^2 = \pi r^2$

$r_1^2 + r_2^2 = r^2$

$5^2 + (12)^2 = r^2$

Option (b)

3. Distance covered in one revolution

$= 2\pi r$

$= 2 \times \frac{22}{7} \times \frac{35}{2}$

$= 110$ cm

Option (b)

4.

Diagonal = BD $= \sqrt{(4 - 0)^3 + (0 - 3)^3}$

$= \sqrt{16 + 9}$

$= \sqrt{25}$

$= 5$

Option (a)

5. Radius $= \frac{18}{2} = 9$ cm

Perimeter $= 2\pi r$

$= 2\pi r\ (9)$

$= 18\pi r$ cm

Option (c)

WORKSHEET 1

Section A

1. Arc length $= \frac{\theta}{360°} 2\pi r$

$3\pi = \frac{\theta}{360} 2\pi \times 6$

$\Rightarrow 3\pi = \frac{\theta\pi}{30}$

$\Rightarrow \theta = \frac{3\pi \times 30}{\pi}$

$= 90°$

2. Diameter $= 14$ cm

$\Rightarrow$ radius $= \frac{14}{2}$ 7 cm

Perimeter of semi – circle protractior

$= 2r + \frac{1}{2}(2\pi r)$

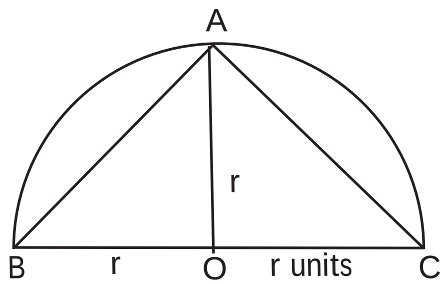
$= 2r + \pi r$

$= 2(7) + \frac{22}{7} \times 7$

$= 14 + 22$

$= 36$ cm

3.



Area of $\Delta BAC = \frac{1}{2} \times BC \times AO$

$= \frac{1}{2} \times 2r \times r$

$= r^2$ sq.units

4. Perimeter of sector

$= 2r + \frac{\theta}{360^\circ} 2\pi r$

$= 2r\left(1 + \frac{\pi\theta}{360^\circ}\right)$

$= 2(10.5)\left(1 + \frac{22}{7} \times \frac{60}{360}\right)$

$= 21\left(1 + \frac{11}{21}\right)$

$= \frac{21 \times 32}{21}$

$= 32$ cm

5. $r = 10$ cm

$\theta = 108^\circ$

area of sector $= \frac{\theta}{360^\circ} \pi r^2$

$= \frac{108}{360} \times \pi(100)$

$= 3\pi 10$

$= 30\pi$ cm^2

6. Distance covered in one revolution

$= 2\pi r$

$= 2 \times \frac{22}{7} \pi$

Number of revolutions in covering a distance of x metres.

$= \frac{x}{2 \times \frac{22}{7} \times r}$

$= \frac{7x}{44r}$

7. Let the diameter and a side be x units.

So, radius of circle $= \frac{x}{2}$ units

$\therefore$ Area of circle $= \pi\left(\frac{x}{2}\right)^2$

$= \frac{\pi x^2}{4}$

Area of equilateral triangle

$= \frac{\sqrt{3}}{4}(\text{side})^2$

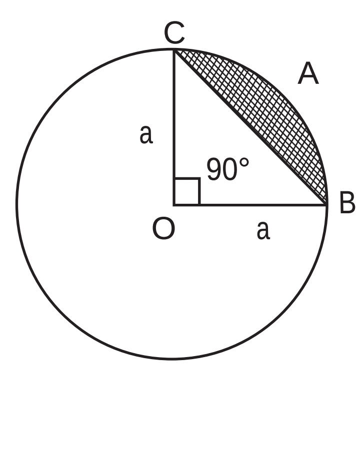
$= \frac{\sqrt{3}}{4} x^2$

$\therefore \frac{\text{Area of circle}}{\text{Area of equilateral triangle}}$

$= \frac{\frac{\pi x^2}{4}}{\frac{\sqrt{3}}{4} x^2}$

$= \frac{\pi}{\sqrt{3}}$

8.



Perimeter of segment ABC

= BC + length of arc $\overset{\frown}{BAC}$

In ΔBOC,

$BC^2 = OC^2 + OB^2$

(By Pythagoras theorem)

$BC^2 = a^2 + a^2$

$= 2a^2$

$\therefore BC = \sqrt{2}\,a$

Also length of arc $\overset{\frown}{BAC}$

$= \frac{90}{360} \times 2 \times \frac{22}{7} \times a$

$= \frac{11a}{7}$

So, Perimeter of segment ABC

$= \sqrt{2}\,a + \frac{11a}{7}$

Section B

9. We know that AD = AF

BD = BE

CE = CF

Let AD = AF = x

BD = BE = y

CE = CF = z

Then x + y = 12

y + z = 8

x + z = 10

On Solving above equation we get x = 7, y = 5, z = 3

So AD = 7, BE = 5, CF = 3

10. BP = AP = 5 cm

(The lengths of tangents drawn from an external point to a circle are equal.)

$\therefore \angle PAB = \angle PBA$

(Angle opposite to equal sides are equal.)

In PAB,

$\angle P + \angle PAB + \angle PBA = 180°$

(Angle sum property)

$60° + 2\angle PAB = 180°$

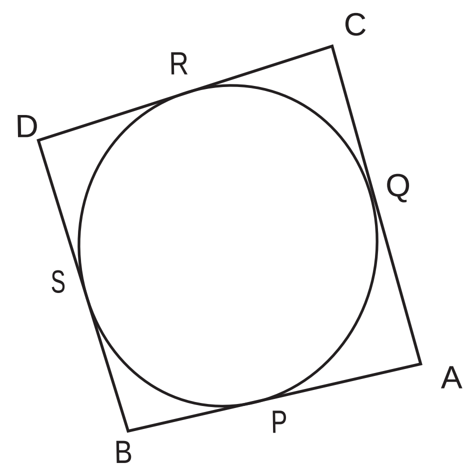
$2\angle PAB = 120°$

$\angle PAB = 60°$

$\therefore \angle PAB = \angle PBA = 60°$

$\therefore$ AB = PA = PB = 5 cm

11.



~~AP~~ + ~~PB~~ + ~~CR~~ + ~~RD~~ = ~~BQ~~ + ~~CQ~~ + ~~AS~~ + ~~DS~~

As we know that the length of tangents drawn from an external point to a circle are equal,

AP = AS (i)

BP = BQ (ii)

CR = CQ (iii)

DR = DS (iv)

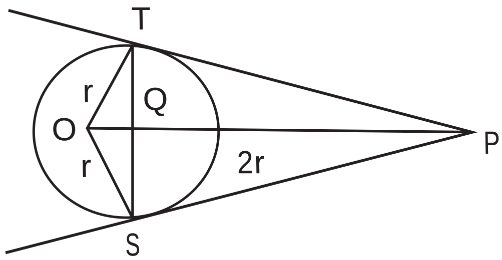
On adding both sides of (i), (ii), (iii) and (iv), we get

AP + BP + CR + DR = AS + BQ + CQ + DS

$\therefore$ (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

AB + CD = BC + AD

12.



$\angle TOP = \theta$

As we know that radius is perpendicular to the tangent at the point of contact.

$\angle OTP = 90°$

So, in ΔOTP

$\cos\theta = \frac{OT}{OP} = \frac{r}{2r} = \frac{1}{2}$

$\therefore \cos = \frac{1}{2} = \cos 60°$

$\Rightarrow \theta = 60°$

$\therefore \angle TOS = 60° + 60° = 120°$

As OT = OS

$\Rightarrow \angle OTS = \angle OST$

(Angles opposite to equal sides are equal.)

In $\angle OTS$,

$\angle TOS + \angle OTS + \angle OST = 180°$

$120° + 2\angle OTS = 180°$

$2\angle OTS = 60°$

$\angle OTS = 30°$

$\therefore \angle OTS = \angle OST = 30°$

13. In ΔOTP and ΔOSP

OT = OS (Radii of same circle)

OP = PO (Common)

PT = PS

(The lengths of tangents drawn from an external point to a circle are equal.)

$\therefore \Delta OTP \cong \Delta OSP$ (SSS)

$\therefore \angle OPS = \angle OPT$

$= \frac{1}{2}\angle TPS$ (CPCT)

$= \frac{1}{2}$ (120°)

$= 60°$

In ΔOSP,

$OS \perp PS$

(Radius is perpendicular to the tangent at the point of contact.)

$\cos(\angle OPS) = \frac{PS}{OP}$

$\Rightarrow \cos 60° = \frac{PS}{OP}$

$\Rightarrow \frac{1}{2} = \frac{PS}{OP}$

$\Rightarrow OP = 2\,PS$

14. In ΔOAP,

OA = 6 cm

AP = 8 cm

$\therefore OP^2 = OA^2 + AP^2$

(By Pythagoras theorem)

$= 6^2 + 8^2$

$= 36 + 64$

$= 100$

$\Rightarrow OP = 10$ cm

Now, In ΔOBP,

$OP^2 = OB^2 + BP^2$

$10^2 = 4^2 + BP^2$

(By Pythagoras theorem)

$100 = 16 + BP^2$

$BP^2 = 100 - 16$

= 84

$\therefore$ BP = $2\sqrt{21}$ cm

15. $\angle OAC = 90°$ (as radius $\perp$ tangent)

$\angle BOC = \angle OAC + \angle ACO$

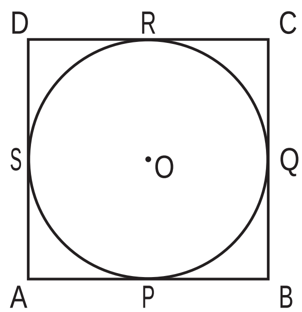
(Exterior angle property)

$130° = 90° + \angle ACO$

$\angle ACO = 130° - 90° = 40°$

Section C

16.



A rhombus is a parallelogram with all equal sides.

In 11 gm ABCD

AB = CO and AD = BC

Hence AP = AS; BP = BQ; CR = CQ; DR = DS

Adding we get AP + BP + CR + DR = AS + BQ + CQ + DS

(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

AB + CD = AD + BC

AB + AB = AD + AD

2 AB = 2 AD

So AB = AD and AB = CD and AD = BC

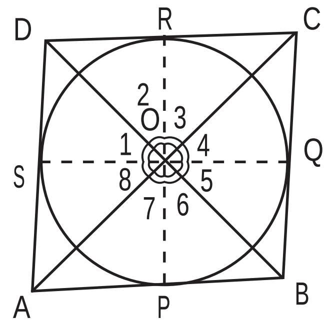
So AB = CD = AD = CD

So ABCD is 11 gm with equal sides.

$\therefore$ ABCD is a rhombus.

$\therefore$ Proved.

17.



Const: Join OP, OQ, OR and OS.

Proof: Since, the two tangents drawn from an external point to a circle subtend equal angles at the centre.

$\therefore \angle 1 = \angle 2, \angle 3 = \angle 4, \angle 5 = \angle 6, \angle 7 = \angle 8$

Since sum of all the angles subtended at a point is 360°.

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8$

$= 360°$

$\Rightarrow 2\angle 2 + 2\angle 3 + 2\angle 6 + 2\angle 7 = 360°$

$\Rightarrow 2(\angle 2 + \angle 3 + \angle 6 + \angle 7) = 360°$

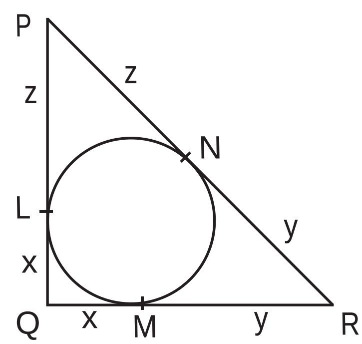
$\Rightarrow \angle 2 + \angle 3 + \angle 6 + \angle 7 = 180°$

$\Rightarrow (\angle 6 + \angle 7) + (\angle 2 + \angle 3) = 180°$

$\Rightarrow \angle AOB + \angle COD = 180°$

Similarly, we can prove $\angle AOD + \angle BOC = 180°$

18.



QL = QM

RM = RN

PL = PN

We know that the tangents drawn to a circle from an external point are equal in length.

Let QL = QM = x

Let RM = RN = y

Let PL = PN = z

Consider PQ + QR + PR = 60

$\Rightarrow$ x + z + x + y + z + y = 60

$\Rightarrow$ 2x + 2y + 2z = 60

$\Rightarrow$ x + y + z = 30

PQ = 20

x + z = 20

$\therefore$ RN = 10 cm

Also, QR = 16

x + y = 16

$\therefore$ z = 30 – (x + y)

= 30 – 16

= 14 cm

$\therefore$ PL = 14 cm

Again, PR = 24

y + z = 24

$\therefore$ x = 30 – (y + z)

= 30 – 24

= 6

$\therefore$ QM = 6 cm

19. Given: a circle with centre o to which XY and XY' are tangents.

TP – $\angle AOB = 90°$

Const: Join OY, OY' and OC

Proof: In ΔOYA and ΔOCA

OY = OC [radii]

OA=OA [common]

AY = AC [tangents]

$\Rightarrow$ By SSS

$\Delta OYA \approx \Delta OCA$

$\Rightarrow \angle OY'A = \angle OCA$ [CPCT]---------------(1)

|| ly $\Delta OY'B \approx \Delta OCB$

$\Rightarrow \angle Y'BO = \angle CBO$ [CPCT]---------------(2)

$\angle YAB + \angle Y'BA = 180°$[co-interior angles]

$2\angle OAB + 2\angle OBA = 180°$ ---[from (1) and (2)]

$2(\angle OAB + \angle OBA) = 180°$

$\angle OAB + \angle OBA = 90°$---------------------(3)

In ΔAOB

$\angle OAB + \angle OBA + \angle AOB = 180°$
[Angle sum property of a triangle]

$90° + \angle AOB = 180°$

$\angle AOB = 180° – 90°$

$\angle AOB = 90°$

$\therefore$ Hence Proved

20. GIVEN: a circle with centre o to which XY and XY' are tangents

TP – $\angle AOB = 90°$

CONST.: join OY, OY' and OC

PROOF: In ΔOYA and ΔOCA

OY = OC [radii]

OA=OA [common]

AY = AC [tangents]

$\Rightarrow$ By SSS

$\Delta OYA \approx \Delta OCA$

$\Rightarrow \angle OY'A = \angle OCA$ [CPCT]---------------(1)

|| ly $\Delta OY'B \approx \Delta OCB$

$\Rightarrow \angle Y'BO = \angle CBO$ [CPCT]-----------------(2)

$\angle YAB + \angle Y'BA = 180°$[co-interior angles]

$2\angle OAB + 2\angle OBA=180°$-------[from (1) and (2)]

$2(\angle OAB + \angle OBA) = 180°$

$\angle OAB + \angle OBA = 90°$---------------------(3)

In ΔAOB

$\angle OAB + \angle OBA + \angle AOB = 180°$

[Angle sum property of a triangle]

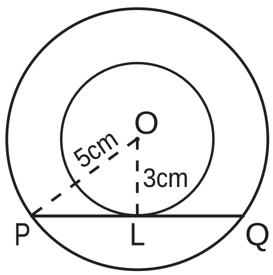
$90° + \angle AOB = 180°$

$\angle AOB = 180° - 90°$

$\angle AOB = 90°$

$\therefore$ Hence Proved

21.



Let O be the common centre of the two concentric circle.

Let PQ be a chord of the larger circle which touches the smaller circle at L.

Join OL and OP.

Since, the tangent at any point of a circle is perpendicular to the radius through the point jof contact.

Therefore,

$\angle OLP = 90°$

Now,

In ΔOLP, we have

$OP^2 = OL^2 + PL^2$

[Using Pythagoras theorem]

$\Rightarrow (5)^2 = (3)^2 + PL^2$

$\Rightarrow 25 = 9 + PL^2$

$\Rightarrow PL^2 = 16$

$\Rightarrow PL = 4$ cm

Since, the perpendicular from the centre of a circle to a chord bisects the chord.

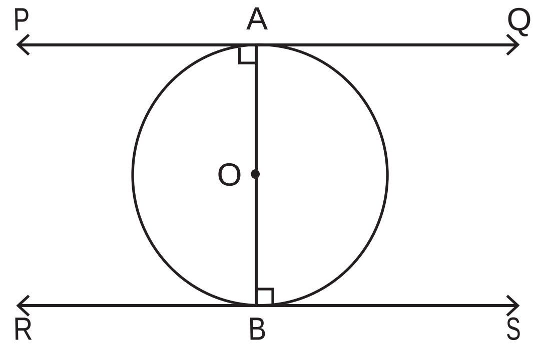
Therefore,

PL = LQ = 4 cm

$\therefore$ PQ = 2 PL = $2 \times 4 = 8$ cm

Hence, the required length = 8 cm.

22.



Let AB be the diameter of a circle, with centre O. The tangents PQ and RS are drawn at point A and B, respectively.

We know that a tangent at any point of a circle is perpendicular to the radius through the point of contact.

$\therefore OA \perp PQ$ and $OB \perp RS$

$\Rightarrow \angle OBR = 90°$

$\angle OBS = 90°$

$\angle OAP = 90°$

$\angle OAQ = 90°$

We can observe the following:

$\angle OBR = \angle OAQ$ and $\angle OBS = \angle OAP$

Also, these are the pair of alternate interior angles.

Since alternate angles are equal, the lines PQ and RS are parallel to each other.

Hence, proved.

Section D

23. Clearly $\angle OPT = 90°$

Applying Pythagoras in ΔOTP, we have

$\Rightarrow OT^2 = OP^2 + PT^2$

$\Rightarrow 13^2 = 5^2 + PT^2$

$\Rightarrow PT^2 = 169 - 25 = 144$

$\Rightarrow PT = 12$ cm

Since, lengths of tangents drawn from a point to a circle are equal. Therefore,

$AP = AE = x$(say)

$\Rightarrow AT = PT - AP = (12 - x)$ cm

Since AB is the tangent to the circle E. Therefore, $OE \perp AB$.

$\Rightarrow \angle OEA = 90°$

$\Rightarrow \angle AET = 90°$

$\Rightarrow AT^2 = AE^2 + ET^2$ [

[Applying Pythagoras Theorem in ΔAET]

$\Rightarrow (12 - x)^2 = x^2 + (13 - 5)^2$

$\Rightarrow 144 - 24x + x^2 = x^2 + 64$

$\Rightarrow 24x = 80$

$\Rightarrow x = \frac{10}{3}$ cm

Similarly, $BE = \frac{10}{3}$ cm

$\therefore AB = AE + BE = \left(\frac{10}{3} + \frac{10}{3}\right)$ cm $= \frac{20}{3}$ cm

24. (i) $PA \cdot PB = (PN - AN)(PN + BN)$

$= (PN - AN)(PN + AN)$ (As $AN = BN$)

$= PN^2 - AN^2$

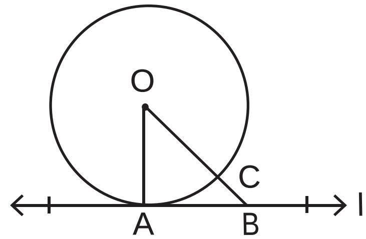
(ii) $PN^2 - AN^2 = (OP^2 - ON^2) - AN^2$ (As $ON \perp PN$)

$= OP^2 - (ON^2 + AN^2)$

$= OP^2 - OA^2$ (As $ON \perp AN$)

$= OP^2 - OT^2$ (As $OA = OT$)

25.



Given: A circlr C (0, r) and a tangent l at point A.

To prove: $OA \perp l$

Construction: Take a point B, other than A. On the tangent l. Join OB. Suppose OB meets the circle in C.

Proof: We know that, among all line segment joining the point O to a point on l, the perpendicular is shortest to l.

OA = OC (Radius of the same circle)

Now, OB = OC + BC

$\therefore OB > OC$

$\Rightarrow OB > OA$

$\Rightarrow OA > OB$

B is an arbitrary point on the tangent l. Thus, OA is shorter than any other line segment joining O to any point on l.

Here, $OA \perp l$

26. We know that $\angle ADO = 90°$ (Since O'D is perpendicular to AC)

$\angle ACO = 90°$ (OC (radius) perpendicular to AC (tangent))

In triangles ADO' and ACO,

$\angle ADO = \angle ACO$ (each 90°)

$\angle DAO = \angle CAO$ (common)

by AA criterion, triangles ADO' and ACO are similar to each other.

$\frac{AO'}{AO} = \frac{DO'}{CO}$

(corresponding sides of similar triangles)

AO = AO' + O'X + OX

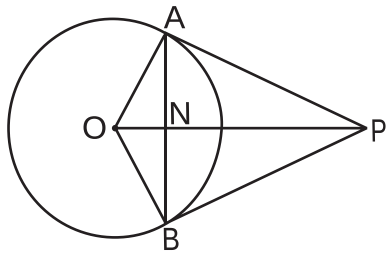
= 3AO' (since AO' = O'X = OX because radii of the two circles are equal)

$\frac{AO'}{AO} = \frac{AO'}{3AO} = \frac{1}{3}$

$\frac{DO'}{CO} = \frac{AO'}{AO} = \frac{1}{3}$

$\frac{DO'}{CO} = \frac{1}{3}$

27.



OA = 10 cm

ON ⊥ AB

$AN = NB = \frac{16}{2} = 8$ cm

Pythagoras Theorem

In ΔONA,

$ON^2 + NA^2 = OA^2$

$ON^2 = OA^2 - NA^2$

$ON^2 = 10^2 - 8^2$

$ON^2 = 36$ cm

ON = 6 cm

$\tan\angle AON = \frac{AN}{ON} = \frac{8}{6} = \frac{4}{3}$

ΔOAP

$\tan\angle AON = \frac{PA}{OA}$

$\frac{4}{3} = \frac{PA}{10}$

$\Rightarrow PA = \frac{40}{3}$ cm

28. Given ∠RPQ = 30° and PR and PQ are tangents drawn from P to the same circle.

Hence PR = PQ [Since tangents drawn from an external point to a circle are equal in length]

∴∠PRQ = ∠PQR [Angles opposite to equal sides are equal in a Δ]

In ΔPQR

∠RQP + ∠QRP + ∠RPQ = 180°

[Angle sum property of a Δ]

2∠RQP + 30° = 180°

2∠RQP = 150°

∠RQP = 75°

So ∠RQP = ∠QRP = 75°

∠RQP = ∠RSQ = 75°

[By Alternate Segment Theorem]

Given, RS || PQ

∴∠RQP = ∠SRQ = 75° [Alternate angles]

∠RSQ = ∠SRQ = 75°

∴ QRS is also an isosceles triangle.

[Since sides opposite to equal angles of a triangle are equal.]

∠RSQ + ∠SRQ + ∠RQS = 180°

[Angle sum property of a triangle]

75° + 75° + ∠RQS = 180°

150° + ∠RQS = 180°

∴∠RQS = 30°

29. Given: ABCD is a quadrilateral such that ∠D = 90°.

BC = 38 cm, CD = 25 cm and BP = 27 cm

BP = BQ = 27 cm

[Tangents from an external point]

BC = 38

⇒ BQ + QC = 38

⇒ 27 + QC = 38

⇒ QC = 38 – 27

$\Rightarrow$ QC = 11 cm

$\therefore$ QC = 11 cm = CR

[Tangents from an external point]

CD = 25 cm

CR + RD = 25

$\Rightarrow$ 11 + RD = 25

$\Rightarrow$ RD = 25 – 11

$\Rightarrow$ RD = 14cm

Also,

RD = DS = 14cm

$\therefore$ OR and OS are radii of the circle.

From tangents R and S, $\angle ORD = \angle OSD = 90°$

Now, ORDS is a square.

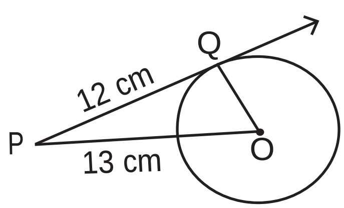
$\therefore$ OR = DS = 14 cm

Thus, radius , r = 14 cm

WORKSHEET 2

Section A

1.



As we know that tangent to a circle is perpendicular to the radius through the point of contact.

$\therefore$ OQ $\perp$ PQ

$\therefore$ $\angle OQP = 90°$

So, In ΔOQP,

$OP^2 = OQ^2 + PQ^2$

$13^2 = OQ^2 + 12^2$

$169 = OQ^2 + 144$

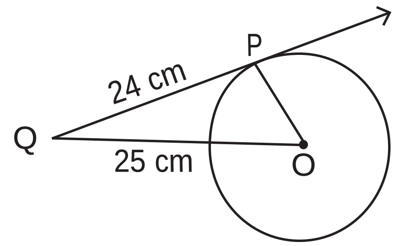
$OQ^2 = 169 - 144$

$= 25$

$\therefore$ OQ = 5 cm

So, radius of circle = 5 cm

2.



As we know that tangent to a circle is perpendicular to the radius through the point of contact.

$\therefore$ OP $\perp$ PQ

i.e. $\angle OPQ = 90°$

In ΔOPQ,

$OQ^2 = OP^2 + PQ^2$

$25^2 = OP^2 + 24^2$

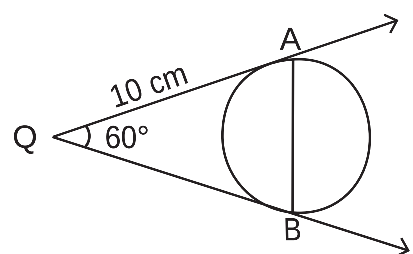
$625 = OP^2 + 576$

$OP^2 = 625 - 576$

$= 49$

$\therefore$ OP = 7 cm

3.



PB = PB

= 10 cm

(Length of tangents drawn from an external point to a circle are equal.)

$\Rightarrow$ $\angle PAB = \angle PBA$

(Angles opposite to equal sides are equal)

In ΔPBA,

$\angle P + \angle PAB + \angle PBA = 180°$

(Angle sum property)

$60° + \angle PAB + \angle PBA = 180°$

$\angle PAB + \angle PBA = 180° - 60°$

$= 120°$

$\Rightarrow \angle PAB = \angle PBA = 60°$

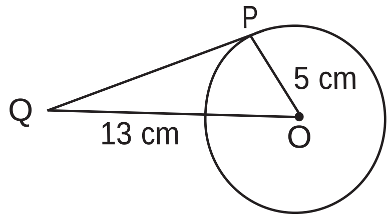
So, $\angle PAB = \angle PBA = \angle P = 60°$

$\Rightarrow$ ΔAPB is equilateral

$\Rightarrow$ AB = AP = 10 cm

(Sides of equilateral triangle are equal.)

4.



As we know that tangent to a circle is perpendicular to the radius through the point of contact.

$\therefore$ $OP \perp PQ$

i.e. $\angle OPQ = 90°$

So, In ΔOPQ,

$OQ^2 = OP^2 + PQ^2$

$13^2 = 5^2 + PQ^2$

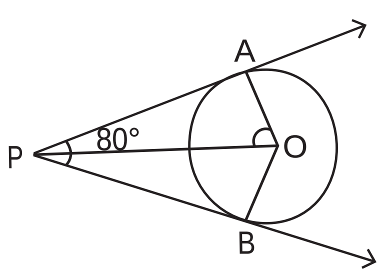
$169 = 25 + PQ^2$

$\therefore PQ^2 = 169 - 25$

$= 144$

$\Rightarrow PQ = 12$ cm

5.



In ΔPOA and ΔPOB,

PA = PB

(Length of tangents drawn from an external point to a circle are equal.)

OP = PO (Common)

OA = OB (Radii of same circle)

$\therefore \Delta POA \cong \Delta POB$

(SSS congruence creteria)

$\therefore \angle APO = \angle BPO$ (CPCT)

$= \frac{1}{2} \angle APB$

$= \frac{1}{2} (80°)$

$= 40°$

Also, $OA \perp AP$

i.e. $\angle OAP = °90$

(As tangent is perpendicular to radius through point of contact.)

In ΔOAP,

$\angle OAP + \angle APO + \angle AOP = 180°$

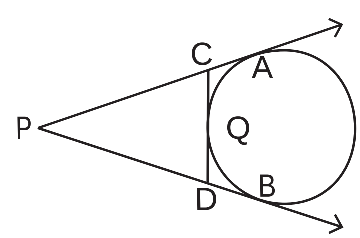
(Angle sum property)

$\therefore 90° + 40° + \angle AOP = 180°$

$\Rightarrow 130° + \angle AOP = 180°$

$\Rightarrow \angle AOP = 180° - 130° = 50°$

6.



PA = PB

(Tangents from external point P)

$\Rightarrow$ PC + CA = 10

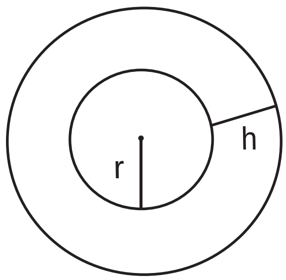
$\Rightarrow$ PC + CQ = 10

[∵ CA = CQ (Tangents from external point C)]

$\Rightarrow$ PC + 2 = 10

$\Rightarrow$ PC = 8 cm

7.



Radius of inner circle = r

Area of inner circle = πr^2

Radius of outer circle = r + h

Area of outer circle = $\pi (r + h)^2$

So, area of circular path

= area of outer circle

– area of inner circle

$= \pi (r + h)^2 - \pi r^2$

$= \pi (r^2 + h^2 + 2rh - r^2)$

$= \pi (h^2 + 2rh)$

$= \pi h (h + 2r)$

$= \pi h (2r + h)$

8. Area of sector $= \frac{\theta}{360} \pi r^2$

$\therefore \quad 20\pi = \frac{\theta}{360} \pi r^2$

$\Rightarrow \quad 20 = \frac{\theta}{360} \pi r^2$

$\Rightarrow \quad = \frac{\theta}{360} \frac{20}{r^2}$ ①

Also, arc length = 5π

$\Rightarrow \quad \frac{\theta}{360} 2\pi r = 5\pi$

$\Rightarrow \quad \frac{2r\theta}{360} = 5$

$\Rightarrow \quad \frac{\theta}{360} = \frac{5}{2r}$ ②

From ①, ②

$\frac{\theta}{360} = \frac{20}{r^2} = \frac{5}{2r}$

$\Rightarrow \quad 40r = 5r^2$

$\Rightarrow \quad 5r (r - 8) = 0$

$\Rightarrow \quad r = 8$

So, radius of circle = 8 cm

9. A circle can have infinitely many tangents

10. Remark: If AB and CD are two common tangents to the two circles of unequal radii then they will always intersect each other.

Given: Two circles with centre's O_1 and O_2. AB and CD are common tangents to the circles which intersect in P.

To prove: AB = CD

Proof:

AP = PC ...(i) (Length of tangents drawn from an external point to the circle are equal)

PB = PD ...(ii)(Length of tangents drawn from an external point to the circle are equal)

Adding (i) and (ii), we get

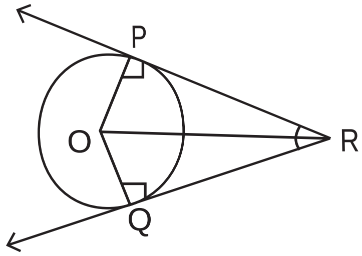
AP + PB = PC + PD

$\Rightarrow$ AB = CD

Section B

11. $\angle ABQ = \frac{1}{2} \angle AOQ = \frac{1}{2} (58°) = 29°$

(∵ Angle subtended by an arc at the centre is double the angle subtended by it on the remaining part of the circle.)

12. 

Join OP and OQ

PR and RQ are tangents to circle at points P and Q respectively.

$\Rightarrow$ OP $\perp$ PR and OQ $\perp$ QR

(As tangent is perpendicular to radius through point of contact.)

In ΔOPR and ΔOQR

OP = OQ

(Radii of same circle)

OR = OR (Common)

$\angle$OPR = $\angle$OQR = 90° (Proved above)

$\therefore$ $\Delta OPR \cong \Delta OQR$

(RHS congruence criteria)

$\Rightarrow$ $\angle ORP = \angle ORQ = \frac{1}{2}\angle PRQ$

$= \frac{1}{2}(120°)$

$= 60°$

In ΔPRO,

$\cos 60° = \frac{PR}{OR}$

$\frac{1}{2} = \frac{PR}{OR}$

$\Rightarrow$ $PR = \frac{1}{2} OR$ (i)

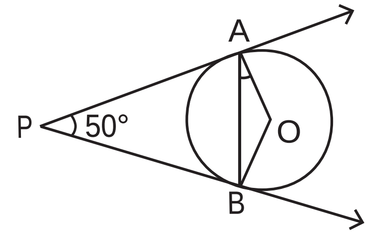
In ΔQRO

$RQ = \frac{1}{2} OR$ (ii)

On adding (i) and (ii), we get

$PR + RQ = \frac{1}{2} OR + \frac{1}{2} OR$

$= RO$

13. 

$\Rightarrow$ PA = PB

(Length of tangents drawn from an external point to a circle are equal.)

$\Rightarrow$ $\angle PAB = \angle PBA$ (i)

(Angles opposite to equal sides are equal.)

In ΔAPB,

$\angle P + \angle PAB + \angle PBA = 180°$

(Angle sum property)

$50° + \angle PAB + \angle PAB = 180°$ (From (i))

$2\angle PAB = 130°$

$\therefore$ $\angle PAB = \angle PBA = 65°$

OA $\perp$ AP

(As tangent is perpendicular to radius through point of contact.)

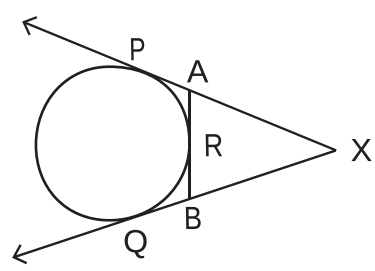
$\Rightarrow$ $\angle OAP = 90°$

$\Rightarrow$ $\angle OAB + \angle PAB = 90°$

$\Rightarrow$ $\angle OAB + 65° = 90°$

$\Rightarrow$ $\angle OAB = 90° - 65°$

$= 25°$

14. 

As we know that lengths of tangents drawn from an exterior point to a circle are equal.

$\therefore$ XP = XQ, AP = AR and BQ = BR

$\Rightarrow$ XP + AP = XB + BQ

$\Rightarrow$ XP + AR = XB + BR

15. As we know that lengths of tangents draw from an exterior point to a circle are equal.

CE = CD = 9 cm

BF = BD = 6 cm

AE = AF = x cm

Also, $OE \perp AC$, $OD \perp BC$ and $OF \perp AB$

(As tangent is perpendicular to radius through point of contact.)

Area of ΔBOC $= \frac{1}{2} \times BC \times OD$

$= \frac{1}{2} \times (9 + 6) \times 3$

$= \frac{1}{2} \times 15 \times 3$

$= \frac{45}{2}$ cm²

Area of ΔAOC $= \frac{1}{2} \times AC \times OE$

$= \frac{1}{2} \times (9 + x) \times 3$

$= \frac{3}{2}(9 + x)$ cm²

Area of ΔAOB $= \frac{1}{2} \times AB \times OF$

$= \frac{1}{2} \times (x + 6) \times 3$

$= \frac{3}{2}(x + 6)$ cm²

Area of ΔABC = area of ΔBOC

+ area of ΔAOC

+ area of ΔAOB

$54 = \frac{45}{2} + \frac{1}{2}(9 + x) \times \frac{3}{2}(x + 6)$

$54 = \frac{45}{2} + \frac{27}{2} + \frac{18}{2} + \frac{3}{2}x + \frac{3}{2}x$

$54 = 45 + 3x$

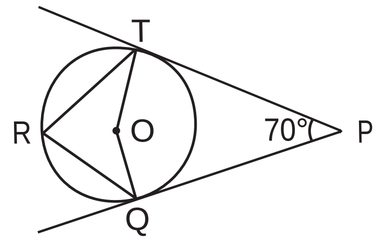
$9 = 3x$

$\therefore$ = 3

So, AB = x + 6 = 3 + 6 = 9 cm

AC = x + 9 = 3 + 9 = 12 cm

16.



As we know that tangent is perpendicular to the radius through the point of contact.

$\therefore$ $OT \perp PT$ and $OQ \perp PQ$

i.e. $\angle OTP = \angle OQP = 90°$

In quadrilateral TOQP

$\angle TOQ + \angle OQP + \angle QPT + \angle PTO = 360°$

(Angle sum property of quadrilateral.)

$\Rightarrow$ $\angle TOQ + 90° + 70° + 90° = 360°$

$\Rightarrow$ $\angle TOQ + 250° = 360°$

$\Rightarrow$ $\angle TOQ = 360° - 250°$

$= 110°$

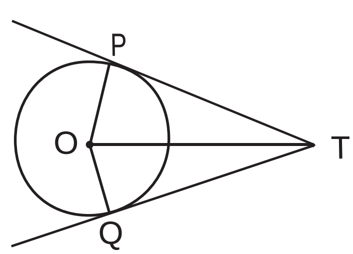
$\angle TRQ = \frac{1}{2} \angle TOQ$

(Angle subtended an arc at the centre is double the angle subtended by it on the remaining part of the circle.)

$= \frac{1}{2}(110)$

$= 55°$

17.



Join OT

$OP \perp PT$

(As tangent is perpendicular to the radius through point of contact)

i.e. $\angle OPT = 90°$

In $\angle OPT$,

$$OT^2 = OP^2 + PT^2$$

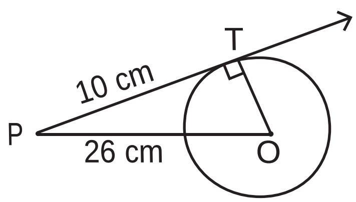
$$= 5^2 + 8^2$$

$$= 25 + 64$$

$$= 89$$

$\therefore$ $OT = \sqrt{89}$ cm

18.



$OT \perp PT$ i.e. $\angle OTP = 90°$

(Tangent is perpendicular to radius through point of contact.)

In ΔOTP,

$$OP^2 = OT^2 + PT^2$$

(By Pythagoras theorem)

$$26^2 = OT^2 + 10^2$$

$$676 = OT^2 + 100$$

$$OT^2 = 576$$

$$OT = 24 \text{ cm}$$

$\therefore$ Radius of the circle = 24 cm

19. AE and CE are tangents to the circle with center O,

$\therefore$ $AE = CE$ (i)

($\because$ Lengths of tangents drawn from an exterior point to a circle are equal.)

Also, DE and BE are tangents to the circle with centre O²

$\therefore$ $BE = DE$ (ii)

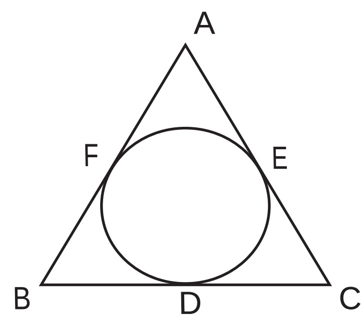
($\because$ Lengths of tangents drawn from an exterior point to a circle are equal.)

On adding (i) and (ii), we get

$$AE + BE = CE + DE$$

$\therefore$ $AB = CD$

20.



$AF = AE$ (i)

(Lenghts of tangents drawn from an exterior point to a circle are equal.)

ALso, $AB = AC$ (ii)
(Given)

On subtracting (i) from (ii), we get

$AB - AF = AC - AE$

$BF = CE$ (iii)

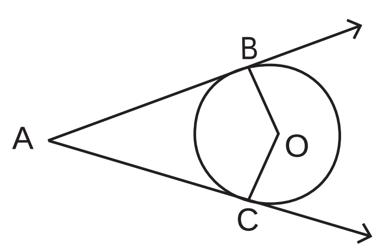
But $BF = BD$ and $CE = CD$

(Lengths of tangents drawn from an exterior point to a circle are equal.)

$\therefore$ $BD = CD$

Section C

21.



AB and AC are tangents to a circle.

$OB \perp AB$ and $OC \perp AC$

(Tangent is perpendicular to the radius through the point of contact.)

i.e. $\angle OBA = \angle OCA = 90°$ (i)

In quadrilateral ABOC,

$\angle A + \angle B + \angle O + \angle C = 360°$

(Angle sum property of quadrilateral)

$\Rightarrow \angle A + \angle O + \angle B + \angle C = 360°$

$\Rightarrow \angle A + \angle O + 90° + 90° = 360°$

From (i)

$\Rightarrow \angle A + \angle O + 180 = 360°$

$\Rightarrow \angle A + \angle O = 360° - 180°$

$= 180°$

22. BP and BQ are tangents to the circle

$\therefore$ BP = BQ (i)

(Lenghts of tangents drawn from an exterior point to a circle are equal.)

Also, CP = CR (ii)

(Lenghts of tangents drawn from an exterior point to a circle are equal.)

Consider

Perimeter of ΔABC = AB + BC + AC

= AB + (BP + CP) + AC

= AB + (BQ + CR) + AC

From (i) and (ii),

= AQ + AR

= AQ + AQ

= 2AQ

$\because$ AQ = AR as lengths of tangents drawn from an exterior point to a circle are equal.

$\therefore AQ = \frac{1}{2}$ (Perimeter of ΔABC)

23. Consider ΔOAP and ΔOBP,

AP = BP

(Lenghts of tangents drawn from an exterior point to a circle are equal.)

OP = OP (Common)

AO = BO (Radii of same circle)

$\therefore \Delta OAP \cong \Delta OBP$ (SSS congruence creteria)

$\Rightarrow \angle APO \ \angle BPO$ (CPCT)

Now, Consider ΔACP and ΔBCP,

AP = BP

PC = CP (Common)

$\angle APC = \angle BPC$ (Proved above)

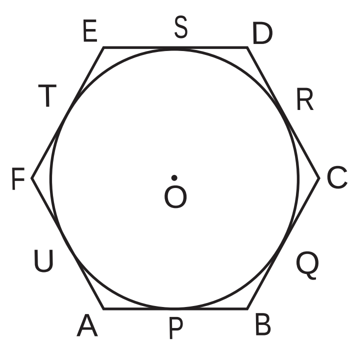
$\therefore \Delta APC \cong \Delta BCP$ (SSS congruence creteria)

$\Rightarrow$ AC = BC and $\angle ACP = \angle BCP = 90°$

(CPCT)

So, OP is the perpendicular bisector of AB

24.



As we know that lengths of tangents drawn from an external point to a circle are equal

$\therefore$ AP = AU

BP = BQ

CQ = CR

DS = DR

ES = ET

FU = FT

Consider

AB + CD + EF

= (AP + BP) + (CR + DR) + (ET + TF)

= (AU + <u>B</u>Q) + (<u>C</u>Q + <u>D</u>S) + (<u>E</u>S + UF)

= (BQ + QC) + (DS + ES) + (AU +FU)

= BC + DE + AF

25. PR and CR are tangents to circle with centre A

$\therefore$ PR = CR (i)

(Lenghts of tangents drawn from an exterior point to a circle are equal.)

QR and CR are tangent to circle with center B

$\therefore$ QR = CR (ii)

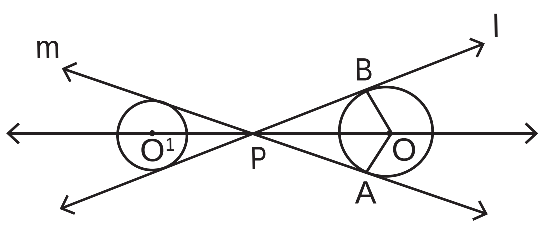
(Lenghts of tangents drawn from an exterior point to a circle are equal.)

From (i) and (ii), we get

PR = QR

$\therefore$ RC bisects PQ

26.



In ΔOPA and ΔOBP,

OA = OB (Radii of circle)

PA = PB

(Lengths of tangents from an external point to a circle are equal.)

OP = PO (Common)

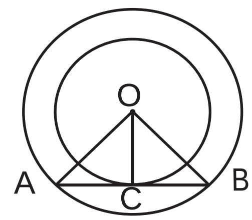
$\therefore$ $\Delta AOP \cong \Delta OBP$ (SSS congruence criteria)

$\Rightarrow$ $\angle APO \cong \angle BPO$ (CPCT)

$\Rightarrow$ OP is the bisector of $\angle APB$

$\therefore$ O lies on the bisector of the angle between l and m.

27.



We know that the radius and tangent are perpendicular at their point of contact.

$\therefore$ $\angle OCA = \angle OCB = 90°$

Now, In ΔOCA and ΔOCB

$\angle OCA = \angle OCB = 90°$

OA = OB (Radii of the larger circle)

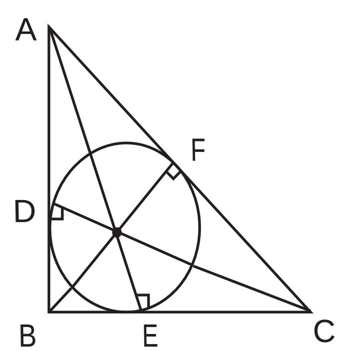
OC = OC (Common)

By RHS congruency

$\Delta OCA \cong \Delta OCB$

$\therefore$ CA = CB

28.



In ΔABC, right angles at B

$AC^2 = AB^2 + BC^2$

$= 24^2 + 10^2$

$= 576 + 100$

$= 676$

$\therefore$ AC $= 26$ cm

Area of $\Delta ABC = \frac{1}{2} \times BC \times AB$

$= \frac{1}{2} \times 10 \times 24$

$= 120$ cm²

Also, $OF \perp AC$, $OE \perp BC$ and $OD \perp AB$

($\because$ Tangent is perpendicular to the radius through point of contact.)

Area of $\Delta BOC = \frac{1}{2} \times BC \times OE$

$= \frac{1}{2} \times 10 \times r$

$= 5r$

a rea of $\Delta AOC = \frac{1}{2} \times 26 \times r$

$= 13r$

area of $\Delta AOB = \frac{1}{2} \times AB \times OD$

$= \frac{1}{2} \times 24 \times r$

$= 12r$

Area of ΔABC = area of ΔBOC

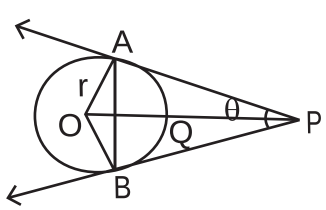
+ area of ΔAOC

+ area of ΔAOB

$\therefore$ 120 $= 5r + 13r + 12\ r$

120 $= 30r$

4 $= r$

29. 

AP is tangent to the circle

$\therefore$ $OA \perp AP$

i.e. $\angle OAP = 90°$

(Tangent is perpendicular to radius through point of contact.)

In ΔOAP,

$\sin\theta = \frac{OA}{OP} = \frac{r}{2r}$

$= \frac{1}{2}$

(OP = Diameter $= 2r$)

$\therefore$ $\theta = 30°$

$\Rightarrow$ $\angle OPA = 30°$

Similarly, $\angle OPB = 30°$

$\therefore$ $\angle APB = 30° + 30°$

$= 60°$

Also, AP = BP

(Lengths of tangent drawn from an external point to a circle are equal.)

So, In ΔAPB,

$\angle PAB = \angle PBA$ (i)

(Angles opposite to equal sides are equal.)

In ΔAPB,

$\Rightarrow$ $\angle PAB + \angle PBA + \angle APB = 180°$

(Angle sum property)

$\Rightarrow$ $\angle PAB + \angle PAB + 60° = 180°$

$\Rightarrow$ $2\angle PAB = 180° - 60°$

$= 120°$

$\Rightarrow$ $\angle PAB = \frac{120°}{2}$

$= 60°$

So, $\angle PAB = \angle PBA = \angle APB = 60°$

$\Rightarrow$ ΔAPB is equilateral.

30. As we know that lengths of tangents drawn from an external point to a circle are equal,

$\therefore$ PD = PF, RF = RE, QD = QE

Consider

Perimeter of ΔPQR

$= PQ + QR + PR$

$= (\underline{PD} + DQ) + (QE + ER) + (\underline{PF} + FR)$

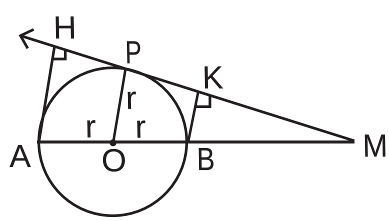
$= (PD + PF) + (RF + RE) + (QD + QE)$

$= (PF + PF) + (RE + RE) + (QD + QD)$

$= 2\,PF + 2\,RE + 2\,QD$

$= 2\,(PF + ER + QD)$

Section D

31. 

PM is to circle

$\therefore$ $\angle MPO = 90°$

(Tangent is perpendicular to radius through point of contact)

Let AH = x, BK = y, BM = z

Let r be the radius of circle

In ΔMKB and ΔMHA

$\angle M = \angle M$ (Common)

$\angle MKB = \angle MHA = 90°$

$\therefore$ $\Delta MKB \cong \Delta MHA$

(AA similarity creteria.)

$\Rightarrow = \dfrac{MK}{MH} = \dfrac{KB}{HA} = \dfrac{MB}{MA}$

(Corresponding sides of similar triangles are propertional)

$\Rightarrow \dfrac{BK}{AH} = \dfrac{MB}{MA}$

$\Rightarrow \dfrac{x}{y} = \dfrac{z}{2r + z}$

$\Rightarrow 2r\,y + y\,z + = x\,z$

$\Rightarrow Z = \dfrac{2ry}{x - y}$ (i)

Now, In ΔMKB and ΔMPO,

$\angle M = \angle M$ (Common)

$\angle MKB = \angle MPO = 90°$

$\therefore$ ΔMKB and ΔMPO, (AA similarity creteria.)

$\Rightarrow \dfrac{MK}{MP} = \dfrac{BK}{OA} = \dfrac{MB}{MO}$

(Corresponding sides of similar triangles are proportional.)

$\Rightarrow \dfrac{BK}{PO} = \dfrac{BM}{OM}$

$\Rightarrow \dfrac{y}{r} = \dfrac{z}{r + z}$

$\Rightarrow yr + yz = rz$

$\Rightarrow z = \dfrac{ry}{r - y}$ (ii)

From (i) and (ii), we get

$z = \dfrac{2ry}{x - y} = \dfrac{ry}{r - y}$

$\Rightarrow \dfrac{2ry}{x - y} = \dfrac{ry}{r - y}$

$\Rightarrow \dfrac{2y}{x - y} = \dfrac{y}{r - y}$

$\Rightarrow \dfrac{2}{x - y} = \dfrac{1}{r - y}$

$\Rightarrow 2r - 2y = x + y$

$\Rightarrow x + y = 2r$

$\Rightarrow AH + BK = AB$ ($\because$ AB = 2r)

32. Consider ΔOEA and ΔOEP

OA = OP (Radii of same circle)

OE = OE (Common)

AE = PE (OE bisects AP)

$\therefore$ $\Delta OEA \cong \Delta OEP$

(SSS congruence creteria)

$\Rightarrow \quad \angle OEA = \angle OEP \quad$ (CPCT)

$\therefore \quad \angle OEA = \Delta OEP = 90°$ (i)

($\angle OEA$ and $\angle OEP$ are linear pair)

Also, $AB \perp BC$ as BC is a tangent to the circle

(Tangent is perpendiculer to the radius through the point of contact.)

$\Rightarrow \quad \angle ABC = 90°$ (ii)

Now, In ΔAEO and ΔABC

$\angle EAO = \angle BAC$ (Common)

$\angle AEO = \angle ABC$

$= 90°$ (From (i) and (ii))

$\Rightarrow \quad \Delta AEO \sim \Delta ABC$

(By SS Similarity creteria)

33. Given: d_1, d_2 $(d_2 > d_1)$ be the diameters of two concentric circles and C be the length of a chord of a circle which is tangent to the circle.

To prove: $d_2 = d_1^2 + c^2$

Now,

$OQ = \frac{d_2}{2}$, $OR = \frac{d_1}{2}$ and $PQ = c$

Since PQ istangent to the circle therefore OR is perpendicular to PQ

$\Rightarrow QR = \frac{PQ}{2} = \frac{c}{2}$

Using pythagorus theorm in triangle OQR

$OQ^2 = OR^2 + QR^2$

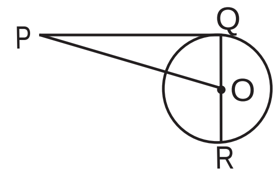
$\Rightarrow \left(\frac{d_2}{2}\right)^2 = \left(\frac{d_1}{2}\right)^2 + \left(\frac{c}{2}\right)^2$

$\Rightarrow \frac{1}{4}(d_2)^2 = \frac{1}{4}(d_1)^2 + (c)^2$

$\Rightarrow d_2^2 = d_1^2 + c^2$

Hence Proved.

34.



$OQ : PQ = 3 : 4$

Let $OQ = 3k$, $PQ = 4k$

PQ is tangent to the circle

$\therefore \quad OQ \perp PQ$

i.e. $\angle OQP = 90°$

(Tangent is perpendicular to radius through point of contact.)

In ΔOQP,

$OP^2 = OQ^2 + PQ^2$

(Pythagoras theorem)

$= (3k)^2 + (4k)^2$

$= 9k^2 + 16k^2$

$= 25k^2$

$\therefore \quad OP = 5k$

Also, Perimeter of $\Delta POQ = 60$ cm

$\Rightarrow \quad PO + OQ + PQ = 60$

$\Rightarrow \quad 5k + 3k + 4k = 60$

$\Rightarrow \quad 12k = 60$

$\Rightarrow \quad k = \frac{60}{12}$

$= 5$

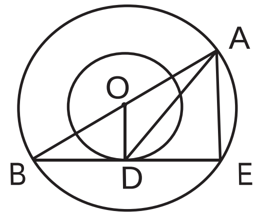
So, $PQ = 4k = 4 \times 5 = 20$ cm

$QR = 20\ Q = 2(3k) = 6k$

$= 6 \times 5 = 30$ cm

$OP = 5k = 5 \times 5 = 25$ cm

35.



BE is tangent to circle

$\therefore$ OD $\perp$ BE

i.e. $\angle ODB = 90°$

(Tangent is perpendicular to radius through point of contact)

$\Rightarrow$ BD = DE

(As perpendicular from centre to the chord bisects the chord)

$\Rightarrow$ D is a midpoint of BE

Also, O being the centre is a midpoint of AB

So, By midpoint theorem,

OD || AE and OD $= \frac{1}{2}$ AE

$\therefore$ AE = 20 D

= 2 (8)

= 16 cm

In ΔODB, $\angle ODB = 90°$

$\therefore$ $OB^2 = OD^2 + BD^2$

(By Pythagoras theorem)

$\Rightarrow$ $13^2 = 8^2 + BD^2$

$\Rightarrow$ $169 = 64 + BD^2$

$\Rightarrow$ $BD^2 = 169 - 64$

$\Rightarrow$ $BD^2 = 105$

$\Rightarrow$ $BD = \sqrt{105}$ cm

$\Rightarrow$ $DE = \sqrt{105}$ cm

($\because$ BD = DE)

In ΔAED, $\angle AED = 90°$

$\therefore$ $AD^2 = AE^2 + DE^2$

$= (16)^2 + (\sqrt{105})^2$

$= 256 + 105$

$= 361$

$\therefore$ AD = 19 cm

36. BD is tangent to the circle

$\therefore$ OC $\perp$ BD

i.e. $\angle OCD = 90°$

(Tangent is perpendicular to radius through point of contact.)

$\Rightarrow$ $\angle OCA + \angle ACD = 90°$ (i)

Now, OA = OC

(Being radii of same circle.)

$\therefore$ In ΔAOC,

$\angle OCA = \angle OAC$

(Angles opposite to equal sides are equal.)

$\Rightarrow$ $\angle OCA = \angle BAC$ (ii)

From (i) and (ii), we get

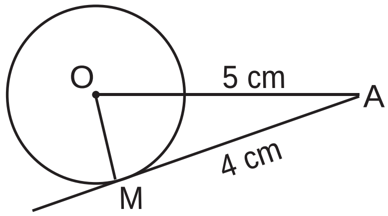
$\angle BAC + \angle ACD = 90°$

Chapter 11

Constructions

MULTIPLE CHOICE QUESTIONS

1.



$\therefore$ ΔOMA $\angle M = 90°$

By Pythagoras Theorem,

$OM^2 = OA^2 - AM^2$

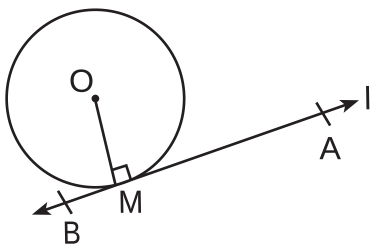
$= 5^2 - 4^2$

$\therefore OM^2 = 9 = 3^2$

$\therefore OM = r = 3$ cm

Option (c)

2.



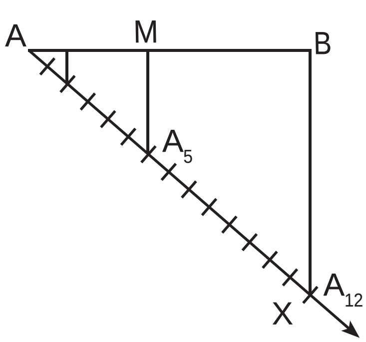
Line AB is a tangent of circle with centre O.

$\therefore OM \perp AR$

$\therefore \angle OMA = 90°$

Option (c)

3.



For 5 : 7

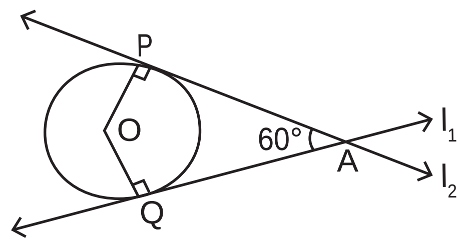
Given, $\angle BAX$ $\angle 90°$

$\therefore A_5M \parallel BA_{12}$

$\therefore$ Total equal arc lengths are (5 + 7) = 12

Option (d)

4.



Given, $\angle PAQ = 60°$

Where AP and AQ are two tangents of the circle with centre O.

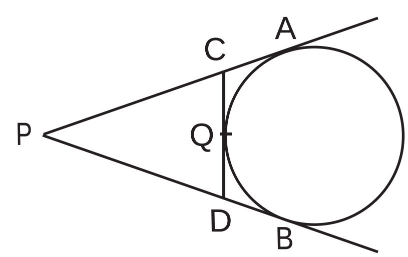
$\therefore \angle POQ = 360° - (90° + 90° + 60°)$

$= 360° - (240°)$

$\angle POQ = 120°$

Option (d)

5.



Given, PB = 12 cm, CQ = 3 cm

PA and PB are two tangents of the circle with centre O and CD is a third tangent.

We know that the two tangents draw to a circle from an external point are equal.

$\therefore$ PA = PB = 12 cm

Same, CQ = CA = 3 cm

$\therefore$ P – C – A,

$\therefore$ PA = PC + CA

$\therefore$ 12 = PC + 3

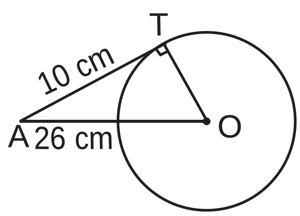
$\therefore$ 12 – 3 = PC = 9 cm

Option (a)

WORKSHEET 1

Section A

1.



Since the tangent to a circle is perpendicular to the radius through the point of contact.

$\therefore \angle OTA = 90°$

In right ΔOTA, we have

$OA^2 = OT^2 + AT^2$

$\Rightarrow OT^2 = 676 - 100$

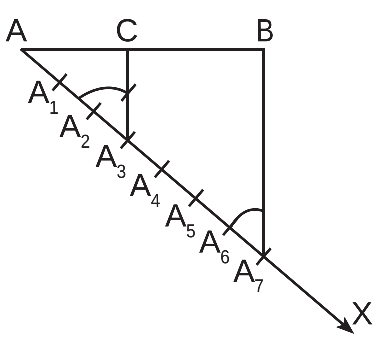
$\Rightarrow 576$

$\Rightarrow (24)2$

$\Rightarrow OT = 24$

Hence the radius of the circle is 24 cm.

2.



(i) Draw AB = 6 cm.

(ii) Draw a ray AX making an acute $\angle BAX$.

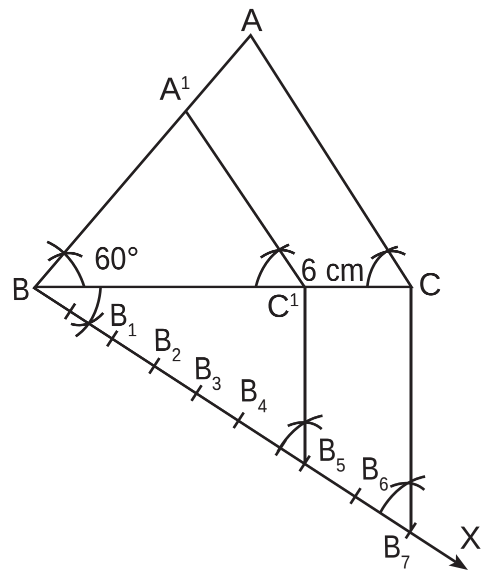
(iii) Along AX, mark point $A_1, A_2, A_3, ..., A_7$. Such that $AA_1 = A_1A_2 ... = A_6A_7$.

(iv) Join A_7B.

(v) Through A_3 draw a line $A_3C \parallel A_7B$ intersecting AB at C.

Thus, points C so obtained is the required point which divides internally in the ratio 3:4.

3.



Given, BC = 6 cm, AB = 5 cm, $\angle ABC = 60°$ and $\frac{5}{7}$ of the corresponding sides of the triangle ABC.

Steps of construction:

(i) BC = 6 cm is drawn

(ii) At point B, AB = 5 cm is draw angle $\angle ABC = 60°$ with BC.

(iii) AC is joined to form ΔABC.

(iv) A ray BX is drawn making an acute angle with BC opposite to vertex A.

(v) 7 Points $B_1, B_2, B_3 ..., B_7$ at equal distance are marked on BX.

(vi) B_5 joined with C^1 to form B_5C^1.

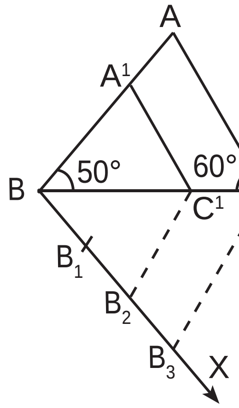
(vii) C^1A^1 is draw parallel to CA.

Thus A^1BC^1 is the required triangle.

4. Given that

Construct triangle of given data, BC = 6 cm, $\angle B = 50°$ and $\angle C = 60°$ and then a triangle similar to it whose sides are (2/3)rd of the corresponding sides of ΔABC.

We follow the following steps to construct the given



Steps of construction

(i) First of all we draw a line segment BC = 60°.

(ii) With B as centre draw an angle $\angle B$ = 50°.

(iii) With C as centre draw an angle $\angle C$ = 60° which intersecting the line drawn in step ii at A.

(iv) Join AB and AC to obtain ΔABC.

(v) Below BC, makes an acute angle $\angle CBX$ = 60°.

(vi) Along BX, mark off three points B_1, B_2 and B_3 such that $BB_1 = B_1B_2 = B_2B_3$.

(vii) Join B_3C.

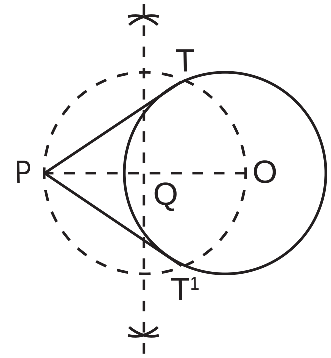
(viii) Since we have to construct a triangle each of whose sides is two-third of the corresponding sides of ΔABC.

So, we take two parts out of three equal parts on BX from point B_2 draw $B_2C \parallel B_3C$ and meeting BC at C.

(ix) From C^1 draw $C^1A^1 \parallel AC$ and meeting AB at A^1.

Thus, ΔABC is the required triangle, each of whose sides is two third of the corresponding sides of ΔABC.

5.



(i) Take a point O in the plane of the paper and draw a circle of radius 3 cm.

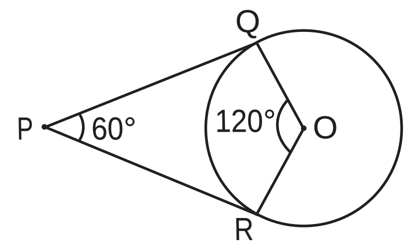
(ii) Mark a point P at a distance of 5.5 cm from the centre O and join OP.

(iii) Draw the right bisector of OP, intersection OP at Q.

(iv) Taking Q as centre and OQ = PQ as radius, draw a circle to intersect the given circle at T and T^1.

(v) Join PT and PT^1 to get the required tangents.

6.



Given angle between tangents is 60°

i.e. $\angle QPR = 60°$

Since Angle at centre is double the angle between tangents

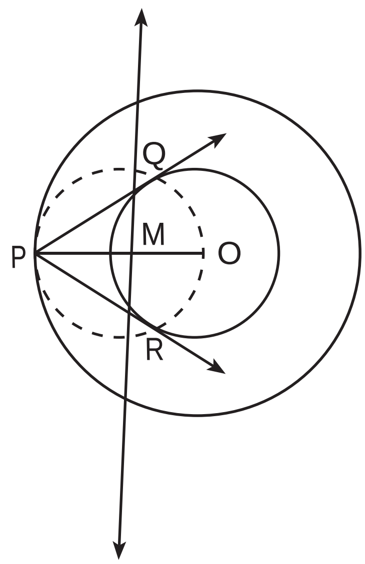
$\therefore \angle QPR = 2 \times 60° = 120°$

So, we need to draw $\angle QPR = 120°$

$\therefore$ We draw a radius, then second radius at 120° from first.

Section B

7.



Steps of construction

(i) Draw a concentric circle two with circles radii are 3 cm and 5 cm.

(ii) Let P is a point on the circumference of center circle. Joined O to P, then OP formed.

(iii) Perpendicular bisector of OP. M is a mid point of the OP.

(iv) Draw a circle M is a centre and OM radius. This circle is intersect with three circles in Q and P points.

(v) Joined P to Q and P to R.

Thus, PQ and PR are required two tangents.

OQ = 3 cm and OP = 5 cm,

In ΔOPQ,

$PQ^2 = OP^2 - OQ^2$

$= (5)^2 - (3)^2$

$= 25 - 9$

$PQ^2 = 16 - 4$

$\therefore$ PQ = 4 ...(i)

8. (i) Make an line BC 5.5 cm

(ii) Take its bisector and now cut it 3 cm.

(iii) Join BA and CA.

(iv) Now from B make an circle ($\angle 60°$) angle.

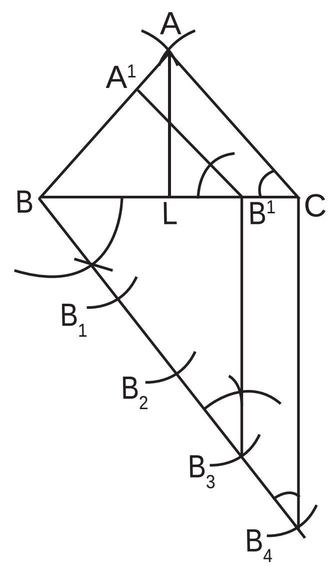
(v) Extend that line.

(vi) Cut 4 arcs.

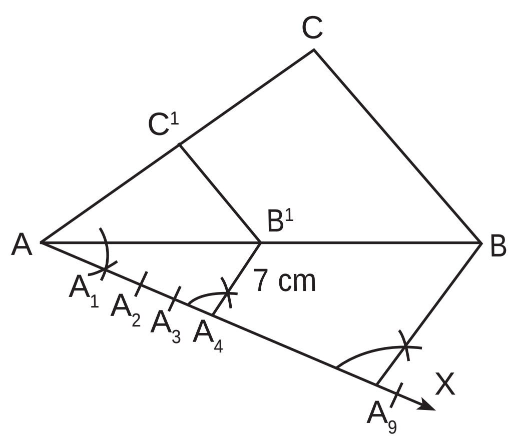
(vii) From B_3 make a parallel line to CB_4.

(viii) Do same formation inside the triangle.

(ix) Your triangle is ready.



9.



Steps of construction

(i) AB = 7 cm drawn

(ii) With A is centre and radius = 5 cm on arc is drawn and same with B is centre and radius = 6 cm, on arc is drawn.

(iii) Both an arc.

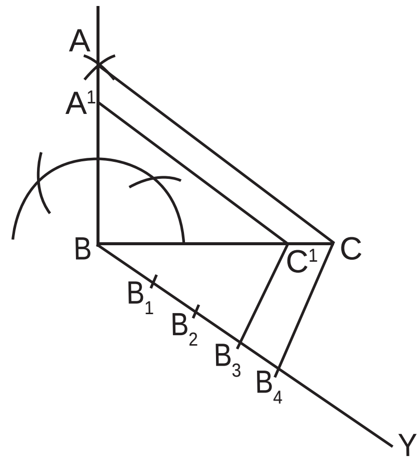
(iv) Join AC and BC to form ΔABC.

(v) Ray AX is drawn making an acute angle with AB below it.

(vi) 9 equal points is marked on AX as $A_1, A_2, \ldots A_9$.

(vii) A_9B is joined. A_4B^1 is drawn parallel to A_9B and B^1C^1 is drawn parallel to BC, ΔAB^1C^1 is the required triangle.

10.



Steps of construction

(i) Draw a line segment BC = 8 cm

(ii) Draw line segment BX making an angle of 90° at the point B of BC.

(iii) From B mark an arc on BX at a distance of 6 cm. Let it is A.

(iv) Join A to C.

(v) Making an acute angle draw a line segment BY from B.

(vi) Mark B_1, B_2, B_3, B_4 on BX such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.

(vii) Join B_4 to C.

(viii) Draw a line segment B_3C^1 || to B_4C to meet BC at C^1.

(ix) Draw line segment CA^1 || to CA to meet AB at A^1.

A^1BC^1 is the required triangle.

11. Steps of construction

(i) Take AB = 7 cm

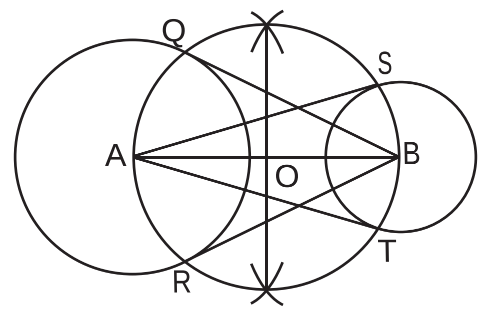
(ii) With A as centre and 3 cm as radius, draw a circle.

(iii) Similarly, with B as centre and 2 cm as radius, draw a circle.

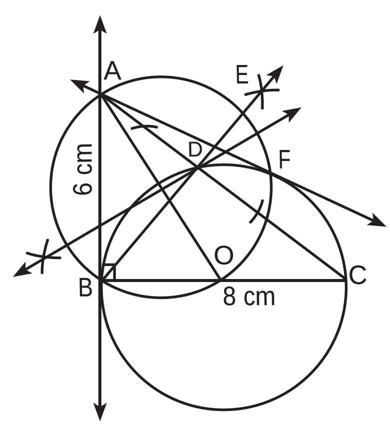
(iv) Now, draw the perpendicular bisector of AB and mark the point of intersection O.

(v) With O as centre and OA as radius, draw a circle. Mark the 2 points where the circles with centre O and A meet as Q and R. Similarly, mark the points where the circles with centres O and B meet as S and T respectively.

(vi) Join BR and BQ as well as AS and AT. Now, BR, BQ, AS and AT are the required tangents.



12.



Steps of construction

(i) Construct the triangle as per given measurements.

(ii) Take any arbitrary radius and draw two arcs of circle from point B on AC intersecting AC at X and Y.

(iii) Taking X and Y as centre, draw two arcs of circles to intersect each other at point

E. Join B and E. BE is the perpendicular from B on AC.

(iv) DBDC is a right angled. Hence, BC the hypotenuse will form the diameter of the circle passing through the vertices of ΔBDC.

(v) BC = 8 cm, OC = 4 cm, draw a circle of radius equal 4 cm, passing through B, D and C.

(vi) Join O and A. Obtain the mid-point P of segment OA by drawing perpendicular bisector to OA.

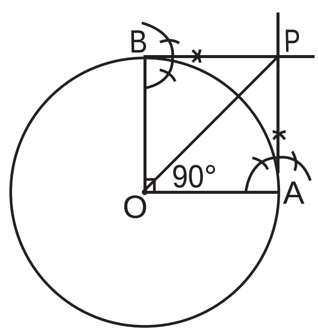
(vii) Draw a circle with centre P and radius AP.

(viii) Let B and F be the points of intersection of these two circles. Hence, AB and AF are the required tangents.

WORKSHEET 2

Section A

1.



Steps of construction

(i) Draw a circle of any convenient radius with O as centre.

(ii) Take a point A on the circumference of the circle and join OA. Draw a perpendicular to OA at point A.

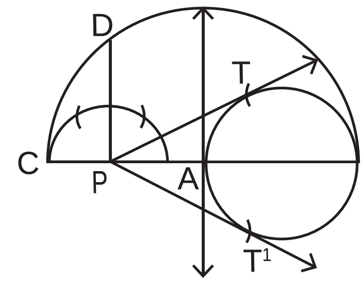
(iii) Draw a radius OB, making an angle of 90° with OA.

(iv) Draw a perpendicular to OB at point B. Let both the perpendicular intersect at point P.

(v) Join OP.

PA and PB are the required tangents, which make an angle of 45° with OP.

2.



Steps of construction

(i) Draw a circle of radius 4 cm.

(ii) Take a point P outside the circle and draw a secant PAB, intersecting the circle at A and B.

(iii) Produce AP to C such that AP = CP.

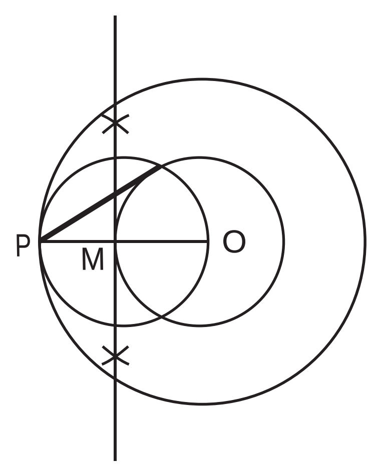
(iv) Draw a semi-circle with CB as diameter.

(v) Draw PD and CB intersecting the semi-circle at D.

(vi) With P as centre and PD as radius draw arcs to intersect the given circle at T and T^1.

(vii) Join PT and PT^1. Then PT and PT^1 are the required tangents.

3.



Consider the attached construction diagram

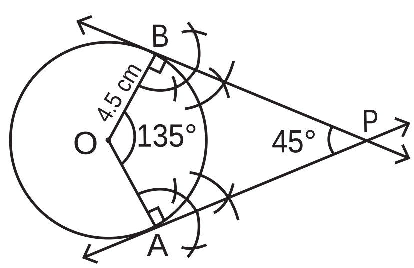
(i) Draw a concentric circles of radius 6 cm, 12 cm center O and P be the point on the circle.

(ii) M is the midpoint formed by the perpendicular bisector of OP with O, P as centers, and radius more than 6 cm draw arc and join the intersection arc to get perpendicular bisector cutting OP at M.

(iii) With M as center draw a circle that passes through O, P.

(iv) This circle (center M) cuts the original circle (center O) the thick line is tangent needed.

4.



(i) Draw circle of radius 4.5 cm with centre O.

(ii) Take any points A on the circle. Join OA. Mark another point B on the circle such that $\angle AOB = 135°$, supplementary to the angle between the tangents.

Since the angle between the tangents to be constructed is 45°.

$\therefore \angle AOB = 180° - 45° = 135°$

(iii) Construct angles of 90° at A and B extend the lines so as to intersect at point P.

(iv) Thus AP and BP are the required tangents to the circle.

5. Steps of construction

(i) Taking point O as a centre draw a circle of radius 4 cm.

(ii) Now taking O as centre draw a concentric circle of radius 6 cm.

(iii) Taking any point P on the outer circle join OP.

(iv) Draw a perpendicular bisector of OP.

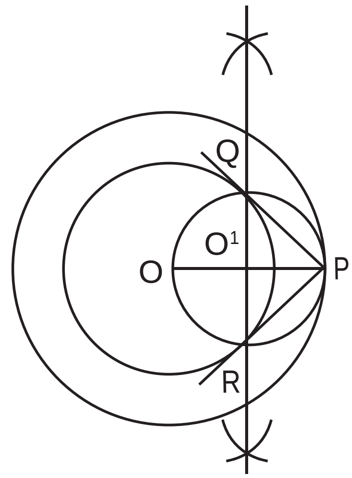
(v) Name the intersection of bisector and OP as O^1.

(vi) Now, draw a circle taking O^1 as centre and O^1P as radius.

(vii) Name the intersection point of two circles as R and Q.

(viii) Join PR and PQ. These are the required tangents.

(ix) Measure the lengths of the tangents. PR = 4.47 cm and PQ = 4.47 cm.



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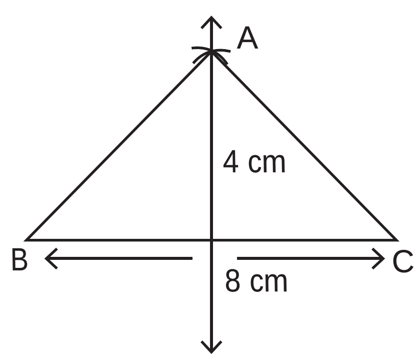


Steps of construction

(i) BC = 8 cm is drawn.

(ii) Perpendicular bisector of BC is drawn and it intersect BC at M.

(iii) At a distance of 4 cm a point A is marked on perpendicular bisector of BC.

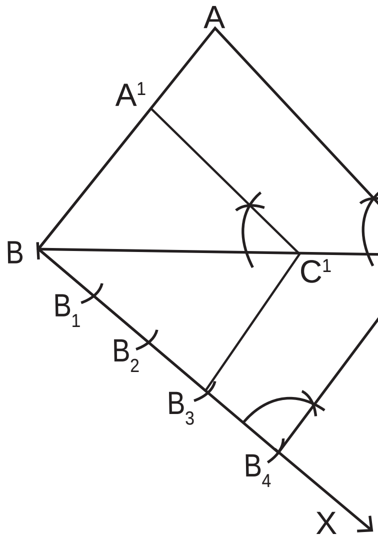
(iv) AB and AC are joined to form ΔABC.

(v) Ray BX is drawn making an acute angle with BC apposite to vertex A.

(vi) 4 points B_1, B_2, B_3 and B_4 are marked on BX.

(vii) B_4 is joined with C to form B_4C.

(viii) B_4C^1 is drawn parallel to B_4C and C^1A^1 is drawn parallel to CA. Thus A^1BC^1 is the required triangle formed.



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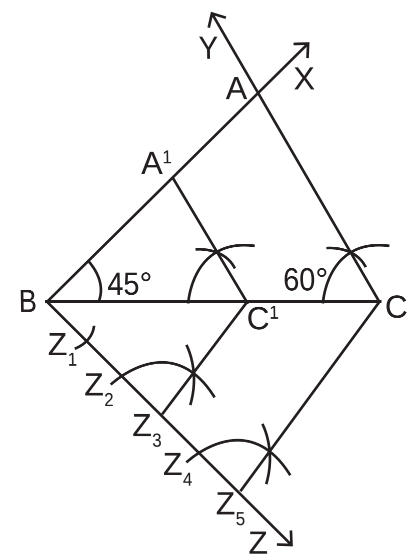


Steps of construction

(i) Draw a line segment BC = 8 cm.

(ii) At B, draw $\angle XBC = 45°$.

(iii) At C, draw $\angle YCB = 60°$. Suppose BX and CY intersect at A.

Thus, ΔABC is the required triangle.

(iv) Below BC, draw an acute angle $\angle ZBC$.

(v) Along BZ, mark five points Z_1, Z_2, Z_3, Z_4 and Z_5 such that $BZ_1 = Z_1Z_2 = Z_2Z_3 = Z_3Z_4 = Z_4Z_5$

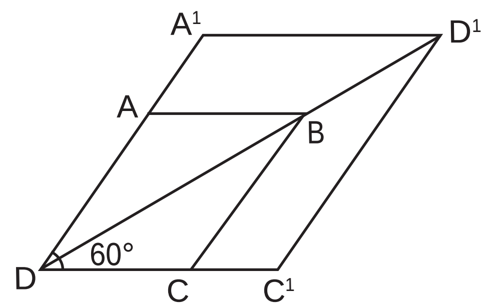
(vi) Join CZ_5.

(vii) From Z_3, draw $Z_3C^1 \parallel CZ_5$ meeting BC at C^1.

(viii) From C^1, draw $A^1C^1 \parallel AC$ meeting AB in A^1.

Here, ΔA^1BC^1 is the required triangle whose sides are $\frac{3}{5}$ of the corresponding sides of ΔABC.

8.



Given ABCD is a parallelogram with BD as diagonal.

$\angle ABC = 60°$

ΔBD^1C^1 has been drawn similar to ΔBDC by a scale factor $\frac{4}{3}$ which is greater than 1 i.e. D^1 lies on extended BD and C^1 lies on extended BC.

To determine - whether $A^1BC^1D^1$ is a parallelogram or not.

Solution-

ABCD is a parallelogram i.e. $AB \parallel DC$ ...(i)

Now ΔBDC and ΔBD^1C^1 are similar

(by construction)

$\therefore \angle BDC = \angle BD^1C^1$

But they are corresponding angles.

$\therefore DC \parallel D^1C^1$

$\Rightarrow D^1C^1 \parallel AB$ or A^1B, since a^1 lies on extended

BA ...(from (i))

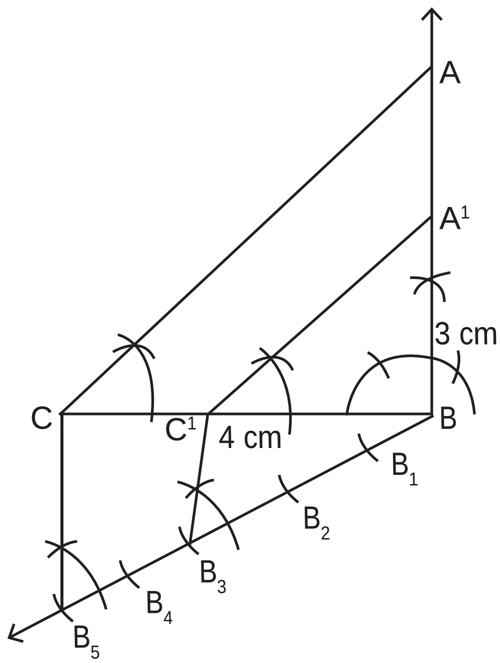
$A^1D^1 \parallel BC^1$ ...(by construction) ...(iii)

So, from (ii) and (iii)

$A^1BC^1D^1$ is a parallelogram.

Ans-Yes.

9.



Steps of construction

(i) BC = 4 cm is drawn.

(ii) At B, a ray making an angle of 90° with BC is drawn.

(iii) With B is centre and radius equal to 4 cm, an arc is made on provision ray intersecting it at point A.

(iv) AC is joined to form ABC.

(v) Ray BX is drawn making acute angle with BC opposite to vertex A.

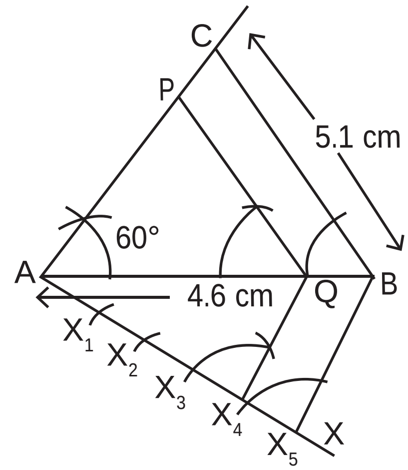
(vi) 5 points $B_1, B_2, \dots, B_5$ at equal distance are marked on BX.

(vii) B_5C is joined and B_3C^1 is made parallel to B_5C.

(viii) A^1C^1 is joined together.

Thus, A^1BC^1 is the required triangle.

10.



Steps of construction

(i) Draw a line segment AB of 4.6 cm.

(ii) At B draw an angle of 60°.

(iii) With centre B and radius 5.1 cm, an arc which intersect line of angle at C.

(iv) Joined BC.

(v) At A draw and angle BAX of any measure.

(vi) Starting from A, cut 5 equal parts on AX.

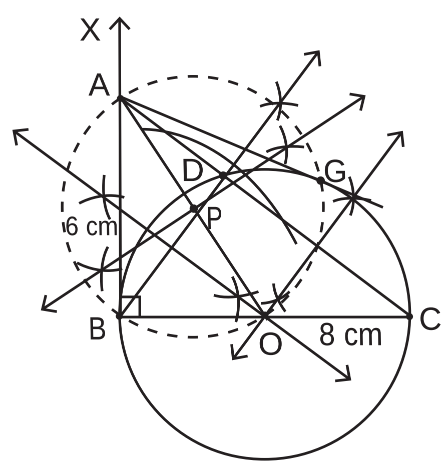
(vii) Joined X_5B.

(viii) Through X_4, draw $X_4Q \parallel X_5B$.

(ix) Through Q, draw $AP \parallel BC$.

$\therefore \Delta PAQ \sim \Delta CAB$.

11.



Steps of construction

(i) Draw a line BC of 8 cm length.

(ii) Draw BX perpendicular to BC.

(iii) Mark an arc at the distance of 6 cm on BX. Mark it as A.

(iv) Joine A and C. Thus ΔABC is the required triangle.

(v) With B as the centre, draw an arc on AC.

(vi) Draw the bisector of this arc and join it with B. Thus, BD is perpendicular to AC.

(vii) Now, draw the perpendicular bisector of BD and CD. Take the point of intersection as O.

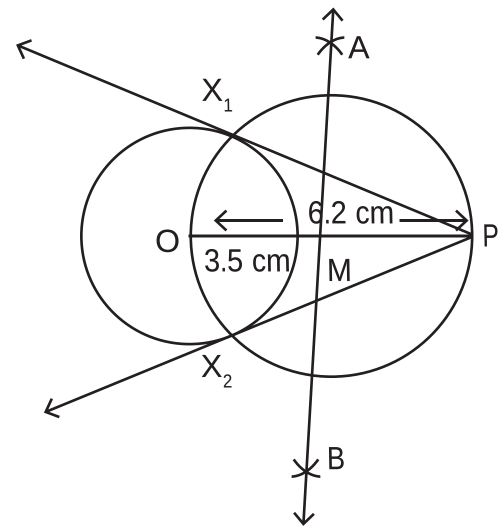
(viii) With O as the centre and OB as the radius, draw a circle passing through points B, C and D.

(ix) Join A and O and bisect it. Let P be the mid point of AO.

(x) Taking P as the centre and PO as its radius. draw a circle which will intersect the circle at point B and G. Join A and G.

Here, AB and AG are the required tangents to the circle from A.

12.



Steps of construction

(i) Draw the circle with centre O and radius 3.5 cm.

(ii) Joint P from centre to outside the circle. OP = 6.2 cm.

(iii) Construct mid point of OP, M is the mid point of OP

(iv) Draw a circle with centre M and radius OM intersect the given circle at X_1 and X_2.

(v) Joined PX_1 and PX_2.

Thus, PX_1 and PX_2 are required two tangents from point P.

Chapter 12 Area Related to Circles

MULTIPLE CHOICE QUESTIONS

1. $2\pi r \;\; r = 37$

$r\left(2 \times \frac{22}{7} - 1\right) = 37$

$r = \frac{37 \times 7}{44 - 7}$

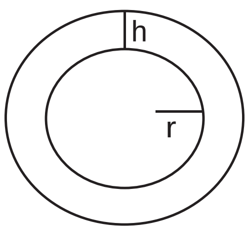
$= 7$ cm

$\therefore$ Area $= \frac{22}{7} \times 7 \times 7$

$= 154$ cm²

Option (d)

2.



Area of circular path

$= \pi (r + h)^2 - \pi r^2$

$= \pi (r^2 + h^2 + 2rh) - \pi r^2$

$= \pi (h^2 + 2rh)$

$= \pi h (h + 2rh)$

Option (b)

3. $2\pi r = 22$

$2 \times \frac{22}{7} \times r = 22$

$r = \frac{22 \times 7}{2 \times 22}$

$= 3.5$ cm

Area $= \frac{22}{7} \times 3.5 \times 3.5$

$= 38.5$ cm²

Option (a)

4. Perimeter of circle = 2 (perimeter of square)

$\Rightarrow 2\pi r = 2(4x)$

$\Rightarrow \pi r = 4x$

$\Rightarrow r = \frac{4x}{\pi}$

Ratio of areas $= \frac{\pi r^2}{x^2}$

$= \pi \frac{16x^2}{\pi^2} \times \frac{1}{x^2}$

$= 16 : \pi$

Option (c)

5. $\frac{\text{Area of Section } 5_1}{\text{area of Section } 5_2}$

$= \frac{\frac{120}{360}\pi r^2}{\frac{150}{360}\pi r^2}$

$= 4 : 5$

Option (d)

WORKSHEET 1

Section A

1. arc length = 3.5 cm

$\frac{\theta}{360} 2\pi r = 3.5$

$\Rightarrow \frac{\theta \pi r}{360} = \frac{3.5}{2}$ (i)

Area of sector $= \frac{\theta}{360} \pi r^2$

$= \frac{3.5}{2} \pi r \; r$

$= \frac{3.5}{2} \pi$ (From (i))

$= \frac{3.5}{2} \times 5$

$= 8.75 \text{ cm}^2$

2. Length of arc $= \frac{\theta}{360} 2\pi r$

$= \frac{45}{360} \times 2 \times \pi \times 5$

$= \frac{45\pi}{36}$

$= \frac{5\pi}{4}$ cm

3. We know

Area of sector of circle $= \frac{\text{given angle}}{360°} \times \pi r^2$

here r = 21 cm

Given angle = 120°

$= \frac{120°}{360°} \times \frac{22}{7} \times 21^2$

$= \frac{1}{3} \times \frac{22}{7} \times 21 \times 21$

$= \frac{22 \times 21 \times 21}{7 \times 3}$

$= 22 \times 21$

$= 462 \text{ cm}^2$

4. Area of circle = Circumference of circle

$\Rightarrow \quad \pi r^2 = 2\pi r$

$\Rightarrow \quad r = 2$

5. Circumference = $2\pi r$ metres

Distance covered = 5 m

So, no. of revolutions $= \frac{\textit{Distance Covered}}{\textit{Circumference}}$

$= \frac{5}{2\pi r}$

6. Let r_1 = 19 cm and r_2 = 9 cm.

Circumference of circle

= Sum of circumferences of the two circles

$\Rightarrow \quad 2\pi r = 2\pi r_1 + 2\pi r_2$

$\Rightarrow \quad r = r_1 + r_2$

$= 19 + 9$

$= 28$ cm

7. Let r_1 = 12 cm, r_2 = 5 cm

Area of circle = Sum of areas of the two circles

$\Rightarrow \quad \pi r^2 = \pi r_1^2 + \pi r_2^2$

$\Rightarrow \quad r^2 = r_1 + r_2$

$= 12^2 + 5^2$

$= 144 + 25$

$= 169$

r = 13 cm = 13 cm

Diameter = 2r

= 26 cm

8. Circumference = 582 cm

$\Rightarrow \quad 2\pi r = 582$

$\Rightarrow \quad 2 \times \frac{22}{7} r = 582$

$\Rightarrow \quad r = \frac{291 \times 7}{2 \times 11}$

$= \frac{2037}{22} \text{ cm}^2$

$\therefore$ area of circle $= \pi r^2$

$= \frac{22}{7} \times \frac{2037}{22} \times \frac{2037}{22}$

$= 36943.95 \text{ cm}^2$

Section B

9. Let r_1 = 8 cm, r_2 = 6 cm

Area of circle = Sum of area of 2 circles

$\pi r^2 = \pi r_1^2 + \pi r_2^2$

$r^2 = r_1^2 + r_2^2$

$= 8^2 + 6^2$

$= 64 + 36$

$= 100$

$\therefore \quad r = 10$ cm

$\therefore$ Circumference of circle

$= 2\pi r$

$= 2 \times \frac{22}{7} \times 10$

$= \frac{440}{7}$ cm

10. Time = 10 minutes

$= 10 \times 60$

= 600 seconds

Speed = 66 km/hr

$= \frac{\cancel{66}^{11} \times \cancel{1000}^{5}}{\cancel{3600}_{\cancel{6}\ 3}}$

$= \frac{55}{3}$ m/s

$\therefore$ Total Distance covered = speed x time

$= \frac{55}{3} \times 600$

= 11000 m

Distance covered in one revolution

$= 2\pi r$

$= 2 \times \frac{22}{7} \times \frac{40}{100} = \frac{88}{35}$ m

$\therefore$ Number of revolutions

$= \frac{11000 \times 3}{88}$

= 4375

11. Circumference = 22

$\Rightarrow \quad 2\pi r = 22$

$\Rightarrow \quad 2 \times \frac{22}{7} r = 22$

$\Rightarrow \quad r = \frac{22 \times 7}{2 \times 22}$

$= \frac{7}{2}$

= 3.5 cm

area of quadrant $= \frac{1}{4} \pi r^2$

$= \frac{1}{4} \times \frac{22}{7} \times 3.5 \times 3.5$

$= 9.625 \text{ cm}^2$

12. Angle subtended in 60 minutes = 360°

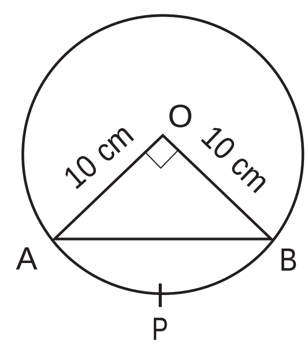
$\therefore$ Angle subtended in 10 minutes $= \frac{360}{60} = 6^\circ$

Area $= \frac{\theta}{360} \pi r^2$

$= \frac{60}{360} \times \frac{22}{7} \times 16 \times 16$

$= 134.095 \text{ cm}^2$

13.



area of sector OAPB $= \frac{90}{360} \times \frac{22}{7} \times (10)^2$

$= \frac{550}{7} \text{ cm}^2$

area of sector $\Delta AOB = \frac{1}{2} \times 10 \times 10 = 50 \text{ cm}^2$

$\therefore$ area of minor segment

$= \frac{550}{7} - 50$

$= \frac{550 - 350}{7}$

$= \frac{200}{7}$ cm²

$= 28.6$ cm²

Area of major segment

= area of circle – area of minor segment

$= \pi (10)^2 - \frac{200}{7}$

$= \frac{22}{7} \times 100 - \frac{200}{7}$

$= \frac{2000}{7} - \frac{200}{7}$

$= \frac{2000}{7}$ cm²

$=$ 285.7 cm²

14. Area cleaned at each sweep of the blades

$= \frac{\theta}{360} \pi r^2$

$= \frac{115}{360} \times \frac{22}{7} \times 2.5 \times 2.5$

$= 6.27$ cm²

$\therefore$ Total area cleaned $= 2 \times 6.27$

$= 12.54$ cm²

15. Area of each semi circle

$= \frac{1}{2} \times \frac{22}{7} \times \left(\frac{14}{2}\right)^2$

$= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$

$= 11 \times 7$

$= 77$ cm²

Area of square $= (14)^2$

$= 196$ cm²

$\therefore$ area of shaded region

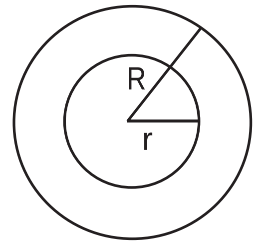
= area of square – 2x area of semi circle

$= 196 - 2\,(77)$

$= 196 - 154$

$=$ 42 cm²

16.



r $= \frac{17.5}{2}$ 8.75 cm

with of path = 3.5 cm

$\Rightarrow$ R $= 8.75 + 3.5$

$= 12.25$ cm

area of path $= \pi (R^2 - R^2)$

$= \frac{22}{7} [(12.25)^2 - (8.75)^2]$

$= \frac{22}{7} [150.0625 - 76.5625]$

$= 231$ cm²

Section C

17. Cost of fencing at the rate of ` 24 per metre

= ` 5280

$\Rightarrow$ Perimeter of circular field x 24 = 5280

$\Rightarrow$ Perimeter of circular field $= \frac{5280}{24}$

$= 220$ m

$\Rightarrow$ $2\pi r = 220$

$\Rightarrow$ $2 \times \frac{22}{7} \times r = 220$

$\Rightarrow$ $r = \frac{220 \times 7}{2 \times 22}$

$= 35$ m

$\therefore$ Area of field $= \pi r^2$

$= \frac{22}{7} \times 35 \times 35$

$= 3850\ m^2$

Cost of ploughing the field $= 0.05 \times 3850$

= ` 1925

18. Given :

PQ = 24 cm, PR = 7 cm

We know that any angle made by the diameter QR in the semicircle is 90°.

$\angle RPQ = 90°$

In right angled ΔRPQ

$RQ^2 = PQ^2 + PR^2$

[By pythagoras theorem]

$RQ^2 = 24^2 + 7^2$

$RQ^2 = 576 + 49$

$RQ^2 = 625$

$RQ = \sqrt{625 \text{ cm}}$

RQ= 25 cm

radius of the circle (OQ) $= \frac{25}{2}$cm

Area of right Δ RPQ $= \frac{1}{2} \times$ Base $\times$ height

Area of right Δ RPQ $= \frac{1}{2} \times$ RP $\times$ PQ

Area of right Δ RPQ $= \frac{1}{2} \times 7 \times 24 = 7 \times 12$
$= 84\ cm^2$

Area of right Δ RPQ = 84 cm^2

Area of semicircle $= \frac{\pi r^2}{2}$

$= \left(\frac{22}{7}\right) \times \frac{\left(\frac{25}{2}\right)^2}{2}$

$= \frac{(22 \times 25 \times 25)}{(7 \times 2 \times 2 \times 2)}$

$= 11 \times \frac{625}{28} = \frac{6875}{28}\ cm^2$

Area of semicircle $= \frac{6875}{28}\ cm^2$

Area of the shaded region = Area of semicircle – Area of right Δ RPQ

$= \left(\frac{6875}{28} - 84\right) cm^2$

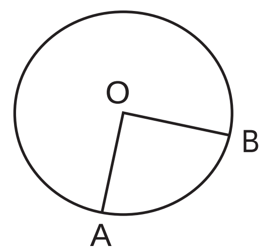
$= \frac{(6875 - 2532)}{28}$

Area of the shaded region $= \frac{4523}{28}$

$= 161.54\ cm^2$

Hence, the area of the shaded region
$= 161.54\ cm^2$

19.



r = 5.7 cm

Perimeter of sector of circle = 27.2 cm

OA + OB + length of arc $\widehat{AB}$ = 27.2

5.7 + 5.7 + length of $\widehat{AB}$ = 27.2

length of $\widehat{AB}$ = 27.2 – 5.7 – 5.7 = 15.8 cm

$\Rightarrow \frac{\theta}{360}\ 2\pi r = 15.8$

$\Rightarrow \frac{\theta}{360}\ \pi r = \frac{15.8}{2}$

$= 7.9$ cm

Area of sector OAB $= \frac{\theta}{360}\ \pi r^2$

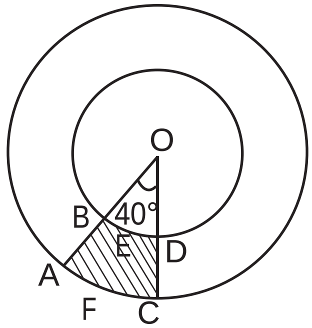
$= \frac{\theta}{360}\ \pi r\ \ r$

$= 7.9\ r$

$= 7.9\ (5.7)$

$= 45.03\ cm^2$

20.



Radius of inner circle = 7 cm

Radius of outer circle = 14 cm

Area of shaded region =

Area of sector OAFC – Area of sector OBED

$= \frac{40°}{360°} \times \pi\ (14)^2 - \frac{40°}{360°} \times \pi (7)^2$

$= \frac{1}{9} \times \frac{22}{7} \times 14 \times 14 - \frac{1}{9} \times \frac{22}{7} \times 7 \times 7$

$= \frac{616}{9} - \frac{154}{9} = \frac{462}{9}$

$= \frac{154}{9}$ cm²

21. Area of shaded region = Area of square ABCD – Area of 4 quadrant – Area of circle with diameter2 cm

Area of square $= 4 \times 4 = 16$ cm²

Area of sector $= \frac{\theta}{360} \times \pi \times r^2$

Area of 4 quadrant $= 4 \times \frac{90}{360} \pi \times 1 \times 1$

$= 3.14$ cm²

Area of circle $= \pi \times r^2 = \pi \times 1 \times 1$

$= 3.14$ cm²

∴ Area of shaded region $= 16 - 6.28$

$= 9.72$ cm²

22. Area of quadrant OAB $= \frac{1}{4} \pi (21)^2$

$= \frac{441}{4} \pi$ cm²

Area of quadrant ODC $= \frac{1}{4} \pi (14)^2$

$= \frac{196}{4} \pi$ cm²

∴ Area of shaded region

$= \frac{441}{4} \pi - \frac{196}{4} \pi$

$= \frac{245}{4} \pi$

$= \frac{245}{4} \times \frac{22}{7}$

$= 192.5$ cm²

23. Angle subtended by minute hard

in 60 minutes = 360°

in 1 minutes $= \frac{360°}{6}$

= 6°

in 5 minutes = 5 x 6

= 30°

So, area swept by minutes hard

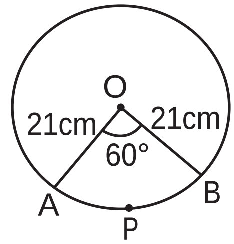
in 5 minutes $= \frac{30}{360} \pi\ (14)^2$

$= \frac{\pi}{12} \times 196$

$= \frac{196}{12} \times \frac{22}{7}$

$= 51.3$ cm²

24. (i) the length of the arc



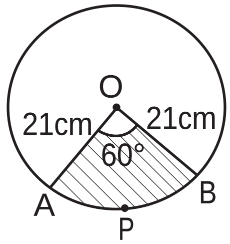
Length of arc APB $= \frac{\theta}{360} \times (2\pi r)$

$= \frac{40°}{360°} \times 2 \times \frac{22}{7} \times 21$

$= \frac{1}{6} \times 2 \times \frac{22}{7} \times 21$

= 22 cm

(ii) area of the sector formed by the arc



Area of sector OAPB $= \frac{\theta}{360} \times \pi r^2$

$= \frac{60}{360} \times \frac{22}{7} \times 21 \times 21$

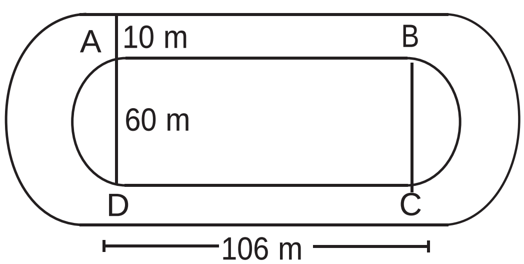
$= \frac{1}{6} \times \frac{22}{7} \times 21 \times 21$

$= \frac{1}{6} \times 22 \times 3 \times 21$

= 231 cm^2

Section D

25.



(i) Distance around the track along its inner edge = AB + CD + 2(semi-perimeter of inner circles ends)

$= 106 + 106 + 2\left(\frac{22}{7} \times 30\right)$

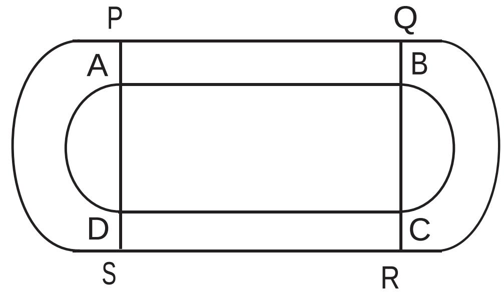
$= 212 + \frac{1320}{7}$

$= \frac{1484 + 1320}{7}$

$= \frac{2804}{7}$

= 400.57 cm

(ii)



Area of track

= area of rectangle PQRS

– area of rectangle ABCD

+ 2 [Area of semi-circle with radius 40 cm]

[area of semicircle with radius 30 cm]

= (106 x 80) – (106 x 60)

$+ 2\left[\frac{\pi}{2}(40)^2 - \frac{\pi}{2}(30)^2\right]$

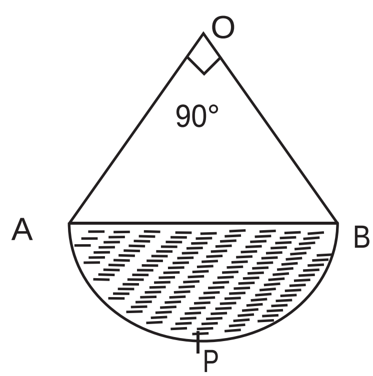
= 8480 – 6360 + π (1600 – 900)

$= 8480 - 6360 + \frac{22}{7} \times 700$

= 8480 – 6360 + 2200

= 4320 m^2

26.



ABCD is a square

$\therefore$ AC and BD bisect each other and are equal

$\therefore$ AO = $\therefore$ OC = DO = BO

In ΔAOB,

$AB^2 = OA^2 + OB^2$

$(56)^2 = OA^2 + OA^2$ [$\therefore$ OA = OB]

$3136 = 2OA^2$

$1568 = OA^2$

$OA = \sqrt{1568}$

$= 28\sqrt{2}$ m

So, area of sector OAPB

$= \frac{90}{360} \times \frac{22}{7} \times (28\sqrt{2})^2$

$= \frac{1}{4} \times \frac{22}{7} \times 784 \times 2$

$= 1232\ m^2$

Also, area of ΔOAB

$= \frac{1}{2} \times OA \times OB$

$= \frac{1}{2} \times 28\sqrt{2} \times 28\sqrt{2}$

$= 784\ m^2$

So, area of shaded part

$= 2\ [1232 - 784]$

$= 896\ m^2$

Also, area of square lawn + area of flower bets

$= 896 + 3136$

$= 4032\ m^2$

27. Area of square $= 8^2$

$= 64\ cm^2$

area of 1 quadrant

$= \frac{1}{4} \pi (1.4)^2$

$= \frac{1}{4} \times \frac{22}{7} \times \frac{14}{10} \times \frac{14}{10}$

$= 1.54$ cm

$\therefore$ Area of the shaded portion of the square

= Area of square – area of circle – 2 (area of a quadrant)

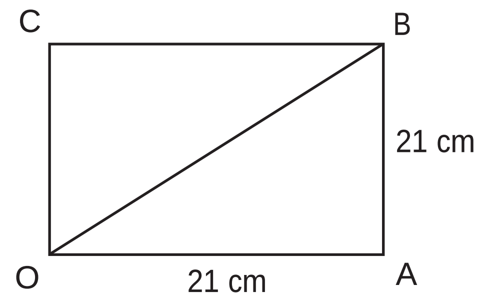
$= 64 - 55.44 - 2\ (1.54)$

$= 8.56 - 3.08$

$= 5.48\ cm^2$

28. area of square OABC $= (21)^2$

$= 411\ cm^2$



In ΔOAB, right angled at A

$OB^2 = OA^2 + AB^2$

$= (21)^2 + (21)^2$

$= 441 + 441 = 882\ cm^2$

$\therefore\ OB = 21\sqrt{2}$

So, area of quadrant OPBQ with OB as radius

$= \frac{90}{360} \times \frac{22}{7} \times (21\sqrt{2})^2$

$= \frac{1}{4} \times \frac{22}{7} \times 441 \times 2$

$= 693\ cm^2$

$\therefore$ area of shaded part

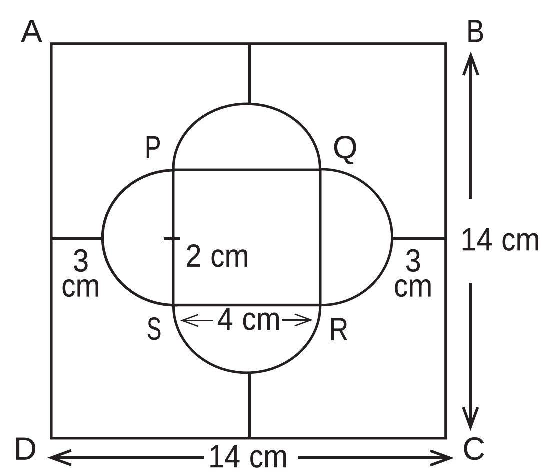
= area of quadrant OPBQ

– area of square OABC

$= 693 - 441$

$= 252\ cm^2$

29.



Area of square ABCD = $(14)^2$

$= 196 \text{ cm}^2$

area of square PQRS = 4^2

$= 16 \text{ cm}^2$

area of 4 semi-circles with radius 2 cm

$= 4\left(\frac{1}{2}\right)\frac{22}{7} \times 2 \times 2$

$= \frac{176}{7} \text{ cm}^2$

So, area of shaded part

$= 196 - \left(16 + \frac{176}{7}\right)$

$= 196 - 16 - \frac{176}{7}$

$= 180 - \frac{176}{7}$

$= \frac{1260 + 176}{7}$

$= \frac{1084}{7}$

$= 154.85 \text{ cm}^2$

30. Area of circle $= \pi r^2$

$= \pi (7)^2$

$= 49\pi \text{ cm}^2$

$= \frac{49 x 22}{7} \text{ cm}^2$

$= 154 \text{ cm}^2$

Area of sector

$= \frac{60}{360} \times \frac{22}{7} \times 7 \times 7$

$= \frac{154}{6} \text{ cm}^2$

$= \frac{77}{3} \text{ cm}^2$

$= 25.67 \text{ cm}^2$

Area of $\Delta ABC = \frac{\sqrt{3}}{4}(14)^2$

$= \frac{\sqrt{3}}{4} \times 196$

$= 84.87 \text{ cm}^2$

So, area of shaded part

= Area of circle + Area of triangle

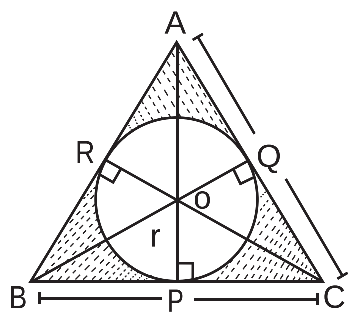
– 2 Area of sector

$= 154 + 84.87 - 2\,(25.67)$

$= 154 + 84.87 - 51.34$

$= 187.53 \text{ cm}^2$

31.



$AP \perp BC$, $BQ \perp AC$ and $CR \perp AB$

[As tangent is perpendicular to radius through point of contact.]

Area of $\Delta ABC = \frac{\sqrt{3}}{4}(\text{side})^2$

$= \frac{\sqrt{3}}{4}(12)^2$

$= \sqrt{3}\,(36)$

$= 36\sqrt{3} \text{ cm}^2$

Also, area of ΔABC

= area of ΔAOC + area of ΔBOC + area of ΔAOB

$36\sqrt{3} = \frac{1}{2}(12)\,r + \frac{1}{2}(12)\,r\ \frac{1}{2}(12)\,r$

$36\sqrt{3} = 6r + 6r + 6r$

$\frac{36\sqrt{3}}{18} = r$

$r = 2\sqrt{3}$ cm

$\therefore$ area of circle $= \pi r^2$

$= \frac{22}{7}(2\sqrt{3})^2$

$= \frac{22}{7} \times 12$

$= 37.71$ cm²

So, area of shaded part

= area of ΔABC

– area of circle

$= 36\sqrt{3} - 37.71$

$= 62.352 - 37.71$

$= 24.642$ cm²

32. Area of square OABC

$= 7^2$

$= 49$ cm²

Area of square OAPC

$= \frac{90}{360} \times \frac{22}{7} \times 7 \times 7$

$= \frac{154}{4}$ cm²

$= 38.5$ cm²

So, area of shaded region

$= 49 - 38.5$

$= 10.5$ cm²

WORKSHEET 2

Section A

1. Area of section $= \frac{p}{360}\pi r^2$

2. Perimeter of circle = Perimeter of square

(radius=r) (side=x)

$\Rightarrow 2\pi r = 4x$

$\Rightarrow \pi r = 2x$ (i)

So, $\frac{\text{area of circle}}{\text{area of square}} = \frac{\pi r^2}{x^2}$

$= \frac{\pi\left(\frac{2x}{\pi}\right)^2}{x^2}$

$= \frac{\pi 4x^2}{\pi^2} \times \frac{1}{x^2}$

$= \frac{4}{\pi}$

$= \frac{4 \times 7}{22} = 14 : 11$

3. Area of sector $= \frac{\theta}{360}\pi r^2$

$= \frac{\not{60}}{\not{360}_{\not{6}\,3}} \times \frac{22}{7} \times \overset{5}{\not{10}} \times 10$

$= \frac{1100}{21}$

$= 52\frac{8}{11}$ cm²

4. Circumference = 100 cm

Let radius of circle be r cm

Now, $2\pi r = 100$

$r = \frac{100}{2\pi} = \frac{50}{\pi}$

Diagonal of square = Diameter of circle =

$2r = \frac{100}{\pi}$

Let the side of the square = a cm

Now, diagonal of square $= \sqrt{a^2 + a^2} = \sqrt{2}\,a$

$\Rightarrow \frac{100}{\pi} = \sqrt{2}\,a$

$\Rightarrow a = \frac{100}{\sqrt{2}\pi}$

$\Rightarrow a = \frac{100\sqrt{2}}{2\pi}$

$\Rightarrow a = \dfrac{50\sqrt{2}}{2\pi}$ cm

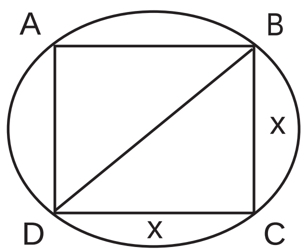
5. Area of circle = 220

$\Rightarrow \quad \pi r^2 = 220$

$\Rightarrow \quad r = 2\sqrt{\dfrac{220}{\pi}}$

$\Rightarrow$ Diameter $= 2r$

$=2\sqrt{\dfrac{220}{\pi}}$ cm



In ΔABC,

$BD^2 = BC^2 + CD^2$

$\left(2\sqrt{\dfrac{220}{\pi}}\right)^2 = 2x^2$

$4\left(\dfrac{220}{\pi}\right) = 2x^2$

$x^2 = \dfrac{880}{\pi(2)}$

$= \dfrac{440}{\pi}$

So, area of square $= x^2$

$= \dfrac{440}{\pi}$

$= \dfrac{440}{\pi} \times 7$

$= 140$ cm²

6. $r = 0.25$ m

Distance covered in one revolution

$= 2\pi r$

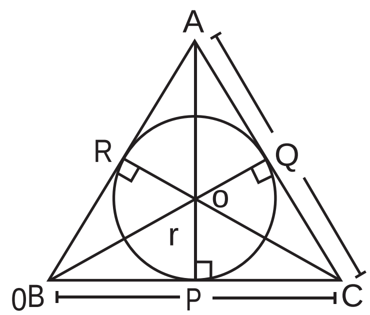
$= 2 \times \dfrac{22}{7} \times 0.25$

So, number of revolutions

$= \dfrac{11\times1000\times7}{2\times22\times0.25}$

$= 7000$

7.



Area of $\Delta ABC = \dfrac{\sqrt{3}}{4}$ (side)²

$= \dfrac{\sqrt{3}}{4} \times 42 \times 42$

$= 441\sqrt{3}$ cm²

Also, $AP \perp BC$, $BQ \perp AC$ and $CR \perp AB$

[As tangent is perpendicular to radius through point of contact.]

So,

area of ΔABC = area of ΔBOC

+ area of ΔAOC

+ area of ΔAOB

$\Rightarrow \quad 441\sqrt{3} = \dfrac{1}{2} \times 42 \times r + \dfrac{1}{2} \times 42 \times r + \dfrac{1}{2} \times 42 \times r$

$\Rightarrow \quad 441\sqrt{3} = 63 \quad r$ cm²

$\Rightarrow \quad r = \dfrac{441\sqrt{3}}{63}$

$= 7\sqrt{3}$ cm

So, area of circle $= \pi r^2$

$= \dfrac{22}{7} \times 7\sqrt{3} \times 7\sqrt{3}$

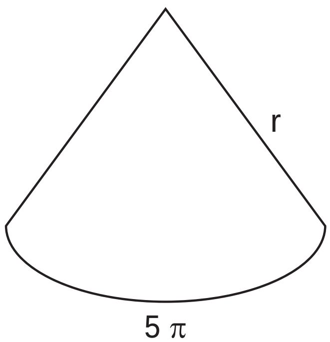
$= 462$ cm²

8. Area of sector

$$= \frac{90}{360} \times \frac{22}{7} \times 2 \times 2$$

$$= \frac{22}{7} \text{ cm}^2$$

9.



arc length $= 5\pi$ cm

$$\frac{\theta}{360} 2\pi r = 5$$

$$\frac{2\pi}{360} = 5$$

$$r\theta = 900 \quad \text{(i)}$$

Also, area of sector $= 20\pi$

$$\frac{\theta}{360} \pi r^2 = 20\pi$$

$$\frac{\theta r^2}{360} = 20$$

$$r\theta^2 = 7200 \quad \text{(ii)}$$

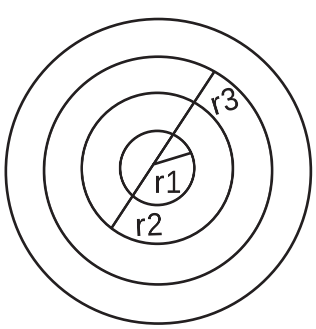
From (i) and (ii), we get

$$r^2 \frac{900}{r} = 7200$$

$$900r = 7200$$

$$r = 8 \text{ cm}$$

10.



According to question,

$$\pi r^2 - \pi r_3^2 = \frac{1}{4} \pi r^2$$

$$\Rightarrow \pi r^2 - \frac{1}{4} \pi r^2 = \pi r_3^2$$

$$\Rightarrow \frac{3}{4} \pi r^2 = \pi r_3^2$$

$$\Rightarrow \frac{3}{4} r^2 = r_3^2$$

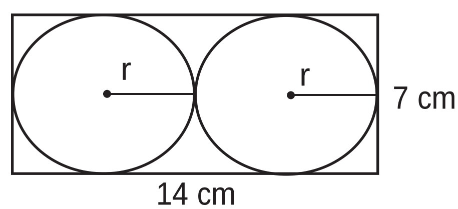
$$\Rightarrow r^3 = \frac{\sqrt{3}}{2} r$$

$$= \frac{\sqrt{3}}{2} (20)$$

$$= 10\sqrt{3}$$

Section B

11.



Area of rectangle $= 14 \times 7$

$= 98 \text{ m}^2$

Area of circle $= \pi \left(\frac{7}{2}\right)^2$

$$= \frac{22}{7} \times \frac{49}{7}$$

$$= \frac{77}{2} \text{ m}^2$$

So, area of remaining portion

= Area of rectangle

– 2 (area of circle)

$$= 98 - 2\left(\frac{7}{2}\right)$$

$$= 98 - 77$$

$$= 21 \text{ cm}^2$$

12. $R - r = 7$ (i)

$\pi R^2 - \pi r^2 = 286$

$(R^2 - r^2)\frac{22}{7} = \frac{286 \times 7}{22}$

$= 91$

$\Rightarrow (R - r)(R + r) = 91$

$\Rightarrow 7 (R + r) = 91$

$\Rightarrow R + r = 13$ (ii)

On solving (i) and (ii), we get

$$R - r = 7$$
$$R + r = 13$$
$$2r = 20$$
$$r = 10\ cm$$

$\therefore R = 13 - r$

$= 13 - 10$

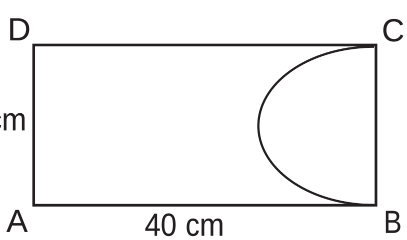
$= 3$ cm

So, sum of radii $= R + r$

$= 10 + 3$

$= 13$ cm

13.



Area of semi-circle $= \frac{1}{2}\pi r^2$

$= \frac{1}{2}\pi (14)^2$

$= \frac{1}{2} \times \frac{22}{7} \times 196$

$= 308\ cm^2$

Area of rectangle ABCD

$= AB \times BC$

$= 40 \times 28$

$= 1120\ cm^2$

So, area of remaining paper

$= 1120 - 308$

$= 812\ cm^2$

14. Perimeter of the top of the table

$= OA + OB + \frac{270}{360}(2\pi)\ 42$

$= 42 + 42 + 63\pi$

$= 84 + 63\pi$

$= 84 + 198$

$= 84 + 198$

$= 282$ cm

15. Circumference of circle $= 2\pi r$

$44 = 2 \times \frac{22}{7} \times r$

$r = \frac{44 \times 7}{2 \times 22}$

$= 7$ cm

So, area of quadrant $= \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \frac{22}{7} \times 7^2$

$= 38.5\ cm^2$

16. Perimeter of the shaded region = Perimeter of bigger semi-circle + 2 (Perimeter of smaller semi-circle)

$= \pi D + 2(\pi d)$

$= \frac{22}{7} \times 14 + 2\left(\frac{22}{7} \times 7\right)$

$= 44 + 2 \times 22$

$= 44 + 44$

$= 88$ cm

17. It is given that the area of trapezium ABCD is $24.5\ cm^2$

And AD || BC

$DAB = 90^o$

$AD = 10$ cm

$BC = 4$ cm

As we know that area of trapezium = $\frac{1}{2}$ + (sum of parallel sides) x height

$\Rightarrow 24.5 = \frac{1}{2}(AD + BC) \times AB$

$\Rightarrow 24.5 \times 2 = (10 + 4) \times AB$

$\Rightarrow 49 = 14 \times AB$

$\Rightarrow AB = \frac{49}{14}$

$\Rightarrow AB = 3.5$ cm

Now,

Area of quadrant

$= \frac{1}{4}\pi r^2$

$= \frac{1}{4}\pi AB^2$

$= \frac{1}{4} \times \frac{22}{7} \times 3.5 \times 3.5$

$= \frac{1}{2} \times 11 \times 0.5 \times 3.5$

$= 9.625$

Area of quadrant = 9.625 cm^2

Now,

Area of shaded region = area of trapezium – area of quadrant

= 24.5 – 9.625

= 14.875

Hence, the area of shaded region is 14.875 cm^2

18. Area of quadrant OACB

$= \frac{1}{4} \times \frac{22}{7} \times 7 \times 7$

= 38.5 cm^2

area of $\Delta AOD = \frac{1}{2} \times 7 \times 4$

= 14 cm^2

$\therefore$ area of shaded part = 38.5 – 14

= 24.5 cm^2

19. Length of arc = 8.8 cm

$2\pi \times r \times 30 \div 360 = 8.8$

$= 2 \times 22 \div 7 \times r \times 1 \div 12 = 8.8$

$r = 8.8 \times 7 \times 12 \div 2 \times 22$

$r = 739.2 \div 4$

$r = 16.8$ cm

So required length of pendulum is 16.8 cm

= 16.23 cm

20. Cost of fencing 1 metre

Circular field = ` 12

Total cost of fencing a circular field = ` 2640

$\therefore$ Circumference of circular field

$= \frac{2640}{12}$

= 220 m

$\Rightarrow 2\pi r = 220$

$2 \times \frac{22}{7} \times r = 220$

$r = \frac{220 \times 7}{2 \times 22}$

= 35 m

Area of circular field = $\frac{22}{7}(35)^2$

= 3850 m^2

$\therefore$ cost of ploughing the field

= 3850 x 2

= ` 7700

Section C

21. Area of 1 and 3 = $(10 \times 10 - 2 \times \frac{1}{2} \times 3.14 \times 5 \times 5)$

[Area of semi circle = $\frac{1}{2}\pi r^2$]

$= (100 - 3.14 \times 25)$

$= (100 - 78.5)$

$= 21.5\text{ cm}^2$

So,

Even the Area of 2 and 4 is equal to 21.5cm^2

So,

Area of shaded region = Area of ABCD – Area 0f (1 + 2 + 3 + 4)

$= 100 - (21.5 + 21.5)$

$= 100 - 43$

Area of shaded region = 57cm^2

22. In ΔQPR,

$\angle QPR = 90°$ (Angle in a semi circle is aright angle.)

$\therefore$ $QR^2 = PQ^2 + PR^2$ (Pythagoras theorem)

$= (12)^2 + 5^2$

$= 144 + 25$

$= 169$

$\Rightarrow$ QR = 13 cm

area of semi-circle with QR as diameter

$$= \frac{1}{2} \times \frac{22}{7} \times \left(\frac{13}{2}\right)^2$$

$$= \frac{\cancel{22}^{11}}{14} \times \frac{169}{\cancel{4}_2}$$

$$= \frac{1859}{28}\text{ cm}^2$$

area of $\Delta QPR = \frac{1}{2} \times PR \times PQ$

$$= \frac{1}{2} \times 5 \times 12$$

$$= 30\text{ cm}^2$$

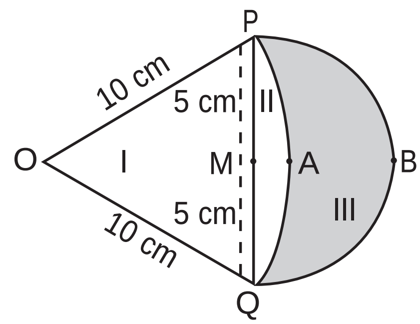
$\therefore$ area of shaded part

$$= \frac{1859 - 840}{28}$$

$$= \frac{1019}{28}\text{ cm}^2$$

$$= 36.4\text{ cm}^2$$

23.



Given: OP = OQ = 10 cm

It is known that tangents drawn from an external point to a circle are equal in length.

So,

OP = OQ = 10 cm

Therefore, ΔABC is an equilateral triangle.

$\Rightarrow \angle POQ = 60^o$

Now

Area of part II = Area of the sector – Area of the equilateral triangle POQ

$$= \frac{\angle POQ}{360^o} \times \pi r^2 - \frac{\sqrt{3}}{4} \times (10)^2$$

$$= \frac{60^o}{360^o} \times \pi(10)^2 - \frac{\sqrt{3}}{4} \times (10)^2$$

$$= 100\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)$$

Area of the semicircle on diameter PQ = Area of part II + Area of part III

$$\frac{1}{2} \times \pi (5)^2 = \frac{25}{2}\pi$$

$\therefore$ Area of the shaded region (part III)

$= \frac{25}{2}\pi - 100\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)$

$= \frac{25}{2}\pi - \frac{100}{6}\pi + 25\sqrt{3}$

$= 25\sqrt{3} - \frac{25}{6}\pi$

$= 25\left(\sqrt{3} - \frac{\pi}{6}\right)$

Hence proved.

24. Area of sector (with radius 14 cm)

$= \frac{60}{360} \times \frac{22}{7} \times 14 \times 14$

area of sector (with radius 28 cm)

$= \frac{60}{360} \times \frac{22}{7} \times 28 \times 28$

$= \frac{11}{21}(784 - 196)$

$= \frac{11}{21}(588)$

$= 308 \text{ cm}^2$

25. Let AO = OB = x

$\Rightarrow$ AB = 2x

Perimeter of semi-circle (with AO as diameter)

$= \frac{1}{2} \times 2 \times \frac{22}{7}\ \frac{x}{2}$

$= \frac{11x}{7}$ cm

Perimeter of semi-circle (with AB as diameter)

$= \frac{1}{2} \times 2 \times \frac{22}{7} x$

$= \frac{22x}{7}$ cm

Given : Perimeter of figure = 40 cm

$\Rightarrow \frac{11x}{7} + \frac{22x}{7} + OB = 40$ cm

$\Rightarrow \frac{33x}{7} + x = 40$

$\Rightarrow \frac{40x}{7} = 40$

$\Rightarrow x = 7$ cm

Area of semi-circle (with AO as diameter)

$= \frac{1}{2} \times \frac{22}{7}\left(\frac{7}{2}\right)^2$

$= \frac{11}{7} \times \frac{49}{4}$

$= \frac{77}{4} \text{ cm}^2$

Area of semi-circle (with AB as diameter)

$= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$

$= 77 \text{ cm}^2$

So, area of shaded region

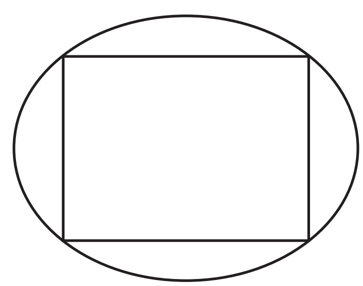
$= \frac{77}{4} + 77$

$= \frac{77 + 308}{4}$

$= \frac{308}{4} \text{ cm}^2$

$= 96.25 \text{ cm}^2$

26.



As all the vertices of a rhombus lie on a circle

$\therefore$ it must be a square

$\Rightarrow$ both the diagonals must be equal

area of circle = 1256 cm²

$3.14 r^2 = 1256$

$\Rightarrow r^2 = \frac{1256}{3.14}$

$= 400$

$\Rightarrow \quad r = 20$ cm

$\Rightarrow$ Diameter of circle $= 2r$

$= 40$ cm

Diameter must be same as diagonal of the square

$\Rightarrow$ Diagonal of square $= 40$ cm

So, area of rhombus

$= \frac{1}{2} \times 40 \times 40$

$= 800$ cm²

27. Radius of long hand = 60 cm

Distance travelled by long hand in 1 round

$= 2\pi (6)$

$= 12\pi$

Number of rounds made by long hand

In 24 hours = 24

So, Total distance travelled by long hand in 24 hours $= 24 \times 12\pi$

$= 288\pi$

Radius of short hand = 4 cm

Distance travelled by short hand in 1 round

$= 2\pi (4)$

$= 8\pi$

Number of rounds made by short hand

In 24 hours = 2

So, Total distance travelled by short hand in 24 hours $= 8\pi \times 2$

$= 16\pi$

So, Sum of distances $= 228\pi + 16\pi$

$= 304\pi$

$= 304 \times 3.14$

$= 954.56$ cm

28. Area of trapozium

$= \frac{1}{2}$ (Sum of parallel sides) x Distance between the parallel sides

$= \frac{1}{2} (AB + DC) \times 14$

$= \frac{1}{2} (18 + 32) \times 14$

$= 350$ cm²

area of a quadrant $= \frac{1}{4} \pi (7)^2$

$= \frac{1}{4} \times \frac{22}{7} \times 49$

$= \frac{154}{4}$ cm²

So, area of shaded region

= area of trapezium

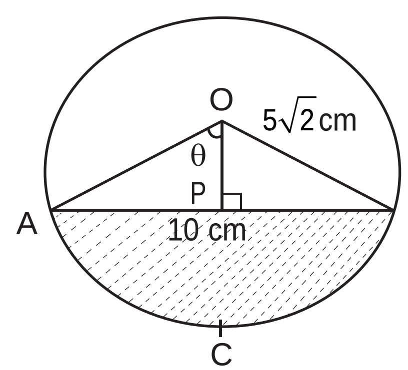
– 4 (area of a quadrant)

$= 350 - 4\left(\frac{154}{4}\right)$

$= 350 - 154$

$= 196$ cm²

29.



Draw $OP \perp AB$

In ΔOPA and ΔOPB

OA = OB (radii of same circle)

OP = OP (common)

$\angle OPA = \angle OPB = 90°$ (By construction)

$\therefore \Delta OPA \cong \Delta OPB$ (RHS)

$\Rightarrow \quad AP = BP \quad (CPCT)$

$\Rightarrow \quad AB = 2AP$

$\Rightarrow \quad AP = \frac{10}{2} = 5$ cm

In ΔOPA,

$OA^2 = OP^2 + AP^2$

$(5\sqrt{2})^2 = OP^2 + 5^2$

$50 - 25 = OP^2$

$25 = OP^2$

$\therefore \ OA^2 = 5$ cm

So, area of $\Delta OAB = \frac{1}{2} AB \times OP$

$= \frac{1}{2} 10 \times 5$

$= 25 \text{ cm}^2$

In ΔAOP,

$\tan\theta = \frac{AP}{OP}$

$= \frac{5}{5}$

$= 1$

$\Rightarrow \quad \theta = 45°$

So, $\angle AOB = 2\,(45°)$

$= 90°$

$\therefore$ area of sector AOBC

$= \frac{90}{360} \times \frac{22}{7} \times 25 \times 2$

$= \frac{275}{7} \text{ cm}^2$

So, area of shaded part

= area of sectior AOBC

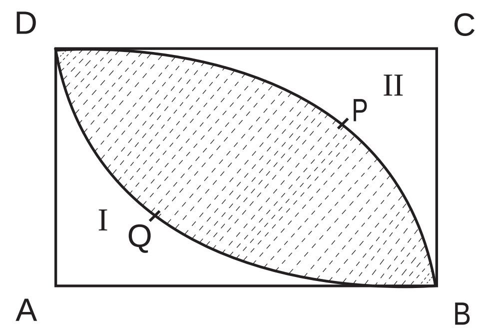
– area of ΔAOB

$= \frac{275}{7} - 25$

$= \frac{275 - 175}{7}$

$= \frac{100}{7} \text{ cm}^2$

30.



Area of square ABCD $= 7^2$

$= 49 \text{ cm}^2$

Area of quadrant ABPD

$= \frac{90}{360} \pi (7)^2$

$= \frac{\pi}{4} (49)$

$= \frac{49}{4} \times \frac{22}{7}$

$= \frac{77}{2} \text{ cm}^2$

$= 38.5 \text{ cm}^2$

So, area of part II

$= 49 - 38.5$

$= 10.5 \text{ cm}^2$

Similarly, area of part I $= 10.5 \text{ cm}^2$

$\therefore$ Area of the shaded region

= area of square ABCD

– area of 1 – area of 11

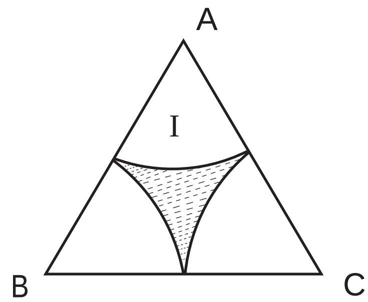
$= 49 - 10.5 - 10.5$

$= 49 - 21$

$= 28 \text{ cm}^2$

Section D

31.



Area of $\triangle ABC$ $= \frac{\sqrt{3}}{4}$ (side)^2

$= \frac{1.732}{4}$ $(8)^2$

$= 27.712$ cm^2

area of sector 1

$= \frac{60}{360} \times 3.142 \times 4^2$

$= \frac{60}{360} \times 3.142 \times 16$

$= 8.38$ cm^2

So, area of shaded part

= Area of $\triangle ABC$

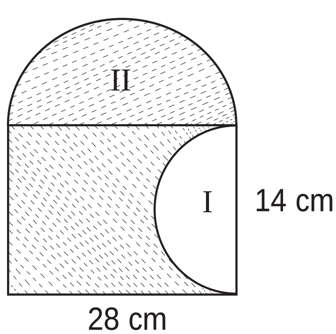
– 3 (area of sector 1)

= 27.712 – 3 (8.38)

= 27.712 – 25.14

= 2.572 cm^2

32.



Area of ractangle piece = 28 x 14

= 392 cm^2

area of part 1 $= \frac{1}{2} \times \frac{22}{7} \times 7^2$

= 77 cm^2

area of part II $= \frac{1}{2} \times \frac{22}{7} \times (14)^2$

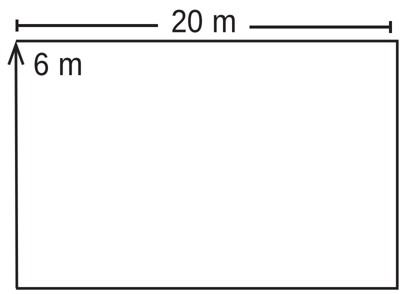
308 cm^2

So, area of shaded region

= area of rectangular piece – area of part I + area of part II

= 392 – 77 + 308 = 623 cm^2

33.



Area of the grassy lawn in which the calf can graze initially

$= \frac{90}{360} \times \frac{22}{7} \times 6 \times 6$

= 28.286 m^2

area of gassy lawn in which the calf can graze if the length of rope is increased by 5.5 m

$= \frac{90}{360} \times \frac{22}{7} \times 11.5 \times 11.5$

= 103.911 m^2

$\therefore$ Increase in the area of the grassy lawn in which the calf can graze

= 109.911 – 28.286

= 75.625 cm^2

34. Area of square = $(28)^2$

= 784 cm^2

area of part of circle inside square

= area of sector

$= \frac{90}{360} \times \frac{22}{7} \times 14 \times 14$

= 154 cm²

area of circle with center O^1

$= \frac{22}{7} \times 14 \times 14$

= 616 cm²

So, area of shaded part

= Area of square – 2 (area of sector) + (area of circle)

= 784 – 2 (154) + 2 (616)

= 784 – 308 + 1232

= 1708 cm²

35. Area of trapezium

$= \frac{1}{2}$ (AD + BC) AB

$\Rightarrow$ 24.5 $= \frac{1}{2}$ (10 + 4) AB

$\Rightarrow \frac{24.5 \times 2}{14}$ = AB

$\Rightarrow$ 3.93 cm = AB

area of quadrant ABE

$= \frac{1}{4} \times \frac{22}{7} \times 3.93 \times 3.93$

= 112.135 cm²

So, area of shaded region

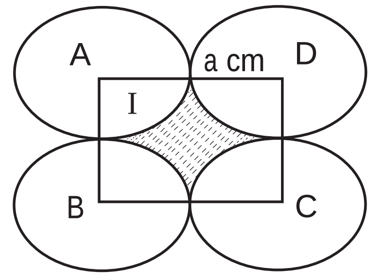
= area of trapezium

– area of quadrant ABC

= 24.5 – 12.135

= 12.365 cm²

36.



To find : area of shaded region

area of part 1

$= \frac{90}{360} \times \frac{22}{7} \times a^2$

$= \frac{11}{14} a^2$ cm²

area of square ABCD

(with side a + a = 2a cm)

= (2a)²

= 4a² cm²

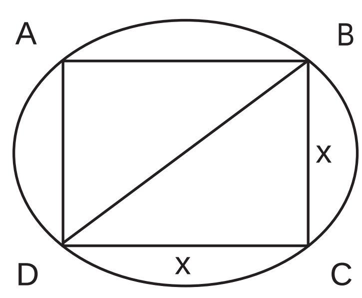
So, area of shaded region

= area of square

$- 4a^2 - 4\left(\frac{11}{14}a^2\right)$

$= 4a^2 - \frac{22}{7}a^2 = \frac{6}{7}a^2$

37.



Circumference of circle = 650 m

$\Rightarrow 2\pi r = 650$

$\Rightarrow 2 \times \frac{22}{7} r = 650$

$\Rightarrow r = \frac{\cancel{650}^{325} \times 7}{\cancel{2} \times 22}$

$= \frac{2275}{22}$ m

$\therefore$ Diameter $= 2$ (radius)

$= \frac{2275}{22}$ m

In ΔBCD,

$BC^2 + CD^2 = BD^2$

$x^2 + x^2 = \left(\frac{2275}{11}\right)^2$

$2x^2 = \left(\frac{2275}{11}\right)^2$

$\Rightarrow x^2 = \frac{1}{2}\left(\frac{2275}{11}\right)^2$

Area of square ABCD $= x^2$

$= \frac{1}{2}\left(\frac{2275}{11}\right)^2$ m²

$= 21386.88$ m²

38. GIVEN:

Side of a square (AB)= 22 cm

Lets the radius of the central part be r cm

Area of the central part $= \frac{1}{5} \times$ area of the square

$\pi r^2 = \frac{1}{5}(22 \times 22)$

$\frac{22}{7} \times r^2 = \frac{(22 \times 22)}{5}$

$r^2 = \frac{(22 \times 7)}{5} = \frac{154}{5}$

$r^2 = 30.8$

$r = \sqrt{30.8} = 5.549 \approx 5.55$ cm

i) Circumference of the central part $= 2\pi r$

$= 2 \times \left(\frac{22}{7}\right) \times 5.55 = 34.88$ cm

ii) Let O is the centre of the central part and O is also the centre of the square.

Diagonal of square AC= $\sqrt{2}a$

$OA = OC = \frac{1}{2}AC$ (diagonals of a square are equal in length and bisect each other)

$OA = \frac{\sqrt{2}a}{2} = \frac{\sqrt{2} \times 22}{2} = 11\sqrt{2}$

$AE = BF = OA - OE = 11\sqrt{2} - 5.55 = 11 \times 1.41 - 5.55 = 15.51 - 5.55 = 9.96$ cm

$[\sqrt{2} = 1.41]$

$EF = \frac{1}{4}$ (circumference of the circle)

$= \frac{1}{4}(2\pi r) = \frac{\pi r}{2} = \frac{1}{2} \times \frac{22}{7} \times 5.55 = 8.72$ cm

Perimeter of part ABEF = AB + AE + EF + BF = 22 + 9.96 + 8.72 + 9.96 = 50.64 cm

Hence, the circumference of the central part = 34.88 cm and the Perimeter of part ABEF = 50.64 cm.

39. Length of a rectangle (AB) = DC = 20 cm

Breadth of a rectangle(BC) = AD=15 cm

AE = 9 cm, ED = 12 cm

Area of rectangle = length x breadth
= 20 × 15 = 300 cm²

Diameter of Semicircle = Breadth of a rectangle = 15 cm

Radius of Semicircle $= \frac{\text{diameter}}{2} = \frac{15}{2}$ cm

Area of semicircle $= \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times (3.14)\left(\frac{15}{2}\right)^2$

$= \frac{(1.57 \times 225)}{4}$

$= \frac{353.25}{4} = 88.31$ cm²

Area of right angled $\Delta = \frac{1}{2} \times$ Base × height

Area of right angled $\Delta AED = \frac{1}{2} \times AE \times ED$

$= \frac{1}{2} \times 9 \times 12 = 9 \times 6 = 54$ cm²

Area of shaded region = Area of rectangle – Area of right angled ΔAED + Area of semicircle

Area of shaded region = 300 – 54 + 88.31

= 246 + 88.31

= 334.31 cm²

Hence, the Area of shaded region is 334.31 cm²

40. Area of semicircle (with diameter CD)

$= \frac{1}{2} \times \frac{22}{7} \times 7^2$

= 77 cm²

area of rectangle ABCD

= AB x BC

= 14 x 7

= 98 cm²

area of semi-circle (with diameter BC)

$= \frac{1}{2} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2$

$= \frac{1}{2} \times \frac{22}{7} \times \frac{49}{4}$

= 19.25 cm²

Similarly, area of semicircle (with diameterAD)

= Area of rectangle ABCD

– area of semicircle (with diameter CD)

+ area of semicircle (with diameter BC)

+ area of semicircle (with diameter AD)

= 98 – 77 + 19.25 + 19.25

= 59.5 cm²

Chapter 13 Surface Areas & Volumes

MULTIPLE CHOICE QUESTIONS

1. a) Volume of piece of iron = (49 × 33 × 24) cm²

Volume of sphere $= \frac{4}{3}\pi r^3$

A T Q;

Volume of iron = Volume of sphere

$49 \times 33 \times 24 = \frac{4}{3}\pi r^3$

$49 \times 33 \times 24 = \frac{4}{3} \times \frac{22}{7} \times \pi r^3$

$r^3 = \frac{49 \times 33 \times 24 \times 3 \times 7}{4 \times 22}$

$r^3 = 9261$

$r = 3\sqrt{9261}$

$r = 21$ cm

2. a) A.T. Q;

Volume of cone = Volume of cylinder

$\frac{1}{3}\pi r^2 h_1 = \pi r^2 h_2$

$\frac{1}{3} h_1 = h_2$

$\frac{1}{3} h_1 = 5$

$h_1 = 15$ cm

3. a) A T Q,

Volume of cylinder = Volume of cone

$\pi r_1^2 h_1 = \frac{1}{3}\pi r_2^2 h_2$

$(18)^2 \ (32) = \frac{1}{3} \times (r_2)^2 \times 24$

$(r_2)^2 = \frac{324 \times 32 \times 3}{24}$

$(r_2)^2 = (18 \times 2)^2$

$r_2 = 36$ cm

4. a) C.S.A. of cylinder = 264 m²

Volume of cylinder = 924 m³

$\frac{C.S.A \text{ of cylinder}}{\text{Volume of cylinder}} = \frac{264}{924}$

$\frac{2\pi r h}{\pi r^2 h} = \frac{264}{924}$

$\frac{2}{r} = \frac{264}{924}$

$r = \frac{2 \times 924}{264}$

$r = 7$ cm (i)

We know;

C.S.A of cylinder = 264 cm²

$2\pi rh = 264$ cm²

$2 \times \frac{22}{7}\ 7 \times h = 264$

$h = \frac{264 \times 7}{2 \times 22 \times 7} = 6$ m

Radio of Diameter to height $= \frac{2r}{h} = \frac{2 \times 7}{6}$

$= \frac{7}{3}$

5. a) In the right circle cone, the cross section made by a plane parallel to its base is a circle.

WORKSHEET 1

Section A

1. A.T. Q;

Radius of cylinder = Radius of sphere

Diameter of sphere $= 2\pi r$

$= 2r$

2. Surface area of cube $= 6a^2$

Surface area of sphere $= 4\pi r^2$

A T Q;

Surface area of cube = Surface area of sphere

$$6a^2 = 4\pi r^2$$

$$3a^2 = 2\pi r^2$$

$$\left(\frac{r}{a}\right)^2 = \frac{3}{2\pi}$$

$$\frac{r}{a} = \frac{\sqrt{3}}{\sqrt{2}\sqrt{\pi}}$$

$$\Rightarrow \frac{\text{Volume of sphere}}{\text{Volume of cube}} = \frac{\frac{4}{3}\pi r^3}{a3}$$

$$= \frac{4}{3}\pi\left(\frac{r^3}{a^3}\right)$$

$$= \frac{4}{3}\pi\left(\frac{\sqrt{3}}{\sqrt{2}\sqrt{\pi}}\right)^2$$

$$= \frac{\cancel{4}^{2}}{\cancel{3}}\pi\left(\frac{\cancel{3}\sqrt{3}}{\cancel{2}\sqrt{2}\,\pi\sqrt{\pi}}\right)$$

$$\frac{\text{Volume of sphere}}{\text{Volume of cube}} = \frac{\sqrt{6}}{\sqrt{\pi}}$$

3. Volume of sphere $= n \times$ Volume of cones

$$\frac{4}{3}\cancel{\pi}r^3 = n \times \frac{1}{3}\cancel{\pi}r^2h$$

$$4 \times (10.5)^2 = n \times (3.5)^2\,(3)$$

$$n = \frac{4 \times (10.5)^3}{(3.2)^2 \times 3} = 126.\text{ cones}$$

4. Let the radius of the cone = R

and Height of a cone = r [Given]

Volume of cone $= 1 \div 3\pi^2 h$

Now,

Volume of sphere = Volume of cone

$$=)\ \frac{4}{3}\pi^3 = \frac{1}{3}^2 h$$

$=)\ 4r^2 = r^2\,(r)$

$=)\ r = 2$

Thus, the radius of the base of cone = 2r

5. Volume of frustum $= \left(\frac{\pi h}{3}\right) \times (R^2 + r^2 + Rr)$

[R= 812 = 4 m; r = 412 = 2 m]

$$= \frac{\pi(6)}{3} \times ((4)^2 + (2)^2 + 4 \times 2)$$

$= 2\pi\ (28)$

$= 56 \times \frac{22}{7}$

$= 8 \times 22$

$= 176\ m^3$

6. Total surface area of canvas

= Sum of curved surface area of and curved surface area of cylinder

C. S.A of cone $= \pi rl$

$= (\pi)\,(105)\,(40)$

$= 4200\pi\ m^2$

C. S.A of cylinder $= 2\pi rh$

$= (2)\,(\pi)\,(105)\,(4)$

$= 840\,\pi\ m^2$

Total surface area of canvas $= 4200\pi + 840\pi$

$= 5040\,\pi$

$= 5040\,\pi$

$= 5040 \times \frac{22}{7}$

$= 720 \times 22$

$= 15840\ m^2$

7. Surface area of Hemisphere = Surface area of cone

$3\pi r^2 = \pi r l + \pi r^2$

$2\,\pi\, r^2 = \pi\, r\, l$

$2r = \sqrt{r^2 + h^2} \quad [\, l = \sqrt{r^2 + h^2}\ ]$

Squaring b/s :-

$4r^2 = r^2 + h^2$

$3r^2 = h^2$

$\frac{r}{h} = \frac{1}{\sqrt{3}}$

8. Volume of hemisphere

= Surface area of hemisphere

$\frac{2}{3}\pi r^3 = 3\pi r^2$

$\frac{2}{3} r = 3$

$r = \frac{9}{2}$

$2r = 9$ cm

Diameter of hemisphere = 9 cm-

Section B

9. Width of canal = 30 m

Depth of canal = 12 m

Flow velocity = 10 km/hr = 10,000 m/hr

Standing water required = 8 cm = 0.8 m

Time = 30 minutes = $\frac{1}{2}$ hr = 0.5 hr.

Area irrigated by 0.08 m

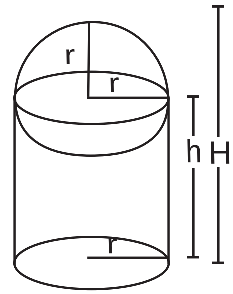
$= 12 \times 30 \times 10{,}000 \times 0.5$

$A \times 0.08 = 1{,}800{,}000$

$A = \frac{1{,}800{,}000}{0.08}$

$A = 22{,}500{,}000\ m^2$

10.



Given,

Volume of air $= 41\frac{19}{21}\ m^3$

$2r = H$ (i)

Total height of the building

= Height of cylinder

Total height of the building

= Height of hemisphere

$H = h + r$ (ii)

From (i) and (ii) :

$2r = h + r$

$h = r$

Volume of building = Volume of cylinder

+ Volume of hemisphere

$\frac{880}{21} = \pi r^2 h + \frac{2}{3}\pi r^3$

$= \pi r^2\,[\,h + \frac{2}{3} r\,]$

$= \pi r^2\,[\,\frac{5}{3} r\,]$

$\frac{880}{21} = \pi r^2 \left[\frac{5}{3}\right]$

$r^3 = \frac{880 \times 3 \times 7}{321 \times 5 \times 22} = 8$

h = 2 m

H = h + r

H = 2 + 2

H = 4 m

11. Let radius cone y = r

So, radius of cone x = 3r

Let volume of cone y = V

So, volume of cone x = 2V

Let height of cone x be h_1 &

height of cone y be h_2

$$\frac{\text{Volume of cone x}}{\text{Volume of cone y}} = \frac{\frac{1}{3}\pi(3r)^2(h_1)}{\frac{1}{3}\pi(r)^2(h_2)}$$

$$\frac{2V}{V} = \frac{9h_1}{h_2}$$

$$\frac{h_1}{h_2} = \frac{2}{9}$$

12. $l \times b = x$

$b \times h = y$

$h \times l = z$

$(l \times b) \times (b \times h) \times (h \times l) = x\,y\,z$

$(l\,b\,h)^2 = x\,y\,z$

$l\,b\,h = \sqrt{xyz}$

Volume of cuboid $= \sqrt{xyz}$

13. Total area without dimples $= \pi r^2 \times n$

$= \frac{22}{7}(0.2)^2 \times 150$

$= 18.857\ \text{cm}^2$

Total area of without dimples $= 4\pi r^2$

$= 4 \times \frac{22}{7} \times \left(\frac{4.1}{2}\right)^2$

$= 52.83\ \text{cm}^2$

Area where there are no dimples

$= (52.83 - 18.857)\ \text{cm}^2$

$= 33.714\ \text{cm}^2$

Surface area exposed to surroundings

$= (33.973 + 37.714)\ \text{cm}^2$

$= 71.687\ \text{cm}^2$

14. Volume of resulting spheres

= Volume of three spheres

$$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 + \frac{4}{3}\pi r_3^3$$

$$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(r_1^3 + r_2^3 + r_3^3)$$

$r^3 = (6^3 + 8^3 + 10^3)$

$r^3 = (216 + 512 + 1000)$

$r^3 = 1728$

$r = 12$ cm

15. Let the height of platform be 'h' metres.

Volume of mud dug out from the well

= Volume of platform

$770 = 22 \times 14 \times h$

$h = \frac{770}{22 \times 14}$

$h = 2.5$ m

16. $2\pi R = 18$ cm

$2 \times \frac{22}{7} \times R = 18$ cm

$R = \frac{18 \times 7}{2 \times 22}$

$R = \frac{9}{\pi}$ cm

$2\pi r = 6$

$r = \frac{6}{2\pi}$

$r = \frac{3}{\pi}$

Given; $l = 4$ cm

Curved surface area of frustum

$= \pi (R + r) l$

$= \pi \left(\frac{9}{\pi} + \frac{3}{\pi}\right) (4)$

$= \not{\pi} \left(\frac{12}{\not{\pi}}\right) (4)$

$= 48 \text{ cm}^2$

17. Given,

Area of valley $= 97280 \text{ km}^2$

Rainfall $= 10$ cm $= 0.00010$ km

Volume of rainwater

= Area of valley × Rainfall

$= 97280 \times 0.00010 \text{ km}^3$

$= 0.97280 \text{ km}^3$

Volume in 1 day $= \frac{0.97280}{14} = 0.7 \text{ km}^3$

Volume of a river $= l \times b \times h$

$= (1072 \times \frac{75}{1000} \times \frac{3}{1000}) \text{ km}^3$

$= 0.2412 \text{ km}^3$

Volume of 3 rivers $= 3 \times 0.2412$

$= 0.7236 \text{ km}^3$

18. h of cone = 12 cm (Given)

r of cone = 4.5 cm

slant height (l) $= \sqrt{h^2 + r^2}$

$= \sqrt{(12)^2 + (4.5)^2}$

$= \sqrt{144 + 20.25}$

$= \sqrt{164.25}$

$l = 12.81$ cm

Section C

19. Capacity of drinking glass

$= \frac{1}{3} \pi h (r_1^2 + r_2^2 + r_1 r_2)$

$= \frac{1}{3} \pi (14) (2^2 + 1^2 + 2 \times 1)$

$= \frac{1}{3} \times \frac{22}{7} \times 14 (7)$

$= \frac{308}{3} = 102.6 \text{ cm}^3$

20. Radius of sphere $= \frac{6 \text{ cm}}{2} = 3$ cm

Radius of wire $= \frac{2 \text{ cm}}{2} = 1$ cm

Volume of sphere = Volume of cylinder (wire)

$\frac{4}{3} \not{\pi} r^2 h = \not{\pi} r^2 h$

$\frac{4}{3} (3) = (1)^2 h$

$4 \times 9 = h$

$h = 36$ cm

∴ length of wire = 36 cm

21. Height of cone = 9 cm

Radius of cone $= \frac{24}{2} = 12$ cm

Volume of cone $= \frac{1}{3} \pi r^2 h$

$= \frac{1}{3} \pi (12)^2 (9)$

$= 432 \pi \text{ cm}^3$

Height of cylinder = 110 cm

Radius of cylinder = 12 cm

Volume of cylinder $= \pi r^2 h$

$= \pi (12)^2 (110)$

$= 15840 \pi \text{ cm}^3$

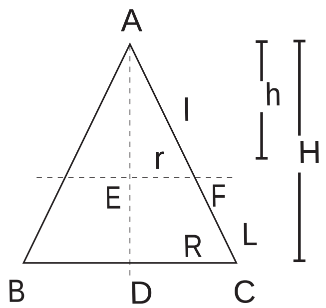
Volume of irom pole $= 432\pi + 15840\pi$

$= 16272\pi$

$= 51140.5\ cm^3$

Mass of pole $= 409124$ g

22.



Let R, H and L be the radius, height and slant height of the larger cone. & Let r, h and l be the radius, height and slant height of smaller cone.

Consider ΔADC & ΔAEF

$$\frac{r}{R} = \frac{h}{H} = \frac{l}{L} \quad \text{(i)}$$

C. S. A. of smaller cone $= \pi rl$

C. S. A. of larger cone $= \pi RL$

$$RL - \pi rl = \frac{8}{9}$$

$$\Rightarrow \frac{1}{9}\pi RL = \pi rl$$

$$\Rightarrow \frac{L}{r} \times \frac{R}{r} = 9$$

$$\frac{H}{h} \times \frac{H}{h} = 9 \quad \text{(From (i)}$$

$$\frac{H}{h} = 3$$

$$\Rightarrow \frac{H-h}{h} = \frac{3h-h}{h} = 2$$

Required ratio $= h : (H - h)$

$= 1 : 2$

23. Volume of sphere = Volume of cylinder

$$\frac{4}{3}\not{\pi} r^3_5 = \not{\pi} r^3_c h \quad (rs = 4.2\ cm) \ (rc = 6\ cm)$$

$$\frac{4}{3}(4.2)^3 = (6)^2 h$$

$$\frac{4}{3} \times \frac{(4.2)^3}{(6)^2} = h$$

$$2.744\ cm = h$$

24. Radius of cylinder = 6 cm (Given)

Height of cylinder = 15 cm

Radius of cone and hemisphere = 3 cm (Given)

Height of cone = 12 cm

A. T. Q. ;

Volume of cylinder = Volume of cone + volume of hemisphere

$$\not{\pi} r^2_{cy} h = \left(\frac{1}{3}\not{\pi} r^2_c h + \frac{2}{3}\not{\pi} r_h 3\right) \times n$$

$$(6)^2 \times 15 = \left(\frac{1}{3} \times (3)^2 \times 12 + \frac{2}{3}(3)^3\right) \times n$$

$$540 = (36 + 18) \times n$$

$$\frac{540}{54} = n$$

$$n = 10 \text{ cones.}$$

25. Diameter of copper wire = 3 mm or 0.3 cm

Number of rounds of copper wire around cylinder

$$= \frac{\text{Height of cylinder}}{\text{Diameter of wire}} = \frac{12}{0.3} = 40 \text{ rounds}$$

Wire required in round

$= 2\pi r$ (Circumference of base cylinder)

$= 2 \times \pi \times 5$

$= 10\pi$

Length required in 40 rounds

$= 40 \times 10\pi = 400\pi$

$$400 \times \frac{22}{7}$$

$$\frac{8800}{7} = 1257.14\ cm$$

Radius of wire $= \frac{0.3}{2} = 0.15$ cm

Volume of wire

= Area of wire × lenght of wire

$= \pi r^2 \times 1257.14$

$= \frac{22}{7} \times (0.15)^2 \times 1257.14$

$= 88.898$ cm³

Mass of wire = Density x Volume

$= 8.88 \times 88.898$

Mass of wire = 789.41 g.

26. Radius of hemisphere $= \frac{14}{2} = 7$ cm

Curved surface area of hemisphere

$= 2\pi 6^2$

$= 2 \times \frac{22}{7} \times (7)^2$

$= 308$ cm²

Height of cylinder = Total height – Height of hemisphere

= 13 – 7 = 6 cm

Curved surface area of cylinder

$= 2\pi rh$

$= 2 \times \frac{22}{7} \times 7 \times 6$

$= 264$ cm²

Inner surface area of the

= C S A of cylinder
C S A of hemisphere

= 264 + 308

= 572 cm²

Section D

27. For the given statement first draw a diagram,

In this diagram, we can observe that

Height (h_1) of each conical part = 2 cm

Height (h_2) of cylindrical part 12 – 2 – 2 = 8 cm

Radius (r) of cylindrical part = Radius of conical part $= \frac{3}{2}$ cm

Volume of air present in the model = Volume of cylinder + 2 x Volume of cone

$= \pi r^2 h_2 + 2 \times \pi r^2 h_1$

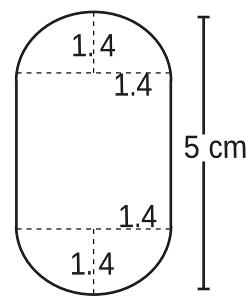
$= \pi \left(\frac{3}{2}\right)^2 \times 8 + 2 \times \frac{1}{3} \pi \left(\frac{3}{2}\right)^2 (2)$

$= \pi \times \frac{9}{4} \times 8 + \frac{2}{3} \pi \times \frac{9}{4} \times 2$

$= 18\pi + 3\pi = 21\pi$

$= 21 \times \frac{22}{7} = 66$ cm²

28.



Total Volume of gulab-jamun

= Volume of cylinder + Volume of 2 hemispheres

Volume of cylinder $= \pi r_c^2 h_c$

r_c = Radius of cylinder $= \frac{2.8}{2} = 1.4$ cm

h_c = Height of cylinder $= 5 - 2 \times (1.4)$

$= 5 - 2.8$

$= 2.2$ cm

Volume of cylinder $= \frac{22}{7} \times (1.4)^2 \times (2.2)$

$= 13.55$ cm³

Volume of 2 hemispheres

$$= 2 \times \left(\frac{2}{3}\pi r^3_h\right)$$

$$= 2 \times \frac{2}{3} \times \pi \times (1.4)^3$$

$$= \frac{2}{3} \times 2 \times \frac{22}{7} \times (1.4)^3$$

$$= 11.50 \text{ cm}^3$$

Volume of gulab-jamun = (11.50 + 13.55) cm^3

$$= 25.05 \text{ cm}^3$$

Volume of sugar syrup in 1 gulabjamun

$$= \frac{30}{100} \times 25.05$$

$$= 7.51 \text{ cm}^3$$

Volume of sugar syrup for 45 gulabjamuns

$$= 45 \times 7.51 = 337.95$$

$$= 338 \text{ (approx)}$$

29. Radius of cylindrical tank $= \frac{10}{2} = 5$ m

$= 500$ cm

Height of cylindrical tank = 200 cm (2 m)

Volume of cylindrical tank $= \pi r^2 h$

$$= \pi (500)^2 (200)$$

$$\text{Time taken} = \frac{\text{Volume of cylindrical tank}}{\text{Volume of water flowing in 1hr}}$$

$$\text{Time taken} = \frac{\pi(500)^2 (200)}{10 \times 300000 \times \pi \times 10}$$

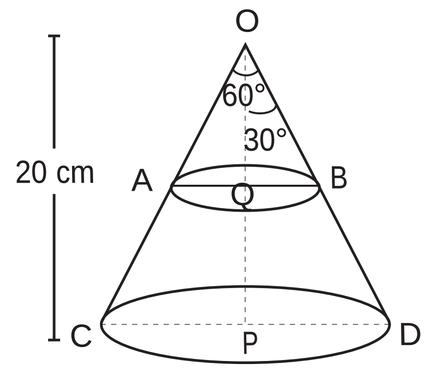
$$= \frac{500 \times 500 \times 200}{10 \times 300000 \times 10}$$

$$= \frac{50}{30} = \frac{5}{3} \text{ hr}$$

$$= \frac{5}{3} \times 60$$

Time taken = 100 minutes

30.



Let, OCD be the metallic cone and ABDC be the required frustum.

Since, frustum is drawn into wire of volume of frustum ABCD = Volume of cylindrical cone

OQ = QP = 10 cm

$\angle QOB = 30^o$

Height of frustum = h = QP = 10 cm

$\tan\theta = \frac{PD}{OP}$ $\qquad$ $\tan\theta = \frac{QB}{OB}$

$\tan 30^o = \frac{1}{\sqrt{3}} = \frac{r_1}{20}$ $\qquad$ $\tan 30^o = \frac{1}{\sqrt{3}} = \frac{r_2}{10}$

$r_1 = \frac{20}{\sqrt{3}}$ cm $\qquad$ $r_2 = \frac{0}{\sqrt{3}}$ cm

$\therefore$ Volume of frustum $= \frac{\pi h}{3}(r^2_1 + r^2_2 + r_1 r_2)$

$$= \frac{\pi \times 10}{3}\left(\left(\frac{20}{\sqrt{3}}\right)^2 + \left(\frac{10}{\sqrt{3}}\right)^2 + \frac{20}{\sqrt{3}} \times \frac{10}{\sqrt{3}}\right)$$

$$= \frac{10\pi}{3}\left(\frac{900}{3} + \frac{100}{3} + \frac{200}{3}\right)$$

$$= \frac{10\pi}{3}\left(\frac{700}{3}\right) = \frac{7000\pi}{3} \text{ cm}^3$$

Given diameter $= \frac{1}{12}$ cm

Volume of wire = volume of frustum

$$\pi r^2 h = \frac{7000\pi}{3}$$

$$h = \frac{7000}{3} \times \left(\frac{1}{\frac{1}{24}}\right)^2 = 7964.4 \text{ m}$$

$\therefore$ h = 7964.4 m

31. Amount of water required to fill the conical vessel

= volume of conical vessel

$= \frac{1}{3}\pi(20)^2 \times 24 = 3200\pi$ cu.cm ...(i)

Amount of water that flows out of cylindrical pipe in 1 minute

$= \pi \times \left(\frac{5}{20}\right)^2 \times 10 \times 100$

$= 62.5\pi$ cu.cm ...(ii)

From (i) and (ii)

Time required to fill the vessel $= \frac{3200\pi}{62.5\pi}$

= 51.2 minutes.

32. Volume of water left = Volume of cylinder

- (Volume of cone + Volume of hemisphere)

Volume of cylinder $= \pi r_c^2 h_c$

r_c = 5 cm

h_c = 10.5 cm

Volume of cylinder $= \pi (5)^2 (10.5)$

$= 262.5\pi$ cm³

Volume of cone $= \frac{1}{3}\pi (3.5)^2 (4)$

$= 16.33\pi$ cm³

Volume of hemisphere $= \frac{2}{3}\pi r_h^3$

r_h^3 = 3.5 cm

Volume of hemisphere $= \frac{2}{3}\pi (3.5)^3$

$= 28.58\pi$ cm³

Volume of water left $= 262.5\pi - (16.33\pi\ 28.58\pi)$

$= \pi (217.59)$ cm³

$= \frac{22}{7} \times 217.59$ cm³

= 683.854 cm³

33. Radius of cylinder $= \frac{4.3}{7} = 2.15$ m

Height of cylinder = 3.8 m

C.S.A. of cylinder $= 2\pi rh$

$= 2 \times \frac{22}{7} \times 2.15 \times 3.8$

$= \frac{359.48}{7}$ m²

= 51.3543 m²

As vertical angle of cone is a right angle.

Let ABC be a triangle

$l = AB = AC$

$l^2 + l^2 = BC^2$

$2l^2 = 4.3^2$

$l^2 = 9.245$

$l = 3.04$ m

We know,

$l^2 = r^2 + h^2$

$(3.04) = (2.15)^2 + h^2$

$9.2416 = 4.6225 + h^2$

$h^2 = 9.2416 - 4.6225$

$h = \sqrt{4.6191}$

$h = 2.149$ m

C.S.A. of cone $= \pi rl$

$= \frac{22}{7} \times 2.15 \times 3.04$

= 20.5417 m²

T.S.A. of building = C.S.A. of cylinder + C.S.A. of cone

= 51.3543 + 20.5417

= 71.896 m²

Volume of building = Volume of cone

+ Volume of cylinder

Volume of cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times (2.15)^2 \times 2.15$

$= 10.4116 \text{ m}^3$

Volume of cylinder = $\pi r^2 h$

$= \frac{22}{7} \times (2.15)^2 \times 3.8$

$= 55.2059 \text{ m}^3$

Total Volume of building

$= 10.4116 + 55.2059$

$= 65.6175 \text{ m}^3$

34. Hypotenuse = sum of square of sides by Pythagoras theorem

$\therefore$ Hypotenuse = $\sqrt{15^2 + 20^2} = 25$ cm

By Pythagoras theorem,

$AE^2 + BE^2 = 15^2$ ...(i)

$CE^2 + BE^2 = 20^2$ ...(ii)

$AE + CE = 25$ ...(iii)

Equation (i) – (ii),

$AE^2 - CE^2 = -175$ ...(iv)

Solving (iii) – (iv),

AE = 9 cm, CE = 16 cm, $BE^2 = 144^2$ cm

$\therefore$ Volume of total solid = Volume of cone formed by ABED + Volume of cone formed by CBED

$= \frac{1}{3}\pi \times BE^2 \times AE + \frac{1}{3}\pi \times BE^2 \times CE$

$= \frac{1}{3}\pi BE^2 \times (AE + CE)$

$= \frac{1}{3}\pi \times 144 \times 25 = 3768 \text{ units}^3$

Volume of double cone formed is 3768 cm^3

Surface area of double cone = Curved surface area of ABED + Curved surface area of CBED

$= \pi \times BE \times BA + \pi \times BE \times BC$

$= \pi \times BE \times (BA + BC)$

$= 3.14 \times \sqrt{144} \times (15 + 20)$

$= 1318.8 \text{ cm}^2$

Surface area of required double cone is 1318.8 cm^2.

WORKSHEET 2

Section A

1. Volume of cone are in the ratio 1 : 4 diameters are in the ratio 4 : 5

So, radius are also in the ratio 4 : 5.

$$\frac{\text{Volume of cone 1}}{\text{Volume of cone 2}} = \frac{1}{4}$$

$$\frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \frac{1}{4}$$

$$\left(\frac{r_1}{r_2}\right)^2 \times \frac{h_1}{h_2} = \frac{1}{4}$$

$$\left(\frac{4}{5}\right)^2 \times \frac{h_1}{h_2} = \frac{1}{4}$$

$$\frac{16}{25} \times \frac{h_1}{h_2} = \frac{1}{4}$$

$$\frac{h_1}{h_2} = \frac{25}{64}$$

2. $4\pi r^2 = 616$

$4 \times \frac{22}{7} \times r^2 = 616$

$r^2 = \frac{616 \times 7}{22 \times 4}$

$r^2 = \frac{4312}{88}$

$r^2 = 49$

$r^2 = 7$ cm

3. Radius of hemisphere = Radius of cone = r

Also, Height of cone = Radius of Hemisphere = r

l (Slant height) = $\sqrt{h^2 + r^2} = \sqrt{r^2 + r^2} = \sqrt{2}r$

$$\frac{\text{Curved surface area of hemisphere}}{\text{Curved surface area of cone}} = \frac{2\pi r}{\pi rl}$$

$$= \frac{2\pi r^2}{\pi r\sqrt{2}r}$$

$$= 2 : \sqrt{2}$$

$$= \sqrt{2} : 1$$

4. Since the radius of the cylinder is same as the radius of the circular plate.

So, the radius of the base of the cylinder = 7 cm

Height of the cylinder is same as the thickness of 50 circular plates.

Thickness of 1 plate = $\frac{1}{2}$ cm

Then, thickness of 50 plates $= 50 \times \frac{1}{2}$

$= 25$ cm

Height of the cylinder = 25 cm

Total surface area of cylinder = $2\pi r\,(r + h)$

$= 2 \times \frac{22}{7} \times 7\,(7 + 25)$

$= 44 \times 32$

Total surface area of the cylinder = 1408 sq cm

Volume of the cylinder = πr^2h

$= \frac{22}{7} \times 7 \times 7 \times 25$

= 3850 cu cm

Volume of the cylinder is 3850 cu cm.

5. Volume of water = Volume of conical flask

$$= \frac{1}{3}\pi r^2h$$

As the water is poured into the cylindrical flask,

So, Volume of cylinder = Volume of water

$$\not{\pi}\,(mr)^2H = \frac{1}{3}\not{\pi}\,r^2h$$

$$H = \frac{h}{3m^2}$$

6. Height of cylinder (h) = 12 cm

Radius of cylinder (r) $= \frac{12}{2} = 6$ cm

A.T. Q.,

Surface area of sphere = C. S.A. of cylinder

$$4\not{\pi}\,r_s^2 = 2\not{\pi}\,rh$$

$$2r_s^2 = (12)\,(6)$$

$$r_s^2 = (6)\,(6)$$

$$r_s = 6 \text{ cm}$$

7. Volume of cylinder = πr^2h

r = 1 cm

h = 5

$V = \pi(5)(1)^2$

$V = 5\pi cm^2$

Volume of sphere, at r = 1 cm

$= \frac{4}{3}\pi r^3$

Volume of spher = $\frac{4}{3}\pi cm^3$

8. In the hemisphere,

Height of hemisphere = Radius of hemisphere

$$h = r$$

For the volume of cone,

Radius of cone = Radius of hemisphere

$$R = r$$

Volume of cone = $\frac{1}{3}\pi R^2h$

$= \frac{1}{3}\pi r^2 h$ [R = r]

$= \frac{1}{3}\pi r^3$ [h = r]

9. TSA = 462 cm^2

TSA hemisphere $= 3\pi r^2$

$462 = 3 \times \frac{22}{7} \times r^2$

$\frac{462 \times 7}{22 \times 3} = r^2$

$49 = r^2$

$r = \sqrt{49} = 7$ cm

Volume of hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times 7 \times 7 \times 7$

$= \frac{2 \times 22 \times 7 \times 7}{3}$

$= \frac{2156}{3}$

$= 718.666\ cm^3$

10. Slant height (l) = 5 cm

$R = r_1 - r_2$ = 4 cm

$l^2 = R^2 + h^2$

$5^2 = R^2 + h^2$

$h^2 = 4^2 + h^2$

$h^2 = 5^2 - 4^2$

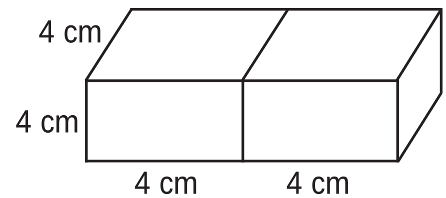
$h^2 = 25 - 16$

$h^2 = 9$

h = 3 cm

Section B

11.



The figure obtained on joining of two cubes of side 4 cm is cuboid of length 8 cm, breadth 4 cm and height 4 cm.

So we know that the surface area of cuboid = 2(lb + bh + lh)

Where l = 8 cm, b = 4 cm, h = 4 cm

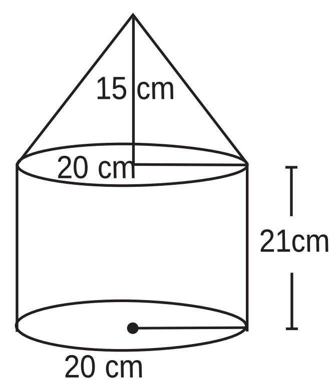
= 2 x [(8 x 4) + (4 x 4) + (4 x 8)]

= 2 x [32 + 16 + 32]

= 2 x [80]

= 160 cm^2

12.



Slant height of cone $= \sqrt{r^2 + h^2}$

$l = \sqrt{(20)^2 + (15)^2}$

$l = \sqrt{400 + 225}$

$l = \sqrt{625}$

= 25 cm

Total surface area of toy

= Curved surface area of cone
+ Curved surface area of cylinder
+ Area of bottom part of cylinder

$= \pi rl + 2\pi r_c h_c + \pi r_c^2$

$= \pi [rl + 2\pi r_c h_c + r_c^2]$

$= \pi [(20)(25) + (2)(20)(21) + (20)^2]$

$= \pi [500 + 840 + 400]$

$= \frac{22}{7} \times 1740$

$= 22 \times 248.57$

$= 5468.54 \text{ cm}^2$

13. Let the original height of cylinder = 10 cm

base radius of cylinder = 3.5 cm

so base radius of hemisphere = 3.5 cm (same as that of cylinder)

The total surface area would be the sum of curved surface area of cylinder and the surface areas of 2 hemispheres.

Surface area of cylinder = $2\pi rh$

Surface area of one hemisphere = $2\pi r^2$

TSA = $2\pi rh + 2(2\ 2\pi r^2)$

TSA = $2\pi rh + 4\pi r^2$

TSA = $2\pi r(h + 2r)$

TSA = $2 \text{ x } \frac{22}{7} \text{ x } 3.5 (10 + 2 \text{ x } 3.5)$

$= 22 \text{ x } (10 + 7)$

TSA = 374 cm^2

14. Height = 18

Radius = R = 18 cm and r = 12 cm

$l = \sqrt{(R - r)^2 + h^2}$

$l = \sqrt{(18 - 12)^2 + 8^2}$

$l = \sqrt{6^2 + 8^2}$

$l = \sqrt{100}$

$l = 10 \text{ cm}$

Total surface area = $\pi (R + r) l + \pi (R^2 + r^2)$

$= \pi [(R + r) l + (R^2 + r^2)]$

$= \pi [(18 + 12)(10) + (18^2 + 12^2)]$

$= \pi [300 + 468]$

$= \pi [768] = \frac{22}{7} \times 768 = 2413.71 \text{ cm}^2$

15. Total surface area of remaining solid

= C.S.A. of cylinder + Area of upper part

+ Curved surface area of cone

$= 2\pi rh + \pi r^2 + (\pi r_c h)$

Radius of cylinder (r) = 6 cm

Height of cylinder (h) = 20 cm

Radius of cone (r_c) = 6 cm

Slant height of cone (l) $= \sqrt{r_c^2 + h}$

$= \sqrt{6^2 + 8^2}$

$= \sqrt{100}$

l = 10 cm

Total surface area of remaining solid

$= 2 \times \frac{22}{7} \times 6 \times 20 + \frac{22}{7} \times 6^2 + \left(\frac{22}{7} \times 6 \times 10\right)$

$= \frac{22}{7} [240 + 36 + 60]$

$= \frac{22}{7} \times 336$

$= 1056 \text{ cm}^2$

16. Water is flowing at 7 m/s

Radius of pipe (r) = 1 cm = 0.01 m

Radius of tank (R) = 40 cm = 0.4 m

Time = $\frac{1}{2}$ hr = 30 min = 1800 seconds

We know,

Volume of cylindrical tank

= Area of cross section

$\times$ speed of flowing water

$\times$ time

$\pi r^2 h = \pi R^2 \times$ Rate of flowing water $\times$ time

$(0.4)^2 h = (0.01)^2 \times 7 \times 1800$

$0.16h = 1.26$

$h = \frac{1.26}{0.16}$

$h = 7.875$ m

17. Let Radius of tank be 'r'

So, Height of tank $= 6r$

Volume of cylindrical tank $= \pi r^2 h$

$= \frac{22}{7} \times r^2 \times 6r$

The cost of painting is given as 237.60

So,

Volume of tank $\times$ 0.6 = 237.60

Volume of tank $= \frac{237.60}{0.6}$

$= 396$ cm²

18. Radius of well (r) = 5 m

Depth of well = 14 m

Volume of Earth taken out

$= \frac{22}{7} \times 5^2 \times 14$

$= 1100$ m³

As the Earth is spread around the embankment ;

Inner radius (r_1) = 5 m

Outer radius (r_2) = (5 + 5) m = 10 m

Height = h

Volume of Earth taken out $= \pi (r_2^2 - r_1^2) h$

$= 1100 = \frac{22}{7} (10^2 - 5^2) h$

$\frac{1100 \times 7}{22 \times (75)} = h$

$h = 4.67$ m

19. Radius of sphere (r) $= \frac{6}{2} = 3$ cm

Radius of cylindrical vessel (R) $= \frac{12}{2} = 6$ cm

Let water be raised by height 'h'

A.T. Q.,

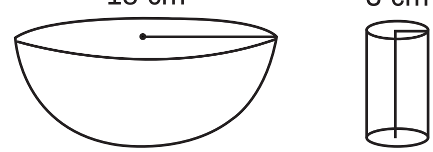
Volume of water raised = Volume of sphere

$\pi R^2 h = \frac{4}{3} \pi r^3$

$R^2 h = \frac{4}{3} r^3$

$36 \times h = \frac{4}{3} \times (3)^3$

$h = \frac{4 \times 27}{36 \times 3} = \frac{108}{108} = 1$ cm

20. 

Volume of liquid in hemispherical bowl $= \frac{2}{3} \pi (18)^3$ cm³

Volume of liquid filled in one cylindrical bottle $= \pi(3)^2 (6)$ cm³

Number of bottles required to empty the bowl

$= \frac{\frac{2}{3} \pi (18)^3}{\pi (3)^2 6} = 72$

Section C

21. Let the radius of base of cylinder be 'r'

A.T. Q.,

Volume of cylinder = Volume of two cones

$\pi r^2 h = \frac{1}{3} \pi 6_1^2 h + \frac{1}{3} \pi r_2^2 h$

$r^2 = \frac{1}{3} r_1^2 + \frac{1}{3} r_2^2$

$$r^2 = \frac{r_1^2 + r_2^2}{3}$$

$$r = \sqrt{\frac{r_1^2 + r_2^2}{3}}$$

22. Volume of bucket = $\frac{1}{3}\pi h (R^2 + r^2 + Rr)$

$$5390 = \frac{1}{3}\pi (15)(196 + r^2 + 14r)$$

$$5390 \times 3 = \frac{22}{7} \times 15 (196 + r^2 + 14r)$$

$$\frac{5390 \times 3 \times 7}{22 \times 15} = 196 + r^2 + 14r$$

$$343 = 196 + r^2 + 14r$$

$$r^2 + 14r + 196 - 343 = 0$$

$$r^2 + 14r - 147 = 0$$

$$r^2 - 7r + 21r - 147 = 0$$

$$r (r - 7) + 21 (r - 7) = 0$$

$$r = -21 \text{ or } r = 7$$

As Radius can't be negative, So $r = 7$ cm

23. Given: Radius of lower circular end = r = 12 cm

Radius of upper circular end = R = 18 cm

Height of frustum = h = 8 cm

Formula: Total surface area of frustum = $\pi r^2 + \pi R^2 + \pi(R + r)l$ cm^2

Where l = slant height

For slant height we have $1 = \sqrt{(R - r)^2 + h^2}$ cm

$\therefore 1 = \sqrt{(18 - 12)^2 + 8^2} = \sqrt{36 + 64} = 10$ cm

$\therefore 1 = 10$ cm

$\therefore$ Total surface area of frustum = $\pi \times 12^2 + \pi \times 18^2 + \pi \times (18 + 12) \times 10$ cm^2

$= 3.14 \times (144 + 324 + 300)$ cm^2

$= 3.14 \times 768$ cm^2

$= 2411.52$ cm^2

$\therefore$ Total surface area of frustum = 2411.52 cm^2

24. Volume of water in tank = $l \times b \times h$

$$= 50 \times 44 \times \frac{21}{100}$$

$$= 462 \text{ m}^3$$

For cylindrical pipe,

$$r = \frac{14}{2} = 7 \text{ cm} = 0.07 \text{ cm}$$

Water is flowing at the rate of 15 km/hour.

Volume of cylindrical pipe = $\pi r^2 h$

$$462 = \frac{22}{7} \times (0.07)^2 \times h$$

$$\frac{462 \times 7}{22 \times (0.07)^2} = h$$

$$\text{Time} = \frac{30000}{15000} = 2 \text{ hours}$$

25. Surface area of sphere = $4\pi r^2$

$$1386 = 4\pi r^2$$

$$\frac{1386 \times 7}{4 \times 22} = r^2$$

$$r^2 = 110.25$$

$$r = 10.5 \text{ cm}$$

Volume of sphere $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times (10.5)^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 10.5 \times 10.5 \times 10.5$$

$$= 4851 \text{ cm}^3$$

A.T.Q.,

Volume of sphere = Volume of wire (cylinder)

$$4851 = \pi r^2 h$$

$$4851 = \frac{22}{7} \times r^2 \times 31.5 \times 100$$

[length of wire = 31.5 m]

$$\frac{4851 \times 7}{22 \times 31.5 \times 100} = r^2$$

$r^2 = 0.49$

$r = 0.7$ cm

$\therefore$ Diameter of wire $= 0.7 \times 2$

$= 1.4$ cm

26. Sum of radius and height of cylinder

$= (r + h)$

$= 37$ cm

Total surface area of cylinder $= 2\pi r (r + h)$

$$1628 = 2 \times \frac{22}{7} \times r\,(37)$$

$$\frac{1628 \times 7}{2 \times 22 \times 37} = r$$

We know,

$r = 7$

$r + h = 37$

$7 + h = 37$

$h = 30$ cm

Volume of cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times 7^2 \times 30$$

$$= \frac{22}{7} \times 49 \times 30$$

$= 4620$ cm²

27. For cylindrical tub,

$$r = \frac{12}{2} = 6 \text{ cm}$$

$h = 15$ cm

Volume of cylindrical tube $= \pi r^2 h$

$= \pi \times 6^2 \times 15$

$= \pi \times 36 \times 15$

$= 540\,\pi$

For ice-cream,

We know,

$h = 2 \times$ diameter $= 2 \times 2r = 4r$

Radius of cone = Radius of hemisphere = R

Volume of ice- cream cone

= Volume of cone + Volume of hemisphere

$$= \frac{1}{3}\pi R^2 h + \frac{2}{3}\pi R^3$$

$$= \frac{1}{3}\pi R^2 (h + 2R)$$

$$= \frac{1}{3}\pi R^2 (4R + 2R)$$

$$= \frac{1}{3}\pi\, 6R^3$$

A.T.Q.,

Volume of cylindrical tub

$= n \times$ Volume of ice - cream cones

$$540\cancel{\pi} = \frac{1}{3}\cancel{\pi}\,6R^3 \times 10$$

$540 \times 3 = 6R^3 \times 10$

$54 \times 3 = 6R^3$

$27 = R^3$

$R = 3$ cm

$\therefore$ Diameter of ice - cream cones

$= 3 \times 2 = 6$ cm

28. Radius of hemisphere = Radius of cone

$= r = 3.5$ cm

Volume of total wood used

$$= 166\frac{5}{6} = \frac{1001}{6} \text{ cm}^3$$

Volume of wood used in toy

= Volume of hemisphere + Volume of cone

$$\frac{1001}{6} = \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$$

$$\frac{1001}{6} = \frac{1}{3}\pi r^2 (2r + h)$$

$$\frac{1001}{6} = \frac{22}{7} \times 3.5 \times 3.5 (2 \times 3.5 + h)$$

$$\frac{1001 \times 7}{2 \times 22 \times 3.5 \times 3.5} = 7 + h$$

$$13 = 7 + h$$

$$h = 13 - 7$$

Height of cone = h = 6 cm

Height of toy = Height of cone + Height of hemisphere

$= 6 + 3.5$

$= 9.5$ cm

Curved surface area of hemisphere = $2\pi r^2$

$$= 2 \times \frac{22}{7} \times (3.5)^2$$

$= 77$ cm²

∴ Cost of painting the hemispherical part

$= 77 \times 10$

= ` 770.

29. Radius of cone (r) $= \frac{3.5}{2}$ cm

Height of cone (h) = 3 cm

A.T.Q.,

Volume of 504 cones = Volume of sphere

$$504 \times \frac{1}{3} \times \pi \times r^2 \times h = \frac{4}{3}\pi R^3$$

$$\frac{504}{3} \times \left(\frac{3.5}{2}\right)^2 \times 3 = \frac{4}{3} R^3$$

$$\frac{504 \times 3.5 \times 3.5 \times 3 \times 3}{3 \times 2 \times 4 \times 2} = R^3$$

$$1157.625 = R^3$$

$$R = 10.5 \text{ cm}$$

Total surface area of sphere = $4\pi R^2$

$$= 4 \times \frac{22}{7} \times 10.5 \times 10.5$$

$= 1386$ cm²

30. Height of cone (h) = 60 cm

Radius of cone (r) = 30 cm

Height of cylinder (H) = 180 cm

Radius of cylinder (R) = 60 cm

Volume water left = Volume of cylinder – Volume of cone

Volume of water left = $\pi R^2 H - \frac{1}{3}\pi r^2 h$

$$= \pi \left[(60)^2 (180) - \frac{1}{3} (30)^2 (60) \right]$$

$$= \pi [648000 - 18000]$$

$$= \frac{22}{7} \times 630000$$

$= 1980000$ cm³

$= 1.98$ m³

Section D

31. Radius of well $= \frac{4}{2} = 2$ m

Depth = 14 m

Volume of well = $\pi r^2 h$

$$= \frac{22}{7} \times 2^2 \times 14$$

$= 176$ m³

A.T.Q.,

Volume of well = Volume of embankment

$$176 = \pi (r_1^2 - r_2^2) h$$

[r² = inner radius = 2 m]

$$176 = \frac{22}{7} (r_1^2 - 4) \times \frac{40}{100}$$

$$140 = r_1^2 - 4$$

$$144 = r_1^2$$

$r_1 = 12$ m

Width of embankment $= (12 - 2)$ m

$= 10$ m

32. Radius of the water tank = 40 cm

Increase in water level = 3.15 m = 315 cm

Volume of water flowing in the tank in half on hour $= \pi r^2 h$

$= \frac{22}{7} \times 40 \times 40 \times 315$

$= 1584000$ cm³

Rate of water flow = 2.52 km/hr

Water flow in half an hour $= 2.52 \times \frac{1}{2}$

$= 1.26$ km

$= 126000$ cm

Let internal diameter be 'd'

Water that flows in half an hour through pipe

$$= \left(\frac{d}{2}\right)^2 (126000)$$

We know,

Water flowing through pipe in half an hour

= Volume of water flowing in half an hour

$$\frac{22}{7} \times \frac{d^2}{4} \times 126000 = 1584000$$

$d^2 = 16$

$d = \sqrt{16}$

$d = 4$ cm

33. Radius of hemisphere bowl $= \frac{36}{2} = 18$ cm

Volume of liquid in the bowl $= \frac{2}{3} \pi r^3$

$= \frac{2}{3} \pi r^3$

$= \frac{2}{3} (18)^3$

$= 3888\pi$ cm³

Diameter of the bottle = 60 m

Radius of the bottle $(r_1) = \frac{6}{2} = 3$cm

Volume of each bottle $= \pi r_1^2 h$

$= \pi (3)^2 (h)$

$= 9\pi h$

A.T.Q.,

90 % of volume of liquid in bowl

= 72 × Volume of liquid in each bottle

$\frac{90}{100} \times 3888 \cancel{\pi} = 72 \times 9 \cancel{\pi} h$

$3499.2 = 648$ h

$h = \frac{3499.2}{648}$

$h = 5.4$ cm

34. First, we need to find out the volume of the previous solid metal cylinder.

h = 10 cm

r = 4.2 cm

Volume of cylinder before scooping out $= \pi r^2 h$

Volume of scooped part $= \frac{4}{3} \pi r^3$

$= \frac{4}{3} \times \pi \times (4.2)^3$

$= 98.784\pi = 98.8\pi$ cm³

Therefore volume of the scooped metal cylinder $= 176.4\pi - 98.8\pi = 77.6\pi$ cm³

Now for wire,

Diameter = 1.4 cm

Radius $= \frac{1.4}{2} = 0.7$

The volume of the scooped metal cylinder = The volume of the wire

$77.6\pi = \pi r^2 h$

Cut π from both the sides

$77.6 = (0.7)^2 h$

$h = \frac{77.6}{0.49}$

h = 158.36 cm = 158.4 cm

Therefore, the length of the wire would be approximately 158.4 cm.

35. Volume of water in cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times (5)^2 \times 8$

$= \frac{200}{3}\pi \text{ cm}^3$

Volume of water flows out $= \frac{1}{4} \times \frac{200}{3}\pi$

$= \frac{50}{3}\pi \text{ cm}^3$

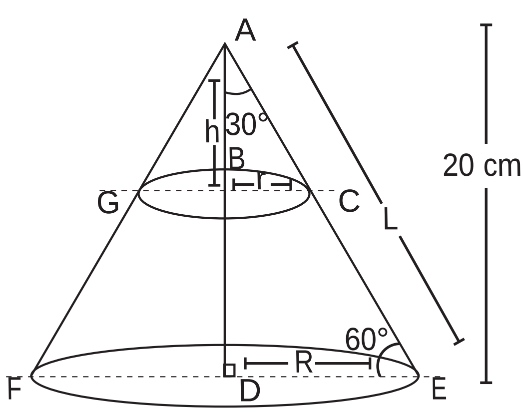
Let the radius of one spherical ball be r cm

$\therefore \quad \frac{4}{3}\pi r^3 \times 100 = \frac{50}{3}\pi$

$r^3 = \frac{50}{4 \times 100} = \frac{1}{8}$

or, $\quad r = \frac{1}{2} = 0.5$ cm

36.



Consider ΔABC & ΔADE;

$\frac{h}{H} = \frac{r}{R} = \frac{l}{L}$

$\frac{h}{20} = \frac{r}{R}$

In ΔADB

$\angle ADE = 90°$ & $\angle DEA = 60°$

$\tan 60 = \frac{20}{DE}$

$\sqrt{3}\, DE = 20$

$DE = \frac{20}{\sqrt{3}}$

$R = \frac{20}{\sqrt{3}}$ cm

In ΔABC

$\tan 30 = \frac{r}{AB}$

$\frac{1}{\sqrt{3}} = \frac{r}{AB}$

From (i)

$r = \frac{10}{\sqrt{3}}$

Volume of frustum $= \frac{\pi h}{3}[R^2 + r^2 + Rr]$

$= \frac{\pi \times 10}{3}\left[\left(\frac{20}{\sqrt{3}}\right)^2 + \left(\frac{10}{\sqrt{3}}\right)^2 + \left(\frac{20}{\sqrt{3}}\right)^2\left(\frac{10}{\sqrt{3}}\right)^2\right]$

$= \frac{10\pi}{3}\left[\frac{400}{3} + \frac{100}{3} + \frac{200}{3}\right]$

$= \frac{10\pi}{3}\left[\frac{700}{3}\right]$

$= \frac{7000\pi}{9}$ cm³

Volume of cylinder $= \pi r^2 h$

$\left[r = \frac{1}{\frac{12}{2}} = \frac{1}{24} \right.$ cm

Volume of cylinder $= \pi\left(\frac{1}{24}\right)^2 h$

$= \frac{\pi h}{24 \times 24}$

$= \frac{\pi h}{576}$ cm³

As Volume of frustum must be equal to volume of cylinder.

$\frac{7000\pi}{9} = \frac{\pi h}{576}$

$h = \frac{7000 \times 576}{9}$

$h = 448000$ cm

or

(Length of wire) h = 4480 m.

37. Length of pond = 80 m

Breadth of pond = 50 m

Let height of pound be 'h'

So, Volume of pond = $l \times b \times h$

$= 80 \times 50 \times h$

$= 4000\ h\ m^3$

Average displacement of water by a person

$= 0.04\ m^3$

For 500 persons, average displacement

$= 500 \times 0.04$

$= 20\ m^3$

A.T.Q.,

Volume of pond = Average displacement by 500 persons

$4000\ h = 20$

$h = \frac{20}{4000} = \frac{1}{200}$

$h = 0.005$ m

or

(Rise in water level) h = 0.5 cm.

38. h = 24 cm

Lower end = r = 8 cm

Upper end = R = 20 cm

Volume $= \frac{\pi h}{3} (R^2 + r^2 + Rr)$

$= \frac{24\pi}{3} [(20)^2 + (8)^2 + (20)(8)]$

$= 8\pi [400 + 64 + 160]$

$= 8\pi [624] = 8 \times \frac{22}{7} \times 624$

$= 15689.142\ cm^2$

$= 15.689142$ l

Cost of milk per liter = ` 27

Total cost $= 15.689142 \times 27$

= ` 423.606

39. ① Volume of bucket = Volume of frustum

$= \frac{1}{3} \pi h (R^2 + r^2 + Rr)$

$= \frac{1}{3} \times \frac{22}{7} \times 24 (14^2 + 7^2 + 14 \times 7)$

$= \frac{1}{3} \times \frac{22}{7} \times 24 (196 + 49 + 98)$

$= \frac{1}{3} \times \frac{22}{7} \times 24 (343)$

$= 8624\ cm^3$

② Slant height (l) $= \sqrt{h^2 + (R - r)^2}$

$= \sqrt{24^2 + (14 - 7)^2}$

$= \sqrt{576 + 49}$

$= \sqrt{625}$

= 25 cm

Area of metal sheet = C.S.A of frustum + Area of base

$= \pi l (R + r) + \pi r^2$

$= \pi [25(14 + 7) + 7^2]$

$= \frac{22}{7} [25 (21) + 49]$

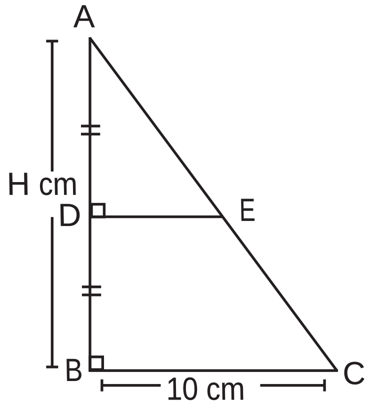
$= \frac{22}{7} [525 + 49]$

$= \frac{22}{7} [574]$

$= 22 \times 82$

$= 1804 \text{ cm}^2$

40.



Given that $AD = DB = \frac{h}{2}$ cm

Consider ΔADE and ΔABC

$\angle ADE = \angle ABC = 90°$

$\angle BAC = \angle DAE =$ (Common)

By Angle - Angle similarity [$\Delta ADE \sim \Delta ABC$]

So, $\frac{AD}{DB} = \frac{DE}{BC} = \frac{r}{10}$ (r is radius of cone)

$$\frac{\frac{H}{2}}{H} = \frac{r}{10}$$

$$= \frac{1}{2} = \frac{r}{10}$$

$r = 5$ cm

$\Rightarrow$ Volume of frustum = Volume of cone ABC – Volume of cone ADE

$$= \frac{1}{3}\pi R^2 H - \frac{1}{3}\pi\left(\frac{R}{4}\right)^2 \frac{H}{2}$$

Here, $\left[\frac{R}{2} = \frac{10}{2} = 5 \text{ cm}\right]$

Volume of frustum of cone

$$= \frac{1}{3}\pi\,(100) \times H - \frac{1}{3}\pi\,(25)\,\frac{H}{2} = \frac{175\pi H}{6}$$

Volume of cone

$$ADE = \frac{1}{3} \times \pi \times 5^2 \times \frac{H}{2} = \frac{25\pi H}{6}$$

$$\text{Ratio of Volume} = \frac{25\pi H/6}{H/6} = \frac{1}{7}$$

Chapter 14

Statistics

MULTIPLE CHOICE QUESTIONS

1. Mode is the most frequency value

   Option (c) is correct

2. $\overline{X} = \frac{\sum x_i}{n}$

   $15 = \frac{1+2+3+\ldots+n}{n}$

   $15n = \frac{n(n+1)}{2}$

   $2 \times 15 = n + 1$

   $n = 29$

   Option (c) is correct

3. Median **Option (b)**

4. Median **Option (a)**

5. **Option (c)** is correct

WORKSHEET 1

Section A

1. Arranging the given data in ascending order, we get,

   4, 4, 5, 5, 5, 6, 6, 8, 8, 8, x

   4 occurred twice

   5 occurred thrice

   6 occurred twice

   and 8 occurred thrice

   Given the mode of the data is 8.

   As 8 and 5 occurred thrice, we have,

   x = 8.

2. Given,

   Observations ⇒ 5, 7, 10, 12, 2x – 8, 2x + 10, 35, 41, 42, 50

   Number of observations ⇒ 10

   Mean of the data ⇒ 25

   $\text{mean} = \frac{\text{sum of observations}}{\text{number of observations}}$

   [Substitute the values]

   $25 = \frac{5+7+10+12+2x-8+2x+10+35+41+42+50}{10}$

   $25 = \frac{202 + 4x}{10}$

   $25 \times 10 = 202 + 4x$

   $250 = 202 + 4x$

   $250 - 202 = 4x$

   $48 = 4x$

   $4x = 48$

   $x = \frac{48}{4}$

   $x = 12$

   Therefore value of x = 12

5. Step-by-step explanation:

   3 MEDIAN = MODE + 2 MEAN

   3 MEDIAN = 15 + 2(30)

   3 MEDIAN = 15 + 60

3 MEDIAN = 75

$$\text{MEDIAN} = \frac{75}{3}$$

MEDIAN = 25

6. Ascending order

0,1,2,2,2,3,3,4,5,6

2 occurred thrice

so 2 is the mode of the data.

7. The first five odd Natural number are 1, 3, 5, 7, 9

$$\text{Average} = \frac{1+3+5+7+9}{5}$$

$$= \frac{25}{5}$$

8. Mean=sigma fixi divide by sigma fi

Therefore

$$3 = 3p + \frac{36}{15}$$

45 = 3p + 36

45 – 36 = 3p

9 = 3p

$$p = \frac{9}{3}$$

p = 3

Section B

9. Modal class = 12 - 15

$l = 12, f_1 = 23, f_o = 10, f_2 = 21, h = 3$

$$\text{Mode} = 1 + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 12 + \frac{23 - 10}{46 - 10 - 21} \times 3$$

$$= 12 + \frac{13}{15} \times 3$$

= 12 + 2.6 = 14.6

10. We know that,

3Median = Mode + 2 Mean

3 (21.2) – 21.4 = 2 Mean

63.6 – 21.4 = 2 Mean

42.2 = 2 Mean

Mean = 21.1

11. Here, the class intervals are of unequal width. If the class intervals are of unequal width the frequencies need not be adjusted to make the class intervals equal.

Calculation of Median

Marks	Number of students (frequency)	CF
0 – 10	5	5
10 – 30	15	20
30 – 60	30	50
60 – 80	8	58
80 – 90	2	60
		$N = \Sigma f_1 = 60$

Here, N = 60

$$\therefore \frac{N}{2} = 30$$

The cumulative frequency just greater than $\frac{N}{2}$ = 30 is 50 and the corresponding class is 30 – 60. Hence, 30 – 60 is the median class.

$\therefore$ l = 30, f = 30, F = 20, h = 30

$$\text{Now, Median} = l + \frac{\frac{N}{2} - F}{f} \times h$$

$$\Rightarrow \text{Median} = 30 + \frac{30 - 20}{30} \times 30 = 40$$

14.

x	f	fx
10	5	50
15	10	150
p	7	7p
25	8	200
30	2	60
	N = 32	**$\Sigma fx = 7p + 460$**

Given

Mean = 18.75

$\Rightarrow \frac{\Sigma fx}{N} = 18.75$

$\Rightarrow \frac{7p + 460}{32} = 18.75$

$\Rightarrow 7p + 460 = 18.75(32)$

$\Rightarrow 7p + 460 = 600$

$\Rightarrow 7p = 600 - 460$

$\Rightarrow 7p = 140$

$\Rightarrow p = \frac{140}{7}$

$\Rightarrow p = 20$

15.

No. of branches (X_1)	No. of plants (f_1)	$u_1 = x_1 - A$ $x_1 - 4$	f_1u_1
2	49	–2	–98
3	43	–1	–43
4	57	0	0
5	38	1	28
6	13	2	85
	N=200		**$\Sigma f_1u_1 = -77$**

Average Number of branches per plant

$= A + \frac{\Sigma f_1 u_1}{N}$

$= 4 + \frac{-77}{200}$

$= 4 - \frac{77}{200}$

$= \frac{800 - 77}{200}$

$= 3.615$

$= 3.62$ (Approx)

16. Let the missing frequency be f, the assumed mean be A = 47.5 and h = 3.

Calculation of Mean

Class Intervals	Mid - Values x_i	f_i	$d_i = x_i - 47.5$	$u_i = \frac{x_i - 47.5}{3}$	f_iu_i
40 – 43	41.5	31	– 6	– 2	–62
43 – 46	44.5	58	– 3	– 1	–58
46 – 49	47.5	60	0	0	0
49 – 52	50.5	f	3	1	f
52 – 55	53.5	27	6	2	54
		$N = \Sigma f_i = 176 + f$			**$\Sigma f_iu_i = f - 66$**

We have,

$\overline{X} = 42.7$, A = 47.5 and h = 3

$\therefore \overline{X} = A + h \left[\frac{1}{N} \Sigma f_iu_i\right]$

$\Rightarrow 47.2 = 47.5 + 3 \times \left[\frac{f - 66}{176 + f}\right]$

$\Rightarrow -0.3 = 3 \times \left[\frac{f - 66}{176 + f}\right]$

$\Rightarrow \quad \frac{-1}{10} = \frac{f - 66}{176 + f}$

$\Rightarrow \quad -176 - f = 10f - 660$

$\Rightarrow \quad 11f = 484 \Rightarrow f = 44$

Hence, the missing frequency is 44.

Section C

17. Let the assumed mean A = 50 and h = 20

Calculation of mean

Class Intervals	Mid - Value (x_i)	Frequency (f_i)	$u_i = \frac{x_i - 50}{20}$	$f_i u_i$
0 – 20	10	17	– 2	– 34
20 – 40	30	f_1	– 1	$-f_1$
40 – 60	50	32	0	0
60 – 80	70	f_2	1	f_2
80 – 100	90	19	2	38
Total		$\Sigma f_i = 68 + f_1 + f_2$		$\Sigma f_i u_i = 4 - f_1 + f_2$

We have, $\sum f_i = 120$ [Given]

$\Rightarrow \quad 68 + f_i + f_2 = 120$

$\Rightarrow \quad f_i + f_2 = 52$ ...(i)

Now, mean $= 50$

$\Rightarrow \quad \overline{X} = A + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$

$\Rightarrow \quad 50 = 50 + 20 \times \left(\frac{4 - f_1 + f_2}{120}\right)$

$\Rightarrow \quad 50 = 50 + \frac{4 - f_1 + f_2}{6}$

$\Rightarrow \quad 0 = \frac{4 - f_1 + f_2}{6}$

$\Rightarrow \quad f_1 - f_2 = 4$ ...(ii)

From equation (i), and (ii), we get

$$f_i + f_2 = 52$$
$$f_1 - f_2 = 4$$
$$2f_1 = 56$$

$\Rightarrow \quad f_1 = 28$

Putting the value of f_1 in equation (i), we get

$$28 + f_2 = 52$$

$\Rightarrow \quad f_2 = 24$

Hence, the missing frequencies f_1 is 28 and f_2 is 24.

18.

Marks	No. of Students (f_i)	Mid - Point (x_i)	$f_i x_i$
1 – 10	20	5	100
10 – 20	24	15	360
20 – 30	40	25	1000
30 – 40	36	35	1260
40 – 50	20	45	900
Total 140			3620

$\therefore$ Mean $\overline{x} = \frac{1}{n} \sum_{n}^{i = l} f_i x_i = \frac{3620}{140} = 25.86$

19. Let the assumed mean A = 55 and h = 10

Marks	Mid - Value	Frequency	$u_1 = \frac{x_1 - 55}{10}$	$f_i u_i$
20 – 30	25	100	–3	–300
30 – 40	35	120	–2	–240
40 – 50	45	130	–1	–130
50 – 60	55	400	0	0
60 – 70	65	200	1	200
70 – 80	75	50	2	100
		N-1000		**$\Sigma f_i u_i = -370$**

We know that mean, $\bar{x} = A + h\left(\frac{1}{N}\Sigma f_i u_i\right)$

Now, we have

$N = \Sigma f_i = 1000, h = 10, A = 55, \Sigma f_i u_i = -370$

$\bar{x} = 55 + 10\left[\frac{1}{1000} \times (-370)\right]$

$= 55 - 3.7$

$= 51.3$ years

20. $150 + x + y = 229$

$x + y = 79$

Median (M) = 46

Total frequency = 229

$f = 65$

$cf = 42 + x$

$L = 40$

$C = 10$

$\frac{n}{2} = \frac{229}{2}$

$\text{Median (M)} = 1 + \frac{\left(\frac{n}{2} - cf\right)}{f} \times C$

$46 = 40 + \left(\frac{229}{2} - 42 - x\right) 65 \times 10$

$(46 - 40) \times 65 = \frac{(229 - 84 - 2x)}{2} \times 10$

$\frac{390}{10} = \frac{(145 - 2x)}{2}$

$39 \times 2 = 145 - 2x$

$78 - 145 = -2x$

$-67 = -2x$

$x = 33.5$

Put value of x in equation (i)

$33.5 + y = 79$

$y = 45.5$

21.

Class Interval	Frequency	Mid Point (x)	cf
0 – 6	4	3	4
6 – 12	x	9	4+x
12 – 18	5	15	9+x
18 – 24	y	21	9+x+y
24 – 30	1	27	10+x+y
Total	**20**		

$\Sigma f = 20$

$x + y = 10$ ...(i)

Median is 14.4

So, Median class is 12 – 18

$$\text{Median} = L + \frac{\frac{n}{2} - c}{f} \times w$$

$$\text{So, } 14.4 = 12 + \frac{\frac{20}{2} - (4 + x)}{5} \times 6$$

$$14.4 \times 5 = 60 + (10 - 4 - x) \times 6$$

$$72 = 60 + 60 - 24 - 6x$$

$$6x = 120 - 24 - 72$$

$$6x = 120 - 96$$

$$6x = 24$$

$$x = 4$$

From equation (i)

$$4 + y = 10 \Rightarrow y = 6$$

22. The given frequency distribution is not continuous. So, we first make it continuous and prepare the cumulative frequency distribution as under.

Age (in year)	**Frequency**	**Age Less than**	**Cumulative Frequency**
– 0.5 – 9.5	5	9.5	5
9.5 – 19.5	15	19.5	20
19.5 – 29.5	20	29.5	40
29.5 – 39.5	23	39.5	63
39.5 – 49.5	17	49.5	80
49.5 – 59.5	11	59.5	91
59.5 – 69.5	9	69.5	100

Now, we plot points (9.5, 5), (19.5), (29.5, 40), (39.5, 63), (49.5, 80), (59.5, 91) and (69.5, 100). and join them by a free hand smooth curve to obtain the required ogive as shown in fig.1.

The cumulative frequency polygon is obtained by joining these points by line segments as shown fig.2.

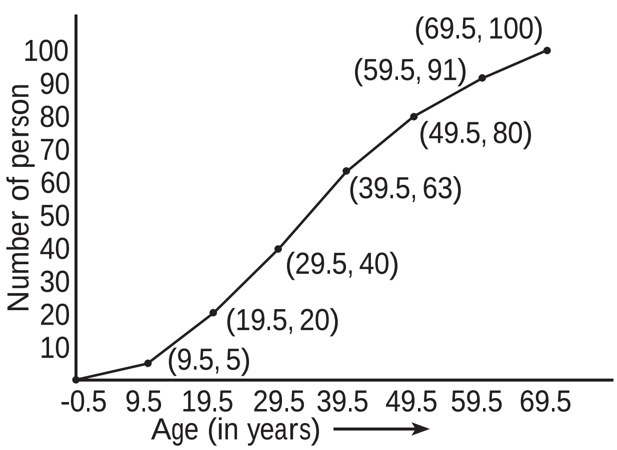


Fig.1 Cumulative frequency curve

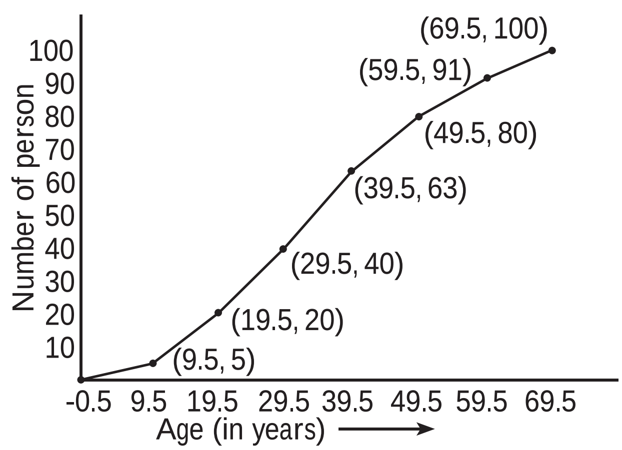


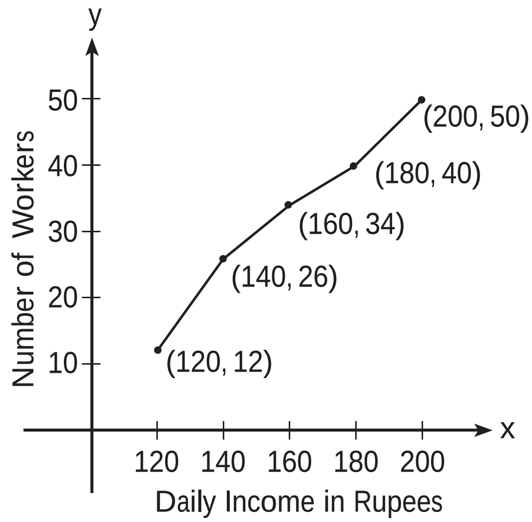
Fig.2 Cumulative frequency polygon

23.

Daily Income (In Rs.)	Number of workers (frequency) f_1	Cumulative frequency less than type
100-120	12	Less than 120 12=12
120-140	14	Less than 140 (12+14)=26
140-160	8	Less than 160 (26+8)=34
160-180	6	Less than 180 (34+6)=40
180-200	10	Less than 200 (40+10)=50
Total	n = 50	

n = 50 given $\frac{n}{2} = 25$

On the graph, we will plot the points (120, 12), (140, 26), (160, 34), (180, 40), (200, 50).



Section D

24.

Monthly Consumption (in units)	Number of Consumer f_i	Cumulative Frequency
65 – 85	4	4
85 – 105	5	9
105 – 125	13	22
125 – 145	20	42
145 – 165	14	56
165 – 185	8	64
185 – 205	4	68
Total	n = 68	

$n = 68$ gives $\frac{n}{2} = 34$

So, we have the median class (125 – 145)

$l = 125, n = 68, f = 20, cf = 22, h = 20$

$$\text{Median} = l + \left\{\frac{\frac{n}{2} - cf}{f}\right\} \times h$$

$$= 125 + \left\{\frac{34 - 22}{-20}\right\} \times 20 = 137 \text{ units}$$

(ii) Modal class is (25 – 145) having maximum frequency $f_m = 20$, f)1 – 13, $f_2 = 14$, $l = 125$ and $h = 20$

$$\text{Mode} = l + \left\{\frac{f_m - f_1}{2f_m - f_1 - f_2}\right\} \times h$$

$$= 125 + \left\{\frac{20 - 13}{40 - 13 - 14}\right\} \times 20 = 125 + \frac{7 \times 20}{13}$$

$$= 125 + \frac{140}{13} = 125 + 10.76 = 135.76 \text{ units}$$

(iii) $n = 68, a = 135, h = 20$, and $\Sigma f_i u_i = 7$

Monthly Consumption (in units)	Number of Consumers	Class mark	$x_i u_i = \frac{x_i - 135}{20}$	$f_i \times u_i$
65 – 85	4	75	–3	–12
85 – 105	5	95	–2	–10
105 – 125	13	115	–1	–13
125 – 145	20	135 = a	0	0
145 – 165	14	155	1	14
165 – 185	8	175	2	16
185 – 205	4	195	3	12
Total	n = 68			7

$n = 68, a = 135, h = 20$, and $\Sigma f_i u_i = 7$

By step-deviation method,

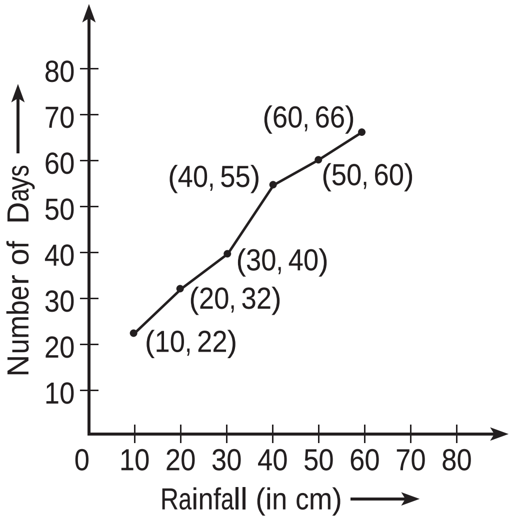
$$\text{Mean} = a + h \times \frac{1}{n} \times \Sigma f_i u_i = 135 + 20 \times \frac{1}{68} \times 7$$

$$= 135 + \frac{35}{17} = 135 + 2.05 = 137.05 \text{ units}$$

26. Prepare a table for less than type.

Rainfall (in cm)	No. of days	Rainfall (Less than)	Cumulative Frequency	Suitable Points
0 – 10	22	10	22	(10, 22)
10 – 20	10	20	32	(20, 32)
20 – 30	8	30	40	(30, 40)
30 – 40	15	40	55	(40, 55)
40 – 50	5	50	60	(50, 60)
50 – 60	6	60	66	(60, 66)

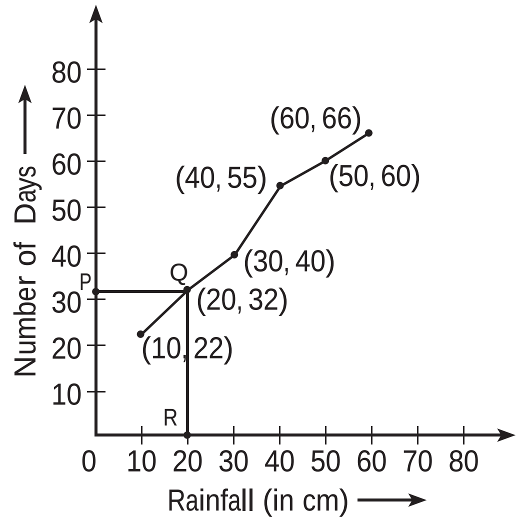
Now, plot the less than ogive using suitable points.



Here, N = 66

$\therefore \frac{N}{2} = 33$

In order to find the median rainfall, we first locate the point corresponding to 33rd day on the y-axis. Let the point be P. From this point draw a line parallel to the x-axis cutting the curve at Q. From this point Q, draw a line parallel to the y-axis and meeting the x-axis at the point R. The x-coordinate of R is 21.25.

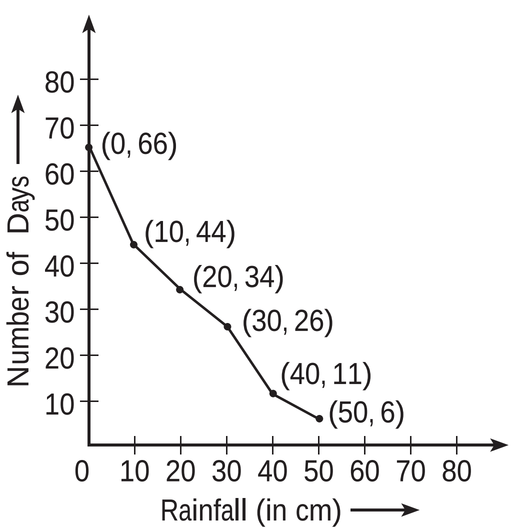


Thus, the median rainfall is 21.25 cm.

Let us now prepare a table for more than type.

Rainfall (in cm)	No. of days	Rainfall (More than)	Cumulative Frequency	Suitable Points
0 – 10	22	0	66 (0, 66)	(0, 66)
10 – 20	10	10	44	(10, 44)
20 – 30	8	20	34	(20, 34)
30 – 40	15	30	26	(30, 26)
40 – 50	5	40	11	(40, 11)
50 – 60	6	50	6	(50, 6)

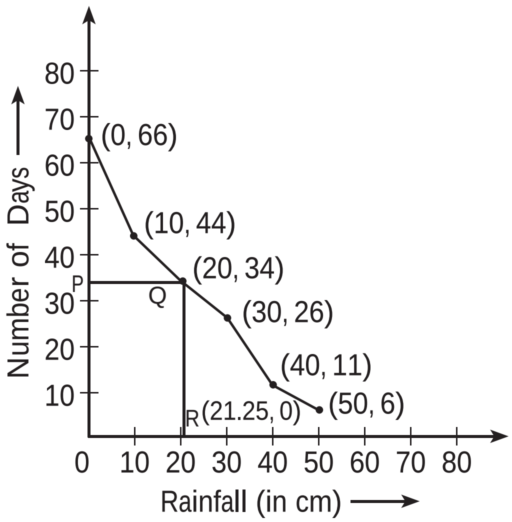
Now, plot the more than ogive with suitable points.



Here, N = 66

$\therefore \frac{N}{2} = 33$

In order to find the median rainfall, we first locate the point corresponding to 33rd day on the y-axis. Let the point be P. From this point draw a line parallel to the x-axis cutting the curve at Q. From this point Q, draw a line parallel to the y-axis and meeting the x-axis at the point R. The x-coordinate of R is 21.25.



Thus, the median rainfall is 21.25 cm.

27. $N = 50 \Rightarrow \frac{N}{2} = 25$

$\therefore$ Median class = 60 – 80

Here, l = 60, f = 12, F = 24, h = 20

$$\text{Median} = l + \left(\frac{\frac{N}{2} - F}{f}\right) \times h$$

$$= 60 + \left(\frac{25 - 24}{12}\right) \times 20$$

$$= 60 + \frac{1}{12} \times 20 = \frac{185}{3} = 61.6$$

Modal class = 60 – 80

$l = 30, f = 12, f_1 = 10, f_2 = 6, h = 20$

$$\text{Mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

$$= 60 + \frac{12 - 10}{2 \times 12 - 10 - 6} \times 10$$

$$= 60 + \frac{2}{8} \times 20 = 65$$

Mode = 3 Median – 2 Mean

65 = 3 × (61.6) – 2 Mean

2 Mean = 184.8 – 65 = 58.68

$$\text{Mean} = \frac{119.8}{2} = 59.9$$

Hence, Mean = 59.9, Mode = 65, Median = 61.6

28.

Number of Mangoes	Number of boxes	x_i	f_ix_i
50 – 52	15	51	765
53 – 55	110	54	5940
56 – 58	135	57	7695
59 – 61	115	60	6900
62 – 64	25	63	1575
	Σf_i=400		Σf_ix_i=22875

$$\bar{x} = \frac{\Sigma f_ix_i}{\Sigma f_i} = \frac{22875}{400} = 57.1875$$

Hence, the mean number of mangoes kept in a packing box is 57.1875.

29.

Class Interval	Mid Value (V)	Frequency ($F = F_i$)	VF
0 – 20	10	17	170
20 – 40	30	f1	30 f1
40 – 60	50	f2	50 f2
60 – 80	70	f3	70 f3
80 – 100	90	19	1710
		SUM(F) = 120	SUM(VF) = Mean = 50

1. $17 + f1 + f2 + f3 + 19 = 120$

$\Rightarrow f1 + f2 + f3$

$= 120 – 36 = 84$

$\Rightarrow f2 : f3 = 4 : 3$

So, Let, f2 and f3 = 4k and 3k

$\Rightarrow f1 + 4k + 3k = 84$

$\Rightarrow \Rightarrow f1 + 7k = 84$ ...(i)

2. $170 + 30f1 + 50f2 + 70f3 + 1710 = 50 \times 120$

$\Rightarrow 170 + 30f1 + 50 \times 4k + 70 \times 3k + 1710 = 6000$

$\Rightarrow 170 + 30f1 + 200k + 210k + 1710 = 6000$

$\Rightarrow 30f1 + 410k = 4120$

$\Rightarrow \Rightarrow 3f1 + 41k = 412$ ...(ii)

$\Rightarrow \Rightarrow f1 + 7k = 84$ ...(i)

= 3 x equation (i) - equation (ii)

$\Rightarrow 21k – 41k = 252 – 412$

$\Rightarrow –20k = –160$

$\Rightarrow k = 8$

Substituting the value of k in equation (ii)

$\Rightarrow f1 + 56 = 84$

$\Rightarrow f1 = 84 – 56$

$\Rightarrow f1 = 28$

$\Rightarrow f2 = 4 \times 8 = 32$

$\Rightarrow f3 = 3 \times 8 = 24$

30. $\text{Mode} = 1 + \left(\frac{f1 - f0}{2f1 - f0 - f2}\right) \times h$

$$65 = 60 + \left(\frac{12 - f1}{2(12) - f1 - 6}\right) \times 20$$

$$65 - 60 = \left(\frac{12 - f1}{24 - f1 - 6}\right) \times 20$$

$$5 = \left(\frac{12 - f1}{18 - f1}\right) \times 20$$

$$1 = \left(\frac{12 - f1}{18 - f1}\right) \times 4$$

$$1 = \frac{4(12 - f1)}{18 - f1}$$

$18 – f1 = 48 – 4f1$

$–f1 + 4f1 = 48 – 18$

$3f1 = 30$

$f1 = 10$

WORKSHEET 2

Section A

1. Given: Mean = 10.5

Median = 9.6

To find: Mode

Solution:

Empirical formula: Mode = 3median – 2mean

Substitute the values

Mode = 3(9.6) – 2(10.5)

Mode = 28.8 – 21

Mode = 7.8

Hence the mode using empirical relation, when it is given that mean and median are 10.5 and 9.6 respectively is 7.8.

4. The total number of frequency (n) is 50.

Now, the median class will be the c. f. of the corresponding class have a greater value of $\left(\frac{n}{2}\right)$ th term.

Therefore, the cf of the class greater than 25 is 30 – 40.

Here, lower limit (l) = 30, class size (h) = 10,

$\left(\frac{n}{2}\right)$ = 25, cumulative frequency of the preceding class (cf) = 16, frequency of median class (f) = 10

Now, median $= l + \left[\frac{\left[\left(\frac{n}{2}\right) - cf\right]}{f}\right] \times h$

$= 30 + \left[\frac{(25-16)}{10}\right] \times 10$

$= 30 + 9$

$= 39$

Hence, the median is 39.

5. Modal class of the distribution is the class which have the highest frequency (number of patients).

   The age group 10 – has the highest no. of patients (27)

   So, 40 – 50 is the modal class and the lower limit of this class is 40.

6. Step by step explanation:

   Number of observation = 50

   Mean of observation = 18

   Let the sum of observation be = x

   Mean $= \frac{x}{50} = 18$

   If each observation is increased by 4, then;

   New Mean $= \frac{4(x)}{50} = ?$

   From old mean; we get x = 50 × 18 = 900

   $\Rightarrow \frac{4(900)}{50}$ = New mean

   ⇒ New mean = 4(18) = 72

7. The intersection point of less than ogive and more than ogive, the x-coordinate is the median.

   Median is 18.

10. $\frac{\text{sum of 10 numbers}}{10} = 12$

    sum of 10 numbers = 120

    $\frac{\text{sum of 20 numbers}}{20} = 9$

    sum of 20 numbers = 180

    Mean of 30 $= \frac{\text{sum of 30 numbers}}{30}$

    $= \frac{120 + 180}{30} = 10$

Section B

12.

Marks	No. of Students (f_i)	Mid point (x_i)	f_ix_i
0 – 10	20	5	100
10 – 20	24	15	360
20 – 30	40	25	1000
30 – 40	36	35	1260
40 – 50	20	45	900
	140		**3620**

$\therefore$ Mean $\overline{x} = \frac{1}{n}\sum_{n}^{i=l} f_ix_i = \frac{3620}{140} = 25.86$

14. The values of the mean from the given data are:

    Fx = 30, 60, 75, 35k, 90

    When the frequencies are added we get:

    N = 15 + k

    Thus, the total frequencies = 255 + 35k

    Mean = Sigma $\frac{fx}{N}$

    $21.5k = 225 + \frac{35k}{N} + k$

When cross multiplied

67.5 = 13.5k

k = 5

15. We know,

If the number of observation (n) is even

Then,

1. First of all find the value at the position $\left\{\frac{n}{2}\right\}$

2. and find the value at the position $\left\{\frac{n}{2}+1\right\}$

3. now find the average of two value to get the median.

e.g, median = $\dfrac{\left\{\frac{n}{2}\right\}\text{th} + \left\{\frac{n}{2}+1\right\}\text{th}}{2}$

Given, 11, 12, 14, 18, (x + 2), (x + 4), 30, 32, 35, 41 are in ascending order.

number of terms = 10 {even}

So, median = $\dfrac{\left\{\left(\frac{n}{2}\right)\text{th} + \left(\frac{n}{2+1}\right)\text{th}\right\}}{2}$

$24 = \dfrac{(5\text{th} + 6\text{th})}{2}$

$24 = \dfrac{\{(x+2)+(x+4)\}}{2}$

24 = (x + 3)

x = 21

Hence, x = 21

16. Arranging data in ascending order, we have

19, 25, 30, 32, 35, 48, 51, 59

So, n = 9

Thus, Median = $\left(\frac{9+1}{2}\right)^{th}$ observation = 5^{th} obs = 35

Now, if 25 is replaced by 52, we have

19, 30, 32, 35, 37, 48, 51, 52, 59

$\therefore$ Median = 37

17. We know that

Mean = $\dfrac{\Sigma f_i x_i}{\Sigma f_i}$

$\Rightarrow \dfrac{104+30+51+76+5p^2+100p+138}{8+2+3+4+5p+6} = 18$

$\Rightarrow \dfrac{399 + 5p^2 + 100p}{23 + 5p} = 18$

$\Rightarrow 399 + 5p^2 + 100p = 414 + 90p$

$\Rightarrow 5p^2 + 10p = 15$

$\Rightarrow p^2 + 2p = 3$

So, p = 1 or p = 3

Since, p is frequency t cannot be negative

So, p = 1

18. $x_1 = 5 + 7 = 12$

$x_2 = 18 - 12 = 6$

$x_3 = 18 + 5 = 23$

$x_4 = 30 - 23 = 7$

19. Mode = $L + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) h$

$67 = 60 + \left(\dfrac{15 - x}{30 - x - 12}\right) 10$

$$7 = \left(\frac{15 - x}{18 - x}\right) 10$$

$$\frac{7}{10} = \frac{15 - x}{18 - x}$$

$7(18 - x) = 10(15 - x)$

$126 - 7x = 150 - 10x$

$3x = 24$

$x = 8$

Section C

21. Let number of boys be x and number of girls be y.

Average score of boys = 71

$\Rightarrow$ Total score of boys = $71x$

Average score of girls = 73

$\Rightarrow$ Total score of girls= $73y$

$\therefore$ Total score of school = $71x + 73y$ ...(i)

Average score of school = 71.8

Total score of school = $71.8 (x + y)$ ...(ii)

From (i) and (ii),

$71x + 73y = 71.8 (x + y)$

$\Rightarrow 71x + 73y = 71.8x + 71.8y$

$\Rightarrow 1.2y = 0.8x$

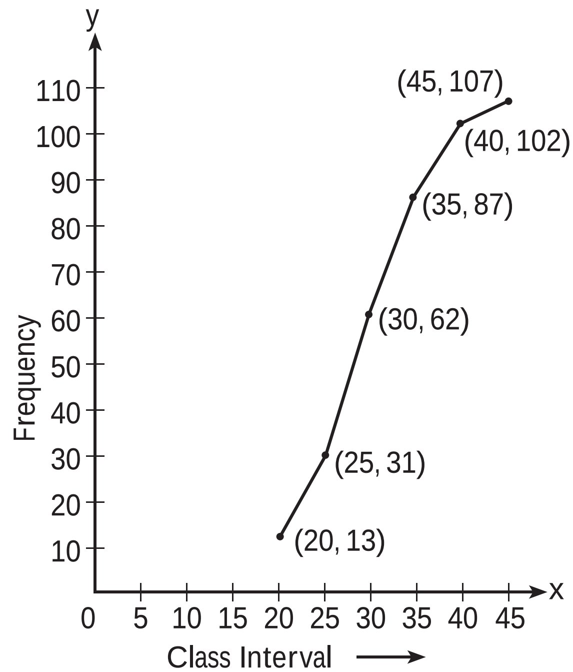
$\Rightarrow 3y = 2x$

$\Rightarrow \frac{x}{y} = \frac{3}{2}$

$\Rightarrow$ Ratio of boys to girls = 3 : 2

22.

C I (Less than)	Frequency
Less than 20	13
Less than 25	31
Less than 30	62
Less than 35	87
Less than 40	102
Less than 45	107



23. Here class length h = 8, Assumed mean = 4

Class	Frequency	Mid Values x_i	$x_iu_i = \frac{(x_i - a)}{h}$	f_iu_i	C. f.
0 – 8	10	8	–5	–50	10
8 – 16	12	12	–4	–48	22
16 – 24	8	20	–3	–24	30
24 – 32	25	28	–2	–50	55
32 – 40	15	36	–1	–15	70
40 – 48	11	44	0	0	81
48 – 56	21	52	1	21	102
56 – 64	30	60	2	60	132
64 – 72	22	68	3	66	154

$\overline{x} = a + h\left(\frac{\Sigma f_iu_i}{\Sigma f_i}\right) \times h$

$= 44 + 8\left(\frac{-40}{154}\right)$

$= 44 - 2.077$

$\overline{x} = 41.92$

Now, for median, N = 154, $\frac{N}{2} = 77$, the median class will be 40-48, then 1 = 40, f = 11, C = 70

$\text{Median} = l + \left(\frac{\frac{N}{2} - C}{f}\right) \times h$

$= 40 + \frac{77 - 70}{11} \times 8$

$= 40 + 5.09$

$= 45.09$

24.

Class Interval	Frequency	Cumulative Frequency
0 – 10	2	2
10 – 20	3	5
20 – 30	x	5 + x
30 – 40	6	11 + x
40 – 50	5	16 + x
50 – 60	3	19 + x
60 – 70	2	21 + x
Total frequency=21+x		

$\Rightarrow N = 21 + x$

$\Rightarrow \frac{N}{2} = \frac{21 + x}{2}$

So, CF corresponding to this is 11 + x

And class corresponds to this is 30 – 40

So, l = 30, F = 5 + x, f = 6, and h = 10

$$\text{Median} = l + \left(\frac{\frac{N}{2} - F}{f}\right) \times h$$

$$\Rightarrow 35 = 30 + \left(\frac{\frac{21 + x}{2} - 5 - x}{6}\right) \times 10$$

$\Rightarrow x = 5$

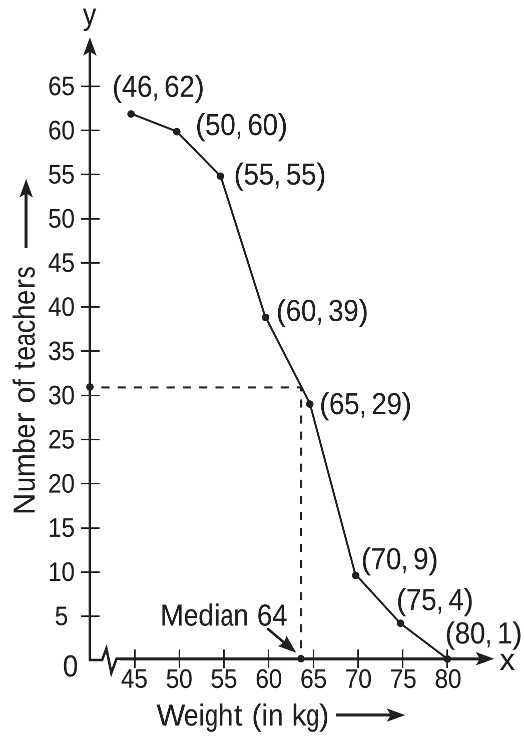
25.

Weight (in kg)	No. of teachers (More than)
More than 45	62
More than 50	60
More than 55	55
More than 60	39
More than 65	29
More than 70	9
More than 75	4
More than 80	1

Here, n = 62, then, $\frac{n}{2} = \frac{62}{2} = 31$

Now, we are going to draw horizontal line from 31 at y-axis. The point at which it touches curve from their draw vertical line towards x-axis.

$\Rightarrow$ We got median = 64



26. Given that Mean = 63.5

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$63.5 = \frac{2047.5 + 52.5x + 57.5y}{40}$$

2540 = 2047.5 + 52.5x + 57.5y

52.5x + 57.5y = 492.5

10.5x + 11.5y = 98.5

2.1x + 2.3y = 19.7

21x + 23y = 197 ...(i)

Total cities are 40

31 + x + y = 40

x + y = 9 ...(ii)

From (i) and (ii)

21x + 23(9 – x) = 197

21x + 207 – 23x = 197

–2x = –10

$x = 5$

By this way

$x + y = 9$

$y = 9 - 5$

$y = 4$

27. $f_1 = 3$ and $f_2 = 12$

Step by step explanation

Mean = 65.6

In continuous distribution, x_i is the mid point of the interval.

x_i in $[a, b] = \frac{a + b}{2}$

$\sum f_i = 50$

$5 + 8 + f_1 + 20 + f_2 + 2 = 50$

$f_1 + f_2 = 15$ (i)

$\sum x_i f_i = 100 + 320 + 60f_1 + 1600 + 100f_2 + 240$

$= 2260 + 60f_1 + 100f_2$

Mean $= \frac{\Sigma x_i f_i}{\Sigma f_i}$

$65.5 = \frac{2260 + 60f_1 + 100f_2}{50}$

$60f_1 + 100f_2 = 1020$

$3f_1 + 5f_2 = 51$ (ii)

Solving (i) and (ii) using elimination method,

we get

$f_1 = 3$ and $f_2 = 12$

Section D

29.

Marks obtains	Number of students (f_i)	Class mark (x_i)	$f_i x_i$
25 – 35	7	30	210
35 – 45	31	40	1240
45 – 55	33	50	1650
55 – 65	17	60	1020
65 – 75	11	70	770
75 – 80	1	80	80
	$\Sigma f_i = 100$		$\Sigma f_i x_i = 4970$

Mean $= \frac{\Sigma f_i x_i}{\Sigma f_i}$

$= \frac{4970}{100}$

$= 49.7$

Thus, mean of the given data is 49.7.

Now, to find the median let us put the data in the table given below:

Class	Frequency (f_i)	Cumulative frequency (cf)
25 – 35	7	7
35 – 45	31	38
45 – 55	33	71
55 – 65	17	88
65 – 75	11	99
75 – 80	1	100
Total	$N = \Sigma f_i = 100$	

Now, $N = 100 \Rightarrow \frac{N}{2} = 50$

The cumulative frequency just greater than 50 is 71 and the corresponding class is 45 – 55.

Thus, the median class is 45 – 55.

$\therefore$ l = 45, h = 10, N = 100, f = 33 and cf = 38.

Now,

$$\text{Median} = 1 + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h$$

$$= 45 + \left(\frac{50 - 38}{33}\right) \times 10$$

$$= 45 + 3.64$$

$$= 48.64$$

Thus, the median is 48.64

We know that,

$$\text{Mode} = 3(\text{Median}) - 2(\text{Mean})$$

$$= 3 \times 48.64 - 2 \times 49.70$$

$$= 145.92 - 99.4$$

$$= 46.52$$

Hence, Mean = 49.70, Median = 48.64 and Mode = 46.52

30. Let $x_1, x_2, x_3, ..., x_n$ be n observations.

Let $\overline{x}$ be the mean of the above observations.

Then,

$$\overline{x} = \frac{x_1 + x_2 + x_3 + ... + x_n}{n}$$

Now, add 1 to x_1, 2 to x_2, 3 to x_3, ..., n to x_n.

Thus the new mean is

$$\overline{y} = \frac{x_1 + 1 + x_2 + 2 + x_3 + 3 ... + x_n + n}{n}$$

$$= \frac{x_1 + x_2 + x_3 + ... + x_n}{n} + \frac{1 + 2 + 3 + ... + n}{n}$$

Since, $1 + 2 + 3 + ... + n = \frac{n(n-1)}{2}$, we have

$$\overline{y} = \overline{x} + \frac{\frac{n(n-1)}{2}}{n}$$

$$= \overline{x} + \frac{n(n-1)}{2}$$

Using the Empirical Formula,

3Median = Mode + 2Mean

$$\Rightarrow \left[\text{Median} = \frac{\text{Mode} + 2\text{Mean}}{3}\right]$$

Putting the value, we get

$$\Rightarrow \frac{7.88 + 2(8.32)}{3}$$

$$\Rightarrow \frac{7.88 + 16.64}{3}$$

$$\Rightarrow \frac{24.52}{3}$$

8.173 (approx) = 8.17

31. To find Median we first need to arrange the data in increasing order:

4, 11, 29, 40, 46, 51

Now, we can see that we have a pair of middle numbers, so our answer will be their average

$$= \frac{29 + 40}{2} = \frac{69}{2} = 34.5$$

Chapter 15 Probability

MULTIPLE CHOICE QUESTIONS

1. **c**) When two dices are thrown together, total number of outcomes are:

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)

(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)

(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)

(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)

(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)

(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

P (Getting the same number)

$$= \frac{\textit{Number of favouable outcomes}}{\textit{Total number of outcomes}}$$

$$= \frac{6}{36} = \frac{1}{6}$$

2. **d**) Total Number of cards = 52

Cards that are not ace = 48

P (card not an arc)

$$= \frac{\textit{Number of favouable outcomes}}{\textit{Total number of outcomes}}$$

$$= \frac{48}{52} = \frac{12}{13}$$

3. When two dices are rolled together, total number of outcomes are:

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)

(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)

(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)

(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)

(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)

(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

P (Getting even number on both dices)

$$= \frac{\textit{Number of favouable outcomes}}{\textit{Total number of outcomes}}$$

Number of favourable outcomes

= (2, 2), (2, 4), (2, 6), (4, 2)

(4, 6), (6, 2), (6, 4), (4, 6)

P (Getting even number on both dices)

$$= \frac{9}{36} = \frac{1}{4}$$

4. **c**) Number from 1 to 15 that are multiple of 4 = 4, 8 , 12

P (Multiple of 4) $= \frac{3}{15} = \frac{1}{5}$

5. **c**) prime number from 1 to 30

= 2, 3, 5, 7, 11, 13, 17, 19, 23, 29

P (Prime number between 1 and 30)

$$= \frac{10}{30} = \frac{1}{3}$$

WORKSHEET 1

Section A

1. If an event cannot occur, then its probability is 0.

2. Total number of face cards = 12 cards

Total number of red face cards = 6 cards

P (red face cards) $= \frac{6}{52} = \frac{3}{26}$

3. Total number of outcomes when a die is thrown = 1, 2, 3, 4, 5, 6

Odd number less than 3 = 1

P (odd number less than 3) = $\frac{1}{6}$

4. If three coins are tossed simultaneously, total number of outcomes are (HHH), (HHT), (HTH), (THH), (HTT), (THT), (TTT), (TTH)

Outcomes for at least two heads

= (HHH), (HHT), (HTH), (THH)

P (atleast two heads) = $\frac{4}{8} = \frac{1}{2}$

5. A non-leap year has 365 days

For 364 days, there are 52 weeks i.e. 52 Sundays

For the remaining 1 day, only one Sunday can exist.

So,

P (Getting 53 Sundays in non ☐leap year) = $\frac{1}{7}$

6. Number of aces in a deck of cards = 4

P (ace) = $\frac{4}{52} = \frac{1}{13}$

7. Given number = 3, 5, 5, 7, 7, 9, 9, 9, 9

Average of the given number

$= \frac{3+5+5+7+7+9+9+9+9}{9}$

$= \frac{63}{9} = 7$

So, 7 comes two times in these numbers

Thus, P (selecting their average) = $\frac{2}{9}$

8. In a single throw of dice, total number of outcomes are 6 namely 1, 2, 3, 4, 5, and 6

Perfect squares = 1, 4

P (Getting perfect square) = $\frac{2}{6} = \frac{1}{3}$

Section B

9. When two coins are tossed together, total number of outcomes are (H, H), (H, T), (T, H) (T, T)

Outcomes for at least 1 head and 1 tail

= (H, T), (T, H)

P (at least 1 head and 1 tail) = $\frac{2}{4} = \frac{1}{2}$

10. Tickets are numbered from 1 to 20

Multiples of 2 between 1 and 20

= 2, 4, 6, 8, 10, 12, 14, 16, 18, 20

Multiples of 7 between 1 and 20 = 7, 14

Outcomes which are multiple of 2 or 7

= 2, 4, 6, 7, 8, 10, 12, 14, 16, 18, 20

P (multiples of 2 or 7) = $\frac{11}{20}$

11. Number of red marble = 3

Number of blue marbles = 2

Total number of marbles = 5

P (blue marble) = $\frac{2}{5}$

12 a) When a die is thrown, total number of outcomes are 6 namely 1, 2, 3, 4, 5 and 6

Outcomes which are multiple of 3 = 3, 6

P (multiple of 3) = $\frac{2}{6} = \frac{1}{3}$

b) Outcomes which are even number or a multiple of 3 = 2, 3, 4, 6

P (even number or multiple of 3)

$$= \frac{4}{6} = \frac{2}{3}$$

13. Let boys be B and girls be G

outcomes can be BBB, GGG, BBG, BGB, GBB, GGB, GBG, BGG

then Probability of 3 girls = $\frac{1}{8}$

Probability of 0 girls = $\frac{1}{8}$

Probability of 2 girls = $\frac{3}{8}$

Probability of 1 girl = $\frac{3}{8}$

14. No, the given statement is false and we do want a higher chance of getting tail in the 4 th because every coin toss has an equal probability of getting head and tail which is $\frac{1}{2}$

∴ There are equal chances of getting head and tail in the 4th toss

15. Prizes available in 1000 tickets = 5

P (winning a prize) = $\frac{5}{1000} = \frac{1}{200}$

16. A number x is chosen from the numbers □3, □2, □1, 0, 1, 2 and 3.

To find: Probability of getting |x| <2

Total numbers are 7

Numbers x such that |x|<2 are –1, 0, 1

Total numbers x such |x|<2 are 3

We know that Probability = $\frac{\text{Number of favourable event}}{\text{Total number of event}}$

Hence, the probability of getting a number x such that |x|<2 is equal to $\frac{3}{7}$

Section C

17. Word 'Assassination' has 6 vowels and 7 consonants

6 Vowels = { A, A, I, A, I, O }

7 Consonants = { S, S, S, S, N, T, N }

i) P (vowels) = $\frac{6}{13}$

ii) P (consonants) = $\frac{7}{13}$

18. (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)

(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)

(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)

(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)

(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)

(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

Sum of number greater than 10 = (5, 6), (6, 5) (6, 6)

P (sum greater than 10) = $\frac{3}{26} = \frac{1}{12}$

19. A leap year has 366 days in which there are 52 weeks and 2 days

These 2 days can be filled as :

{ Monday, Tuesday }

{ Tuesday, Wednesday }

{ Wednesday, Thursday }

{ Thursday, Friday }

{ Friday, Saturday }

{ Saturday, Sunday }

P (53 Sundays & 53 Mondays) = $\frac{1}{7}$

20. Total number of marbles = 225

Let 'x' marbles be green

Probability of green marbles = $\frac{2}{3}$

P (green) $= \frac{2}{3}$

$\frac{x}{225} = \frac{2}{3}$

x $= \frac{225 \times 2}{3}$

x $= 75 \times 2$

x = 150 green marbles

Number of blue marbles = 225 □150

= 75 blue marbles

21. a) Total number of cards = (60 □13) + 1

= 48 cards

Cards divisible by 5 = 15, 20, 25, 30, 35, 40, 45, 50, 55, 60

P (disible by 5) = $\frac{10}{48} = \frac{5}{24}$

b) Cards which are perfect square = 16, 25, 36, 49

P (perfect square) = $\frac{4}{48} = \frac{1}{12}$

22. When two dices are thrown, the total number of outcomes are :

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)

(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)

(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)

(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)

(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)

(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

Getting a number greater than 3 on each dice

= (4, 4) (4, 5), (4, 6)

(5, 4) (5, 5), (5, 6)

(6, 4) (6, 5), (6, 6)

P (greater than 3 on each dice) = $\frac{9}{36} = \frac{1}{9}$

② Getting a total of 6 or 7

= (1, 5), (1, 6), (2, 4), (2, 5),

(3, 3), (3, 4), (4, 2), (4, 3),

(5, 1), (5, 2), (6, 1)

P (total of 6 or 7) = $\frac{11}{36}$

23. As king, queen & jack clubs are removed from the deck of cards, total number of cards becomes 49.

a) Heart,

Number of cards (Heart) = 13

P (heart) $= \frac{13}{49}$

b) Queen,

Number of cards (queen) = 3

P (queen) $= \frac{3}{49}$

c) Clubs

Number of cards (clubs) = 10

P (clubs) $= \frac{10}{49}$

24. Given card number are from 1 to 20

① Number divisible by 2 or 3

= 2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20

P (divisible by 2 or 3) = $\frac{13}{20}$

② Prime number between 1 and 20

= 2, 3, 5, 7, 11, 13, 17, 19

P (prime numbers) = $\frac{8}{20} = \frac{2}{5}$

Section D

25. Total number of red face cards = 6

When red face cards are removed, the total number of cards now becomes 46.

① A red card

Total number of remaining red cards = 20

$$P\text{ (red card)} = \frac{20}{46} = \frac{10}{23}$$

② A face card

Total number of remaining face cards

= 12 □ 6 = 6

$$P\text{ (face card)} = \frac{6}{46} = \frac{3}{23}$$

③ A red card

Total number of remaining clubs cards = 13

$$P\text{ (clubs)} = \frac{13}{46}$$

26 When a dice is thrown two times, total number of outcomes are :

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)

(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)

(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)

(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)

(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)

(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

① $P\text{ (5 will not come either time)} = \frac{25}{36}$

② $P\text{ (5 will come exactly once)} = \frac{10}{36} = \frac{5}{18}$

27. Total number of cards = (45 □ 5) + 1 = 41

a) Odd number cards = 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45

$$P\text{ (odd number)} = \frac{21}{41}$$

b) Perfect square numbers = 9, 16, 25, 36

$$P\text{ (perfect square)} = \frac{4}{41}$$

c) Multiples of 5 = 5, 10, 15, 20, 25, 30, 35, 40, 45

$$P\text{ (multiple of 5)} = \frac{9}{41}$$

28. Cards are numbered as 3, 5, 7, _ _ _ _ 37.

Total number of cards = 19

Prime numbered cards = 3, 5, 7, 11, 13, 17, 17, 23, 29, 31, 37.

$$P\text{ (prime numbers)} = \frac{11}{19}$$

29. a) No. of red or white balls in bag is 13 (5 red and 8 white)

Therefore, ${}^{13}C_1$ (selecting 1 out of 13 items) times out of ${}^{20}C_1$ (selecting 1 out of 20 items) a red or white ball is picked.

Let, E be the event of drawing a red or white ball from bag

We know that, Probability P(E)

$$= \frac{\text{(Number of favourable outcomes)}}{\text{(Total number of possible outcomes)}}$$

$$= \frac{{}^{13}C_1}{{}^{20}C_1} = \frac{13}{20}$$

b) No. of non-black balls in bag is 13 (5 red and 8 white)

Therefore, ${}^{13}C_1$ (selecting 1 out of 13 items) times out of ${}^{20}C_1$ (selecting 1 out of 20 items) a non-black ball is picked.

Let, E be the event of drawing a non-black ball from bag

We know that, Probability P(E)

$$= \frac{\text{(Number of favourable outcomes)}}{\text{(Total number of possible outcomes)}}$$

$= \frac{^{13}C_1}{^{20}C_1} = \frac{13}{20}$

c) No. of neither black nor white balls in bag is 13 (5 red balls)

Therefore, $^5C_1$ (selecting 1 out of 5 items) times out of $^{20}C_1$ (selecting 1 out of 20 items) a neither white nor black ball is picked.

Let, E be the event of drawing a neither white nor black ball from bag

We know that, Probability P(E)

$= \frac{\text{(Number of favourable outcomes)}}{\text{(Total number of possible outcomes)}}$

$= \frac{^5C_1}{^{20}C_1} = \frac{5}{20} = \frac{1}{4}$

30. a) P (queen) = $\frac{1}{5}$

b) i) P (Ace) = $\frac{\text{Number of aces}}{\text{Total number of cards}} = \frac{1}{4}$

ii) P (king)

$= \frac{\text{Number of Kings in second draw}}{\text{Total number of cards in second draw}}$

$= \frac{0}{4} = 0$

31. Number of red balls = 4

Number of black balls = 5

Number of white balls = 6

Total number of balls = 4 + 5 + 6 = 15

a) P (white) $= \frac{6}{15} = \frac{2}{5}$

b) P (red) $= \frac{4}{15}$

c) P (not black) = P (red and white)

$= \frac{4+6}{15} = \frac{10}{15} = \frac{2}{3}$

d) P (red or white) $= \frac{4+6}{15} = \frac{10}{15} = \frac{2}{3}$

32. a) Total number of cards = 52

Number of black kings = 2

P (black king) = $\frac{2}{52} = \frac{1}{26}$

b) Cards which are neither red nor or queen = 24

P (neither red nor queen) = $\frac{24}{52} = \frac{6}{13}$

c) Cards which are neither king nor queen = 44

P (neither king nor queen) = $\frac{44}{52} = \frac{11}{13}$

d) Cards which are either black or a king = 28

P (neither a black card or a king) = $\frac{28}{52} = \frac{7}{13}$

WORKSHEET 2

Section A

1. Total number of discs = 90

Prime number less than 23 = 2, 3, 4, 5, 7, 11, 13, 17, 19

P (prime number less than 23) = $\frac{8}{90} = \frac{4}{45}$

2. If two dice are thrown together, the total number of outcomes are :

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)

(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)

(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)

(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)

(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)

(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

P (even number on both dice) = $\frac{9}{36} = \frac{1}{4}$

3. Let B be Boy and G be Girl

Total number of outcomes = GGG, GGB, GBB,

GBG, BBB, BBG, BGB, BGG

Favourable outcomes for at least 1 boy = 7

Total number of outcomes = 8

P (at least 1 boy) = $\frac{7}{8}$

4. Cards are numbered from 1 to 25

Total number of outcomes = 25

Cards divisible by both 2 and 3 = 6, 12, 18, 24

P (divisible by both 2 and 5) = $\frac{4}{25}$

5. S = {□3, □2, □1, 0, 1, 2, 3}

n(S) = 7

Let A be the event that getting number less than 2

A = {□3, □2, □1, 0, 1}

n (A) = 5

Required probability = $\frac{5}{7}$

6. We have a set of natural numbers from 1 to 10 where a and b are two variables which can take values from 1 to 10.

So, total number of possible combination of a and b so that $\left(\frac{a}{2}\right)$ is a fraction without replacement are:

$\left(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots \frac{1}{10}\right)$

Similarly we have 9 such sets of 10 elements each. So total number of possible combination,

= (9) (10)

= 90

Now the possible combination which makes $\left(\frac{a}{b}\right)$ an integer without replacement are:

$= \left(\frac{2}{1}, \frac{3}{1}, \frac{4}{1}, \frac{5}{1}, \frac{6}{1}, \frac{7}{1}, \frac{8}{1}, \frac{9}{1}, \frac{10}{1}, \frac{4}{2}, \frac{6}{2}, \frac{6}{3}, \frac{8}{2}, \frac{8}{4}, \frac{9}{3}, \frac{10}{2}, \frac{10}{5}\right)$

= 17

Therefore the probability that $\left(\frac{a}{b}\right)$ is an integer,

$= \frac{\text{Possible combination which } \left(\frac{a}{b}\right) \text{ an integer}}{\text{Total Possible combination of } \left(\frac{a}{b}\right)}$

$= \frac{17}{90}$

7. Number of red balls = 5

Number of green balls = 8

Number of white balls = 7

Total of white balls = 5 + 8 + 7 = 20

P (getting a white balls or green balls)

$= \frac{8+7}{20}$

$= \frac{15}{20} = \frac{3}{4}$

8. When a dice is thrown once, the total number of outcomes is 6.

P (number less than 3) = $\frac{2}{6} = \frac{1}{3}$

9. Total number of alphabets = 26

Number of consonants = 21

Number of vowels = 5

$$P\text{ (consonants)} = \frac{21}{26}$$

10. Probability of two students not having the same birthday = P (B') = 0. 992

Probability of two students having the same birthday = P (B') = 1 □P (B')

= 1 □0. 992

= 0. 008

Section B

11. Total number = □3, □2, □1, 0, 1, 2, 3

Number whose square is less than or equal to 1

i) (□1)□= 1

ii) (0)□ = 0

iii) (1)□ = 1

$$P\text{ (square is less than or equal to 1)} = \frac{3}{7}$$

12. S = { (T, T), (T, H), (H, H), (H, T) }

n (S) = 4

Let A be the event of getting at least on tail

A = { (T, H), (H,T), (T, T) }

n (A) = 3

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{4}$$

$$P(A) = \frac{3}{4}$$

13. When two dice are tossed together, total number of outcomes are :

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)

(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)

(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)

(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)

(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)

(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

a) Outcomes for number on each dice

= { 2, 2 }, { 2, 4 }, { 2, 6 },

{ 4, 2 }, { 4, 4 }, { 4, 6 },

{ 6, 2 }, { 6, 4 }, { 6, 6 }

$$P\text{ (both numbers are even)} = \frac{9}{36} = \frac{1}{4}$$

b) Outcomes for sum on two dices is 5

$$P\text{ (sum on two dices is 5)} = \frac{4}{36} = \frac{1}{9}$$

14. When two dices are rolled simultaneously, total number of outcomes are :

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)

(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)

(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)

(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)

(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)

(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

Outcomes for sum on the two dices is 10

= (4, 6), (5, 5), (6, 4)

$$P\text{ (sum on the two dice is 10)} = \frac{3}{36} = \frac{1}{12}$$

15. Total number of outcomes without replacement

= $10 \times 9 = 90$

For a = 1, b = 2, 3, 4, ... 10; $\frac{a}{b}$ not an integer

For a = 2, b = 1, 3, 4, ... 10; (a = 2, b = 1)

For a = 3, b = 1, 2, 4, 5, ... 10; (a = 3, b = 1)

For a = 4, b = 1, 2, 3, 5, 6, .. 10; (a = 4, b = 1)

&

(a = 4, b = 2)

For a = 5, b = 1, 2, 3, 4, 6, . . . 10; (a = 5, b = 1)

For a = 6, b = 1, 2, 3, 4, 5, 7, 8, .10; (a = 6, b = 1) & (a = 6, b = 2) & (a = 6, b = 3)

For a = 7, b = 1, 2, 3, 4, 5, 6, 8, 9, 10 (a = 7, b = 1)

For a = 8, b = 1, 2, 3, 4, 5, 6, 7, 9, 10; (a = 6, b = 1) & (a = 8, b = 2) & (a = 8, b = 4)

For a = 9, b = 1, 2, 3, 4, 5, 6, 7, 8, 10; (a = 9, b = 1) & (a = 9, b = 3)

For a = 10, b = 1, 2, 3, 4, 5, 6, 7, 8, 9; (a = 10, b = 1) & (a = 10, b = 2) & (a = 10, b = 5)

Total number of cases = 17

[(2, 1), (3, 1), (4, 2), (5, 1), (6, 1), (6, 2),

(6, 3), (7, 1), (8, 1), (8, 2), (8, 4), (9, 1),

(9, 3), (10, 1), (10, 2), (10, 5)]

P ($\frac{a}{b}$ is an integer) = $\frac{17}{90}$

16. When three coins are tossed simultaneously, the total outcomes are :

(H H H), (H H T), (H T H), (T H H),

(T T T), (T T H), (T H T), (H T T)

Outcomes for exactly 2 heads

= (H H T), (H T H), (T H H)

P (exactly 2 heads) = $\frac{3}{8}$

17. No. of cards left = 52 □3 = 49

No. of cards of spade left = 13 □3 = 10 No. of black cards left = 13 + 10 = 23 [∵ Spade is of black colour]

Total no. of ways to draw a card = 49 No. of ways to draw a black card = 23

∴ Required probability = $\frac{23}{49}$

18. Cards are number from 1 to 20

Number which are multiples of 3 or 7

= 3, 6, 7, 9, 12, 14, 15, 18

P (multiple of 3 or 7) = $\frac{8}{20} = \frac{2}{5}$

19. a) Total number of cards = 52

Total number of red king = 2

P (red king) = $\frac{2}{52} = \frac{1}{26}$

b) Total number of queen = 4

Total number of jack = 4

P (queen or jack) = $\frac{4+4}{52} = \frac{8}{52} = \frac{2}{13}$

20. Total number of red cards = 100

Total number of yellow cards = 200

Total number of blue cards = 50

Total number of cards = 100 + 200 + 50 = 350

a) P (blue card) = $\frac{50}{350} = \frac{1}{7}$

b) P (not a yellow card) = P (red and blue card)

$= \frac{100+50}{350} = \frac{150}{350} = \frac{3}{7}$

c) P (neither yellow nor blue card) = P (red card)

$= \frac{100}{350} = \frac{2}{7}$

Section C

21. When two dices are throw together, the total number of outcomes are :

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)

(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)

(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)

(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)

(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)

(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

a) Outcomes for prime number on each dice

= (3, 1), (3, 5) (5, 2)

(5, 2), (5, 5), (3, 3)

(2, 2), (2, 3), (2, 5)

P (prime number on each dice) = $\frac{9}{36} = \frac{1}{4}$

b) Outcomes for total of 9

= (3, 6) (4, 5), (5, 4), (6, 3)

Outcomes for total of 11 = (5, 6) (6, 5)

P (total of 9 or 11) = $\frac{4+2}{36} = \frac{6}{36} = \frac{1}{6}$

22. When two dices are thrown together, the total number of outcomes are :

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)

(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)

(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)

(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)

(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)

(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

a) Outcomes for a number greater than 3 on each dice = (4, 4), (4, 5), (4, 6)

(5, 4), (5, 5), (5, 6)

(6, 4), (6, 5), (6, 6)

P (number greater than 3 on each dice)

$= \frac{9}{36} = \frac{1}{4}$

b) Outcomes for getting a total of 6 on both dice

= (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)

P (getting a total of 6 or 7 on both dice)

$= \frac{5+6}{36} = \frac{11}{36}$

23. Total number of shirts = 100

Shirts which are good = 88

Shirts with minor defects = 8

Shirts with major defects = 4

a) P (Ramesh buys the selected shirt)

$= \frac{\textit{Number of good shirts}}{\textit{Total number of shirts}} = \frac{88}{100} = \frac{22}{25}$

b) P (Kewal buys the selected shirt)

$= \frac{\textit{Number of good shirts} + \textit{shirts with minor defects}}{\textit{Total number of shirts}} = \frac{88+8}{100}$

$= \frac{96}{100} = \frac{24}{25}$

24. When three coins are tossed together, the total number of outcomes are :

(H H H), (H H T), (H T H), (T H H),

(T T T), (T T H), (T H T), (H T T)

a) Outcomes for exactly two heads

= (H H T), (H T H), (T H H)

P (at least two heads) = $\frac{3}{8}$

b) Outcomes for at least two heads

= (H H H), (H H T), (H T H), (T H H)

P (at least to heads) = $\frac{4}{8} = \frac{1}{2}$

c) Outcomes for at least to tails

= (T T T), (T T H), (T H T), (H T T)

P (at least two tails) = $\frac{4}{8} = \frac{1}{2}$

25. Total number of cards = 52

Total number of jack, king = 2 + 2 + 2

And queen of red colour = 6

After removing these 6 cards, the total number of cards become 52 □2 = 46

a) A black king

Total number of black kings = 2

P (black king) = $\frac{2}{46} = \frac{1}{23}$

b) A card of red colour

Number remaining red cards = 26 □6 = 20

P (red cards) = $\frac{20}{46} = \frac{10}{23}$

c) A card of black colour

Number of black cards = 26

P (black king) = $\frac{26}{46} = \frac{13}{23}$

26. Cards are number from 1 to 100

Number divisible by 9 and is a perfect square

= 36, 81

a) P (divisible by 9 and a perfect square)

$= \frac{2}{100} = \frac{1}{50}$

b) Prime number greater than 80 = 83, 89, 97

P (prime number greater than 80) = $\frac{3}{100}$

27. When a coin tossed 3 times, the total number of outcomes are :

(H H H), (H H T), (H T H), (T H H),
(T T T), (T T H), (T H T), (H T T)

Ramesh wins if all the tosses show same result

= P (A) = (H H H), (T T T)

P (A) = $\frac{2}{8}$

P (Ramesh lossing the game) = 1 □P (A)

= 1 □$\frac{2}{8} = \frac{6}{8} = \frac{3}{4}$

28. Eight equal parts of the game are numbered as

= 1, 2, 3, 4, 5, 6, 7, 8,

a) An odd number

Outcomes for odd number = 1, 3, 5, 7

P (odd number) = $\frac{4}{8} = \frac{1}{2}$

b) A number greater than 3

Outcomes for number greater than 3

= 4, 5, 6, 7, 8

P (number greater than 3) = $\frac{5}{8}$

c) A number less than 9

Outcomes for number less than 9

= 1, 2, 3, 4, 5, 6, 7, 8,

P (number less than 9) = $\frac{8}{8} = 1$

29. A number ' x ' can be selected from 1, 2, 3, and 4

A number ' y ' can be selected from 1, 3, 9 and 16

Total number of Outcomes = $4 \times 4 = 16$

Cases for product ' x y ' to be less than 16 :

1) (1, 1) $= 1 \times 1 = 1$

2) (1, 3) $= 1 \times 3 = 3$

3) (1, 9) $= 1 \times 9 = 9$

4) (2, 1) $= 2 \times 1 = 2$

5) (2, 3) $= 2 \times 3 = 6$

6) (3, 1) $= 3 \times 1 = 3$

7) (3, 3) $= 3 \times 3 = 9$

8) (4, 1) $= 4 \times 1 = 4$

9) (4, 3) $= 4 \times 3 = 12$

P (product of x and y less than 16) = $\frac{9}{16}$

30. When three coins are tossed together, the total number of outcomes are :

(H H H), (H H T), (H T H), (T H H),
(T T T), (T T H), (T H T), (H T T)

1) Outcomes for at least 2 head
= (H H H), (H H T), (H T H), (T H H)

P (at least to heads) = $\frac{4}{8} = \frac{1}{2}$

2) Outcomes for at most 2 head

= (H H T), (H T H), (T H H), (T T H),
(T H T), (H T T), (T T T)

P (at most 2 heads) = $\frac{7}{8}$

Section D

31. Probability of selecting red balls P (R) = $\frac{1}{4}$

Probability of selecting blue balls P (B) = $\frac{1}{3}$

Probability of selecting orange balls P (O)

$$= 1 \,\square \left(\frac{1}{4}+\frac{1}{3}\right)$$

$$= 1 \,\square \frac{7}{12}$$

$$= 1 \,\square \frac{12 \,\square 7}{12}$$

$$= 1 \,\square \frac{5}{12}$$

P (O) = $\frac{5}{12}$

Let there be ' n ' balls in a jar.

So,

P (O) = $\frac{5}{12}$

$$\frac{5}{12} \times n = 10$$

$$n = \frac{10 \times 12}{5}$$

$$n = 24 \text{ balls}$$

P (B) = $\frac{1}{3}$

$\frac{1}{3} \times 24 = 8$ balls which are blue in colour

∴ 8 blue balls are present in the jar.

32. Total number of balls in a bag = 18 balls

Total number of red balls = x

Total number of balls which are not red

= 18 □x

1) P (ball is not red) = $\frac{18 \,\square x}{18}$

2) P (ball is red) = $\frac{x}{18}$

As 2 red balls are further added in the bag,

Total number of red balls = x + 2

Total number of balls in the bag = 18 + 2
= 20

P (red ball) = $\frac{x+2}{20}$

A.T. Q., $= \frac{9}{8} \times \frac{x}{18} = \frac{(x-2)}{20}$

$$\frac{x}{16} = \frac{x+2}{20}$$

$20x = 16x + 32$

$4x = 32$

$x = 8$ balls

∴ Initial number of red balls = 8

33. Cards are numbered from 1 to 25

1) Outcomes for number divisible by 3 or 5

= 3, 5, 6, 9, 10, 12, 15, 18, 20, 21, 24, 25

P (numbers divisible by 3 or 5) = $\frac{12}{25}$

2) Outcomes for a perfect square number

= 1, 4, 9, 16, 25

P (perfect square number) = $\frac{5}{25} = \frac{1}{5}$

34. 1) Total number of cards = 52

Total number of spared = 13

Total number of aces excluding spades = 3

P (spade or ace) = $\frac{13+3}{52} = \frac{16}{52} = \frac{4}{13}$

2) P (black king) = $\frac{2}{52} = \frac{1}{26}$

3) Total number of jack = 4

Total number of king = 4

P (Either jack or king) = P (J) = $\frac{4+4}{52}$

$= \frac{8}{52} = \frac{2}{13}$

P (neither jack noe king) = 1 □P (J)

$= 1$ □$\frac{2}{13}$

$= \frac{13-2}{13}$

$= \frac{11}{13}$

4) Number of king = 4

Number of queen = 4

p (Either a king or a queen) = $\frac{4+4}{52}$

$= \frac{8}{52} = \frac{2}{13}$

35. Total number of cards = 49

Total number of outcomes = 49

1) Odd number

Favourable outcomes = 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 37, 39, 41, 43, 45, 47, 49

Number of Favourable outcomes = 25

$$\text{Probability}(E) = \frac{\text{No. of favourable outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{25}{49}$$

2) A multiple of 5

Favourable outcomes = 5, 10, 15, 20, 25, 30, 35, 40, 45

Number of Favourable outcomes = 9

$$\text{Probability}(E) = \frac{\text{No. of favourable outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{9}{49}$$

3) A perfect square

Favourable outcomes = 1, 4, 9, 16, 25, 36, 49

Number of Favourable outcomes = 7

$$\text{Probability}(E) = \frac{\text{No. of favourable outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{7}{49} = \frac{7}{1}$$

4) An even prime number

Favourable outcomes = 2

Number of Favourable outcomes = 1

$$\text{Probability (E)} = \frac{\text{No. of favourable outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{1}{49}$$

36. Throwing a die twice and throwing two dice simultaneously are treated as the same experiment.

Sample space = { (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) }

(i) P (5 will not come up either time) = Number of times 5 does not show divided by total number of outcomes
P(5 will not come up either time) = $\frac{25}{36}$

(ii) P (5 will come up at least once) = Number of times 5 shows up at least once divided by total number of outcomes
P (5 will come up at least once) = $\frac{11}{36}$

37. Total number of persons = 12

Number of persons who are extremely patient = 3

Number of persons who are extremely honest = 6

Number of persons who are extremely kind

= 12 □(3 + 6)

= 12 □9

= 3

a) P (person who is extremely patient) = $\frac{3}{12}$

$= \frac{1}{4}$

b) P (persons who are extremely kind or honest)

$$= \frac{3+6}{12}$$

$$= \frac{9}{12}$$

$$= \frac{3}{4}$$