ADDITIONAL™ PRACTICE MATHEMATICS 10

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1. Real Numbers

MULTIPLE CHOICE QUESTIONS

1. (a) Here, a = Dividend, b = Divisor, q = Quotient and r = Remainder
Using Euclid's Divison Lemma,
$a = bq + r, 0 \le r < b$
$a = 3q + r$
Here $b = 3;$
So, possible values of r = 0, 1, 2.
$\therefore 0 \le r < 3.$

2. (c) LCM of 23 and 33 = 23 × 33

3. (b)

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n + 7 | 2n + 13 | 2
        2n + 14
        -    -
        --------
           -1 | n + 7 | -n - 7
                n
               -
               -------
                0 + 7
                  + 7
                  -
                -------
                    0
```

$\therefore$ HCF = –1

4. (d) The two numbers 51 and 34
Their L. C. M is
102 = 17 x 3 x 2
51 = 17 x 3
34 = 17 x 2

5. (a) 70 – 5 = 65
(b) 125 – 8 = 117
Now, we will compute the H.C.F. of 65 and 117.

(a) 65

5	65,
13	13
	1

(b) 117

3	117
3	39
13	13
	1

13 is the only common factor of 65 and 117. Hence H.C.F. of 65 and 117 is 13.
Therefore, 13 is the largest number which divides 70 and 125 leaving the remainder of 5 and 8 respectively.
Let us check it.

(a) 70 ÷ 12
Quotient = 5
Remainder = 5

(b) 125 ÷ 12
Quotient = 9
Remainder = 8

WORKSHEET 1

Section A

1. Here, a is a dividend.
2. Number 13233343563715 is a composite number as it has more than two factors and a number which has more than two factors is a composite number and it is also divisible by 5 besides 1 and the number itself.
3. Dividend = Divisor × Quotient + Remainder
 = 53 × 34 + 21
 = 1802 + 21
 = 1823
4. $y = 5 \times 13 = 65$
 $x = 3 \times 195 = 585$
5. HCF (k, 2k, 3k, 4k, 5k) = k
6. Smallest composite no. = 4
 Smallest prime no. = 2
 ∴ HCF (2, 4) = 2
7. $6n = (2 \times 3)n$
 We know that a number ends with digit 0 only if it has both 2 and 5 as factors. As 6n does not have 5 as a prime factor, so, 6n does not end with digit 0.
8. $P = ab^2 \quad Q = a^3b$
 FACTORS OF $P(ab^2)$ $= a \times b \times b$
 FACTORS OF $Q(a^3b)$ $= a \times a \times a \times b$
 so, LCM OF PQ $= a \times a \times a \times b \times b$
 $= a^3b^2$
9. HCF of $a = x^3y^2$
 $b = xy^3$
 $a = x\,x\,\,x\,\,y\,y$
 $b = x\,y\,y\,y$
 The highest common factors of a and b are
 xy^2
10. $$\text{LCM}(a, b) = \frac{a \times b}{\text{HCF}(a, b)}$$
 $$= \frac{1800}{12} = 150$$

Section B

11. Let a be a given positive number.
 On dividing a by 4, let q be the quotient and r be the remainder.
 Then,by Euclid's algorithm,we have:
 $a = 4q + r$ where $0 < = r < 4$
 $a = 4q + r$ where $r = 0, 1, 2, 3$
 $a = 4q + 2 = 2(2q + 1)$
 It is clearly shown that 2q +1 is divisible by 2. Therefore, 4q + 2 is a positive integer.
12. Using Euclid's Algorithm,
 $240 = 228 \times 1 + 12$

228 = 12 × 19 + 0
Here, remainder = 0, Divisor = 12
So, HCF (240, 228) = 12

13. If a, b are any two positive numbers
Their HCF (a, b) = h and LCM (a, b) = l then
a × b = h × l
Given
a = 253, b = 440
h = 11
l = 253 × R
Therefore
$$h \times l = a \times b$$
$$11 \times 253 \times R = 253 \times 440$$
$$R = \frac{(253 \times 440)}{(11 \times 253)}$$
$$R = 40$$

14. 3 × 12 × 101 + 4
= 4 × (3 × 3 × 101 + 1)
So, 4 is also a factor of 3 × 12 × 101 + 4 besides 1 and the no. itself.
So, 3 × 12 × 101 + 4 is a composite number.

15. Step by step explanation:
We have 1200,
We fist factorise the number "1200".
1200 = 2 × 2 × 2 × 2 × 3 × 5 × 5
We can see that 3 has no pair.
To make it a perfect square, we will multiply by 3 on both the sides.
1200 × 3 = 2 × 2 × 2 × 2 × 3 × 3 × 5 × 5
$$\sqrt{3600} = \sqrt{2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5}$$
$$\sqrt{3600} = 2 \times 2 \times 3 \times 5$$
$$\sqrt{3600} = 60$$
Hence, 3 is the smallest natural number by which 1200 should be multiplied so that to make it a perfect square.

16. Factors of 1 to 10 numbers
1 = 1
2 = 1 × 2
3 = 1 × 3
4 = 1 × 2 × 2
5 = 1 × 5
6 = 1 × 2 × 3
7 = 1 × 7
8 = 1 × 2 × 2 × 2
9 = 1 × 3 × 3
10 = 1 × 2 × 5
LCM of number 1 to 10 = LCM (1, 2, 3, 4, 5, 6, 7, 8, 9, 10)
= 1 × 2 × 2 × 2 × 3 × 3 × 5 × 7 = 2520

17. Let x and x + 1 be two consecutive positive integers.
If x is even, x + 1 is odd, so, x (x + 1) is even

If x is odd, x + 1 is even, so, x (x + 1) is even.
Therefore, the product of two consecutive positive integers is always divisible by 2.

18. $3 \times 5 \times 13 \times 46 + 23 = 23 \times (3 \times 5 \times 13 \times 2 + 1)$
So, 23 is a factor of $3 \times 15 \times 13 \times 46 + 23$ besides 1 and the no. itself.
Therefore, $3 \times 5 \times 13 \times 46 + 23$ is a composite number.

19. As least prime factor of a is 3, a is an odd no. (because if a is even then it's least prime factor must be 2).
Also, as least prime factor of b is 5, b is an odd no.
Therefore, a + b is even such that it's least prime factor is 2.

20. No, two numbers can not have 15 as their HCF and 175 as their LCM because 15 is not a factor of 175.
(HCF of two numbers is always the factor of their LCM)

Section C

21. Using Euclid's Division lemma.
$a = 6q + r; o < r < 6$
$r = 0, a = 6q = 2 (3q)$, even
$r = 1, a = 6q + 1 = 2 (3q) + 1$, odd
$r = 2, a = 6q + 2 = 2 (3q + 1)$, even
$r = 3, a = 6q + 3 = 2 (3q + 1) + 1$, odd
$r = 4, a = 6q + 4 = 2 (3q + 2)$, even
$r = 5, a = 6q + 5 = 2 (3q + 2) + 1$, odd
So, any positive even integer can be written in the form of 6q, 6q + 2 or 6q + 4.

22. We know that any positive odd integer (say a) is of form 4q + 1 or 4q + 3
Case 1
$a = 4q + 1$
$a^2 = (4q + 1)^2 = 16q^2 + 1 + 8q = 8 (2q^2 + q) + 1$
$= 8m + 1 \qquad (m = 2q^2 + q)$
Case 2
$a = 4q + 3$
$a^2 = (4q + 3)^2 = 16q^2 + 9 + 24q = 8 (2q^2 + 3q + 1) + 1$
$= 8m + 1 \qquad (m = 2q^2 + 3q + 1)$
So, square of an odd positive integer is of form 8m + 1.

23. Consider 252 and 324.
Here, a = 324 and b = 252
by euclid's division lemma
$a = bq + r, \qquad 0 \leq \text{or} = r < b$
$324 = 252 \times 1 + 72$
$252 = 72 \times 3 + 36$
$72 = 36 \times 2 + 0$
Therefore, HCF (252, 324) = 36
Now consider 36 and 180.
Here a = 180 and b = 36.
by euclid's division lemma- $a = bq + r, 0 < \text{or} = r < b$
$180 = 36 \times 5 + 0$
Therefore, HCF (180, 36) = 36
Hence, HCF (180, 252, 324) = 36

24. Using Euclid's Division lemma,
$a = 5q + r; 0 < r < 5$

$r = 0, a = 5q, a^2 = 25q^2 = 5m \ (m = 5q^2)$
$r = 1, a = 5q + 1, a^2 = 25q^2 + 1 + 10q$
$= 5\,(5q^2 + 2q) + 1$
$= 5m + 1 \ (m = 5q^2 + 2q)$
$r = 2, a = 5q + 2, a^2 = 25q^2 + 4 + 20q$
$= 5\,(5q^2 + 4q) + 4$
$= 5m + 4 \ (m = 5q^2 + 4q)$
$r = 3, a = 5q + 3, a^2 = 25q^2 + 9 + 30q$
$= 5\,(5q^2 + 6q + 1) + 4$
$= 5m + 4 \ (m = 5q^2 + 6q + 1)$
$r = 4, a = 5q + 4, a^2 = 25q^2 + 16 + 40q$
$= 5\,(5q^2 + 8q + 3) + 1$
$= 5m + 1 \ (m = 5q^2 + 8q + 3)$
So, square of positive integer cannot be of form 5m + 2 or 5m + 3.

25. Minimum distance each should walk so that each can cover the same distance.
= LCM (40, 42, 45)
= 2520 cm

2	40,	42,	45
2	20,	21,	45
5	10,	21,	45
2	2,	21,	9
3	1,	21,	9
3	1,	7,	3
7	1,	7,	1
	1	1	1

26. $7 \times 19 \times 11 + 11 = 11\,(7 \times 19 \times 1 + 1)$
So, 11 is also a factor of $7 \times 19 \times 11 + 11$besides 1 and number itself.
So, $(7 \times 19 \times 11 + 11)$ is a composite number.
$7 \times 6 \times 4 \times 3 \times 2 \times 1 + 3 = 3\,(7 \times 6 \times 4 \times 2 \times 1 + 1)$
So, 3 is also a factor of $7 \times 6 \times 4 \times 3 \times 2 \times 1 + 3$ besides 1 and number itself.
So, $7 \times 6 \times 4 \times 3 \times 2 \times 1 + 3$ is a composite number.

27. Here, we have to find LCM (12, 15, 18) which indicates after how long they all again toll together.
LCM (12, 15, 18) = 180
So, three bells will toll together after 180 minutes i.e. 3 hours.

2	12,	15,	18
3	6,	15,	9
2	2,	5,	3
3	1,	5,	3
5	1,	5,	1
	1	1	1

28. Using Euclid's division algorithm,
$1170 = 650 \times 1 + 520$
$650 = 520 \times 1 + 130$
$520 = 130 \times 4 + 0$
So, HCF (650, 1170) = 130
Therefore, the largest number which divides 650 and 1170 exactly is 130.

29. Consider $\frac{1}{3 + 2\sqrt{2}} = \frac{1}{3 + 2\sqrt{2}} \times \frac{3 - 2\sqrt{2}}{3 - 2\sqrt{2}}$

$$= \frac{3 - 2\sqrt{2}}{3^2 - \left(2\sqrt{2}\right)^2} = \frac{3 - 2\sqrt{2}}{1} = 3 - 2\sqrt{2}$$

Let if possible $3 - 2\sqrt{2}$ is rational

$3 - 2\sqrt{2} = \frac{p}{q}$, p and q are integers and $q \neq 0$

$\frac{1}{2}\left(3-\frac{p}{q}\right)=\sqrt{2}$

Here, $\frac{1}{2}\left(3-\frac{p}{q}\right)$ is rational but $\sqrt{2}$ is irrational which is not possible.

So, we get a contradiction.

Therefore, $3-2\sqrt{2}$ is irrational.

i.e. $\frac{1}{3+2\sqrt{2}}$ is irrational.

30. Using Euclid's division lemma,
$117 = 65 \times 1 + 52$
$65 = 52 \times 1 + 13$
$52 = 13 \times 4 + 0$
Here, remainder = 0, divisor = 13
So, HCF (117, 65) = 13
To find : m, n
$13 = 65 - 52(1)$
$= 65 - (117 - 65(1))$
$= 65(2) + 117(-1)$
$= 65m + 117n$
So, m = 2, n = –1

Section D

31. Using Euclid's Division Algorithm, we get
$256 = 36 \times 7 + 4$
$36 = 4 \times 9 + 0$
Here, remainder = 0, divisor = 4
So, HCF (256, 36) = 4
LCM (256, 36) $= 2^8 \times 3^2$
$= 2304$

2	256,	36
2	128,	18
2	64,	9
2	32,	9
2	16,	9
2	8,	9
2	4,	9
2	2,	9
3	1,	9
3	1,	3
	1,	1

Now,
HCF × LCM $= 4 \times 2304$
$= 9216$
Product of numbers $= 256 \times 36$
$= 9216$

So, HCF × LCM = Product of numbers

32. We know that every positive even integer is of form $2q$ and every positive odd integer is of form $2q + 1$.

Case 1 $n = 2q$

Consider $n^2 - n = 4q^2 - 2q = 2(2q^2 - q)$

$\therefore n^2 - n$ is divisible by 2

Case 2 $n = 2q + 1$

Consider $n^2 - n = (2q + 1)^2 - (2q + 1)$

$= 4q2 + 1 + 4q - 2q - 1$

$= 4q2 + 4q - 2q$

$= 2(2q2 + 2q - q)$

$= 2(2q2 + q)$

$\therefore n^2 - n$ is divisible by 2.

From Case 1, Case 2, we get $n^2 - n$ is divisible by 2 for every positive integer n.

33. According to Euclid's Division lemma,

$a = 3q + r, 0 \le r < 3$

For $r = 0$

$a = 3q \Rightarrow a^3 = 27q^3 \Rightarrow a^3 = 9(3q^3)$

$= 9m \ (m = 3q^3)$

For $r = 1$

$a = 3q + 1 \Rightarrow a^3 = 27q^3 + 1 + 27q^2 + 9q$

$= 9(3q^3 + 3q^2 + q) + 1$

$= 9m + 1 \ (m = 3q^3 + 3q^2 + 2)$

For $r = 2$

$a = 3q + 2 \Rightarrow a^3 = 27q^3 + 8 + 54q^2 + 36q$

$= 9(3q^3 + 6q^2 + 4q) + 8$

$= 9m + 8 \ (m = 3q^3 + 6q^2 + 4q)$

Therefore, cube of any positive integer is of form $9m$, $9m + 1$ or $9m + 8$ for some integer m.

34. (i) Greatest possible length of each plank

= HCF (42, 49, 56)

$= \text{HCF}(2 \times 3 \times 7, 7^2, 2^3 \times 7)$

= 7

So, greatest possible length of each plank is 7m.

(ii) HCF (182, 169)

$= \text{HCF}(2 \times 7 \times 13, 13^2)$

= 13

2	182
7	91
13	13
	1

13	169
13	13
	1

35. To find the no. of required baskets such that Each basket contains only one of the two fruits but equal in number.

We will find the H.C.F i.e. highest common factor

H.C.F : The largest common factor of two or more numbers is called the highest common factor

Thus $990 = 2 \times 3 \times 3 \times 4 \times 5$

$945 = 3 \times 3 \times 3 \times 5 \times 7$

Thus HCF $= 3 \times 3 \times 5 = 45$

Thus the no. of fruits to be put in each basket in order to have minimum no. of baskets = 45.

36. Let the three consecutive positive integers be n, $n + 1$ and $n + 2$.

If number is divided by 3, remainder can be 0, 1 or 2. i.e. $n = 3q + r, 0 \le r < 3$

If $r = 0$, $n = 3q$ divisible by 3

If $r = 1$, $n + 2 = 3q + 1 + 2$

$= 3q + 3$

$= 3(q + 1)$ divisible by 3

If $r = 2$, $n + 1 = 3q + 2 + 1 = 3(q + 1)$ divisible by 3

So, one of numbers n, n + 1, and n + 2 must be divisible by 3 i.e. n (n + 1) (n + 2) is divisible by 3

Now, if a number is divided by 2, remainder is 0 or 1

i.e. $n = 2q + r$; $0 \le r < 2$

$r = 0$, $n = 2q$ divisible by 2

Also, $n + 2 = 2q + 2 = 2(q + 1)$ divisible by 2

So, one of n, n + 1 or n + 2 is divisible by 2 i.e. n (n + 1) (n + 2) is divisible by 2.

Since, n (n + 1) (n + 2) is divisible by 2 and 3 implies n (n + 1) (n + 2) is divisible by 6.

37. (a)

2	420
2	210
5	105
3	21
7	7
	1

2	180
2	90
3	45
3	15
5	5
	1

2	378
3	189
3	63
3	21
7	7
	1

So, HCF of 378, 180 and 420 is $2 \times 3 = 6$

And LCM of 378, 180 and 420 is $2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 7 = 3780$

2	378,	180,	420
3	189,	90,	210
3	63,	30,	70
7	21,	10,	70
2	3,	10,	10
5	3,	5,	5
3	3,	1,	1
	1	1	1

So, let us check whether LCM (378, 180 and 420) × HCF (378, 180 and 420) is equal to the product of the three numbers

$3780 \times 6 = 378 \times 180 \times 420$

$22680 \ne 28576800$

Hence LCM × HCF is not equal to the product of the three numbers.

Now HCF × LCM

$= 6 \times 3780 = 22680$

Product of numbers $= 378 \times 180 \times 420$

$= 28576800$

So, HCF × LCM ≠ Product of numbers.

(b) Let if possible $2\sqrt{2}$ is rational.

$$2\sqrt{2} = \frac{p}{q}, \text{ p and q are integers, } q \ne 0$$

$$\Rightarrow \quad \sqrt{2} = \frac{p}{q}$$

Here, $\frac{p}{2q}$ is rational but $\sqrt{2}$ is irrational

So, we get a contradiction.

$\therefore 2\sqrt{2}$ is irrational.

38. (i) Let if possible $\frac{2\sqrt{3}}{5}$ is rational.

$$\frac{2\sqrt{3}}{5} = \frac{p}{q};\ \text{p and q are integers, } q \neq 0$$

$$\sqrt{3} = \frac{5p}{2q}$$

Here, $\frac{5p}{2q}$ is rational but $\sqrt{3}$ is irrational which is not possible, so we get a contradiction.

$\therefore \frac{2\sqrt{3}}{5}$ is irrational.

(ii) 3 rational numbers between 1.12 and 1.13 are 1.1210, 1.1211, 1.1213.
3 irrational numbers between 1.12 and 1.13 are 1.121121112111..., 1.1221222..., 1.123123312333...

WORKSHEET 2

Section A

1. Here, denominator = $2^2 \cdot 5^7 \cdot 7^2$. As denominator is not of the form $2^m \times 5^n$, so, the given rational number has a nonterminating repeating decimal expansion.

2. $$\frac{2\sqrt{45} + 2\sqrt{20}}{2\sqrt{5}} = \frac{6\sqrt{5} + 4\sqrt{5}}{2\sqrt{5}}$$

$$= \frac{10\sqrt{5}}{2\sqrt{5}}$$

$= 5$ which is rational.

3. HCF (a, b) × LCM (a, b) = a × b

$15 \times \text{LCM} = 45 \times 105$

$$\text{LCM} = \frac{\cancel{45} \times 105}{\cancel{15}} = 315$$

4. Decimal expansion will terminate after 4 places of decimal.

5. HCF × LCM = 100 × 170 = 17000.

6. Here, denominator = $1500 = 2^2 \times 3 \times 5^3$
As denominator is not of the form $2^m \times 5^n$, so, it has non-terminating repeating decimal expansion.

7. HCF (a, b) × LCM (a, b) = a × b

$$9 \times 360 = a \times 45$$

$$\frac{9 \times 360}{45} = a$$

$$72 = a$$

8. $\frac{7}{625} = 0.0112$

9. $$\frac{95}{40} + \frac{15}{4} = \frac{95 + 150}{40}$$

$$= \frac{245}{40} = 6.125$$

10. Decimal expansion will terminate after 5 places of decimal.

Section B

11.

$$\begin{array}{r|l} 8 & 3 \\ & -0 \\ \hline & 30 \\ & -24 \\ \hline & 60 \\ & -56 \\ \hline & 40 \\ & -40 \\ \hline & 0 \end{array} \quad \text{Quotient } 0.375$$

$\therefore \frac{3}{8} = 0.375$

12. Let if possible $5\sqrt{6}$ is rational.

$$5\sqrt{6} = \frac{p}{q}; \text{p, q are integers, } q \neq 0$$

$$\sqrt{6} = \frac{p}{5q}$$

Here, $\frac{p}{5q}$ is rational but $\sqrt{6}$ is irrational which is not possible. So, we get a contradiction i.e. $5\sqrt{6}$ is irrational.

13. Let $x = 1.\overline{41}...$ (1)

$x \times 100 = 1.\overline{41} \times 100$

$100x = 141.1.\overline{41}$ (2)

On subtracting (1) from (2) , we get

$99x = 140$

$x = \frac{140}{99}$

14. Maximum capacity $= \text{HCF }(850, 680)$

$= \text{HCF }(2 \times 5^2 \times 17, 2^3 \times 5 \times 17)$

$= 2 \times 5 \times 17$

$= 170\ l.$

15. (i) $(-1) + (-1)^{2n} + (-1)^{2n} + 1 + (-1)^{4n+1}$

$= (-1) + (1) + (-1) + (-1)$

$= -2$

(ii) $\left(2^3\right)^{\frac{-5}{3}} = 2^{3\times\frac{-5}{3}} = 2^{-5} = \frac{1}{2^5} = \frac{1}{32}$

16. The given rational number is $\frac{13}{64}$

Now $\frac{13}{64} = \frac{13}{2^6} = \frac{13}{2^6 \times 5^0}$

The denominator of the given rational number is of the form

$2^m \times 5^n$, i.e. $2^6 \times 5^0$

$\therefore$ The decimal expansion of $\frac{13}{64}$ is of the form of terminating.

The decimal expansion of $\frac{13}{64}$ terminates after 6 places of decimal.

$$\begin{array}{r} 0.203125 \\ 64\overline{)13.000000} \\ 128 \\ \hline 200 \\ 192 \\ \hline 80 \\ 64 \\ \hline 160 \\ 128 \\ \hline 320 \\ 320 \\ \hline 0 \\ \hline \end{array}$$

17. Using Euclid's Algorithm.

$$\begin{aligned} 4052 &= 420 \times 9 + 272 \\ 420 &= 272 \times 1 + 148 \\ 272 &= 148 \times 1 + 124 \\ 148 &= 124 \times 1 + 24 \\ 124 &= 24 \times 5 + 4 \\ 24 &= 4 \times 6 + 0 \end{aligned}$$

Here remainder = 0, Divisor = 4

So, HCF (4052, 420) = 4

18. Let if possible $\frac{3}{\sqrt{5}}$ is rational

$\frac{3}{\sqrt{5}} = \frac{p}{q}$, p, q are integers, $q \neq 0$

$\sqrt{5} = \frac{3q}{p}$

Here, $\frac{3q}{p}$ is rational but 5 is irrational which is not possible, so we get a contradiction.

$\therefore \frac{3}{\sqrt{5}}$ is irrational.

19. Using Euclid's Division Algorithm,

$$\begin{aligned} 180 &= 144 \times 1 + 36 \\ 144 &= 36 \times 4 + 0 \end{aligned}$$

Here, remainder = 0, divisor = 36

So, HCF (144, 180) = 36

We can write

$$\begin{aligned} 36 &= 180 - 144\ (1) \\ &= 36 \\ &= 39 - 3 \\ &= 13\ (3) - 3 \\ &= 13\ m - 3 \\ m &= 3 \end{aligned}$$

20. $9^n = (3 \times 3)^n$

Since, prime factorization does not contain 2 and 5, so, it cannot end with digit 0.

Section C

21. Let if possible $\sqrt{3} + \sqrt{5}$ is rational

$$\sqrt{3} + \sqrt{5} = \frac{p}{q} \text{ p and q are integers and } q \neq 0$$

$$\sqrt{3} = \frac{p}{q} - \sqrt{5}$$

$$\Rightarrow \quad \left(\sqrt{3}\right)^2 = \left(\frac{p}{q} - \sqrt{5}\right)^2$$

$$3 = \frac{p^2}{q^2} + 5 - \frac{2p}{q}\sqrt{5}$$

$$\frac{2p}{q}\sqrt{5} = \frac{p^2}{q^2} + 2$$

$$\sqrt{5} = \frac{q}{2p}\left(\frac{p^2}{q^2} + 2\right)$$

Here, $\frac{q}{2p}\left(\frac{p^2}{q^2} + 2\right)$ is rational but $\sqrt{5}$ is irrational, which is not possible.

Therefore, $\sqrt{3} + \sqrt{5}$ is irrational.

22. Let if possible $2\sqrt{3} + \sqrt{7}$ is rational

$$2\sqrt{3} + \sqrt{7} = \frac{p}{q}, \text{ q are integers, } q \neq 0$$

$$\sqrt{7} = \frac{p}{q} - 2\sqrt{3}$$

$$7 = \frac{p^2}{q^2} + 12 - \frac{4p}{q}\sqrt{3}$$

$$\frac{4p}{q}\sqrt{3} = \frac{p^2}{q^2} + 5$$

$$\sqrt{3} = \frac{q}{4p}\left(\frac{p^2}{q^2} + 5\right)$$

Here, $\frac{q}{4p}\left(\frac{p^2}{q^2} + 5\right)$ is rational but $\sqrt{3}$ is irrational which is not possible. So, we get a contradiction

$\therefore 2\sqrt{3} + \sqrt{7}$ is irrational.

$$\left(2\sqrt{3} + \sqrt{7}\right)\left(2\sqrt{3} - \sqrt{7}\right) = \left(2\sqrt{3}\right)^2 - \left(\sqrt{7}\right)^2$$

$$= 12 - 7 = 5 \text{ which is rational}$$

23. 5 × 7 × 13 × 17 + 289 = 17 (5 × 7 × 13 × 1 + 17)
Here, 17 is also a factor of 5 × 7 × 13 × 17 + 289 besides 1 and number itself. So, it is a composite number.
Also, 7 × 11 × 13 × 15 + 225 = (7 × 11 × 13 × 1 + 15) 15
Here, 15 is also a factor of 7 × 11 × 13 × 15 + 225 besides 1 and number itself. So, it is a composite number.

24. LCM (20, 30, 40) = 120
So, all the three bells will toll together after 120 minutes i.e. 2 hours.

2	20,	30,	40
2	10,	15,	20
5	5,	15,	10
2	1,	3,	2
3	1,	3,	1
	1	1	1

25. Using Euclid's Division algorithm.
2058 = 378 × 5 + 168
378 = 168 × 2 + 42
168 = 42 × 4 + 0
Here, remainder = 0, divisor = 42
So, HCF (2058, 378) = 42

26. Let HCF = x

$\therefore$ LCM = 14x

$$LCM + HCF = 600$$

$$14x + x = 600$$

$$15x = 600\ x = 40$$

We know that HCF (a, b) × LCM (a, b) = a × b

$$40 \times 14 \times 40 = a \times 280$$

$$a = \frac{40 \times 14 \times 40}{280}$$

$$= 80$$

27. According to Euclid's division lemma,

a = bq + r and 0 ≤ r < b

let a = Some integer

b = 4

r = 0, 1, 2, 3

a = 4q, 4q + 1, 4q + 2, 4q + 3

Therefore, a is a positive integer if

a = 4q + 1, 4q + 3

28. Let if possible $7 - 2\sqrt{3}$ is rational.

$$7 - 2\sqrt{3} = \frac{p}{q}, \text{ p and q are integers and } q \neq 0$$

$$2\sqrt{3} = 7 - \frac{p}{q}$$

$$\sqrt{3} = \frac{1}{2}\left(7 - \frac{p}{q}\right)$$

Here, $\frac{1}{2}\left(7 - \frac{p}{q}\right)$ is rational but $\sqrt{3}$ is irrational which is not possible.

So, we get a contradiction

$\therefore 7 - 2\sqrt{3}$ is irrational.

Section D

29. (i) Let if possible $\frac{1}{\sqrt{2}}$ is rational

$$\frac{1}{\sqrt{2}} = \frac{p}{q}, \text{ p and q are integers and } q \neq 0$$

$$\sqrt{2} = \frac{q}{p}$$

Here, $\frac{q}{p}$ is rational but $\sqrt{2}$ is irrational which is not possible. So, we get a contraction.

$\therefore \frac{1}{\sqrt{2}}$ is irrational

(ii) Let if possible $7\sqrt{5}$ is rational

$$7\sqrt{5} = \frac{p}{q}, \text{ p and q are integers and } q \neq 0$$

$$\sqrt{5} = \frac{p}{7q}$$

Here, $\frac{p}{7q}$ is rational but $\sqrt{5}$ is irrational which is not possible. So, we get a contraction.

$\therefore 7\sqrt{5}$ is irrational.

30. Using Euclid's division algorithm

$$237 = 81 \times 2 + 75$$
$$81 = 75 \times 1 + 6$$
$$75 = 6 \times 12 + 3$$
$$6 = 3 \times 2 + 0$$

So, HCF (237, 81) = 3

Consider $3 = 75 - 6\,(12)$
$= (81 - 6) - 6\,(12)$
$= 81 - 13\,(6)$
$= 81 - 13\,(81 - 75)$
$= 81 - 81(13) + 13\,(237 - 81(2))$
$= 81\,(1 - 13 - 26) + 237\,(13)$
$= 81\,(-38) + 237\,(13)$
$= 81x + 237\,y$

where $x = -38, y = 13$

31. HCF (96, 240, 336)
$= \text{HCF}\,(2^5 \times 3, 2^4 \times 3 \times 5, 2^4 \times 3 \times 7)$
$= 2^4 \times 3$
$= 48$

So, number of stacks of English books = $\frac{96}{48} = 2$

Number of stacks of Hindi books = $\frac{240}{48} = 5$

Number of stacks of Mathematics books = $\frac{336}{48} = 7$

32. (i) ⇨ AS we have to find 5 rational numbers between 1 and 2
⇨ We can consider = 1.1, 1.2, 1.3, 1.4, 1.5
$= \frac{11}{10}, \frac{12}{10}, \frac{13}{10}, \frac{14}{10}, \frac{15}{10}$
$= \frac{11}{10}, \frac{6}{5}, \frac{13}{10}, \frac{7}{5}, \frac{3}{2}$

(ii) HCF (70 – 5, 125 – 8)
= HCF (65, 117)
= HCF (5 × 13, 32 × 13)
= 13

33. Let if possible $\sqrt{3}$ is rational

$\sqrt{3} = \frac{p}{q}$, p and q are integers and $q \neq 0$

HCF (p, q) = 1

$q\sqrt{3} = p$

$3q^2 = p^2$

3 divides p^2 ⇨ 3 divides p

$p = 3c$

$p^2 = 9c^2$ ⇨ $3q^2 = 9c^2$

$q^2 = 3c^2$

$\Rightarrow$ 3 divides $q^2 \Rightarrow$ 3 divides q

So, p and q have atleast 3 in common which is a contradiction to the fact that HCF (p, q) = 1

So, our supposition was wrong,

$\sqrt{3}$ is irrational.

34. According to Euclid's division lemma, for any positive integer n, we have

$n = bq + r, \quad 0 \le r < b$

Take b = 5

$n = 5q + r, 0 < r < 5$

For r = 0

$n = 5q$, divisible by 5

$n + 4 = 5q + 4$, not divisible by 5

$n + 8 = 5q + 8$, not divisible by 5

$n + 12 = 5q + 12$, not divisible by 5

$n + 16 = 5q + 16$, not divisible by 5

So, for r = 0, only n is divisible by 5

For r = 1

$n = 5q + 1$, not divisible by 5

$n + 4 = 5q + 1 + 4$

$= 5q + 5$

$= 5(q + 1)$, divisible by 5

$n + 8 = 5q + 1 + 8$

$= 5q + 9$, not divisible by 5

$n + 12 = 5q + 1 + 12$

$= 5q + 13$, not divisible by 5

$n + 16 = 5q + 1 + 16$

$= 5q + 17$, not divisible by 5.

So, for r = 1, only n + 4 is divisible by 5

For r = 2,

$n = 5q + 2$, not divisible by 5

$n + 4 = 5q + 6$, not divisible by 5

$n + 8 = 5q + 10$

$= 5(q + 2)$, divisible by 5

$n + 12 = 5q + 14$, not divisible by 5

$n + 16 = 5q + 18$, not divisible by 5

So, for r = 2, only n + 8 is divisible by 5

For r = 3

$n = 5q + 3$, not divisible by 5

$n + 4 = 5q + 7$, not divisible by 5

$n + 8 = 5q + 11$, not divisible by 5

$n + 12 = 5q + 15$

$= 5(q + 3)$, divisible by 5

$n + 16 = 5q + 19$, not divisible by 5.

So, for r = 3, only n + 12 is divisible by 5.

For r = 4

$n = 5q + 4$, not divisible by 5

$n + 4 = 5q + 8$, not divisible by 5

$n + 8 = 5q + 12$, not divisible by 5

$n + 12 = 5q + 16$, not divisible by 5

$n + 16 = 5q + 20$
$= 5(q + 4)$, divisible by 5

So, for $r = 4$, only $n + 16$ is divisible by 5.

35. Let if possible $n + \sqrt{m}$ is rational

$n + \sqrt{m} = \frac{p}{q}$, p and q are integers and $q \neq 0$

$\therefore \quad \sqrt{m} = \frac{p}{q} - n$

Here, $\frac{p}{q} - n$ is rational (as p, q are integers and n is rational) but $\sqrt{m}$ is irrational.

So, we get a contradiction.

Therefore, $n + \sqrt{m}$ is irrational.

36. To prove: $\sqrt{p} + \sqrt{q}$ is irrational.

Let if possible $\sqrt{p} + \sqrt{q}$ is rational

$\sqrt{p} + \sqrt{q} = \frac{a}{b}$, a and b are integers and $b \neq 0$

$\sqrt{p} = \frac{a}{b} - \sqrt{q}$

On squaring both sides, we get

$$p = \frac{a^2}{b^2} + 2 - \frac{2a}{b}\sqrt{q}$$

$$\frac{2a}{b}\sqrt{q} = \frac{a^2}{b^2} + q - p$$

$$\sqrt{q} = \frac{b}{2a}\left(\frac{a^2}{b^2} + q - p\right)$$

Here, $\frac{b}{2a}\left(\frac{a^2}{b^2} + q - p\right)$ is rational but $\sqrt{q}$ is irrational (as square root of a prime number is irrational) which is not possible.

So, we get a contradiction.

Therefore, $\sqrt{p} + \sqrt{q}$ is irrational.

37. = 72/24 km/h = 3 km/h

Time = Distance/Speed

Time for 1^{st} cyclist = $\frac{360}{2}$ = 180 hrs

Time for 2^{nd} cyclist = $360 \div \frac{60}{24}$ = 144 hrs

Time for 3^{rd} cyclist = $\frac{360}{3}$ = 120 hrs

L.C.M of 180, 144, 120 = 720 hrs

Total Time taken in days = $\frac{720}{24}$ = 30 days

38. (i) In order to find the maximum number of columns in which they can march, we will find HCF (32, 616).

$32 = 2^5$

$616 = 2^3 \times 7 \times 11$

So, HCF (32, 616) = 2^3 = 8

Hence, maximum number of columns = 8

(ii) We know that for any two positive integers a and b,

LCM (a, b) × HCF (a, b) = a × b

LCM (306, 657) × HCF (306, 657)

= 306 × 657

LCM (306, 657) × 9 = 306 × 657

$$\text{LCM }(306, 657) = \frac{306 \times 657}{9} = 22338$$

39. (i) According to Euclid's Division lemma,

$a = bq + r; \quad 0 \le r < b$

Take $b = 6$

$a = 6q + r; 0 < r < 6$

For $r = 0$

$a = 6q$

$= 2(3q)$ which is even

For $r = 1$

$a = 6q + 1$

$= 2(3q) + 1$ which is odd

For $r = 2$

$a = 6q + 2$

$= 2(3q + 1)$ which is even

For $r = 3$

$a = 6q + 3$

$= 6q + 2 + 1$

$= 2(3q + 1) + 1$ which is odd

For $r = 4$

$a = 6q + 4$

$= 2(3q + 2)$ which is even

For $r = 5$

$a = 6q + 5$

$= 6q + 4 + 1$

$= 2(3q + 2) + 1$ which is odd

Therefore, every positive integer is of form $6q + 1$ or $6q + 3$ or $6q + 5$.

(ii) LCM $(x^3 y^3, x^3 y^5) = x^3 y^5$

40. (i) 135 and 225

225 = 135 × 1 + 90

135 = 90 × 1 + 45

90 = 45 × 2 + 0

So, HCF (135, 225) = 45

(ii) 196 and 38220

38220 = 196 × 195 + 0

So, HCF (196, 38220) = 196

(iii) 867 and 255

867 = 255 × 3 + 102

255 = 102 × 2 + 51

102 = 51 × 2 + 0

So, HCF (867, 255) = 51

Chapter 02

Polynomials

MULTIPLE CHOICE QUESTIONS

1. (a) Let α, β be the zeroes of f(x)

$\therefore \quad \alpha\beta = 3$

$\Rightarrow \quad \frac{K}{1} = 3$

$\Rightarrow \quad K = 3,$

2. (c) $\alpha + \beta = \frac{-3}{4}, \quad \alpha\beta = \frac{7}{4}$

So, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{\frac{-3}{4}}{\frac{7}{4}} = \frac{-3}{7}$

3. (d) both a and c.

4. (b) Let $p(x) = 2x^2 + 2ax + 5x + 10$

As (x + a) is a factor of p(x),

$\therefore \quad p(-a) = 0$

$2a^2 - 2a^2 - 5a + 10 = 0$

$5a = 10$

$a = 2$

5. (b) If c = 0,

Discriminant (D) = $b^2 - 4a(0)$

< 0 (as f(x) has no real zeroes)

$\Rightarrow \quad b^2 < 0$ not possible

So, $c \neq 0$

If $c > 0$

In the discriminant, b^2 is positive. Discriminant (D) will be negative only if a > 0

Consider $a + b + c < 0$

$\Rightarrow b < -a - c$

$\Rightarrow -b > a + c$

$\Rightarrow b^2 > (a + c)^2 = a^2 + c^2 + 2ac$

$\Rightarrow b^2 - 4ac > a^2 + c^2 - 2ac$

$\Rightarrow b^2 - 4ac > (a - c)^2 \geq 0$

Discriminant cannot be negative and positive simultaneously.

$\therefore$ a cannot be greater than 0.

$\Rightarrow$ c cannot be greater than 0.

So, only possibility is c < 0

WORKSHEET 1

Section A

1. $b^2 - 4ac = 0$

f(x) has two equal zeroes

2. A quadratic polynomial is of form

k {x^2 – (sum of zeroes) x + product of zeroes

$= k\{x^2 - \left(\frac{-1}{2}\right)x + (-3)\}$

$= \frac{k}{2}\{2x^2 + x - 6\}$

3. Let $p(x) = x^4 + x^3 - 2x^2 + x + 1$

Remainder is $p(1) = 1 + 1 - 2 + 1 + 1 = 2$

4. A binomial of degree 6 is $x^6 + 4x^2$

5. $3x^3 - x^2 - 3x + 1$

$= x^2(3x - 1) - 1(3x - 1)$

$= (x^2 - 1)(3x - 1)$

$= (x + 1)(x - 1)(3x - 1)$

6. a + b = 11, ab = 30

$a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$= (a + b) [a + b)^2 - 3ab]$

$= 11 (121 - 90)$

$= 11 (31)$

$= 341$

7. $f(x) = 6x^2 - 3 - 7x$

$= 6x^2 - 7x - 3$

$= 6x^2 - 9x + 2x - 3$

$= 3x (2x - 3) + 1 (2x - 3)$

$= (2x - 3) (3x + 1)$

Now, $f(x) = 0 \Rightarrow x = \frac{3}{2}, \frac{-1}{3}.$

So, zeroes are $x = \frac{3}{2}, \frac{-1}{3}$

8. $p(x) = 4x^2 - 5x - 1$

$\alpha + \beta = \frac{5}{4}, \quad \alpha\beta = \frac{-1}{4}$

$\alpha^2\beta + \alpha\beta^2 = \alpha\beta (\alpha + \beta) = \frac{-1}{4}\left(\frac{5}{4}\right) = \frac{-5}{16}$

9. $f(x) = 6x^3 + 3x^2 - 5x + 1$

$\alpha + \beta + \gamma = \frac{-3}{6} = \frac{-1}{2}$

$\alpha \beta \gamma = \frac{-1}{6}$

So, $\alpha^{-1} \beta^{-1} \gamma^{-1} = \frac{1}{\alpha\beta\gamma} = -6$

$\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2$

$= \alpha\beta\gamma (\alpha + \beta + \gamma)$

$= \frac{-1}{6}\left(\frac{-1}{2}\right) = \frac{1}{12}$

Section B

10. Dividend = Divisor × Quotient + Remainder

$x^3 + 2x^2 + 4x + b = (x + 1) (x^2 + ax + 3) + (2b - 3)$

$= (x^3 + ax^2 + 3x + x^2 + ax + 3 + 2b - 3)$

On comparing coefficients of x^2 and constant terms we get,

$a + 1 = 2 \Rightarrow a = 1$

$b = 3 + 2b - 3 \Rightarrow b = 0$

11. $p(x) = 3x^2 - 6x + 4$

$\alpha + \beta = 2, \quad \alpha\beta = \frac{4}{3}$

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{2} + \frac{1}{\beta}\right) + 3\alpha\beta$

$= \frac{\alpha^2 + \beta^2}{\alpha\beta} + 2\left(\frac{\alpha + \beta}{\alpha\beta}\right) + 3\alpha\beta$

$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} + 2\left(\frac{\alpha + \beta}{\alpha\beta}\right) + 3\alpha\beta$

$= \frac{4 - \frac{8}{3}}{\frac{4}{3}} + 2\left(\frac{2}{\frac{4}{3}}\right) + 3\left(\frac{4}{3}\right)$

$= 1 + 3 + 4$

$= 8$

12. Let the zeroes be $\alpha, \frac{1}{\alpha}$

$\alpha + \frac{1}{\alpha} = \frac{-13}{a^2 + 9}, \quad \alpha\frac{1}{\alpha} = \frac{6a}{a^2 + 9}$

So, $1 = \frac{6a}{a^2 + 9} \Rightarrow a^2 - 6a + 9 = 0$

$a^2 - 3a - 3a + 9 = 0$

$a (a - 3) - 3 (a - 3) = 0$

$(a - 3) (a - 3) = 0$

$a = 3$

13. Let the two zeroes of the $f(t) = kt^2 + 2t + 3k$ α and β.

Sum of zeroes $(\alpha + \beta)$

Product of the zeroes $\alpha\beta$

$\frac{-2}{k} = \frac{3k}{k}$

$-2k = 3k^2$

$2k + 3k^2 = 0$

$k(3k + 2) = 0$

$k = 0$

$k = \frac{-2}{3}$

14.

$$
\begin{array}{r|l}
 & 2x^2 + 2x - 1 \\
\hline
4x^2 + 3x - 2 & 8x^4 + 14x^3 - 2x^2 + 8x - 12 \\
 & 8x^4 + 6x^3 - 4x^2 \\
 & - \quad - \quad + \\
\hline
 & 8x^3 + 2x^2 + 8x - 12 \\
 & 8x^3 + 6x^2 - 4x \\
 & - \quad - \quad + \\
\hline
 & -4x^2 + 12x - 12 \\
 & -4x^2 - 3x + 2 \\
 & + \quad + \quad - \\
\hline
 & 15x - 14 \\
\hline
\end{array}
$$

15. Cubic polynomial is of form

$\{x^3 - (\alpha + \beta + \gamma) x^2 + (\alpha\beta + \beta\gamma + \alpha\gamma) x - \alpha\beta\gamma\}$

$= k \{x^3 - (5 + 6 - 1) x^2 + (30 - 6 - 5) x - 30\}$

$= \quad k \{x^3 - 10x^2 + 19x + 30\}$

16. Let the zeros of the polynomial be:

$a - d$, a and $a + d$, so that the roots are in AP.

$f(x) = x^3 + 3px^2 + 3qx + r.$

The standard form of a cubic equation is:

$x^3 (a + b + c)x^2 + (ab + bc + ca)x - abc - 0.$

Comparing this equation with the given polynomial:

We find:

$3p = -(a - d + a + a + d)$

$\Rightarrow \; 3p = -3a$

$\Rightarrow \; p = -a$

$3q - (a - d) a + a (a + d) + (a + d)(a - d)$

$\Rightarrow 3q = a^2 - ad + a^2 + ad + a^2 - d^2$

$\Rightarrow 3q = 3a^2 - d^2$

$\Rightarrow d^2 = 3a^2 - 3q$

or, $d^2 = 3p^2 - 3q$

And, $r = -(a - d)a(a + d)$

Or, $r = ad^2 - a^3$

Or, $r = (-p)\,(3p^2 - 3q) - (-p)^3$

Or, $r = 3p^3 + 3pq + p^3$

Thus, $r = 3pq - 2p^3$.

Section C

17. $\alpha^2 + \beta^2$ can be written as $(\alpha + \beta)^2 - 2\alpha\beta$

$p(x) = 2x^2 - 5x + 7$

$a = 2$, $b = -5$, $c = 7$

α and β are the zeros of $p(x)$

We know that ,

Sum of zeros $= \alpha + \beta = \dfrac{-b}{a} = \dfrac{5}{2}$

Product of zeros $= \dfrac{c}{a} = \dfrac{7}{2}$

$2\alpha + 3\beta$ and $3\alpha + 2\beta$ are zeros of a polynomial.

Sum of zeros $= 2\alpha + 3\beta + 3\alpha + 2\beta$

$= 5\alpha + 5\beta$

$= 5\,[\alpha + \beta]$

$= 5 \times \dfrac{5}{2}$

$= \dfrac{25}{2}$

Product of zeros $= (2\alpha + 3\beta)(3\alpha + 2\beta)$

$= 2\alpha\,[3\alpha + 2\beta] + 3\beta\,[3\alpha + 2\beta]$

$= 6\alpha^2 + 4\alpha\beta + 9\alpha\beta + 6\beta^2$

$= 6\alpha^2 + 13\alpha\beta + 6\beta^2$

$= 6\,[\,\alpha^2 + \beta^2\,] + 13\alpha\beta$

$= 6\,[\,(\alpha + \beta)^2 - 2\alpha\beta\,] + 13\alpha\beta$

$= 6\left[\left(\dfrac{5}{2}\right)^2 - 2 \times \dfrac{7}{2}\right] + 13 \times \dfrac{7}{2}$

$= 6\left[\dfrac{25}{4} - 7\right] + \dfrac{91}{2}$

$= 6\left[\dfrac{25}{4} - \dfrac{28}{4}\right] + \dfrac{91}{2}$

$= 6\left[\dfrac{-3}{4}\right] + \dfrac{91}{2}$

$= \dfrac{-18}{4} + \dfrac{91}{2}$

$= \dfrac{-9}{2} + \dfrac{91}{2}$

$= \dfrac{82}{2}$

$= 41$

$$\frac{-18}{4} = \frac{-9}{2} \quad \text{[Simplest form]}$$

a quadratic polynomial is given by :-

k { x^2 – (sum of zeros)x + (product of zeros)}

$k\{x^2 - \frac{5}{2x} + 41\}$

k = 2

$2\{x^2 - \frac{5}{2x} + 41]$

$2x^2 - 5x + 82$ is the required polynomial.

18. Dividend = Divisor × Quotient + Remainder

$x^4 + 2x^3 - 2x^2 + x - 1 = (x^2 + 2x - 3)$ Quotient + Remainder

$$\begin{array}{r|l} & x^2 + 1 \\ \hline x^2 + 2x - 3 & x^4 + 2x^3 - 2x^2 + x - 1 \\ & x^4 + 2x^3 - 3x^2 \\ & - \quad - \quad + \\ \hline & x^2 + x - 1 \\ & x^2 + 2x - 3 \\ & - \quad - \quad + \\ \hline & -x + 2 \\ \hline \end{array}$$

So, $x^4 + 2x^3 - 2x^2 + x - 1 = (x^2 + 2x - 3)(x^2 + 1) + (-x + 2)$

So, $-(-x + 2) = x - 2$ must be added to the polynomial f(x).

19.

$$\begin{array}{r|l} & 2x^2 + 5 \\ \hline 3x^2 + 4x + 1 & 6x^4 + 8x^3 + 17x^2 + 21x + 7 \\ & 6x^4 + 8x^3 + 2x^2 \\ & - \quad - \quad - \\ \hline & 15x^2 + 21x + 7 \\ & 15x^2 + 20x + 5 \\ & - \quad - \quad - \\ \hline & x + 2 \\ \hline \end{array}$$

On comparing x + 2 with ax + b, we get

a = 1, b = 2

20. Let the quotient be $q(x) = ax^2 + bx + c$ and remainder $r(x) = px + q$

Using division algorithm,

f(x) = g (x) q(x) + r(x)

$$3x^4 + 5x^3 - 7x^2 + 2x + 2$$

$$= (x^2 + 3x + 1)(ax^2 + bx + c) + px + q$$

$$= ax^4 + bx^3 + cx^2 + 3ax^3 + 3bx^2 + 3cx + ax^2 + bx + c + px + q$$

$a = 3$

$5 = b + 3a \Rightarrow b = 5 - 3a \Rightarrow b = -4$

$-7 = c + 3b + a$

$-7 = c - 12 + 3 \Rightarrow c = 2$

$2 = 3c + b + p$

$2 = 6 - 4 + p \Rightarrow p = 0$

$2 = q + c \Rightarrow q = 2 - 2 = 0$

So, Remainder = px + q = 0

As remainder is zero, g(x) is a factor of p(x).

21. Let $f(x) = x^3 + 2x^2 + kx + 3$

Remainder = f(3) = 21

$3^3 + 2(3)^2 + 3k + 3 = 21$

$27 + 18 + 3k + 3 = 21$

$3k = 21 - 48 = -27$

$k = -9$

Now, we will find the quotient.

Dividend $= x^3 + 2x^2 + kx + 3$

$= x^3 + 2x^2 - 9x + 3$

Divisor = x – 3

$$\begin{array}{r|l} & x^2 + 5x + 6 \\ \hline x - 3 & x^3 + 2x^2 - 9x + 3 \\ & x^3 - 3x^2 \\ & - \quad + \\ \hline & 5x^2 - 9x + 3 \\ & 5x^2 - 15x \\ & - \quad + \\ \hline & 6x + 3 \\ & 6x - 18 \\ & - \quad + \\ \hline & 21 \\ \hline \end{array}$$

So, quotient = $x^2 + 5x + 6$

22. Zeroes are $-\sqrt{3}$ and $\sqrt{3}$

So, factors are $(x+\sqrt{3})$, $(x-\sqrt{3})$

i.e. $(x+\sqrt{3})(x-\sqrt{3})$ is also a factor

i.e. $x^2 - 3$ is a factor of given polynomial.

$$\begin{array}{r|l} & 2x + 1 \\ \hline x^2 - 3 & 2x^3 + x^2 - 6x - 3 \\ & 2x^3 \quad\; - 6x \\ & -\qquad\quad + \\ \hline & \quad x^2 - 3 \\ & \quad x^2 - 3 \\ & \quad - \quad + \\ \hline & \qquad 0 \end{array}$$

For the remaining zero,

put $2x + 1 = 0$

$$x = \frac{-1}{2}$$

23. As $\sqrt{2}$ is a zero of given polynomial, $x-\sqrt{2}$ is a factor of the polynomial.

$$\begin{array}{r|l} & 6x^2 + 7\sqrt{2}x + 4 \\ \hline x-\sqrt{2} & 6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2} \\ & 6x^3 - 6\sqrt{2}x^2 \\ & - \qquad + \\ \hline & \quad 7\sqrt{2}x^2 - 10x \\ & \quad 7\sqrt{2}x^2 - 14x \\ & \quad - \qquad\quad + \\ \hline & \qquad\qquad 4x - 4\sqrt{2} \\ & \qquad\qquad 4x - 4\sqrt{2} \\ & \qquad\qquad - \quad + \\ \hline & \qquad\qquad\quad 0 \end{array}$$

For other zeroes,

$6x^2 + 7\sqrt{2}x + 4 = 0$

$6x^2 + 3\sqrt{2}x + 4\sqrt{2}x + 4 = 0$

$3x(2x+\sqrt{2}) + 4\sqrt{2}x + 4 = 0$

$3\sqrt{2}x(\sqrt{2}x+1) + 4(\sqrt{2}x+1) = 0$

$(3\sqrt{2}x+4)(\sqrt{2}x+1) = 0$

$$x = \frac{-4}{3\sqrt{2}} = \frac{-4\sqrt{2}}{6} = \frac{-2\sqrt{2}}{3}$$

$$\text{and } x = \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$$

24. According to division algorithm,

Dividend = Divisor × Quotient + Remainder

$x^3 - 3x^2 + x + 2 = g(x)(x-2) + (-2x+4)$

$$g(x) = \frac{x^3 - 3x^2 + x + 2 + 2x - 4}{x-2}$$

$$= \frac{x^3 - 3x^2 + 3x - 2}{x-2}$$

$$\begin{array}{r|l} & x^2 - x + 1 \\ \hline x-2 & x^3 - 3x^2 + 3x - 2 \\ & x^3 - 2x^2 \\ & - \quad\; + \\ \hline & \quad - x^2 + 3x - 2 \\ & \quad - x^2 + 2x \\ & \quad + \qquad - \\ \hline & \qquad\quad x - 2 \\ & \qquad\quad x - 2 \\ & \qquad\quad - \quad + \\ \hline & \qquad\qquad 0 \end{array}$$

So, $g(x) = x^2 - x + 1$

Section D

25. $f(x) = x^2 - px + q$

$\alpha + \beta = p, \quad \alpha\beta = q$

Consider

LHS

$$\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2} = \frac{\alpha^4 + \beta^4}{\alpha^2\beta^2}$$

$$= \frac{(\alpha^2)^2 + (\beta^2)^2}{\alpha^2\beta^2}$$

$$= \frac{[\alpha^2 + \beta^2]^2 - 2\alpha^2\beta^2}{(\alpha\beta)^2}$$

$$= \frac{[(\alpha+\beta)^2 - 2\alpha\beta]^2 - 2(\alpha\beta)^2}{(\alpha\beta)^2}$$

$$= \frac{[p^2 - 2q]^2 - 2(q)^2}{(q)^2}$$

$$= \frac{p^4 + 4q^2 - 4p^2q - 2q^2}{q^2}$$

$$= \frac{p^4}{q^2} - \frac{4p^2}{q} + 2 \quad = \text{RHS}$$

26. Let $p(x) = x^3 - 2x^2 + qx - r$

$\alpha + \beta + \gamma = 2$

For $\alpha + \beta = 0 \Rightarrow 0 + r = 2$

$\Rightarrow r = 2$

Also,

$$\alpha\beta\gamma = r$$
$$2\alpha\beta = r$$
$$\alpha\beta + \beta\gamma + \alpha\gamma = q$$
$$\alpha\beta + \gamma(\alpha + \beta) = q$$
$$\alpha\beta + \gamma(0) = q \quad [\text{As } \alpha + \beta = 0]$$
$$\alpha\beta = q$$
$$\frac{r}{2} = q$$
$$2q = r$$

27.

$$\begin{array}{r|l} & 2x^2 - 3x + (-8 - 2k) \\ x^2 + 2x + k & 2x^4 + x^3 - 14x^2 + 5x + 6 \\ & \underline{\underset{-}{2x^4} + \underset{-}{4x^3} + \underset{-}{2kx^2}} \\ & -3x^3 + x^2(-14 - 2k) + 5x + 6 \\ & \underline{\underset{+}{-3x^3} \underset{+}{- 6x^2} \quad \underset{+}{- 3kx}} \\ & x^2(-8 - 2k) + x(5 + 3k) + 6 \\ & \underline{\underset{-}{x^2(-8 - 2k)} + \underset{-}{x(-16 - 4k)} + \underset{-}{k(-8-2k)}} \\ & \underline{x(5 + 3k + 16 + 4k) + 6 + 8k + 2k^2} \end{array}$$

Remainder = $(21 + 7k)x + 6 + 8k + 2k^2$

As $x^2 + 2x + k$ is a factor of

$2x^4 + x^3 - 14x^2 + 5x + 6$,

So, Remainder should be zero

$(21 + 7k)x + 6 + 8k + 2k^2 = 0$

$= 0x + 0$

On comparing coefficient of x, we get
$21 + 7k = 0$

$k = -3$

Now, we will find zeroes of the two polynomials.

$2x^4 + x^3 - 14x^2 + 5x + 6$

$= (x^2 + 2x + k)[2x^2 - 3x + (-8 - 2k)]$

$= (x^2 + 2x - 3)(2x^2 - 3x - 2)$

$= (x^2 + 3x - x - 3)(2x^2 - 4x + x - 2)$

$= [x(x + 3) - 1(x + 3)][2x(x - 2) + 1(x - 2)]$

$= (x + 3)(x - 1)(2x + 1)(x - 2)$

So, zeroes are $-3, 1, \frac{-1}{2}, 2$

Again consider $x^2 + 2x + k$

$= x^2 + 2x - 3$

$= x^2 + 3x - x - 3$

$= x(x + 3) - 1(x + 3)$

$= (x - 1)(x + 3)$

So, zeroes are 1, – 3.

28. $f(x) = x^2 - 2x + 3$

$\alpha + \beta = 2$

$\alpha\beta = 3$

(a) Roots are $(\alpha + 2, \beta + 2)$

Polynomial is

k {x^2 – (sum of zeroes) x + product of zeroes}

$= k\{x^2 - (\alpha + 2 + \beta + 2)x + (\alpha + 2)(\beta + 2)\}$

$= k\{x^2 - (\alpha + \beta + 4)x + \alpha\beta + 2(\alpha + \beta) + 4\}$

$= k\{x^2 - (2 + 4)x + 3 + 2(2) + 4\}$

$= k\{x^2 - 6x + 11\}$

(b) Sum of zeroes

$$= \frac{\alpha - 1}{\alpha + 1} + \frac{\beta - 1}{\beta + 1}$$

$$= \frac{(\alpha - 1)(\beta + 1) + (\alpha + 1)(\beta - 1)}{(\alpha + 1)(\beta - 1)}$$

$$= \frac{\alpha\beta + \alpha - \beta - 1 + \alpha\beta - \alpha + \beta - 1}{\alpha\beta + \alpha + \beta + 1}$$

$$= \frac{2\alpha\beta - 2}{\alpha\beta + \alpha + \beta + 1}$$

$$= \frac{6 - 2}{3 + 2 + 1}$$

$$= \frac{4}{6} = \frac{2}{3}$$

As $\alpha + \beta = 2$, $\alpha\beta = 3$

$$\text{Product of zeroes} = \left(\frac{\alpha-1}{\alpha+1}\right)\left(\frac{\beta-1}{\beta+1}\right)$$

$$= \frac{(\alpha-1)(\beta-1)}{(\alpha+1)(\beta+1)}$$

$$= \frac{\alpha\beta-(\alpha+\beta)+1}{\alpha\beta+(\alpha+\beta)+1}$$

$$= \frac{3-2+1}{3+2+1}$$

$$= \frac{2}{6}$$

$$= \frac{1}{3}$$

A quadratic polynomial is of form

k {x^2 – (sum of zeroes) x + product of zeroes}

$$= \left\{x^2 \ \frac{-2}{3}x\frac{+1}{3}\right\}$$

$$= \frac{k}{3}\left\{3x^2-2x+1\right\}$$

29.

$$\begin{array}{r|l} & x^2 - 2\sqrt{5}x + 3 \\ \hline x-\sqrt{5} & x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5} \\ & x^3 - \sqrt{5}x^2 \\ & - \quad + \\ \hline & -2\sqrt{5}x^2 + 13x - 3\sqrt{5} \\ & -2\sqrt{5}x^2 + 10x \\ & + \quad - \\ \hline & 3x - 3\sqrt{5} \\ & 3x - 3\sqrt{5} \\ & - \quad + \\ \hline & 0 \end{array}$$

For other zeroes,

Consider $x^2 - 2\sqrt{5}x + 3 = 0$

$$x = \frac{2\sqrt{5}\pm\sqrt{20-12}}{2}$$

$$= \frac{2\sqrt{5}\pm\sqrt{8}}{2}$$

$$= \frac{2\sqrt{5}\pm2\sqrt{2}}{2}$$

$$= \sqrt{5}\pm\sqrt{2}$$

30. $ax^3 + 3x^2 - bx - 6$

$x = -1, -2$

Put the values of x in Equation

We get,

$a(-1)^3 + 3(-1)^2 - b(-1) - 6 = 0$

$\Rightarrow -a + 3 + b - 6 = 0$

$\Rightarrow b - a = 3$ --------(1)

Now $x = -2$

$a(-2)^3 + 3(-2)^2 - b(-2) - 6 = 0$

$\Rightarrow -8a + 12 + 2b - 6 = 0$

$\Rightarrow 2b - 8a + 6 = 0$

$\Rightarrow b - 4a = -3$ --------(2)

from (1) and (2)

$$\begin{array}{r} b - a = 3 \\ b - 4a = -3 \\ - \quad + \qquad + \\ \hline 3a = 6 \end{array}$$

$\Rightarrow a = 2$ put in --------(1)

$b - 3 = 2$

$\Rightarrow$ b = 5 now put this value in Equation

$ax^3 + 3x^2 - bx - 6 = 0$

$\Rightarrow 2x^3 + 3x^2 - 5x - 6 = 0$

Two zeroes are given (–1, –2)

$(x + 1)(x + 2) = x^2 + 2x + x + 2 = 0$

$\Rightarrow x^2 + 3x + 2 = 0$

$$\begin{array}{r|l} & 2x - 3 \\ \hline x^2+3x+2 & 2x^3 + 3x^2 - 5x - 6 \\ & 2x^3 + 6x^2 + 4x \\ & - \quad - \quad - \\ \hline & -3x^2 - 9x - 6 \\ & -3x^2 - 9x - 6 \\ & + \quad + \quad + \\ \hline & 000 \end{array}$$

Hence, another zeroes is

$2x - 3 = 0$

$\Rightarrow x = \frac{3}{2}$

31. As zeroes of q(x) are also the zeroes of p(x), so, remainder should be zero. (As q(x) is a

factor of p(x)

$$\begin{array}{r|l} & x^2 - 3x + 2 \\ x^3 + 2x^2 + a & x^5 + x^4 - 4x^3 + 3x^2 + 3x + b \\ & x^5 + 2x^4 + ax^2 \\ & - \quad - \quad - \\ \hline & - 3x^4 - 4x^3 + (3-a)x^2 + 3x + b \\ & - 3x^4 - 6x^3 \qquad - 3ax \\ & + \quad + \qquad + \\ \hline & 2x^3 + (3-a)x^2 + (3+3a)x + b \\ & 2x^3 + 4x^2 \qquad + 2a \\ & - \quad - \qquad - \\ \hline & (-a-1)x^2 + (3+3a)x + (b-2a) \\ \hline \end{array}$$

Remainder = 0

$(-a-1)x^2 + (3 + 3a)x + (b - 2a) = 0$

$\Rightarrow \quad -a - 1 = 0, \quad b - 2a = 0$

$\Rightarrow \quad a = -1, \quad b + 2 = 0$

$\Rightarrow \quad a = -1, \quad b = -2$

Now,

$$p(x) = (x^3 + 2x^2 + a)(x^2 - 3x + 2) + 0$$
$$= (x^3 + 2x^2 - 1)(x^2 - 3x + 2)$$

For other zeroes of p(x),

Put $x^2 - 3x + 2 = 0$

$$x^2 - 2x - x + 2 = 0$$
$$x(x - 2) - 1(x - 2) = 0$$
$$(x - 1)(x - 2) = 0$$
$$x = 1, 2$$

So, x = 1, 2 are zeroes of p(x) but not of q(x)

32. (i) $p(x) = x^3 - 5x^2 - 16x + 80$

Let the two zeroes be $\alpha, -\alpha$ and the third zero be γ.

$$\alpha + (-\alpha) + \gamma = 5$$
$$\gamma = 5$$

Also $\alpha(-\alpha)\gamma = -80$

$$-\alpha^2 (5) = -80$$
$$\alpha^2 = \frac{80}{5} = 16$$
$$\alpha = \pm 4$$

For $\alpha = -4, \quad -\alpha = -(-4) = 4$

For $\alpha = 4, \quad -\alpha = -4.$

So, zeroes are –4, 4, 5

(ii)
$$f(x) = x^2 - p(x + 1) - c$$
$$= x^2 - px - (p + c)$$
$$\alpha + \beta = p, \quad \alpha\beta = -(p + c)$$

Consider

$$(\alpha + 1)(\beta + 1) = \alpha\beta + (\alpha + \beta) + 1$$
$$= -(p + c) + p + 1$$
$$= 1 - c$$

WORKSHEET 2

Section A

1. f(x) has 2 real zeroes.

2. $x^2 + 7x + 12$
$= x^2 + 3x + 4x + 12$
$= x^2 + (x + 3) + 4(x + 3)$
$= (x + 3)(x + 4)$

For zeroes of polynomial,

$x + 3 = 0, \quad x + 4 = 0$

$x = -3, \quad x = -4$

3. Let a, $\frac{1}{\alpha}$ be the zeroes of p(x)

$$\alpha\frac{1}{\alpha} = \frac{-a}{5}$$
$$1 = \frac{-a}{5}$$
$$a = -5$$

4. f(x) has 2 distinct real zeroes

5. Let $p(x) = x^3 + ax^2 + bx + c$

Let α, β, γ be zeroes of p(x)

Such that $\alpha = -1$

$$\alpha\beta\gamma = -C$$
$$(-1)\beta\gamma = -C$$
$$\beta\gamma = C$$

So, product of other two zeroes = C

6. Quadratic polynomial is of form

k {x^2 – (Sum of zeroes) x + product of zeroes}

$$= k\left\{x^2 - \left(\frac{2}{3} - \frac{1}{4}\right)x + \frac{2}{3}\left(\frac{-1}{4}\right)\right\}$$

$$= k\left\{x^2 - \left(\frac{5}{12}\right)x - \frac{1}{6}\right\}$$

$$= k\left\{\frac{12x^2 - 5x - 2}{12}\right\}$$

$$= \frac{k}{12}(12x^2 - 5x - 2)$$

7. $2y^2 + 7y + 5 = 0$

Here a = 2 , b = 7 , c = 5

α, β are 2 zeroes

So sum of the zeroes = $\alpha + \beta = \frac{-b}{ca} = \frac{-7}{2}$

Product of zeroes = $\alpha\beta = \frac{c}{a} = \frac{5}{2}$

Now just put the values

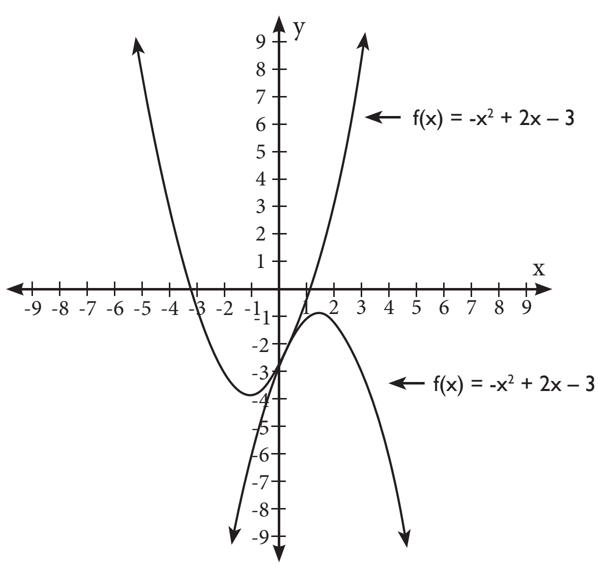
$\alpha + \beta + \alpha\beta = (c + \beta) + \alpha\beta$

$= \frac{-7}{2} + \frac{5}{2}$

$= \frac{-2}{2} = -1$

So required answer is –1

8.



In the above graph, for both the curves we can observe that the sign of c is negative only.

9. $f(x) = (k^2 + 4)x^2 + 13x + 4k$

Let the two zeroes be $\alpha, \frac{1}{\alpha}$

$$1 = \alpha\left(\frac{1}{\alpha}\right) = \frac{4k}{k^2+4}$$

$$k^2 - 4k + 4 = 0$$

$$(k - 2)^2 = 0$$

$$k = 2$$

10. $x^2 + 99x + 127$

$\alpha + \beta = -99, \quad \alpha\beta = 127$

α, β are either both positive or both negative

If α, β are both positive then $\alpha + \beta = -99$ is not possible

So, α and β must be negative.

Section B

11. $f(x) = x^2 - px + q$

$\alpha + \beta = p, \quad \alpha\beta = q$

(i) Consider $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
$= p^2 - 2q$

(ii) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{p}{q}$

12. We know that,

Sum of zeroes = $\frac{-b}{a}$

$\alpha + \beta = 5$ ------(1)

also,

Product of zeroes = $\frac{c}{a}$

$\alpha\beta = k$ ------(2)

Given:

$\alpha - \beta = 1$ ------(3)

From (1) and (3)

$\alpha + \beta = 5$

$\alpha - \beta = 1$ (on adding)

$2\alpha = 6$

$\alpha = 3$

Put this in (3)

$3 - \beta = 1$

$-\beta = -2$

$\beta = 2$

Now put this value in (3)

$\alpha\beta = k$

$2 \times 3 = k$

$6 = k$

13. Quadratic polynomial is of form

$k\{x^2 - (\text{sum of zeroes})x + \text{Product of zeroes}\}$

$$\text{Sum of zeroes} = \frac{4+\sqrt{2}}{2} + \frac{4-\sqrt{2}}{2} = 4$$

$$\text{Product of zeroes} = \left(\frac{4+\sqrt{2}}{2}\right)\left(\frac{4-\sqrt{2}}{2}\right) = \frac{16-2}{4} = \frac{14}{4} = \frac{7}{2}$$

So, quadratic polynomial is

$$k\left\{x^2 - 4x + \frac{7}{2}\right\} = \frac{k}{2}\{2x^2 - 8x + 7\}$$

14.

$$\begin{array}{r|l} & 3x^2 - x \\ \hline 3x^2 + x - 1 & 9x^4 - 4x^2 + 4 \\ & 9x^4 + 3x^3 - 3x^2 \\ & - \quad - \quad + \\ \hline & -3x^3 - x^2 + 4 \\ & -3x^3 - x^2 + 4 \\ & + \quad + \quad - \\ \hline & -x + 4 \\ \hline \end{array}$$

Quotient $= 3x^2 - x$

Remainder $= -x + 4$

15. $f(x) = x^2 - 1 = x^2 + 0x - 1$

$\alpha + \beta = 0, \quad \alpha\beta = -1$

$$\text{Sum of zeroes} = \frac{2\alpha}{\beta} + \frac{2\beta}{\alpha} = 2\left(\frac{\alpha^2 + \beta^2}{\alpha\beta}\right) = 2\left[\frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}\right]$$

$$= \frac{2}{-1}[0 + 2] = -4$$

$$\text{Product of zeroes} = \frac{2\alpha}{\beta}\,\frac{2\beta}{\alpha} = 4$$

A quadratic polynomial is of form $k\{x^2 - (\text{sum of zeroes})\,x + \text{product of zeroes}\}$

$k\{x^2 + 4x + 4\}$

16. $f(x) = ax^2 + bx + c$

$$\alpha + \beta = \frac{-b}{a}, \quad \alpha\beta = \frac{c}{a}$$

Consider
$$\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta} = \frac{(\alpha+\beta)(\alpha^2 + \beta^2 - \alpha\beta)}{\alpha\beta} = \frac{(\alpha+\beta)\left[(\alpha+\beta)^2 - 3\alpha\beta\right]}{\alpha\beta} = \frac{\frac{-b}{a}\left[\frac{b^2}{a^2} - \frac{3c}{a}\right]}{\frac{c}{a}} = \frac{-b}{c}\left(\frac{b^2 - 3ac}{a^2}\right)$$

17. As 1 is a zero of f(x),

so, $(x - 1)$ is a factor of f(x)

$$\begin{array}{r|l} & -x^2 - x + 6 \\ \hline x - 1 & -x^3 + 7x + 6 \\ & -x^3 + x^2 \\ & + \quad - \\ \hline & -x^2 + 7x - 6 \\ & -x^2 + x \\ & + \quad - \\ \hline & 6x - 6 \\ & 6x - 6 \\ & - \quad + \\ \hline & 0 \\ \hline \end{array}$$

For other zeroes of f(x),

put $-x^2 - x + 6 = 0$

$x^2 + x - 6 = 0$

$$x^2 + 3x - 2x - 6 = 0$$
$$x(x + 3) - 2(x + 3) = 0$$
$$(x - 2)(x + 3) = 0$$
$$x = 2, \quad -3$$

So, other zeroes are $x = 2, \quad -3$

18. $f(x) = x^2 - 13x + k$

Let α, β be two zeroes of f(x)

$$\alpha\beta = k = 40$$

So,
$$\begin{aligned} f(x) &= x^2 - 13x + 40 \\ &= x^2 - 5x - 8x + 40 \\ &= x(x - 5) - 8(x - 5) \\ &= -(x - 5)(x - 8) \end{aligned}$$

For zeroes of f(x), put f(x) = 0

i.e. $(x - 5)\quad(x - 8) = 0$

$$x = 5, 8$$

19.

$$\begin{array}{r|l} & 2x^2 - 2x - 1 \\ \hline 4x^2 + 3x - 2 & 8x^4 + 14x^3 - 2x^2 + 7x - 8 \\ & 8x^4 + 6x^3 - 4x^2 \\ & - \quad - \quad + \\ \hline & 8x^3 + 2x^2 + 7x - 8 \\ & 8x^3 + 6x^2 - 4x \\ & - \quad - \quad + \\ \hline & -4x^2 + 11x - 8 \\ & -4x^2 + 4x + 2 \\ & + \quad + \quad - \\ \hline & 15x - 10 \\ \hline \end{array}$$

So, $15x - 10$ must be subtracted from $8x^4 + 14x^3 - 2x^2 + 7x - 8$. So, that the resultant polynomial is exactly divisible by $4x^2 + 3x - 2$.

20. $f(t) = t^2 - 4t + 3$

$\alpha + \beta = 4, \quad \alpha\beta = 3$

Consider

$$\begin{aligned} \alpha^4\beta^3 + \alpha^3\beta^4 &= \alpha^3\beta^3(\alpha + \beta) \\ &= (\alpha\beta)^3(\alpha + \beta) \\ &= 27(4) = 108 \end{aligned}$$

And
$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{4}{3}$$

Section C

21. Let $a - d$, a and $a + d$ be the zeroes of f(x)

$$a - d + a + a + d = 12$$
$$3a = 12$$
$$a = 4$$

Also, $(a - d)\, a\, (a + d) = 28$

$$(4 - d)\, 4\, (4 + d) = 28$$
$$16 - d^2 = 7$$
$$d^2 = 9$$
$$d = \underline{+}3$$

Case 1

$a = 4, \quad d = 3$

So, zeroes are

$a - d, a, a + d = 1, 4, 7$

Case 2

$a = 4, d = -3$

So, zeroes are 7, 4, 1

Therefore, zeroes of polynomial are 1, 4 and 7.

22.

$$\begin{array}{r|l} & 10x^2 + \frac{19}{3}x - \frac{8}{9} \\ \hline 3x^2 - x + 1 & 30x^4 + 9x^3 + x^2 + 2 \\ & 30x^4 - 10x^3 + 10x^2 \\ & - \quad + \quad - \\ \hline & 19x^3 - 9x^2 + 2 \\ & 19x^3 - \frac{19}{3}x^2 + \frac{19}{3}x \\ & - \quad + \quad - \\ \hline & -\frac{8}{3}x^2 - \frac{19}{3}x + 2 \\ & -\frac{8}{3}x^2 + \frac{8}{9}x - \frac{8}{9} \\ & + \quad - \quad + \\ \hline & -\frac{65}{9}x + \frac{26}{9} \\ \hline \end{array}$$

$$\begin{aligned} \text{Dividend} &= 30x^4 + 9x^3 + x^2 + 2 \\ \text{Divisor} &= 3x^2 - x + 1 \\ \text{Quotient} &= 10x^2 + \frac{19}{3}x - \frac{8}{9} \end{aligned}$$

$$\text{Remainder} = -\frac{65}{9}x + \frac{26}{9}$$

According to divisor algorithm,

Dividend = Divisor × Quotient + Remainder

Consider

Divisor × Quotient + Remainder

$= (3x^2 - x + 1)\left(10x^2 + \frac{19}{3}x - \frac{8}{9}\right) - \frac{8x}{9} - \frac{31}{9}$

$= 30x^4 + 19x^3 - \frac{8}{3}x^2 - 10x^3 - \frac{19}{3}x^2 + \frac{8}{9}x +$

$10x^2 + \frac{19}{3}x - \frac{8}{9} - \frac{65}{9}x + \frac{26}{9}$

$= 30x^4 + x^3(19 - 10) + x^2\left(-\frac{8}{3} - \frac{19}{3} + 10\right) +$

$x\left(\frac{8}{9} + \frac{19}{3} - \frac{65}{9}\right) + \left(-\frac{8}{9} + \frac{26}{9}\right)$

$= 30x^4 + 9x^3 + x^2 + 0x + 2$

$= 30x^4 + 9x^3 + x^2 + 2$

= Dividend Hence verified.

23. $f(x) = 4\sqrt{3}\,x^2 + 5x - 2\sqrt{3}$

Sum $= 5\ (8 - 3)$

Product $= -24 \quad (8 \times -3)$

$f(x) = 4\sqrt{3}\,x^2 + 8x - 3x - 2\sqrt{3}$

$= 4x(\sqrt{3}\,x + 2) - \sqrt{3}\ (\sqrt{3}\,x + 2)$

$= (4x - \sqrt{3})(\sqrt{3}\,x + 2)$

For zeroes of f(x), put f(x) = 0

$(4x - \sqrt{3})(\sqrt{3}\,x + 2) = 0$

$x = \frac{\sqrt{3}}{4}, \frac{-2}{\sqrt{3}}$

$$\text{Sum of zeroes} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$= \frac{\sqrt{3}}{4} - \frac{2}{\sqrt{3}} \qquad = -\frac{5}{4\sqrt{3}}$

$= \frac{3-8}{4\sqrt{3}} \qquad = -\frac{5}{4\sqrt{3}}$

$\therefore$ Sum of zeroes $= \frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of zeroes $= \left(\frac{\sqrt{3}}{4}\right)\left(\frac{-2}{\sqrt{3}}\right)$

$= -\frac{1}{2}$

$\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

$= -\frac{2\sqrt{3}}{4\sqrt{3}}$

$= -\frac{1}{2}$

$\therefore$ Product of zeroes $= \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

Hence, the relationship between zeroes and its coefficient is verified.

24. $f(x) = x^2 - x - 2$

$\alpha + \beta = 1, \quad \alpha\beta = -2$

Sum of zeroes $= 2\alpha + 1 + 2\beta + 1$

$= 2(\alpha + \beta) + 2$

$= 2\ (1) + 2$

$= 4$

Product of zeroes $= (2\alpha + 1)(2\beta + 1)$

$= 4\alpha\beta + 2(\alpha + \beta) + 1$

$= -8 + 2 + 1$

$= -5$

Quadratic polynomial is of form

$k\ \{x^2 - (\text{sum of zeroes})\ x + \text{Product of zeroes}\}$

$= \ k\ \{x^2 - 4x - 5\}$

Now, we need to find $\alpha^3 + \beta^3$

$= (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta)$

$= (\alpha + \beta)[(\alpha + \beta)^2 - 3\alpha\beta]$

$= 4(16 + 15)$

$= 4(31)$

$= 124$

25. $f(x) = 3x^2 - 4x + 1$

$$\alpha + \beta = \frac{4}{3}, \quad \alpha\beta = \frac{1}{3}$$

$$\text{Sum of zeroes} = \frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$$

$$= \frac{\alpha^3 + \beta^3}{\alpha\beta}$$

$$= \frac{(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)}{\alpha\beta}$$

$$= \frac{(\alpha+\beta)\left[(\alpha+\beta)^2 - 3\alpha\beta\right]}{\alpha\beta}$$

$$= \frac{\frac{4}{3}\left(\frac{16}{9} - 1\right)}{\frac{1}{3}}$$

$$= 4\left(\frac{16-9}{9}\right) = \frac{28}{9}$$

$$\text{Product of zeroes} = \frac{\alpha^2\beta^2}{\alpha\beta} = \alpha\beta = \frac{1}{3}$$

Quadratic polynomial is of the form k {x^2 – (sum of zeroes) x + product of zeroes}

$$= k\left\{x^2 - \frac{28}{9}x + \frac{1}{3}\right\}$$

$$= \frac{k}{9}\left\{9x^2 - 28x + 3\right\}$$

26.

```
              x² + x  + 7
       ______________________________
x² + 1 ) x⁴ + x³ + 8x² + ax + b
         x⁴ + x²
         –    –
       ______________________________
              x³ + 7x² + ax + b
              x³       + x
              –        –
       ______________________________
                   7x²  + (a – 1) x + b
                   7x²            + 7
                   –              –
       ______________________________
                        (a – 1)x + (b – 7)
```

As $x^4 + x^3 + 8x^2 + ax + b$ is exactly divisible by $x^2 + 1$

$\therefore$ Remainder = 0

$$(a-1)x + (b-7) = 0$$

$$a = 1, \quad b = 7$$

27. Let α, β be the zeroes of f(x)

$$\alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a}$$

Zeroes of the required polynomial are $\frac{1}{\alpha}, \frac{1}{\beta}$

Quadratic polynomial is of form

$$k\left\{x^2 - \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)x + \frac{1}{\alpha\beta}\right\}$$

$$= k\left\{x^2 - \left(\frac{\alpha+\beta}{\alpha\beta}\right)x + \frac{1}{\alpha\beta}\right\}$$

$$= k\left\{x^2 - \left(\frac{-\frac{b}{a}}{\frac{c}{a}}\right)x + \frac{a}{c}\right\}$$

$$= k\left\{x^2 + \frac{b}{c}x + \frac{a}{c}\right\}$$

$$= \frac{k}{c}\{cx^2 + bx + a\}$$

28. $f(x) = x^3 - 4x^2 - 3x + 12$

As $\sqrt{3}, -\sqrt{3}$ are zeroes of f(x), so $(x - \sqrt{3})$ $(x + \sqrt{3})$ are factors of f(x).

i.e. $(x - \sqrt{3})(x + \sqrt{3}) = x^2 - 3$ is a factor of f(x)

factor of f(x)

```
              x – 4
       ________________________
x² – 3 ) x³  – 4x²  – 3x + 12
         x³       – 3x
       –          +
       ________________________
            – 4x²      + 12
            – 4x²      + 12
            +          –
       ________________________
                  0
```

For third zero, $x - 4 = 0$

$$x = 4$$

29. $p(x) = 2x^2 + 5x + k$

$$\alpha + \beta = -\frac{5}{1}, \quad \alpha\beta = \frac{k}{2}$$

Given: $\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$

$$\Rightarrow (\alpha + \beta)^2 - \alpha\beta = \frac{21}{4}$$

$$\Rightarrow \frac{25}{4} - \frac{k}{2} = \frac{21}{4}$$

$$\Rightarrow \frac{k}{2} = \frac{25}{4} - \frac{21}{4} = 1$$

$$\Rightarrow k = 2$$

30.

$$\begin{array}{r|l} & x^2 + 2x + 3 \\ \hline x^2 + 5 & x^4 + 2x^3 + 8x^2 + 12x + 18 \\ & x^4 + 5x^2 \\ & - \quad - \\ \hline & 2x^3 + 3x^2 + 12x + 18 \\ & 2x^3 + 10x \\ & - \quad - \\ \hline & 3x^2 + 2x + 18 \\ & 3x^2 + 15 \\ & - \quad - \\ \hline & 2x + 3 \\ \hline \end{array}$$

On comparing $2x + 3$ with $px + q$,

we get $p = 2, \quad q = 3$

Section D

31.

$$\begin{array}{r|l} & x^2 - 4x + (8 - k) \\ \hline x^2 - 2x + k & x^4 - 6x^3 + 16x^2 - 25x + 10 \\ & x^4 - 2x^3 + kx^2 \\ & - \quad + \quad - \\ \hline & -4x^3 + (16 - k)x^2 - 25x + 10 \\ & -4x^3 + 8x^2 - 4kx \\ & + \quad - \quad + \\ \hline & (8 - k)x^2 + (-25 + 4k)x + 10 \\ & (8 - k)x^2 - 2(8 - k)x + k(8 - k) \\ \hline & (-9 + 2k)x + (10 - 8k + k^2) \\ \hline \end{array}$$

Remainder $= (-9 + 2k)x + (10 - 8k + k^2)$

$= x + \alpha$

$\frac{[x^4 - 6x^3 + 16x^2 - 25x + 10]}{x^2 - 2x + k}$ and remainder = x+a

On diving the above given equation, we get,

Given the reminder is $(x + a)$

$(4k - 25 + 16 - 2k)x + [10 - k(8 - k)] = x + a$

$(2k - 9)x + [10 - 8k + k^2] = x + a$

On comparing on both sides, we get

$2k - 9 = 1$

$2k = 10$

Therefore, $k = 5$

Also, $10 - 8k + k^2 = a$

$10 - 8(5) - 5^2 = a$

$10 - 40 + 25 = a$

Therefore, $a = -5$

32. $f(x) = x^4 - 6x^3 - 26x^2 + 138x - 35$

Zeroes of f(x) are $2 \pm \sqrt{3}$.

So, $[x - (2 + \sqrt{3})], \quad [x - (2 - \sqrt{3})]$

are factors of f(x)

i.e. $[(x - 2) - \sqrt{3}]\,[(x - 2) + \sqrt{3}]$ is a factor of f(x).

i.e. $(x - 2)^2 - (\sqrt{3})^2$ is a factor of f(x)

i.e. $x^2 + 4 - 4x - 3$ is a factor of f(x)

i.e. $x^2 - 4x + 1$ is a factor of f(x)

$$\begin{array}{r|l} & x^2 - 2x - 35 \\ \hline x^2 - 4x + 1 & x^4 - 6x^3 - 26x^2 + 138x - 35 \\ & x^4 - 4x^3 + x^2 \\ & - \quad + \quad - \\ \hline & -2x^3 - 27x^2 + 138x \\ & -2x^3 + 8x^2 - 2x \\ & + \quad - \quad + \\ \hline & -35x^2 + 140x - 35 \\ & -35x2 + 140x - 35 \\ & + \quad - \quad + \\ \hline & 0 \\ \hline \end{array}$$

For other zeroes

Put $x^2 - 2x - 35 = 0$

$x^2 - 7x + 5x - 35 = 0$

$x(x - 7) + 5(x - 7) = 0$

$$(x + 5)(x - 7) = 0$$
$$x = -5, 7$$

So, other zeroes are –5 and 7.

33. $f(x) = x^3 - 5x^2 - 2x + 24$

Let α, β, γ be the zeroes of f(x).

$$\alpha\beta = 12 \quad ...(i)$$
$$\alpha + \beta + \gamma = 5$$
$$\alpha\beta\gamma = -24 \Rightarrow 12\gamma = -24$$
$$\Rightarrow \gamma = -2$$

Also, $\alpha + \beta + \gamma = 5 \Rightarrow \alpha + \beta - 2 = 5$

$$\alpha + \beta = 7 \quad ...(iii)$$

On solving (i) and (ii), we get

$$\alpha(7 - \alpha) = 12$$
$$7\alpha - \alpha^2 = 12$$
$$\alpha^2 - 7\alpha + 12 = 0$$
$$\alpha^2 - 3\alpha - 4\alpha + 12 = 0$$
$$\alpha(\alpha - 3) - 4(\alpha - 3) = 0$$
$$(\alpha - 3)(\alpha - 4) = 0$$
$$\alpha = 3, 4$$

If $\alpha = 3, \beta = 7 - \alpha = 7 - 3 = 4$

If $\alpha = 4, \beta = 7 - \alpha = 7 - 4 = 3$

So, zeroes of the polynomial are 3, 4 and –2.

34. $f(x) = 2x^4 - 2x^3 - 7x^2 + 3x + 6$

$-\sqrt{\frac{3}{2}}$ and $\sqrt{\frac{3}{2}}$ are zeroes of f(x)

$\left(x + \sqrt{\frac{3}{2}}\right), \left(x - \sqrt{\frac{3}{2}}\right)$ are factors of f(x)

$\left(x + \sqrt{\frac{3}{2}}\right)\left(x - \sqrt{\frac{3}{2}}\right)$ is a factor of f(x)

$\left(x^2 - \frac{3}{2}\right)$ is a factor of f(x)

$(2x^2 - 3)$ is a factor of f(x)

$$\begin{array}{r|l} & x^2 - x - 2 \\ \hline 2x^2 - 3 & 2x^4 - 2x^3 - 7x^2 + 3x + 6 \\ & \underline{\overset{-}{2x^4} \quad \overset{+}{- 3x^2}} \\ & -2x^3 - 4x^2 + 3x + 6 \\ & \underline{\overset{+}{-2x^3} \quad \overset{-}{+ 3x}} \\ & -4x^2 + 6 \\ & \underline{\overset{+}{-4x^2} \quad \overset{-}{+ 6}} \\ & 0 \end{array}$$

For other zeroes of f(x)

Put $x^2 - x - 2 = 0$

$$x^2 - 2x + x - 2 = 0$$
$$x(x - 2) + 1(x - 2) = 0$$
$$(x + 1)(x - 2) = 0$$
$$x = -1, 2$$

35. $f(x) = 6x^2 + x - 2$

$\alpha + \beta = -\frac{1}{6}, \quad \alpha\beta = -\frac{1}{3}$

(i) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$

$$= \frac{\frac{1}{36} + \frac{2}{3}}{-\frac{1}{3}} = \frac{\frac{1+24}{36}}{-\frac{1}{3}}$$

$$= \frac{25}{36} \times -\frac{3}{1} = -\frac{25}{12}$$

(ii) $2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) = 2\left(\frac{\alpha+\beta}{\alpha\beta}\right)$

$$= 2\left(\frac{-\frac{1}{6}}{-\frac{1}{3}}\right)$$

$$= 2\left(\frac{1}{2}\right)$$

$$= 1$$

(iii) $\alpha^3 + \beta^3$

$= (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta)$

$= (\alpha + \beta)[(\alpha + \beta)^2 - 3\alpha\beta)$

$= -\frac{1}{6}\left[\frac{1}{36}+1\right]$

$= -\frac{1}{6}\left[\frac{37}{36}\right]$

$= -\frac{37}{216}$

(iv) $\alpha^3\beta^3 - \alpha^5\beta^5$

$= \alpha^3\beta^3(1 - \alpha^2\beta^2)$

$= \left(-\frac{1}{3}\right)^3\left(1-\frac{1}{9}\right)$

$= -\frac{1}{27}\left(\frac{9-1}{9}\right)$

$= -\frac{1}{27}\left(\frac{8}{9}\right) = -\frac{8}{243}$

36. (i) Let $p(x) = 8$

$g(x) = 3$

$q(x) = 2$

$r(x) = 2$

deg $p(x)$ = deg $q(x) = 0$

(ii) Let $p(x) = 15$

$g(x) = 4$

$q(x) = 2$

$r(x) = 7$

deg $q(x)$ = deg $r(x) = 0$

(iii) Let $p(x) = 20$

$g(x) = 3$

$r(x) = 2$

$q(x) = 6$

Here, deg $r(x) = 0$

37. Let $f(x) = 2x^3 + x^2 - 5x + 2$

$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{8}\right)+\frac{1}{4}-\frac{5}{2}+2$$

$$= \frac{1}{4}+\frac{1}{4}-\frac{5}{2}+2$$

$$= \frac{1}{2}+2-\frac{5}{2}$$

$$= \frac{5}{2}-\frac{5}{2}=0$$

So, $\frac{1}{2}$ is a zero of $f(x)$

$f(1) = 2 + 1 - 5 + 2 = 0$

So, 1 is a zero of $f(x)$

$f(-2) = 2(-8) + 4 + 10 + 2$

$= -16 + 16$

$= 0$

So, –2 is a zero of $f(x)$.

Sum of zeroes $\frac{-\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$

$= \frac{1}{2} + 1 - 2$ $\quad = -\frac{1}{2}$

$= -\frac{1}{2}$

So, Sum of zeroes $= \frac{-\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$

Sum of product of zeroes taken two at a time $\frac{\text{Coefficient of } x}{\text{Coefficient of } x^3}$

$= \frac{1}{2}(1) + 1(-2) + \frac{1}{2}(-2) = -\frac{5}{2}$

$= \frac{1}{2} - 2 - 1$

$= \frac{1}{2} - 3$

$= -\frac{5}{2}$

So, sum of product of zeroes taken two at a time $= \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3}$

Product of zeroes $\frac{-\text{Constant term}}{\text{Coefficient of } x^3}$

$= \frac{1}{2}(1)(-2)$ $\quad = -\frac{2}{2}$

$= -1$ $\quad = -1$

So, product of zeroes = $\dfrac{-\text{Constant term}}{\text{Coefficient of } x^3}$

Hence, relationship between zeroes and the coefficients is verified.

38. $f(x) = x^3 + 13x^2 + 32x + 20$

$f(x) = (-2)^3 + 13(-2)^2 + 32(-2) + 20$

$= -8 + 52 - 64 + 20$

$= 12 - 12$

$= 0$

$\Rightarrow$ x + 2 is a factor of f(x)

$$\begin{array}{r|l} & x^2 + 11x + 10 \\ \hline x+2 & x^3 + 13x^2 + 32x + 20 \\ & x^3 + 2x^2 \\ & - \quad - \\ \hline & 11x^2 + 32x + 20 \\ & 11x^2 + 22x \\ & - \quad - \\ \hline & 10x + 20 \\ & 10x + 20 \\ & - \quad - \\ \hline & 0 \end{array}$$

For other zeroes of f(x),

put $x^2 + 11x + 10 = 0$

$x^2 + 10x + x + 10 = 0$

$x(x + 10) + 1(x + 10) = 0$

$(x + 1)(x + 10) = 0$

$x = -1, -10$

So, zeroes of f(x) are –2, –1, –10

39. $f(x) = ax^2 + bx + c$

$\alpha + \beta = -\dfrac{b}{a}, \quad \alpha\beta = \dfrac{c}{a}$

(i) $\alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha + \beta)$

$= \dfrac{c}{a}\left(-\dfrac{b}{a}\right) = -\dfrac{bc}{a^2}$

(ii) $\alpha^4 + \beta^4 = (\alpha^2)^2 + (\beta^2)^2$

$= [\alpha^2 + \beta^2]^2 - 2\alpha^2\beta^2$

$= [(\alpha + \beta)^2 - 2\alpha\beta]^2 - 2(\alpha\beta)^2$

$= \left(\dfrac{b^2}{a^2} - \dfrac{2c}{a}\right)^2 - \dfrac{2c^2}{a^2}$

$= \dfrac{1}{a^4}\left(b^2 - 2ac\right)^2 - \dfrac{2c^2}{a^2}$

$= \dfrac{1}{a^4}\left(b^4 + 4a^2c^2 - 4ab^2c\right) - \dfrac{2c^2}{a^2}$

$= \dfrac{b^4}{a^4} + \dfrac{4c^2}{a^2} - \dfrac{4b^2c}{a^3} - \dfrac{2c^2}{a^2}$

$= \dfrac{b^4}{a^4} - \dfrac{4b^2c}{a^3} + \dfrac{2c^2}{a^2}$

40. $f(x) = x^3 - 6x^2 + 3x + 10$

$a + (a + b) + (a + 2b) = 6$

$3a + 3b = 6$

$$\boxed{\begin{aligned} a + b &= 2 \\ b &= 2 - a \end{aligned}}$$

$a(a + b)(a + 2b) = -10$

$a(2)(4 - a) = -10$

$2a(4 - a) = -10$

$8a - 2a^2 = -10$

$2a^2 - 8a - 10 = 0$

$a^2 - 4a - 5 = 0$

$a^2 - 5a + a - 5 = 0$

$a(a - 5) + 1(a - 5) = 0$

$(a + 1)(a - 5) = 0$

$a = -1, 5$

For $a = -1$, $b = 2 - a = 2 - (-1) = 3$

For $a = 5$, $b = 2 - a = 2 - 5 = -3$

$a = -1$, $b = 3$

zeroes are a, a + b, a + 2b

= –1, –1 + 3, –1 + 6

= –1, 2, 5

a = 5, b = –3

zeroes are a, a + b, a + 2b

= 5, 5 – 3, 5 – 6

= 5, 2, –1

So, zeroes of the given polynomial are –1, 2 and 5.

Chapter 03

Pair of Linear Equations in Two Variables

MULTIPLE CHOICE QUESTIONS

1. (c) The system of equations has no solution if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

i.e. $\frac{1}{2} = \frac{1}{k} \neq \frac{4}{3}$

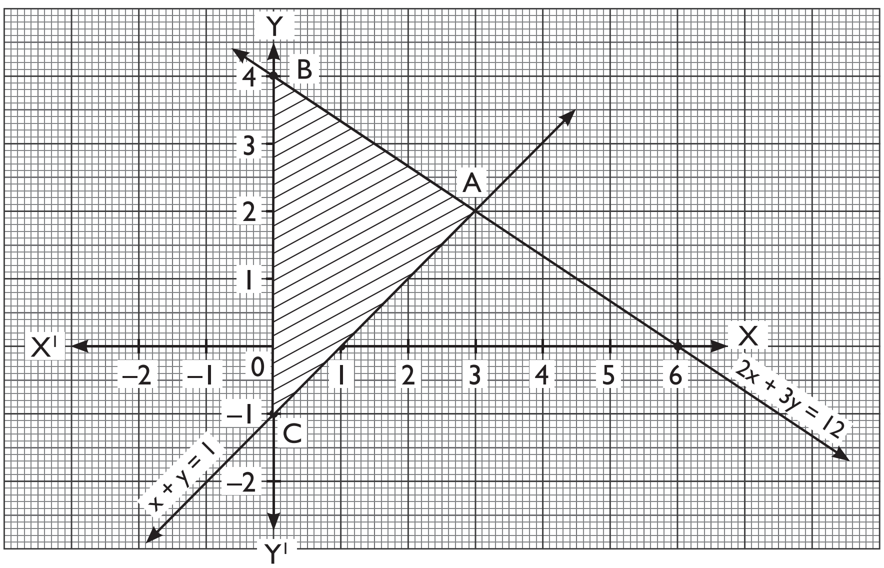
i.e. $k = 2$

2. For no solution, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

i.e $\frac{k}{6} = -\frac{5}{2} \neq \frac{2}{7}$

i.e. $k = -15$

3. (a)



$2x + 3y = 12$

$x - y = 1$

$x = 0$

x	6	0
y	0	4

x	1	0
y	0	–1

area of $\Delta ABC = \frac{1}{2} \times 5 \times 3$

$= \frac{15}{12} = 7.5$ sq. units

4. (c) Let number of coins of ₹ 1 = x

number of coins of ₹ 2 = y

$\therefore \quad x + y = 50$

$\underline{-x \; \underline{+} \; 2y = \underline{-}75}$

$-y = -25$

$y = 25$

So, $x = 50 - y$

$= 50 - 25$

$= 25$

5. (b) Let x be the tens digit and y be the ones digit.

$\therefore \quad x + y = 9 \quad$ ...(i)

and $10x + y + 27 = 10y + x$

$9x - 9y = -27$

$x - y = -3 \quad$ ....(ii)

On solving (i) and (ii), we get

$\therefore \quad x + y = 9$

$x - y = -3$

$2x = 6$

$x = 3$

From (i), $y = 9 - x = 9 - 3 = 6$

So, number is $10x + y$

$= 10(3) + 6 = 36$

WORKSHEET 1

Section A

1. $3x - y + 8 = 0, \quad 6x - ky + 16 = 0$

   The given equations are 3x-y+8=0 and 6x-ky+16 = 0. We have to find the point at which both the equations represent coincident lines.

   For the lines to be coincident,

   Substituting the values, we get

   Either or

   k = 2 or k = 2

   Therefore for k = 2, both the equations represent coincident lines.

2. Let number of girls be x and number of boys be y.

$$x + y = 15 \quad ...(i)$$
$$x = 5 + y \Rightarrow x - y = 5 \quad ...(ii)$$

   On solving equations (i) and (ii), we get

$$x + y = 15$$
$$x - y = 5$$
$$2x = 20$$
$$x = 10$$
$$y = 15 - x = 15 - 10 = 5$$

3. General form of a pair of linear equations in two variables x and y is

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

4. If a pair of linear equations in two variables is consistent, then the lines are either intersecting or coincident.

5. 2x + 3y = 7

   8x + (a + b)y = 28

   Given pair of equations has infinitely many solutions if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{2}{8} = \frac{3}{a+b} = \frac{7}{28}$$

$$a + b = \frac{3 \times 28^4}{7} = 12$$

$$a + b = 12$$

6. The pair of linear equations has infinitely many solutions if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{10}{20} = \frac{5}{10} = \frac{k-5}{k}$$

$$\left(\frac{5}{10} = \right) \frac{1}{2} = \frac{k-5}{k}$$

$$k = 2k - 10$$

$$10 = k$$

7. $2x + 3y = 7$

   (a + b)x + (2a – b)y = 21

   System of equations has infinitely many

   solutions if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

   i.e. $\frac{2}{a+b} = \frac{3}{2a-b} = \frac{7}{21}$

   4a – 2b = 3a + 3b, $\quad 2a - b = \frac{3 \times 21}{7} = 9$

   a = 56, $\quad$ 2a – b = 9

   a = 5(1) $\quad$ 2 (5) – b = 9

   a = 5 $\quad$ b = 10 – 9

   b = 1

8. ax + by = c

   lx + my = n

$$\frac{a}{l} \neq \frac{b}{m} \text{ will give unique Solution}$$

$$\frac{a}{l} = \frac{b}{m} = \frac{c}{n} \quad \text{(infinite solution)}$$

$$\frac{a}{l} = \frac{b}{m} \neq \frac{c}{n} \quad \text{(no solution)}$$

$$\frac{a}{l} \neq \frac{b}{m} \text{ will give unique Solution}$$

Section B

9. The given equation is

$2x + y = 4$

$3y - 2x = 3$

Now, let us find atleast two solutions of each of the above equations, as shown in the following tables.

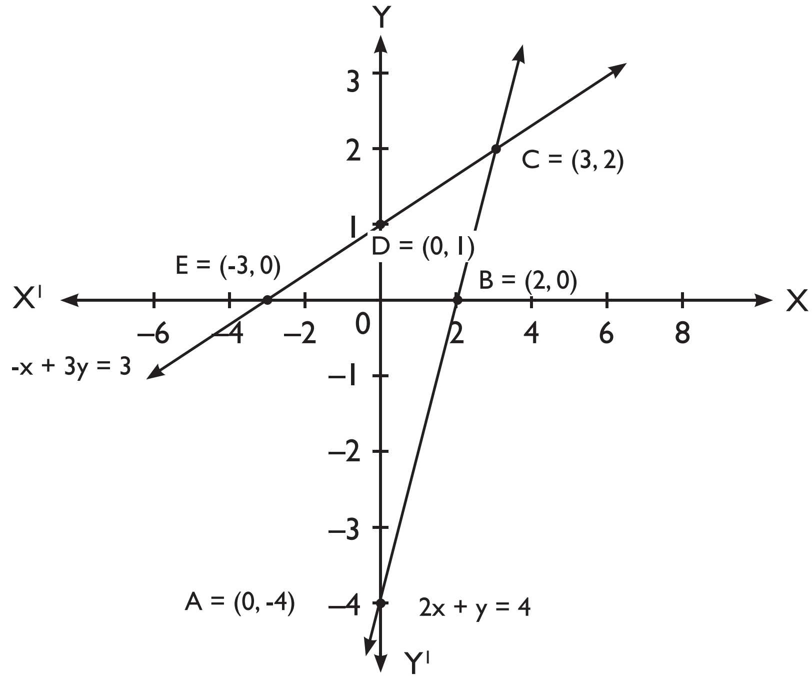
Table for $2x - y = 4$ or $y = 2x - 4$

x	0	2	3
$y = 2x - 4$	–4	0	2

Now table for $3y - 2x = 3$ or

$$y = \frac{x+3}{3}$$

x	0	–3	3
$y = \frac{x+3}{3}$	1	0	2



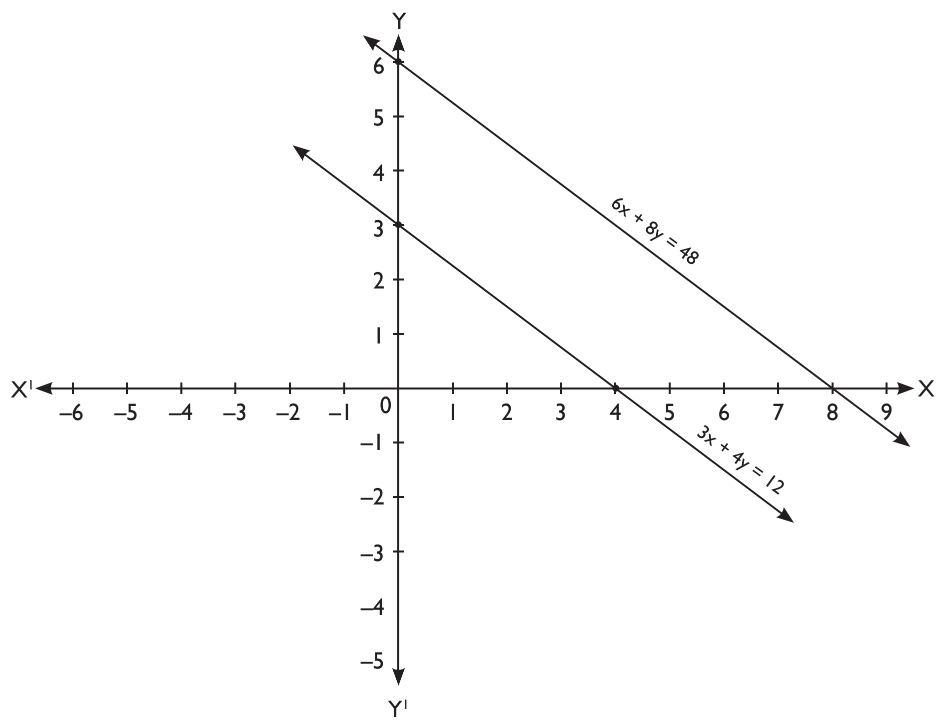
Here, the line intersecting at point C i.e. (3, 2)
The point which intersects at y axis are (0, –4) and D(0, 1)

10. $3x + 4y = 12$

$6x + 8y = 48$

x	4	0
y	0	3

x	8	0
y	0	6



11. (i) $x + 2y = -1$

(ii) $2x - 3y = 12$

From (i), $x = -1 - 2y$

$2(-1 - 2y) - 3y = 12$ (Put in (ii)

$-2 - 4y - 3y = 12$

$-7y = 14$

$y = -2$

So, $x = -1 - 2y$

$= -1 - 2(-2)$
$= -1 + 4$
$= 3$

12. $\frac{2x}{a} + \frac{y}{b} = 2$ ...(i)

$\frac{x}{a} - \frac{y}{b} = 2$ ...(ii)

From (ii), we have $\frac{x}{a} = 4 + \frac{y}{b}$

$x = a\left(4 + \frac{y}{b}\right)$ ...(iii)

Putting this value of x in (i), we get

$$\frac{2a}{a}\left(4 + \frac{y}{b}\right) + \frac{y}{b} = 2$$
$$2\left(4 + \frac{y}{b}\right) + \frac{y}{b} = 2$$
$$8 + \frac{2y}{b} + \frac{y}{b} = 2$$
$$\frac{3y}{b} = -b$$
$$y = \frac{-6b}{3} = -2b$$

From (iii), $x = a\left(4 + \frac{y}{b}\right)$
$= a\left(4 - \frac{2b}{b}\right)$
$= a(4 - 2)$
$= 2a$

13. $28x + 5y = 9$ ...(i)

$3x + 2y = 4$ ...(ii)

On multiplying (i) by 2 and (ii) by 5, we get

$$\begin{array}{rcl} 56x + 10y &=& 18 \\ -\ 15x + 10y &=& 20 \\ \quad - \quad - && - \\ \hline 41x &=& -2 \end{array}$$

$$x = -\frac{2}{41}$$

From (i), $28\left(-\frac{2}{41}\right) + 5y = 9$

$$-\frac{56}{41} + 5y = 9$$
$$5y = 9 + \frac{56}{41} = \frac{425}{41}$$
$$y = \frac{85}{41}$$

14. Let $\frac{1}{x} = p,\ \frac{1}{y} = q$

$2p + \frac{2}{3}q = \frac{1}{6} \Rightarrow 12p + 4q = 1$

Other equation becomes $3p + 2q = 0$

On solving equation $12p + 4q = 1$ and $3p + 2q = 0$, we get

$$12p + 4q = 1$$
$$2(3p + 2q = 0)$$

$$\begin{array}{rcl} 12p + 4q &=& 1 \\ -\ 6p + 4q &=& 0 \\ \quad - && - \\ \hline 6p &=& 1 \end{array}$$

$$p = \frac{1}{6} = x = \frac{1}{p} = 6$$

From equation $3p + 2q = 0$, we get

$$3\left(\frac{1}{6}\right) + 2q = 0$$
$$2q = -\frac{1}{2}$$
$$q = -\frac{1}{4} \Rightarrow y = -4$$

Now, we need to find a

$$y = ax - 4$$
$$-4 = 6a - 4$$
$$6a = 0$$
$$a = 0$$

15. $2x + y = 35$ (i), $3x + 4y = 65$ (ii)

On multiplying equation (i) by 3 and equation (ii) by 2, we get

$$\begin{aligned} 6x + 3y &= 105 \\ 6x + 8y &= 130 \\ - \quad - \quad & \quad - \\ \hline -5y &= -25 \\ y &= 5 \end{aligned}$$

From (i) $\quad 2x + 5 = 35$

$2x = 30$

$x = 15$

16. For unique solution : $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\frac{k}{3} \neq \frac{2}{1}$

$k \neq 6$

For Infinitely many solutions: $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\frac{k}{3} = \frac{2}{1} = \frac{5}{2.5}$

$k = 6$

17. $2x + ky = 11$

$5x - 7y = 5$

For no solution: $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\frac{2}{5} = \frac{k}{-7} \neq \frac{11}{5}$

$5k = -14$

$k = \frac{-14}{5}$

For unique solution:

$\frac{2}{5} \neq \frac{k}{-7}$

$k \neq -\frac{14}{5}$

18. The given system of equation is

$8x + 5y - 9 = 0$

$kx + 10y - 18 = 0$

The system of equation is of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = 8$, $b_1 = 5$, $c_1 = -9$

And $a_2 = k$, $b_2 = 10$, $c_2 = -18$

For a unique solution, we must have

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \quad \frac{8}{k} = \frac{5}{10} = \frac{-9}{-18}$

Now

$\frac{8}{k} = \frac{5}{10}$

$\Rightarrow \quad 8 \times 10 = 5 \times k$

$\Rightarrow \quad \frac{8 \times 10}{5} = k$

$\Rightarrow \quad k = 8 \times 2 = 16$

Hence, the given system f equations will have infinitely many solutions, if k = 16

Section C

19. Let the two no be x and y

Therefore $\frac{x}{y} = \frac{5}{6}$

$6 \times x = 5 \times y$

$6 \times x - 5 \times y = 0 \qquad \times 5$

$30 \times x - 25 \times y = 0 \qquad$ ....(i)

Also $\frac{(x-8)}{(y-8)} = \frac{4}{5}$

$5 \times x - 40 = 4 \times y - 32$

$5 \times x - 4 \times y - 8 = 0 \qquad \times 6$

$30 \times x - 24 \times y - 48 = 0 \qquad$ ....(ii)

Subtracting (i) from (ii)

$y - 48 = 0$

$y = 48$

Putting y = 48 in $\frac{x}{y} = \frac{5}{6}$

$\frac{x}{48} = \frac{5}{6}$

$x = 5 \times \frac{48}{6}$

$x = 5 \times 8$

$x = 40$

Therefore the two numbers are 40 and 48.

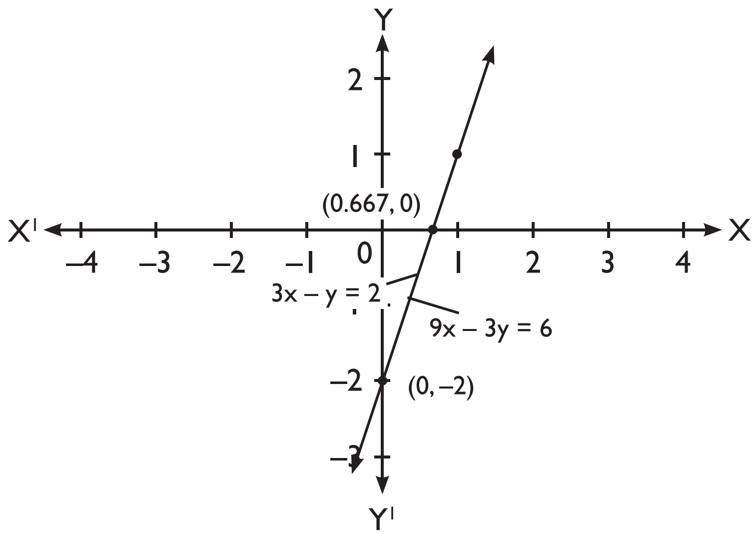
20. $\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$

Thus this equation has infinite solution

x = 0 and y = –2

x = 1 and y = 1

Both lines overlap, thus they are having infinite solutions.



21. $\frac{5}{x+y} - \frac{2}{x-y} = -1, \quad \frac{15}{x+y} + \frac{7}{x-y} = 10$

Let $\frac{1}{x+y} = p$ and $\frac{1}{x-y} = q$

5p – 2q = – 1,   15p + 7q = 10

Using elimination method, we get

$$3(5p - 2q) = -1) \Rightarrow \quad 15p - 6q = -3$$
$$15p + 7q = 10$$
$$- \quad - \quad -$$
$$-13q = -13$$
$$q = 1$$

$$\boxed{x - y = 1}$$

From equation 5p – 2q = –1, we get

$$5p - 2(1) = -1$$
$$5p = 1$$
$$p = \frac{1}{5}$$

$$\boxed{x + y = 5}$$

On solving equations x + y = 5 and x – y = 1, we get

$$x - y = 1$$
$$x + y = 5$$
$$2x = 6$$

$$x = 3 \Rightarrow y = 5 - 3 = 2$$

22. x – 4 = 0

$\Rightarrow$ x = 4

$f(x) = x^3 + ax^2 + 2bx - 24$

$f(4) = 4^3 + a.4^2 + 2.b.4 - 24 = 0$

$\Rightarrow 64 + 16a + 8b = 24$

$\Rightarrow \quad 16a + 8b = 24 - 64$

$\Rightarrow \quad 2a + 1b = 3 - 8$

$\Rightarrow \quad 2a + b = -5$ ....(i)

a – b = 8 ....(ii)

Solving eq.(i) & (ii)

2a + b = –5

a – b = 8

3a = 3

$\Rightarrow$ a = 1

Substituting the value of a in eq.(ii)

1 – b = 8

$\Rightarrow$ b = –7

23. Let number of rows be x and number of students in each row be y. So, total number of students = xy

According to question,

$$(y + 3)(x - 1) = xy$$
$$xy + 3x - y - 3 = xy$$
$$3x - y = 3 \quad ...(i)$$

Again, $(y - 3)(x + 2) = xy$

$$xy + 2y - 3x - 6 = xy$$

$$-3x + 2y = 6 \quad ...(ii)$$

On solving equations (i) and (ii), we get

$$2(3x - y = 3) \Rightarrow \quad 6x - 2y = 6$$
$$-3x + 2y = 6$$
$$\overline{3x = 12}$$
$$x = 4$$

From (i) $\quad y = 3x - 3$
$$= 12 - 3$$
$$= 9$$

So, Total number of students = xy
$$= 4(9)$$
$$= 36$$

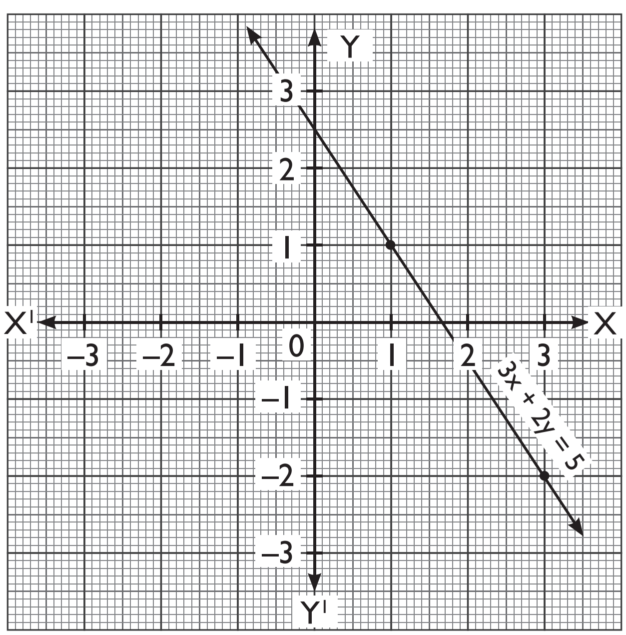
24. $\quad 3x + 2y = 5$
$$3x = 5 - 2y$$
$$x = \frac{5-2y}{3}$$

To check : (1, 1) is a point on the 3x + 2y = 5

$$\text{LHS} = 3x + 2y$$
$$= 3(1) + 2(1)$$
$$= 5$$
$$= \text{RHS}$$

So, (1, 1) is a point on the line 3x + 2y = 5

x	1	3
y	1	–2



25. Let digit at ten's place be x and digit at unit's place be y

So, number = 10x + y

According to question,

$$x + y = 5 \quad ...(i)$$
$$10y + x = 10x + y + 9$$
$$0 = 9x - 9y + 9$$
$$x - y = -1 \quad ...(ii)$$

On solving (i) and (ii), we get

$$x + y = 5$$
$$x - y = -1$$
$$\overline{2x = 4}$$
$$x = 2$$

From (i), $\quad y = 5 - 2 = 3$

So, $\quad$ number $= 10x + y$
$$= 10(2) + 3$$
$$= 23$$

26. Let the adjacent angle be x.

$$\text{Other angle} = \frac{4}{5}x$$

As sum of adjacent angles of a parallelogram is 180°,

$$x + \frac{4}{5}x = 180$$
$$\frac{9x}{5} = 180$$
$$x = \frac{\cancel{180}^{20} \times 5}{\cancel{9}} = 100°$$

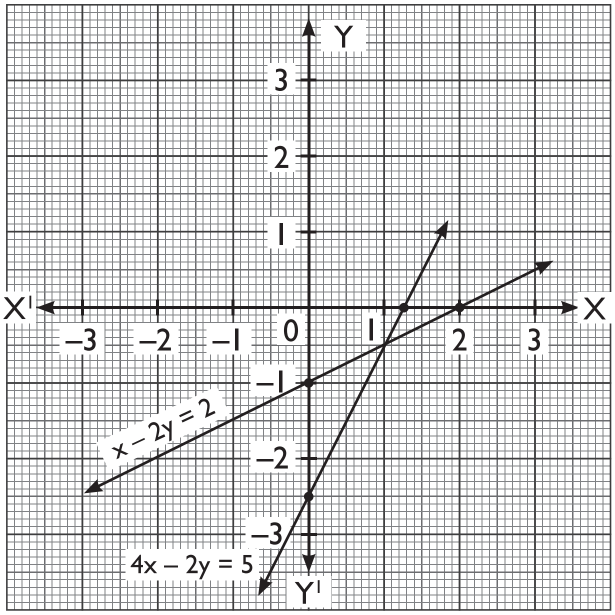
Angles are x, $\frac{4}{5}x$

$$= 100, \frac{4}{5}(100)$$
$$= 100, 80$$

27. (i) $\quad x - 2y = 2, \qquad 4x - 2y = 5$

x	0	2
y	–1	0

x	0	1.25
y	–2.5	0

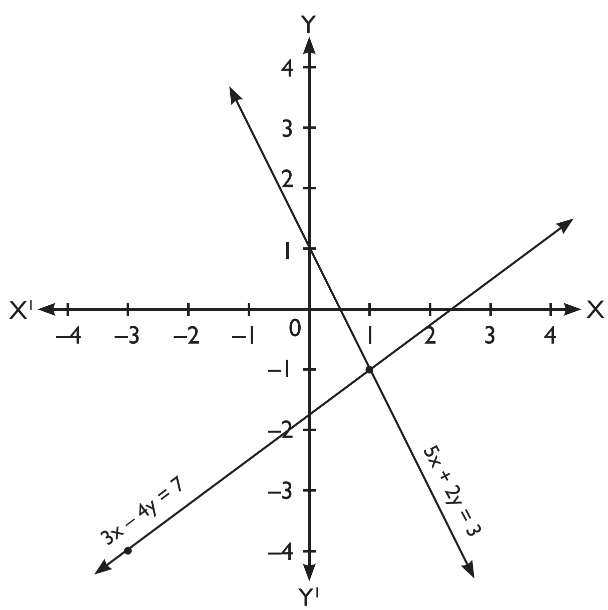


As the lines are intersecting, so the system of equations has unique solution and hence, consistent.

(ii) $3x - 4y = 7$, $5x + 2y = 3$

x	–3	1
y	–4	–1

x	1	2
y	–1	–3.5



28. In ΔABC,

$$\angle A + \angle B + \angle C = 180° \text{ (Angle sum property)}$$

$$x + 3x + y = 180°$$

$$4x + y = 180° \quad ...(i)$$

Given: $3y - 5x = 30 \quad ...(ii)$

From (i), $y = 180 - 4x$

So, eq^n (ii) becomes $3(180 - 4x) - 5x = 30$

$$540 - 12x - 5x = 30$$

$$17x = 510$$

$$x = 30$$

From (i), $y = 180 - 4(30)$

$$= 180 - 120$$

$$= 60°$$

So, $\angle A = x = 30°$

$\angle B = 3x = 90°$

$\angle C = y = 60°$

In ΔABC, $\angle B = 90°$, so it is a right angled triangle

Section D

29. Let speed of boat in still water be x km/hr and that of stream be y km/hr.

So, speed of boat upstream = (x – y) km/hr

Speed of boat downstream = (x + y) km/hr

According to question,

$$\frac{32}{x-y} + \frac{36}{x+y} = 7$$

$$\frac{40}{x-y} + \frac{48}{x+y} = 9$$

Let $\frac{1}{x-y} = p$, $\frac{1}{x+y} = q$

So, we get equations as

$$32p + 36q = 7 \quad ...(i)$$

$$40p + 48q = 9 \quad ...(ii)$$

On multiplying (i) by 5 and (ii) by 4, we get

$$160p + 180q = 35$$

$$\underset{-}{}160p \underset{-}{+} 192q = \underset{-}{}36$$

$$-\ 12q = -1$$

$$q = \frac{1}{12}$$

i.e. $x + y = 12$ ...(iii)

From (i), $32p + 36\left(\frac{1}{12}\right) = 7$

$$32p = 7 - 3 = 4$$

$$p = \frac{1}{8}$$

$$x - y = 8 \quad \text{...(iv)}$$

On solving (iii) and (iv), we get

$$x = 10$$

$$y = 2$$

Speed of boat in still water = 10 km/hr

Speed of stream = 2 km/hr

30. $ax + by = 1$ ...(i)

$$bx + ay = \frac{(a+b)^2}{a^2+b^2} - 1 \quad \text{...(ii)}$$

$$= \frac{a^2+b^2+2ab-a^2-b^2}{a^2+b^2}$$

$$bx + ay = \frac{2ab}{a^2+b^2} \quad \text{...(iii)}$$

On multiplying (i) by b and (iii) by a, we get

$$abx + b^2y = b$$

$$\underset{-}{abx} + \underset{-}{a^2y} = \underset{-}{\frac{2a^2b}{a^2+b^2}}$$

$$y(b^2 - a^2) = b - \frac{2a^2b}{a^2+b^2}$$

$$y(b^2 - a^2) = \frac{a^2b + b^3 - 2a^2b}{a^2+b^2}$$

$$= \frac{b^3 - a^2b}{a^2+b^2}$$

$$= \frac{b(b^2-a^2)}{a^2+b^2}$$

$\therefore$ $$y = \frac{b}{a^2+b^2}$$

From (i), $ax + b\left(\frac{b}{a^2+b^2}\right) = 1$

$$ax = 1 - \frac{b^2}{a^2+b^2}$$

$$= \frac{a^2}{a^2+b^2}$$

$$ax = \frac{a^2}{a^2+b^2}$$

$$x = \frac{a}{a^2+b^2}$$

31. Let speed of X be x km/hr and that of Y be y km/hr

Time taken by X to walk 30 km

$$= \frac{30}{x} \text{ hours}$$

Time taken by Y to walk 30 km

$$= \frac{30}{y} \text{ hours}$$

According to question

$$\frac{30}{x} = \frac{30}{y} + 3$$

$$\frac{30}{x} - \frac{30}{y} = 3$$

$$\frac{1}{x} - \frac{1}{y} = \frac{1}{10} \quad \text{...(i)}$$

Also, $\frac{30}{2x} = \frac{30}{y} - \frac{3}{2}$

$$\frac{15}{x} = \frac{30}{y} - \frac{3}{2}$$

$$\frac{15}{x} - \frac{30}{y} = \frac{-3}{2}$$

$$\frac{1}{x} - \frac{2}{y} = -\frac{1}{10} \quad \text{...(ii)}$$

Let $\frac{1}{x} = p$, $\frac{1}{y} = q$

So, equations (i) and (ii) become

$p - q = \frac{1}{10} \Rightarrow 10p - 10q = 1$ ...(iii)

and $p - 2q = -\frac{1}{10} \Rightarrow 10p - 20q = 1$ ...(iv)

On solving equations (iii) and (iv), we get

$$10p - 10q = 1$$

$$\underset{-}{10p} \underset{+}{-} 20q = \underset{+}{-}1$$

$$10q = 2$$

$$q = \frac{1}{5} \Rightarrow y = 5$$

From (iii), we get $10p - 10\left(\frac{1}{5}\right) = 1$

$$10p = 1 + 2 = 3$$

$$p = \frac{3}{10}$$

$$x = \frac{10}{3}$$

So, Speed of X = $\frac{10}{3}$ km/hr

Speed of Y = 5 km/hr

32. $a(x + y) + b(x - y) = a^2 - ab + b^2$ ...(i)

$a(x + y) - b(x - y) = a^2 - ab + b^2$ ...(ii)

Let $x + y = p$ and $x - y = q$

So, equations (i) and (ii) becomes

$ap + bq = a^2 - ab + b^2$ ...(iii)

$ap - bq = a^2 + ab + b^2$ ...(iv)

On adding (iii) and (iv) we get,

$$2ap = 2(a^2 + b^2)$$

$$p = \frac{1}{a}(a^2 + b^2)$$

From equation (iii),

$$a\frac{1}{a}(a^2 + b^2) + bq = a^2 - ab + b^2$$

$$a^2 + b^2 + bq = a^2 - ab + b^2$$

$$bq = -ab$$

$$q = -a$$

So,

$$x + y = \frac{1}{a}(a^2 + b^2)$$

$$x - y = -a$$

$$2x = \frac{1}{a}(a^2 + b^2) - a$$

$$2x = a + \frac{b^2}{a} - a = \frac{b^2}{a}$$

$$\boxed{x = \frac{b^2}{2a}}$$

So, $y = x + a = \frac{b^2}{2a} + a = \frac{b^2 + 2a^2}{2a}$

33. Let $\frac{1}{2x+3y} = p$ and $\frac{1}{3x-2y} = q$

So, equations become

$$\frac{1}{2}p + \frac{12}{7}q = \frac{1}{2}$$

and $7p + 4q = 2$

i.e. $7p + 24q = 7$ ....(i)

and $7p + 4q = 2$ .... (ii)

On subtracting (ii) from (i), we get

$$20q = 5 \Rightarrow q = \frac{1}{4}$$

From (i), $7p + 24\left(\frac{1}{4}\right) = 7$

$$7p = 1$$

$$p = \frac{1}{7}$$

So, we get $2x + 3y = 7$ ...(iii)

$3x - 2y = 4$ ...(iv)

On multiplying (iii) by 3 and (iv) by 2 and subtracting, we get

$$\begin{array}{r} 6x + 9y = 21 \\ 6x - 4y = 8 \\ - \quad + \quad - \\ \hline 13y = 13 \end{array}$$

$$y = 1$$

From (iii)

$$2x + 3(1) = 7$$

$$2x = 4$$

$$x = 2$$

34. $kx - y = 2$

$6x - 2y = 3$

(i) For unique solution : $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$$\frac{k}{6} \neq \frac{-1}{-2}$$

$$k \neq 3$$

(ii) For no solution : $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$$\frac{k}{6} = \frac{-1}{-2} \neq \frac{2}{3}$$

$$k = 3$$

The system has infinitely many

solutions, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

i.e. $\frac{k}{6} = \frac{-1}{-2} = \frac{2}{3}$

Clearly, $\frac{-1}{-2} \neq \frac{2}{3}$,

So, there is no value of k for which the given system of equations has infinitely many solutions.

35. Income is 8a and 7a expenditure is 19b and 16b

Saving is 1250

$8a - 19b = 1250$

$7a - 16b = 1250$

$19 \times 7b = 1250 \times 7$

$16 \times 8b = 1250 \times 8$

$133b - 128b = 1250$

$5b = 1250$

$b = 250$

$a = 750$

Income of x = 6000

Income of y = 5250

36. $\frac{2}{p+q} = \frac{3}{2p-q} = \frac{7}{21}$

$\frac{2}{p+q} = \frac{3}{2p-q} = \frac{1}{3}$

$\frac{2}{p+q} = \frac{1}{3}$ and $\frac{3}{2p-q} = \frac{1}{3}$

$p+q = 6$ and $2p - q = 9$

$(p + q) + (2p - q) = 6 + 9$

$3p = 15$

$p = 5$

Put $p = 5$ in $p + q = 6$ or $2p - q = 9$, for getting the value of q.

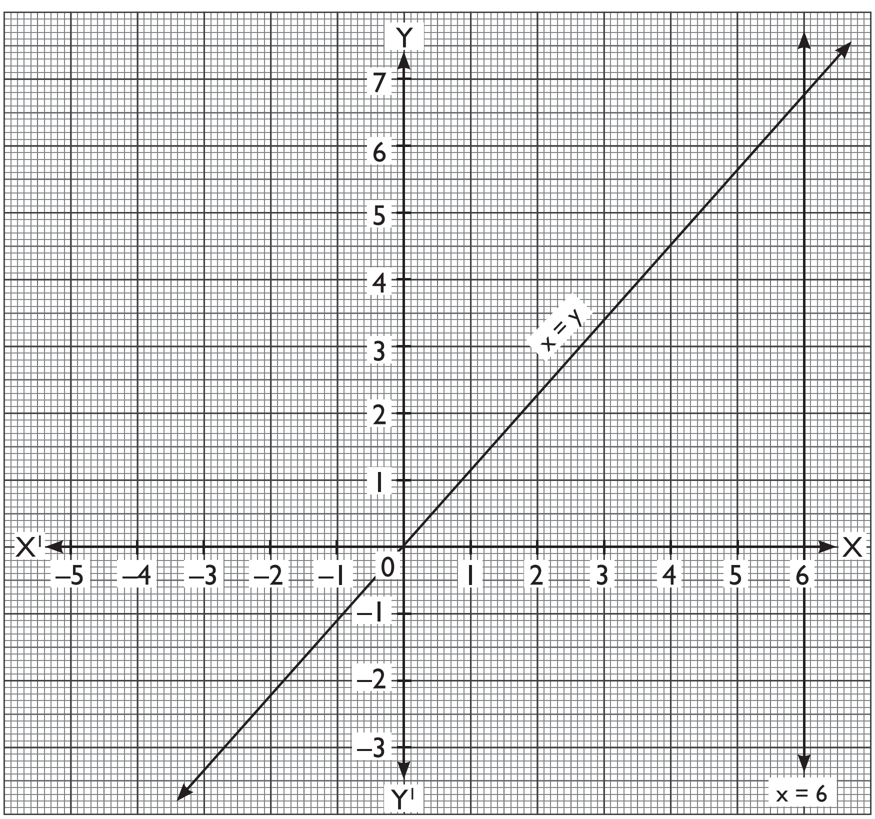
$q = 1$.

Given system of equations will have infinitely many solutions, if $p = 5$ and $q = 1$.

WORKSHEET 2

Section A

1.



Area of $\Delta = \frac{1}{2} \times 6 \times 6$

$= 18$ sq. units

2. The system of equations has no solution if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

i.e. $\frac{1}{3} = \frac{2}{k} \neq \frac{5}{15}$

$k = 6$

3. $3x + y = 1$ and

$(2k - 1)x + (k - 1)y = 2k + 1$

Inconsistent $\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow \frac{3}{(2k-3)} \neq \frac{10}{(k-1)}$

$\Rightarrow 3k - 1 \neq 2k - 1$

$\Rightarrow 3k - 2k \neq -1 + 1$

$\Rightarrow k \neq 1$

4. The system of equations represent intersecting lines if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\frac{2}{k} \neq \frac{5}{7}$

$k \neq \frac{14}{5}$

5. The system of equations has a unique solution if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\frac{k}{6} \neq \frac{-1}{-2}$

$k \neq 3$

6. $x + ky = 0$ $\quad a_1 = 1, \; b_1 = k$

$2x - y = 0$ $\quad a_2 = 2, \; b_2 = -1$

They have unique solution when

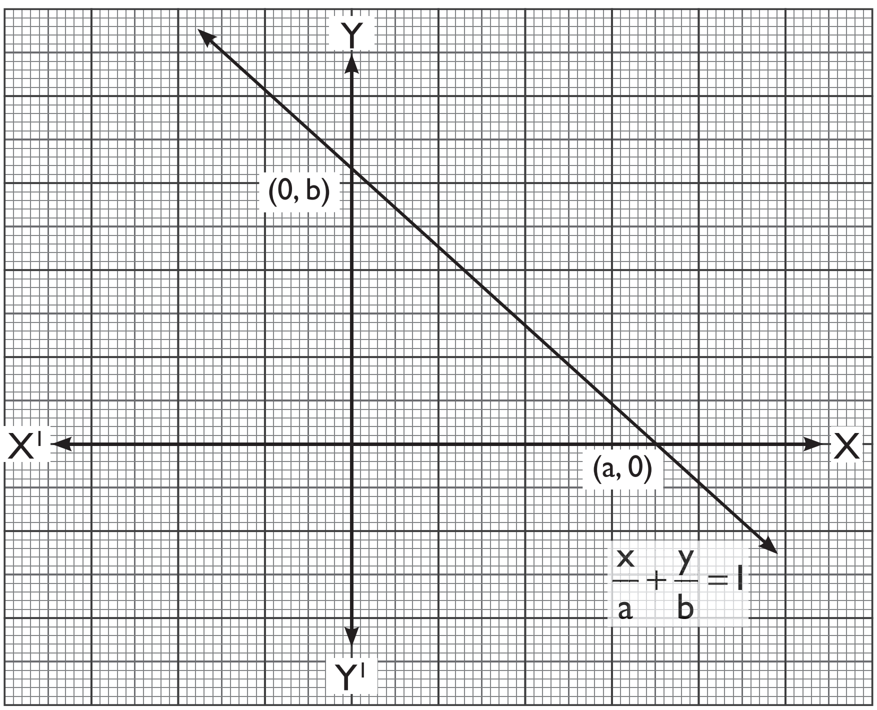
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

i.e. $\frac{1}{2} \neq \frac{k}{-1}$

i.e. $k \neq \frac{-1}{2}$

It means for all values of k except $k = \frac{-1}{2}$, the equation will have unique solution.

7.



Area of triangle $= \frac{1}{2} \times a \times b$

$= \frac{ab}{2}$

8. As (3, a) lies on line $2x - 3y = 5$

$2(3) - 3(a) = 5$

$6 - 3a = 5$

$3a = 1$

$a = \frac{1}{3}$

9. $X + 2y - 8 = 0$

$2x + 4y - 16 = 0$

Here, $a_1 = 1, \; b_1 = 2, c_1 = -8$

$a_2 = 2, b_2 = 4, c_2 = -16$

so, $\frac{a_1}{a_2} = \frac{1}{2}$

$\frac{b_1}{b_2} = \frac{2}{4} = \frac{1}{2}$

$\frac{c_1}{c_2} = \frac{-8}{-16} = \frac{1}{2}$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, the pair of linear equations has infinitely many solutions.

10. $x + y = 14$

$x = 14 - y$ [1]

Now

$x - y = 4$

$x = 4 + y$ [2]

From [1] & [2]

$14 - y = 4 + y$

$10 = 2y$

$y = 5$

Now	&
$x + y = 14$	$x - y = 4$
$x + 5 = 14$	$x - 5 = 4$
$x = 9$	$x = 9$

Hence in both cases value of x and y r same so it is consistent

Section B

11. $-4x + y = 1$ ...(i)

$6x - 5y = 9$ ...(ii)

On multiplying eq^n (i) by 5 and adding both the equations, we get

$5(-4x + y) + 6x - 5y = 5 + 9$

$-20x + 5y + 6x - 5y = 14$

$-14x = 14$

$x = -1$

From (i), $y = 1 + 4x = 1 - 4$

$y = -3$

12. Given,

$2x - 3y + 6 = 0$ ....(i)

$4x - 5y + 2 = 0$ ....(ii)

From eq. (i) and (ii) we have,

$a_1 = 2, b_1 = -3, c_1 = -6$

$a_2 = 4, b_2 = -5, c_2 = 2$

Now, $\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$

$\frac{b_1}{b_2} = \frac{-3}{-5} = \frac{3}{5}$

$\frac{c_1}{c_2} = \frac{6}{2} = 3$

Since, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

∴ The given equation will have a unique solution and the equation will intersect at a point.

13. Given,

$2x = 5y + 4$

$2x - 5y = 4$ ....(i)

$3x - 2y + 16 = 0$

$-3x + 2y = 16$ ......(ii)

Multiplying (i) by 3 and (ii) by 2, we get

$6x - 15y = 12$ .....(iii)

$-6x + 4y = 32$ ......(iv)

Adding (iii) and (iv) we get

$11y = 44$

$y = 4$

Substituting the value of y in (i) we get

$2x - 5y = 4$

$2x - 5 \times 4 = 4$

$2x - 20 = 4$

$2x = 24$

$x = 12$

Hence, $x = 12, y = 4$

14. For unique solution, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

i.e. $\frac{6}{k} \neq \frac{2}{1}$

$k \neq 3$

For infinitely many solutions, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$$\frac{6}{k} = \frac{2}{1} = \frac{3}{b_2}$$

Clearly $\frac{2}{1} \neq \frac{3}{2}$. so, there does not exist any values of k for which the system of equations has infinitely many solutions.

15.
$$99x + 101y = 499 \quad \text{...(i)}$$
$$101x + 99y = 501 \quad \text{...(ii)}$$

On subtracting (i) from (ii), we get

$$2x - 2y = 2$$
$$x - y = 1 \quad \text{...(iii)}$$

On adding (i) and (ii), we get

$$200x + 200y = 1000$$
$$x + y = 5 \quad \text{...(iv)}$$

On adding (iii) and (iv), we get

$$2x = 6$$
$$x = 3$$

From (iv), $y = 5 - x$

$= 5 - 3$

$= 2$

16. The system of equations has infinite solutions if

$x + (k + 1)y = \underline{5}$

$(k + 1)x + 9y = (8k - 1)$

$\Rightarrow$ For infinitely many solutions

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \frac{1}{k+1} = \frac{k+1}{9} = \frac{5}{8k-1}$$

$$\Rightarrow \frac{1}{k+1} = \frac{k+1}{9}$$

$\Rightarrow (k + 1)^2 = 9$

$\Rightarrow k + 1 = \pm\sqrt{9}$

$\Rightarrow k + 1 = \pm 3$

Case - 1	Case - 2
$\Rightarrow k + 1 = +3$	$\Rightarrow k + 1 = -3$
$\Rightarrow k = 2$	$\Rightarrow k = -4$

17. Let the numerator be x and denominator be y.

So, fraction $= \frac{x}{y}$

According to question,

$$\frac{x+1}{y+1} = \frac{7}{8}$$
$$8x + 8 = 7y + 7$$
$$8x - 7y = -1 \quad \text{...(i)}$$

Again, $\frac{x-1}{y-1} = \frac{6}{7}$

$$7x - 7 = 6y - 6$$
$$7x - 6y = 1 \quad \text{...(ii)}$$

On multiplying (i) by 7 and (ii) by 8, we get,

$$56x - 49y = -7$$
$$56x - 48y = 8$$
$$- \quad + \quad -$$
$$-y = -15$$
$$y = 15$$

From (i), $8x - 7(15) = -1$

$8x = -1 + 105 = 104$

$x = \frac{104}{8} = 13$

18. a = 8 and b = 5

Step-by-step explanation:

As, $a_1 = 3,\ b_1 = -a - 1,\ c_1 = 2b - 1$

$a_2 = 5,\ b_2 = 1 - 2a, c_2 = 3b$

As, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Part I

$$\frac{a_1}{a_2} = \frac{b_1}{b_2}$$

$$\frac{3}{5} = \frac{-a-1}{1-2a}$$

$5(-a - 1) = 3(1 - 2a)$

$-5a - 5 = 3 - 6a$

$-5 = 3 - a$

$-8 = -a$

Cancelling the minus sign from both the sides

We get;

$8 = a$

Part II

$\frac{a_1}{a_2} = \frac{c_1}{c_2}$

$\frac{3}{5} = \frac{2b - 1}{3b}$

On cross multiplication

$9b = 10b - 5$

$-b = -5$

By cancelling minus sign from both the sides

We get,

$b = 5$

Therefore, a = 8 and b = 5

19. $2x - 3y + 6 = 0$ ...(i)

$4x - 5y + 2 = 0$ ...(ii)

Here, $\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$

$\frac{b_1}{b_2} = \frac{-3}{-5} = \frac{3}{5}$

$\frac{c_1}{c_2} = \frac{6}{2} = 3$

As $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, so, the system has a unique solution.

On multiplying (i) by 2 and subtracting (ii) from (i), we get

$$\begin{array}{r} 4x - 6y + 12 = 0 \\ 4x - 5y + 2 = 0 \\ - \quad + \quad - \quad \\ \hline -y = -10 \end{array}$$

$y = 10$

From (i), $2x - 3(10) + 6 = 0$

$2x - 24 = 0$

$x = 12$

20. $\frac{x}{10} + \frac{y}{5} + 1 = 15$

$\frac{x}{10} + \frac{y}{5} = 14$

$2x + y = 140$ ...(i)

Again, $\frac{x}{8} + \frac{y}{6} = 15$

$\frac{3x + 4y}{24} = 15$

$3x + 4y = 360$ ...(ii)

From (i), $y = 140 - 2x$

On putting this value of y in (ii), we get

$3x + 4(140 - 2x) = 360$

$3x + 560 - 8x = 360$

$-5x = -200$

$x = 40$

So, $y = 140 - 2x$

$= 140 - 2(40)$

$= 140 - 80$

$= 60$

Section C

21. Let the fixed charge be ₹x and cost of food per day be ₹y.

According to question,

$$\begin{array}{r} x + 20y = 3000 \quad \text{(i)} \\ x + 25y = 3500 \quad \text{(ii)} \\ - \quad + \quad\quad - \quad \\ \hline -5y = -500 \\ y = 100 \end{array}$$

From (i), we get $x = 3000 - 20(100)$

$= 3000 - 2000$

$= 1000$

So, fixed charge = ₹1000

Cost of food per day = ₹100

22. $x + y = 1$

$2x - 3y = 11$

According to cross multiplication method,

$$\frac{x}{-11-3} = \frac{y}{-2+11} = \frac{1}{-3-2}$$

$$\frac{x}{-14} = \frac{y}{9} = \frac{1}{-5}$$

$$\frac{x}{-14} = \frac{1}{-5}, \quad \frac{y}{9} = \frac{1}{-5}$$

$$x = \frac{14}{5}, \quad y = \frac{-9}{5}$$

23. (i) $5x + 6y = 15$

As $\frac{4}{5} \neq \frac{-5}{6} \left(\frac{a_1}{a_2} \neq \frac{b_1}{b_2}\right)$

(ii) $8x - 10y = 30$

As $\frac{4}{8} = \frac{-5}{-10} \neq \frac{10}{30} \left(\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}\right)$

(iii) $8x - 10y = 20$

As $\frac{4}{8} = \frac{-5}{-10} = \frac{10}{20} \left(\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}\right)$

24. For infinite solution

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{2}{4} = \frac{a-4}{a-1} = \frac{-2b+1}{-5b-1}$$

$$2(a - 1) = 4(a - 4)$$

$$2a - 2 - 4a + 16 = 0$$

$$-2a = -14$$

$$a = 7,$$

$$2(-5b + 1) = 4(-2b - 1)$$

$$-10b + 8b + 2 + 4 = 0$$

$$-2b = -6$$

$$b = 3$$

25. Let $\frac{1}{x-1} = p$ and $\frac{1}{y-2} = q$

So, equations become

$$5p + q = 2 \quad \text{(i)}$$

$$6p - 3q = 1 \quad \text{(ii)}$$

On multiplying (i) by 3 and subtracting equations (i) and (ii), we get

$$15p + 3q = 6$$

$$6p - 3q = 1$$

$$21p = 7$$

$$p = \frac{1}{3}$$

$\therefore \quad x - 1 = 3 \Rightarrow x = 4$

From eq^n (i), $\quad q = 2 - 5p$

$$= 2 - 5\left(\frac{1}{3}\right)$$

$$= \frac{1}{3}$$

$\therefore \quad y - 2 = 3 \Rightarrow y = 5$

26. Let $\frac{1}{x} = p$ and $\frac{1}{y} = q$

So, equations become

$$p - 4q = 2$$

$$p + 3q = 9$$

$$-7q = -7$$

$$q = 1$$

From eq^n $\quad p - 4q = 2,$ we get

$$p = 2 + 4(1)$$

$$= 6$$

So, $\quad x = \frac{1}{6}, \quad y = 1$

27. Let father's age be x years and son's age be y years.

According to question,

$$2y + x = 70 \quad \text{(i)}$$

$$2x + y = 95 \quad \text{(ii)}$$

From (i), $x = 70 - 2y$

On putting value of x in (ii), we get

$$2(70 - 2y) + y = 95$$

$$140 - 4y + y = 95$$

$$3y = 45$$

$$y = 15$$

So, $\quad x = 70 - 2y$

$$= 70 - 2(15)$$
$$= 70 - 30$$
$$= 40$$

So, age of father = 40 years

age of son = 15 years

28. Let speed of train be x km/hr and speed of car be y km/hr.

According to question,

$$\frac{160}{x} + \frac{600}{y} = 8$$
$$\frac{240}{x} + \frac{520}{y} = \frac{41}{5}$$

Let $\frac{1}{x} = p, \quad \frac{1}{y} = q$

So, we get equations as

$$160\,p + 600q = 8 \quad ...(i)$$
$$1200p + 2600q = 41 \quad ...(ii)$$

On multiplying (i) by 30 and (ii) by 4, we get

$$\begin{aligned} 4800p + 18000q &= 240 \\ 4800p + 10400q &= 164 \\ - \quad\quad - \quad\quad & \quad - \\ \hline 7600q &= 76 \end{aligned}$$

$$q = \frac{76}{7600} = \frac{1}{100}$$

i.e. $y = 100$

From (i), we get

$$160p + 600\left(\frac{1}{100}\right) = 8$$
$$160p + 6 = 8$$
$$160p = 2$$
$$p = \frac{1}{80}$$

i.e. $x = 80$

So, Speed of train = 80 km/hr

Speed of car = 100 km/hr

29. Let time taken by one man alone be x days.

Let time taken by one boy alone be y days.

According to question,

$$\frac{8}{x} + \frac{12}{y} = \frac{1}{10}$$

$$\frac{6}{x} + \frac{8}{y} = \frac{1}{14}$$

Let $\frac{1}{x} = p$, and $\frac{1}{y} = q$

So, we get equations as

$$8p + 12q = \frac{1}{10}$$
$$80p + 120q = 1 \quad (i)$$

Another equation becomes,

$$6p + 8q = \frac{1}{14}$$
$$84p + 112q = 1 \quad (ii)$$

On multiplying (i) by 21 and (ii) by 20, we get

$$\begin{aligned} 1680p + 2520q &= 21 \\ 1680p + 2240q &= 20 \\ - \quad\quad - \quad\quad & \quad - \\ \hline 280q &= 1 \end{aligned}$$

$$q = \frac{1}{280}$$

So, $y = 280$

From (i),

$$80p + 120\left(\frac{1}{280}\right) = 1$$
$$80p + \frac{3}{7} = 1$$
$$80p = 1 - \frac{3}{7} = \frac{4}{7}$$
$$p = \frac{1}{140}$$

So, $x = 140$

∴ A man can complete the work in 140 days and a boy can complete the work in 280 days.

30. Let father's age = x years

Sum of ages of 2 children = y years

According to question,

$$x = 2y \quad (i)$$

and

$$x + 20 = y + 20 + 20$$
$$x - y = 20 \quad (ii)$$

On putting (i) in (ii), we get

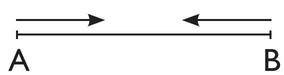
$$2y - y = 20$$
$$y = 20$$
$$\therefore \quad x = 2y = 40$$
So, father's age = 40 years

Section D

31. Let Speed of car A = x km/hr

Speed of car B = y km/hr

According to question,



$$\frac{4}{3}x + \frac{4}{3}y = 80$$
$$x + y = 60 \qquad \text{(i)}$$

$$8x - 8y = 80$$
$$x - y = 10 \qquad \text{(ii)}$$

On adding (i) and (ii), we get

$$2x = 70$$
$$x = 35$$

From (i), $y = 60 - x$

$= 60 - 35$

$= 25$

From (i), $y = 60 - x$

$= 60 - 35$

$= 25$

So, Speed of car A = 35 km/hr

Speed of car B = 25 km/hr

32. Let cost of one chair be $x and cost of one table be $y.

According to question,

$$4x + 3y = 2100 \qquad \text{(i)}$$
$$5x + 2y = 1750 \qquad \text{(ii)}$$

On multiplying eq^n (i) by 5 and (ii) by 4, we get

$$20x + 15y = 10500$$
$$\underset{-}{20x} + \underset{-}{8y} = \underset{-}{7000}$$
$$7y = 3500$$

$$y = \frac{3500}{7} = 500$$

From (i), $4x + 3(500) = 2100$

$$4x = 2100 - 1500$$
$$4x = 600$$
$$x = 150$$

Cost of one chair = $ 150

Cost of one table = $ 500

Therefore,

Cost of five chairs = 5 × 150

= $ 750

Cost of eight tables = 8 × 500

= $ 4000

33. Let father's age = x years

Son's age = y years

According to question,

$$x - 10 = 12(y - 10)$$

i.e. $x - 12y = -110 \qquad \text{(i)}$

For another eq^n,

$$x + 10 = 2(y + 10)$$
$$x - 2y = 10 \qquad \text{(ii)}$$

On subtracting eq^n (ii) from (i), we get

$$x - 12y = -110$$
$$\underset{-}{x} \underset{+}{- 2y} = \underset{-}{10}$$
$$-10y = -120$$
$$y = 12$$

From (ii), $x = 10 + 2y$

$= 10 + 24$

$= 34$

So, Father's age = 34 years

Son's age = 12 years

34. Perimeter of ABCDE = 21 cm

i.e. AB + BC + CD + DE + AE = 21

$$3 + x - y + x + y + x - y + 3 = 21$$
$$3x - y = 15 \qquad \text{(i)}$$

As BE || CD and BC || DE,

BCDE is a parallelogram

$\therefore$ BE = CD (opposite sides of parallelogram)

i.e. $x + y = 5$ (ii)

On adding equations (i) and (ii), we get

$$4x = 20$$
$$x = 5$$

from (i), $3(5) - y = 15$

$$y = 0$$

So, BC = $x - y = 5 - 0 = 5$ cm

CD = $x + y = 5 + 0 = 5$ cm

DE = $x - y = 5 - 0 = 5$ cm

BE = 5 cm

So, perimeter of quadrilateral BCDE

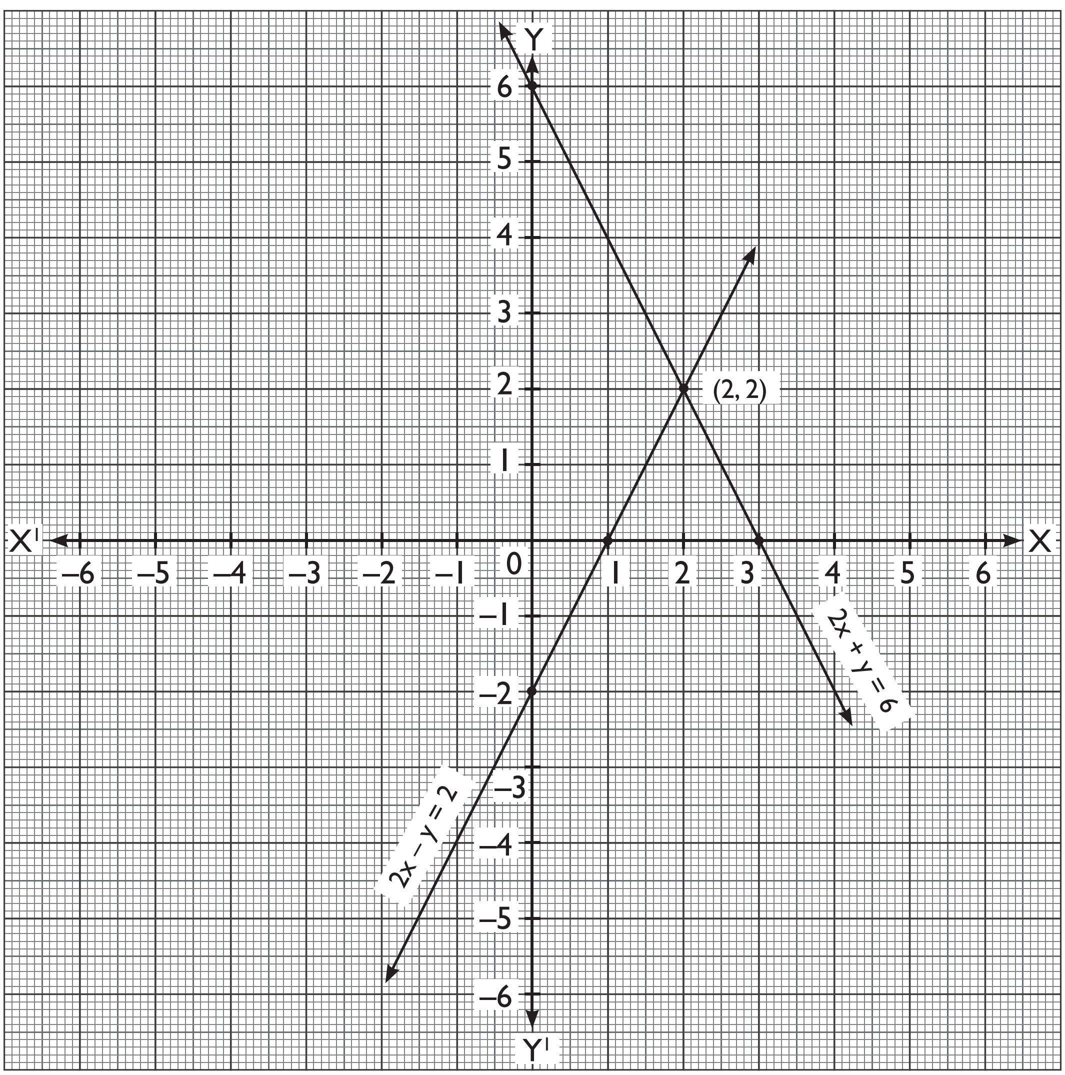
= 4 × 5 (perimeter = 4 × side) = 20 cm

35. Let $\frac{1}{x} = p$ and $\frac{1}{y} = q$.

So, equations become

$$ap - bq = 0$$
$$ab^2p + a^2bq = a^2 + b^2$$
$$\frac{p}{a^2b + b^3 - 0} = \frac{q}{0 + a^3 + ab^2} = \frac{1}{a^3b + ab^3}$$
$$\frac{p}{b(a^2 + b^2)} = \frac{q}{a(a^2 + b^2)} = \frac{1}{ab(a^2 + b^2)}$$
$$\frac{p}{b(a^2 + b^2)} = \frac{1}{ab(a^2 + b^2)}$$
$$\frac{q}{a(a^2 + b^2)} = \frac{1}{ab(a^2 + b^2)}$$
$$p = \frac{1}{a} \qquad q = \frac{1}{b}$$
$$\therefore\ x = a \qquad y = b$$

36.



2x + y = 6,

x	3	0
y	0	6

2x – y = 2

x	0	1
y	–2	0

As the equations intersect at point (2, 2), so, (2, 2) is a solution of given set of equations.

Area of triangle formed by lines representing these equations with the x – axis = $\frac{1}{2} \times 2 \times 2 =$ 2 sq units.

Area of triangle formed by lines representing these equations with the y – axis = $\frac{1}{2} \times 8 \times 2 =$ 8 sq units.

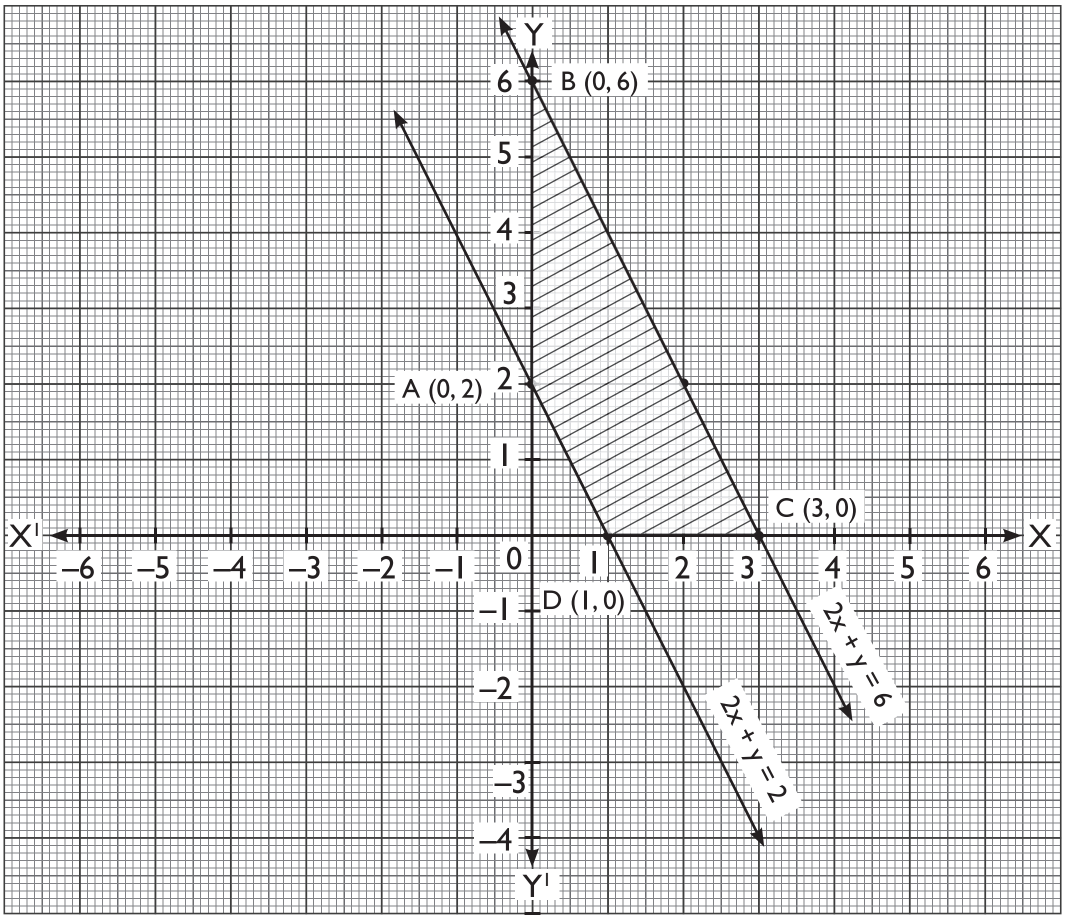
So, Ratio = $\frac{2}{8} = \frac{1}{4}$.

37. $2x + y = 2$

x	0	1
y	2	0

$2x + y = 6$

x	0	3
y	6	0



Vertices of trapezium are A(0, 2), B(0, 6), C(3, 0) and D(1, 0).

Area of trapezium ABCD = area of ΔBOC – area of ΔAOD

$= \frac{1}{2} \times 3 \times 6 - \frac{1}{2} \times 1 \times 2 \quad = 4.5 - 1 = 3.5$ sq. units

38. Let the numerator be x and denominator be y
According to question,

$$y = 5 + 2x$$

$$-2x + y = 5 \qquad \text{(i)}$$

For the other equation,

$$\frac{x-1}{y-1} = \frac{3}{8}$$

$$8x - 8 = 3y - 3$$

$$8x - 3y = 5 \qquad \text{(ii)}$$

From (i), $\quad y = 5 + 2x$

On putting this value of y in (ii), we get

$$8x - 3(5 + 2x) = 5$$
$$8x - 15 - 6x = 5$$
$$2x = 20$$
$$x = 10$$

So, $y = 5 + 2(10)$

$= 25$

So, Fraction $= \frac{x}{y} = \frac{10}{25}$

39. $mx - ny = m^2 + n^2$

$x + y = 2m$

$$\frac{x}{2mn + m^2 + n^2} = \frac{y}{-m^2 - n^2 + 2m^2} = \frac{1}{m+n}$$

$$\frac{x}{(m+n)^2} = \frac{y}{m^2 - n^2} = \frac{1}{m+n}$$

$$\frac{x}{(m+n)^2} = \frac{1}{m+n}$$

$$x = \frac{(m+n)^2}{m+n}$$

$= m + n$

$$\frac{y}{m^2 - n^2} = \frac{1}{m+n}$$

$$y = \frac{m^2 - n^2}{m+n}$$

$= m - n$

40. $(a - b)x + (a + b)y = a^2 - 2ab - b^2$

$(a + b)x + (a + b)y = a^2 - b^2$

$$\frac{x}{(a+b)(-a^2-b^2)-(a+b)(-a^2+2ab+b^2)} = \frac{y}{(a+b)(-a^2+2ab+b^2)-(a-b)(-a^2-b^2)} = \frac{1}{(a-b)(a+b)-(a+b)^2}$$

$$\frac{x}{-a^3 - ab^2 - a^2b - b^3 + a^3 - 2a^2b - ab^2 + a^2b - 2ab^2 - b^3} = \frac{y}{-a^3 + 2a^2b + ab^2 - a^2b + 2ab^2 + b^3 + a^3 + ab^2 - a^2b - b^3} = \frac{1}{a^2 - b^2 - a^2 - b^2 - 2ab}$$

$$\frac{x}{-2b^3 - 2a^2b - 4ab^2} = \frac{y}{4ab^2} = \frac{1}{-2b^2 - 2ab}$$

$$x = \frac{-2b^3 - 2a^2b - 4ab^2}{-2b^2 - 2ab}$$

$$= \frac{-2b(b^2 + a^2 + 2ab)}{-2b(b+a)}$$

$= a + b$

Also, $\frac{y}{4ab^2} = \frac{1}{-2b(a+b)}$

$$y = \frac{-2ab}{a+b}$$

Chapter 04

Quadratic Equations

MULTIPLE CHOICE QUESTIONS

1. (b) As $x = -\frac{1}{2}$ is a solution of $3x^2 + 2kx - 3 = 0$

$$\therefore \quad 3\left(-\frac{1}{2}\right)^2 + 2k\left(-\frac{1}{2}\right) - 3 = 0$$

$$\frac{3}{4} - k - 3 = 0$$

$$k = \frac{3}{4} - 3$$

$$= \frac{3-12}{4}$$

$$= \frac{-9}{4}$$

2. (b) Equation has no real roots if $D < 0$

i.e. $b^2 - 4ac < 0$

i.e. $b^2 - 4\,(1)\,(1) < 0$

i.e. $b^2 - 4 < 0$

i.e. $(b + 2)\,(b - 2) < 0$

i.e. $-2 < b < 2$

3. (d) let α, β be the roots then $\alpha\beta = 3$

$\alpha\beta = 3$

$(1)\,\beta = 3 \qquad (\because\ \alpha = 1)$

$\alpha\beta = 3$

4. (a) $3\sqrt{3}x^2 + 10x + \sqrt{3} = 0$

$D = b^2 - 4ac$

$= (10)^2 - 4\,(3\sqrt{3}/\sqrt{3})$

$= 100 - 36$

$= 64$

5. (a) $x^2 - px - q = 0$

As p is the root

$\therefore \quad p^2 - p^2 + q = 0$

$\Rightarrow \quad q = 0$

Also, q is a root

$\therefore \quad q^2 - pq + q = 0$

$q\,(q - p + 1) = 0$

$q = 0 \quad \text{or} \quad q = p - 1$

$\therefore \quad q = p - 1$

$\Rightarrow \quad p = q + 1$

$= 0 + 1$

$= 1$

So, $p = 1, q = 0$

WORKSHEET 1

Section A

1. $x^2 - 7x + 12$

$x^2 - 3x - 4x + 12$

$x\,(x - 3) - 4\,(x - 3)$

$(x - 3)\,(x - 4)$

2. $2x^2 + 3x - 4 = 0$

$b^2 - 4ac = 9 - 4(2)\,(-4)$

$= 9 + 32$

$= 41 > 0$

As $b^2 - 4ac > 0$

$\Rightarrow$ The equation has real and distinct roots.

3. $3x^2 + 13x + 14 = 0$

LHS $= 3x^2 + 13x + 14$

$= 3(-2)^2 + 13\,(-2) + 14 \qquad$ (put $x = -2$)

$= 12 - 26 + 14$

$= 0$

$= \text{RHS}$

So, $x = -2$ is a root of $3x^2 + 13x + 4 = 0$

4. $x^2 - 3x - 1 = 0$

LHS $= x^2 - 3x - 1$

$= 1^2 - 3(1) - 1$ (Put $x = 1$)

$= 1 - 3 - 1$

$= -3 \neq \text{RHS} (= 0)$

So, $x = 1$ is not a solution of equation $x^2 - 3x - 1 = 0$

5. $x^2 - 3x - 10 = 0$

$D = b^2 - 4ac$

$= (-3)^2 - 4(1)(-10)$

$= 9 + 40$

$= 49$

6. Let $\sqrt{6+\sqrt{6+\sqrt{6+...}}} = x$

$\sqrt{6+x} = x$

On squaring both sides, we get

$6 + x = x^2$

$x^2 - x - 6 = 0$

$x^2 - 3x + 2x - 6 = 0$

$x(x - 3) + 2(x - 3) = 0$

$(x - 3)(x + 2) = 0$

$x = 3, x = -2$

As value of $\sqrt{6+\sqrt{6+\sqrt{6+...}}}$ cannot be negative, so, $x = 3$

7. $3x^2 - kx + 38 = 0$

The quadratic equation has equal roots

if $D = 0$

i.e. $b^2 - 4ac = 0$

i.e. $k^2 - 4(3)(38) = 0$

$k^2 - 456 = 0$

$k^2 = 456$

$k = \pm 2\sqrt{114}$

8. $bx^2 - 2\sqrt{ac}\,x + 6 = 0$

The equation has equal roots if discriminant $= 0$

i.e. $\left(2\sqrt{ac}\right)^2 - 4(b)(b) = 0$

$4ac - 4b^2 = 0$

$b^2 = ac$

Section B

9. Using quadratic formula,

General form $ax^2 + bx + c = 0$

$D = b^2 - 4ac$

Solution is $x = \dfrac{-b \pm \sqrt{D}}{2a}$

Equation is $16x^2 - 24x - 1 = 0$

Where, $a = 16, b = -24, c = -71$

$D = b^2 - 4a$

$D = (-24)^2 - 4(16)(-1)$

$D = 576 + 64$

$D = 640$

Solution is $x = \dfrac{-b \pm \sqrt{D}}{2a}$

$x = \dfrac{-(-24) \pm \sqrt{640}}{2(16)}$

$x = \dfrac{24 \pm 8\sqrt{10}}{16}$

$x = \dfrac{3 \pm \sqrt{10}}{4}$

Therefore, the roots are $x = \dfrac{3+\sqrt{10}}{4}, \dfrac{3-\sqrt{10}}{4}$

10. $\dfrac{1}{x-2} + \dfrac{2}{x-1} = \dfrac{6}{x}$

$\dfrac{x-1+2x-4}{(x-1)(x-2)} = \dfrac{6}{x}$

$x(3x - 5) = 6(x - 1)(x - 2)$

$$3x^2 - 5x = 6(x^2 - 3x + 2)$$
$$3x^2 - 5x = 6x^2 - 18x + 12$$
$$0 = 3x^2 - 13x + 12$$
$$0 = 3x^2 - 9x - 4x + 12$$
$$0 = 3x(x - 3) - 4(x - 3)$$
$$0 = (3x - 4)(x - 3)$$
$$3x - 4 = 0, \qquad x - 3 = 0$$
$$x = \frac{4}{3}, \qquad x = 3$$

11. $x^2 - 2ax + a^2 - b^2 = 0$

$x^2 + [(-a - b) + (-a + b)]x + (a + b)(a - b) = 0$

$x^2 - (a + b)x - (a - b)x + (a + b)(a - b) = 0$

$x[x - (a + b)] - (a - b)[x - (a + b)] = 0$

$[x - (a - b)][x - (a + b)] = 0$

$x - (a - b) = 0$ or $x - (a + b) = 0$

$x = a - b$ or $x = a + b$

12. $4x^2 - 4a^2x + (a^4 - b^4) = 0$

$4x^2 + [-2(a^2 - b^2) - 2(a^2 + b^2)]x + (a^4 - b^4) = 0$

$4x^2 - 2(a^2 - b^2)x - 2(a^2 + b^2)x + (a^2 - b^2)(a^2 + b^2) = 0$

$2x[2x - a^2 - b^2] - (a^2 + b^2)(2x - a^2 - b^2) = 0$

$[2x - (a^2 - b^2)][2x - (a^2 + b^2)] = 0$

$2x - (a^2 - b^2) = 0$ or $2x(a^2 + b^2) = 0$

$x = \frac{-b^2 + a^2}{2}$ or $x = \frac{a^2 + b^2}{2}$

13. Given

$(k - 12)x^2 - 2(k - 12)x + 2 = 0$

Comparing with $a x^2 + b x + c = 0$ we get :-

$a = k - 12, \; b = -2(k - 12), c = 2$

The equation has equal roots!

So $b^2 = 4ac$

$\Rightarrow (-2(k - 12))^2 = 2(k - 12)$

$\Rightarrow (-2)^2 (k - 12)^2 = 2(k - 12)$

$\Rightarrow \quad 2(k^2 + 144 - 24k) = k - 12$

$\Rightarrow \quad 2k^2 + 288 - 48k = k - 12$

$\Rightarrow \quad 2k^2 - 49k + 300 = 0$

$\Rightarrow \quad 2k^2 - 25k - 24k + 300 = 0$

$\Rightarrow k(2k - 25) - 12(2k - 25) = 0$

$\Rightarrow \quad (2k - 25)(k - 12) = 0$

Either :

$k - 12 = 0$

$\Rightarrow \quad k = 12$

or :

$2k - 25 = 0$

$\Rightarrow \quad 2k = 25$

$\Rightarrow \quad k = \frac{25}{2} = 12.5$

The value of k is 12 or 12.5

14. $12abx^2 - (9a^2 - 8b^2)x - 6ab = 0$

$12abx^2 + (-9a^2 + 8b^2)x - 6ab = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-9a^2+8b^2) \pm \sqrt{(-9a^2+8b^2)^2 - 4(12ab)(-6ab)}}{2(12ab)}$$

$$x = \frac{-(-9a^2+8b^2) + \sqrt{(-9a^2+8b^2)^2 - 4(12ab)(-6ab)}}{2(12ab)}$$

$$\Rightarrow \quad x = \frac{3a}{4b}$$

$$x = \frac{-(-9a^2+8b^2) - \sqrt{(-9a^2+8b^2)^2 - 4(12ab)(-6ab)}}{2(12ab)}$$

$$\Rightarrow \quad x = \frac{-2b}{3a}$$

15. Let the two numbers be x and $16 - x$.

According to question,

$$\frac{1}{x} + \frac{1}{16 - x} = \frac{1}{3}$$
$$\frac{16 - x + x}{x(16 - x)} = \frac{1}{3}$$
$$48 = 16x - x^2$$
$$x^2 - 16x + 48 = 0$$
$$x^2 - 12x - 4x + 48 = 0$$

$$x(x-12)-4(x-12)=0$$
$$(x-4)(x-12)=0$$
$$x=4, 12$$

If $x = 4$, Other number = $16 - 4 = 12$

if $x = 12$, Other number = $16 - 12 = 4$

16. $$x+\frac{1}{x} = 11\frac{1}{11}$$
$$\frac{x^2+1}{x} = \frac{122}{11}$$
$$11(x^2+1) = 122x$$
$$11x^2-122x+11 = 0$$
$$11x^2-x-121x+11 = 0$$
$$x(11x-1)-11(11x-1) = 0$$
$$(11x-1)(x-11) = 0$$
$$11x-1=0 \quad \text{or} \quad x-11 = 0$$
$$x=\frac{1}{11} \quad \text{or} \quad x = 11$$

Section C

17. Let D_1 and D_2 be the discriminants of equations $x^2 + 2cx + ab = 0$ and $x^2 - 2(a+b)x + a^2 + b^2 + 2c^2 = 0$ respectively.

$x^2 + 2cx + ab = 0$

$D_1 = (2c)^2 - 4(1)(ab)$

$= 4c^2 - 4ab$

$= 4(c^2 - ab)$

As roots are real and unequal,

so $D_1 > 0$

$c^2 - ab > 0$ (i)

$x^2 - 2(a+b)x + a^2 + b^2 + 2c^2 = 0$

$D_2 = 4(a+b)^2 - 4(1)(a^2 + b^2 + 2c^2)$

$= 8ab - 8c^2$

$= -8(c^2 - ab) < 0$ [From (i)]

So, the given equation has no real roots.

18. $$\frac{1}{x+4}-\frac{1}{x-7} = \frac{11}{30}$$
$$\frac{x-7-x-4}{(x+4)(x-7)} = \frac{11}{30}$$
$$\frac{-11}{x^2-3x-28} = \frac{11}{30}$$
$$\frac{-1}{x^2-3x-28} = \frac{1}{30}$$
$$x^2-3x-28+30 = 0$$
$$x^2-3x+2 = 0$$
$$x^2-2x-x+2 = 0$$
$$x(x-2)-1(x-2) = 0$$
$$(x-1)(x-2) = 0$$
$$x = 1, 2$$

19. Le the smaller side and larger side be x cm and y cm respectively.

Hypotenuse = $3\sqrt{5}$ cm

So, $x^2 + y^2 = \left(3\sqrt{5}\right)^2$

$x^2 + y^2 = 45$ (i)

If smaller side is tripled and the larger side is doubled,

$(3x)^2 + (2y)^2 = (15)^2$

$9x^2 + 4y^2 = 225$ (ii)

From (i), $x^2 = 45 - y^2$

So, we get $9(45 - y^2) + 4y^2 = 225$

$405 - 9y^2 + 4y^2 = 225$

$5y^2 = 180$

$y^2 = \frac{180}{5} = 36$

$y = \pm 6$

For $y = -6$, $x^2 = 45 - 36 = 9$

$x = \pm 3$

For $y = 6$, $x^2 = 45 - 36 = 9$

$x = \pm 3$

As length cannot be negative,

So, $y = -6$, $x = -3$ rejected

$\therefore \quad x = 3, \quad y = 6$

Length of smaller side = 3 cm

Length of larger side = 6 cm

20. As $x = -2$ is a root of equation

$3x^2 + 7x + p = 0$, we have

$$3(-2)^2 + 7(-2) + p = 0$$
$$12 - 14 + p = 0$$
$$p = 2$$
$$x^2 + k(4x + k - 1) + p = 0$$
$$x^2 + k(4x + k - 1) + 2 = 0 \quad \text{(Put } p = 2\text{)}$$
$$x^2 + (4k)x + k^2 - k + 2 = 0$$

As roots are equal,

$$\text{Discriminant (D)} = 0$$
$$(4k)^2 - 4(k^2 - k + 2) = 0$$
$$16k^2 - 4k^2 + 4k - 8 = 0$$
$$12k^2 + 4k - 8 = 0$$
$$3k^2 + k - 2 = 0$$
$$3k^2 + 3k - 2k - 2 = 0$$
$$3k(k + 1) - 2(k + 1) = 0$$
$$(3k - 2)(k + 1) = 0$$
$$3k - 2 = 0 \quad \text{or} \quad k + 1 = 0$$
$$k = \frac{2}{3} \quad \text{or} \quad k = -1$$

21. $x^2(a^2 + b^2) + 2(ac + bd)x + (c^2 + d^2) = 0$

Consider

Discriminant (D)

$= 4(ac + bd)^2 - 4(a^2 + b^2)(c^2 + d^2)$

$= 4(a^2c^2 + b^2d^2 + 2abcd) - 4(a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2)$

$= 8abcd - 4a^2d^2 - 4b^2c^2$

$= -4[(ad)^2 + (bc)^2 - 2abcd]$

$= -4(ad - bc)^2$

≤ 0

For no real roots, $D < 0$

i.e. $D \neq 0$ i.e. $ad \neq bc$

22. As 2 is a root of the quadratic equation $3x^2 + px - 8 = 0$,

$$3(2)^2 + p(2) - 8 = 0$$
$$12 + 2p - 8 = 0$$
$$2p = -4$$
$$p = -2$$

$\therefore$ Other equation becomes

$$4x^2 - 2(-2)x + k = 0$$
$$4x^2 + 4x + k = 0$$

As roots are equal,

$$\text{Discriminant (D)} = 0$$
$$\text{i.e.} \quad 16 - 4(4)(k) = 0$$
$$16 - 16k = 0$$
$$k = 1$$

23. $(x - a)(x - b) + (x - b)(x - c) + (x - c)(x - a) = 0$

$\Rightarrow x^2 - bx - ax + ab + x^2 - cx - bx + bc + x^2 - ax - cx + ac = 0$

$\Rightarrow 3x^2 - 2bx - 2ax - 2cx + ab + bc + ca = 0$

$\Rightarrow 3x^2 - 2(a + b + c)x + (ab + bc + ca) = 0$

Discriminant (D)

$= 4(a + b + c)^2 - 12(ab + bc + ca)$

$= 4(a^2 + b^2 + c^2 + 2ab + 2bc + 2ac - 3ab - 3bc - 3ca)$

$= 4(a^2 + b^2 + c^2 - ab - bc - ca)$

$= 2(2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ac)$

$= 2[(a - b)^2 + (b - c)^2 + (a - c)^2]$

$D = 2[(a - b)^2 + (b - c)^2 + (a - c)^2] \geq 0$

As $D \geq 0$, so roots are real.

Roots are equal if $D = 0$

i.e. $2[(a - b)^2 + (b - c)^2 + (a - c)^2] = 0$

i.e. $a - b = 0, \quad b - c = 0, \quad a - c = 0,$

$a = b, \quad b = c, \quad a = c$

i.e. $a = b = c$.

24. Let the two numbers be x and y such that $x > y$.

$$x - y = 3 \qquad \text{(i)}$$

Also, $$\frac{1}{y} - \frac{1}{x} = \frac{3}{28} \qquad \text{(ii)}$$

From (i), $x = 3 + y$

Putting in (ii), we get

$$\frac{1}{y} - \frac{1}{3+y} = \frac{3}{28}$$

$$\frac{3+y-y}{y(3+y)} = \frac{3}{28}$$

$$\frac{3}{y(3+y)} = \frac{3}{28}$$

$$28 = y^2 + 3y$$

$$y^2 + 3y - 28 = 0$$

$$y^2 + 7y - 4y - 28 = 0$$

$$y(y + 7) - 4(y + 7) = 0$$

$$(y - 4)(y + 7) = 0$$

$$y = 4, -7$$

As y is a natural number,

$y = -7$ is rejected

So, $y = 4$

$\therefore \quad x = 3 + y = 7$

Section D

25. $$\frac{x-2}{x-3} + \frac{x-4}{x-5} = \frac{10}{3}$$

$$\frac{(x-2)(x-5) + (x-3)(x-4)}{(x-3)(x-5)} = \frac{10}{3}$$

$$\frac{x^2 - 7x + 10 + x^2 - 7x + 12}{x^2 - 8x + 15} = \frac{10}{3}$$

$$\frac{2x^2 - 14x + 22}{x^2 - 8x + 15} = \frac{10}{3}$$

$$\frac{x^2 - 7x + 11}{x^2 - 8x + 15} = \frac{5}{3}$$

$$3x^2 - 21x + 33 = 5x^2 - 40x + 75$$

$$0 = 2x^2 - 19x + 42$$

$$0 = 2x^2 - 12x - 7x + 42$$

$$0 = 2x(x - 6) - 7(x - 6)$$

$$0 = (2x - 7)(x - 6)$$

$$(2x - 7) \quad (x - 6) = 0$$

$$2x - 7 = 0 \quad \text{or} \quad x - 6 = 0$$

$$x = \frac{7}{2} \quad \text{or} \quad x = 6$$

26. Let speed of stream be x km/hr

Speed of boat in still water = 18 km/hr

So, Speed of boat downstream = (18 + x) km/hr

Speed of boat upstream = (18 – x) km/hr

According to equation,

$$\frac{24}{18-x} = \frac{24}{18+x} + 1 \qquad \text{up} = \text{D} + 1$$

$$\frac{24}{18-x} - \frac{24}{18+x} = 1$$

$$\frac{18+x-18+x}{(18-x)(18+x)} = \frac{1}{24}$$

$$\frac{2x}{324 - x^2} = \frac{1}{24}$$

$$324 - x^2 = 48x$$

$$x^2 + 48x - 324 = 0$$

$$x^2 + 54x - 6x - 324 = 0$$

$$x(x + 54) - 6(x + 54) = 0$$

$$(x - 6)(x + 54) = 0$$

$$x = 6, -54$$

As speed cannot be negative,

$x = -54$ is rejected.

So, $x = 6$

$\therefore$ Speed of stream = 6 km/hr

27. $$3\left(\frac{3x-1}{2x+3}\right)-2\left(\frac{2x+3}{3x-1}\right) = 5$$

Let $\dfrac{3x-1}{2x+3} = y$

So, equation becomes

$$3y - \frac{2}{y} = 5$$
$$3y^2 - 2 = 5y$$
$$3y^2 - 5y - 2 = 0$$
$$3y^2 - 6y + y - 2 = 0$$
$$3y(y-2) + 1(y-2) = 0$$
$$(3y+1)(y-2) = 0$$
$$3y + 1 = 0 \quad \text{or} \quad y - 2 = 0$$
$$y = -\frac{1}{3} \quad \text{or} \quad y = 2$$

$y = -\frac{1}{3}$	$y = 2$
$\frac{3x-1}{2x+3} = -\frac{1}{3}$	$\frac{3x-1}{2x+3} = 2$
$9x - 3 = -2x - 3$	$3x - 1 = 4x + 6$
$11x = 0$	$x = -7$
$x = 0$	

28. Let original speed of the aircraft be x km/hr.

be x km/hr.

$\therefore$ New speed = (x – 200) km/hr.

Duration of flight at original speed

$$= \frac{600}{x} \text{ hours}$$

Duration of flight at reduced speed

$$= \frac{600}{x-200} \text{ hours}$$

According to question,

$$\frac{600}{x-200} = \frac{1}{2}+\frac{600}{x}$$
$$\frac{600}{x-200} - \frac{600}{x} = \frac{1}{2}$$
$$\frac{1}{x-200} - \frac{1}{x} = \frac{1}{1200}$$
$$\frac{x-x+200}{x(x-200)} = \frac{1}{1200}$$
$$x^2 - 200x = 240000$$
$$x^2 - 200x = 240000 = 0$$
$$x^2 - 600x + 400x - 240000 = 0$$
$$x(x-600) + 400(x-600) = 0$$
$$(x+400)(x-600) = 0$$
$$x = -400 \quad \text{or } x = 600$$

As x, being speed of aircraft can't be negative.

So, x = 600

$\therefore$ Original speed of aircraft = 600 km/hr

Duration of flight = $\dfrac{600}{600} = 1$ hour

29. Let the usual speed of plane be x km / hr

$\therefore$ Time taken = $\dfrac{1500}{x}$ hours

New speed = x + 250 km / hr

$\therefore$ Time taken = $\dfrac{1500}{x+250}$ km / hr

According to question,

$$\frac{1500}{x+250} = \frac{1500}{x} - \frac{1}{2}$$
$$\frac{1500}{x} - \frac{1500}{x+250} = \frac{1}{2}$$
$$\frac{1}{x} - \frac{1}{x+250} = \frac{1}{3000}$$
$$\frac{x+250-x}{x(x+250)} = \frac{1}{3000}$$
$$x^2 + 250x = 750000$$
$$x^2 + 250x - 750000 = 0$$
$$x^2 + 1000x - 750x - 750000 = 0$$
$$x(x+1000) - 750(x+1000) = 0$$
$$(x-750)(x+1000) = 0$$
$$x = 750 \quad \text{or} \quad x = -1000$$

Now, x being the speed of plane cannot be negative,

$x = -1000$ is rejected

So, $x = 750$

∴ Speed of plane = 750 km/hr

30. Let total number of camels be x.

According to question

$$\frac{1}{4}x + 2\sqrt{x} + 15 = x$$

$$2\sqrt{x} + 15 = x - \frac{x}{4}$$

$$2\sqrt{x} + 15 = \frac{3x}{4}$$

$$8\sqrt{x} + 60 = 3x$$

$$3x - 8\sqrt{x} - 60 = 0$$

$$3\left(\sqrt{x}\right)^2 - 8\sqrt{x} - 60 = 0$$

Let $\sqrt{x} = y$

$$3y^2 - 8y - 60 = 0$$

$$3y^2 - 18y + 10y - 60 = 0$$

$$3y(y-6) + 10(y-6) = 0$$

$$(3y+10)(y-6) = 0$$

$$y = -\frac{10}{3} \quad \text{or} \quad y = 6$$

Now,

$y = -\frac{10}{3}$ is rejected as number of camels can not be negative,

So, $y = 6$

i.e. $\sqrt{x} = 6$

∴ $x = 36$

So, total number of camels = 36

31. Let Varun's age be x years and Nihal's age be y years.

According to question.

$$x - 7 = 5(y-7)^2$$

$$x - 7 = 5(y-7)^2 \qquad \text{(i)}$$

For second equation,

$$y + 3 = \frac{2}{5}(x+3)$$

$$5y + 15 = 2x + 6$$

$$2x - 5y = 9 \qquad \text{(ii)}$$

From (ii), $x = \frac{9+5y}{2}$

Putting in (i), we get

$$\frac{9+5y}{2} - 7 = 5(y-7)^2$$

$$9 + 5y - 14 = 10(y^2 + 49 - 14y)$$

$$5y - 5 = 10(y^2 + 49 - 14y)$$

$$y - 1 = 2(y^2 + 49 - 14y)$$

$$y - 1 = 2y^2 + 98 - 28y$$

$$2y^2 - 29y + 99 = 0$$

$$y = \frac{29 \pm \sqrt{841 - 8(99)}}{4}$$

$$y = \frac{29 \pm \sqrt{49}}{4}$$

$$y = \frac{29 \pm 7}{4}$$

$$y = \frac{29+7}{4}, \qquad y = \frac{29-7}{4}$$

$$y = 9, \qquad y = \frac{11}{2}$$

Now, $y = \frac{11}{2}$ is rejected

So, $y = 9$

∴ Nihal's age = 9 years

$$\text{Varun's age} = \frac{9+5y}{2} = \frac{9+45}{2} = 27 \text{ years}$$

32.
$$\frac{1}{a+b+x} - \frac{1}{x} = \frac{1}{a} + \frac{1}{b}$$

$$\frac{x-(a+b+x)}{x(a+b+x)} = \frac{a+b}{ab}$$

$$\frac{-(a+b)}{x(a+b+x)} = \frac{a+b}{ab}$$

$$\frac{-1}{x(a+b+x)} = \frac{1}{ab}$$

$$x(a + b + x) + ab = 0$$

$$xa + xb + x^2 + ab = 0$$

$$x^2 + xa + xb + ab = 0$$

$$x(x + a) + b(x + a) = 0$$

$$(x + a)(x + b) = 0$$

$$x = -a \quad \text{or} \quad x = -b$$

WORKSHEET 2

Section A

1. $$\text{LHS} = x^2 - 3\sqrt{3}x + 6$$

$$= (-2\sqrt{3})^2 - 3\sqrt{3}\,(-2\sqrt{3}) + 6$$

$$= 12 + 18 + 6$$

$$= 36$$

$$\neq \text{RHS} (= 0)$$

So, $x = -2\sqrt{3}$ is not a solution of the given equation.

2. As $x = -\frac{1}{2}$ is a solution of $3x^2 + 2kx - 3 = 0$,

$$3\left(-\frac{1}{2}\right)^2 + 2k\left(-\frac{1}{2}\right) - 3 = 0$$

$$\frac{3}{4} - K - 3 = 0$$

$$K = \frac{3}{4} - 3 = \frac{-9}{4}$$

3. Let the two consecutive positive integers be x, x + 1.

According to question,

$$x(x + 1) = 240$$

$$x^2 + x - 240 = 0$$

4. $$x^2 + 6x + 5 = 0$$

$$x^2 + 5x + x + 5 = 0$$

$$x(x + 5) + 1(x + 5) = 0$$

$$(x + 1)(x + 5) = 0$$

$$x + 1 = 0 \quad \text{or} \quad x + 5 = 0$$

$$x = -1 \quad \text{or} \quad x = -5$$

5. $$x + \frac{2}{x} = 3$$

$$x^2 + 2 = 3x$$

$$x^2 - 3x + 2 = 0$$

$$x^2 - 2x - x + 2 = 0$$

$$x(x - 2) - 1(x - 2) = 0$$

$$(x - 1)(x - 2) = 0$$

$$x = 1, 2$$

6. $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$

$$\text{Discriminant} = (-2\sqrt{2})^2 - 4(\sqrt{3})(-2\sqrt{3})$$

$$= 8 + 24$$

$$= 32$$

7. $2x^2 + 5\sqrt{3}x + 6 = 0$

$$\text{Discriminant (D)} = (5\sqrt{3})^2 - 4(2)(6)$$

$$= 75 - 48$$

$$= 27 > 0$$

So, the given equation has real roots.

8. $$abx^2 + (b^2 - ac)x - bc = 0$$

$$abx^2 + b^2x - acx - bc = 0$$

$$bx(ax + b) - c(ax + b) = 0$$

$$(bx - c)(ax + b) = 0$$

$$bx - c = 0 \quad \text{or} \quad ax + b = 0$$

$$x = \frac{c}{b} \quad \text{or} \quad x = -\frac{b}{a}$$

9. Compare given Quadratic equation $2x^2 - kx + k = 0$ with $ax^2 + bx + c = 0$, we get

$a = 2, b = -k, c = k$

Discriminant (D) = 0

[Given roots are equal]

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow (-k)^2 - 4 \times 2 \times k = 0$

$\Rightarrow k^2 - 8k = 0$

$\Rightarrow k(k - 8) = 0$

$\Rightarrow k = 0 \text{ or } k = 8$

10. $$x^2 + \left(a + \frac{1}{a}\right)x + 1 = 0$$

$$ax^2 + (a^2 + 1)x + a = 0$$

$$ax^2 + a^2x + x + a = 0$$

$$ax(x + a) + 1(x + a) = 0$$

$$(ax + 1)(x + a) = 0$$

$$ax + 1 = 0 \quad \text{or} \quad x + a = 0$$

$$x = -\frac{1}{a} \quad \text{or} \quad x = -a$$

Section B

11. As $x = \frac{2}{3}$ is a root of equation

$$ax^2 + 7x + b = 0$$

$$a\left(\frac{2}{3}\right)^2 + 7\left(\frac{2}{3}\right) + b = 0$$

$$\frac{4}{9}a + \frac{14}{3} + b = 0$$

$$\frac{4a + 42 + 9b}{9} = 0$$

$$4a + 9b = -42 \qquad \text{(i)}$$

As $x = -3$ is a root of equation

$$ax^2 + 7x + b = 0$$

$$9a - 21 + b = 0$$

$$9a + b = 21 \qquad \text{(ii)}$$

From (ii), $b = 21 - 9a$

Putting in (i), we get

$$4a + 9(21 - 9a) = -42$$

$$4a + 189 - 81a = -42$$

$$189 + 42 = 81a - 4a$$

$$231 = 77a$$

$$a = 3$$

So, $b = 21 - 9(3)$

$$= 21 - 27$$

$$= -6$$

12. As -5 is a root of equation

$$px^2 + px + k = 0$$

$$p(-5)^2 + p(-5) + k = 0$$

$$25p - 5p + k = 0$$

$$20p + k = 0 \qquad \text{(i)}$$

Also, as equation has equal roots,

$$\text{Discriminant} = 0$$

$$p^2 - 4pk = 0$$

$$p(p - 4k) = 0$$

$$p = 0 \quad \text{or} \quad p = 4k$$

if $p = 0$, $20(0) + k = 0$

$$k = 0$$

if $p = 4k$, $20(4k) + k = 0$

$$k = 0$$

13. $$\sqrt{2x + 9} + x = 13$$

$$\sqrt{2x + 9} = 13 - x$$

Squaring both sides

$$2x + 9 = 169 + x^2 - 26x$$

$$x^2 - 28x + 160 = 0$$

$$x^2 - 20x - 8x + 160 = 0$$

$$x(x - 20) - 8(x - 20) = 0$$

$$(x - 8)(x - 20) = 0$$

$$x = 8 \quad \text{or} \quad x = 20$$

If $x = 20$

LHS $= \sqrt{40 + 9} + 20 = 27 \neq$ RHS $(= 13)$

So, $x = 20$ is rejected

If $x = 8$,

$$\text{LHS} = \sqrt{16+9} + 8 = 5 + 8 = 13 = \text{RHS}$$

Therefore, $x = 8$

14. $9x^2 - 6b^2x - (a^4 - b^4) = 0$

$9x^2 + [-3(b^2 - a^2) - 3(b^2 + a^2)]x + (-a^4 + b^4) = 0$

$9x^2 - 3(b^2 - a^2)x - 3(b^2 + a^2)x + (a^2 + b^2)(-a^2 + b^2) = 0$

$3x[3x - (b^2 - a^2)] - (a^2 + b^2)[3x - (b^2 - a^2)] = 0$

$[3x - (a^2 + b^2)][3x - (b^2 - a^2)] = 0$

$x = \dfrac{a^2+b^2}{3}$ or $x = \dfrac{b^2-a^2}{3}$

15.
$$\frac{4}{x} - 3 = \frac{5}{2x+3}$$
$$\frac{4-3x}{x} = \frac{5}{2x+3}$$
$$(4 - 3x)(2x + 3) = 5x$$
$$8x + 12 - 6x^2 - 9x = 5x$$
$$6x^2 + 6x - 12 = 0$$
$$x^2 + x - 2 = 0$$
$$x^2 + 2x - x - 2 = 0$$
$$x(x + 2) - 1(x + 2) = 0$$
$$(x - 1)(x + 2) = 0$$
$$x = 1, -2$$

16.
$$\sqrt{2}y^2 + 7y + 5\sqrt{2} = 0$$
$$\sqrt{2}y^2 + 2y + 5y + 5\sqrt{2} = 0$$
$$\sqrt{2}y(y + \sqrt{2}) + 5(y + \sqrt{2}) = 0$$
$$(y + \sqrt{2})(\sqrt{2}y + 5) = 0$$
$$y + \sqrt{2} = 0 \quad \text{or} \quad \sqrt{2}y + 5 = 0$$
$$y = -\sqrt{2} \quad \text{or} \quad y = -\frac{5}{\sqrt{2}}$$

17. Roots of the equation are equal if Discriminant (D) = 0

$$mx(6x + 10) + 25 = 0$$
$$6mx^2 + 10mx + 25 = 0$$
$$D = 0$$
$$(10m)^2 - 4(6m)25) = 0$$
$$100m^2 - 600m = 0$$
$$100m(m - 6) = 0$$
$$m = 0, 6$$

For $m = 0$, equation will become $25 = 0$, which is not possible

So, $m = 6$

18. Given 4 roots 3x square + 5x – 2 root 3

The above given education can be written as under:

$$4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$$
$$4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3} = 0$$
$$4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2) = 0$$
$$(\sqrt{3}x + 2)(4x - \sqrt{3}) = 0$$
$$(\sqrt{3}x + 2) = 0 \quad \text{or} \quad (4x - \sqrt{3}) = 0$$
$$x = \frac{2}{\sqrt{3}} \quad \text{or} \quad x = \frac{\sqrt{3}}{4}$$

19. Let x be the side of square.

So, area of square = x^2

Number of students = $x^2 + 24$

If side of a square is increased by one student, side = $x + 1$

So, number of students = $(x + 1)^2 - 25$

According to question,

$$x^2 + 24 = (x + 1)^2 - 25$$
$$x^2 + 24 = x^2 + 1 + 2x - 25$$
$$48 = 2x$$
$$x = 24$$

$\therefore$ Number of students $= x^2 + 24$

$= (24)^2 + 24$

$= 576 + 24$

$= 600$

20. $4x^2 + 3x + 5 = 0$

$\Rightarrow x^2 + \frac{3}{4}x + \frac{5}{4} =$

$\Rightarrow x^2 + \frac{3}{4}x + \left(\frac{3}{8}\right)^2 = \frac{-5}{4} + \frac{9}{64}$

$\Rightarrow \left(x + \frac{3}{8}\right)^2 = \frac{-71}{64}$

$\Rightarrow x + \frac{3}{8} = \sqrt{\frac{-71}{64}}$ not a real no.

Hence, QE has no real roots.

Section C

21. $(x-5)(x-6) = \frac{25}{(24)^2}$

$x^2 - 11x + 30 = \frac{25}{576}$

$x^2 - 11x + 30 - \frac{25}{576} = 0$

$x^2 - 11x + \frac{17255}{576} = 0$

$576x^2 - 6336x + 17255 = 0$

$576x^2 - 2856x - 3480x + 17255 = 0$

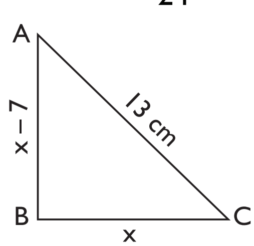
$24x(24x - 119) - 145(24x - 119) = 0$

$(24x - 145)(24x - 119) = 0$

$24x - 145 = 0$ or $24x - 119 = 0$

$x = \frac{145}{24}$ or $x = \frac{119}{24}$

22.



Let the base of Δ ABC=x cm

$\therefore$ Altitude of Δ ABC $= (x - 7)$ cm

We know that,

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

$AC^2 = AB^2 + BC^2$

$(13)^2 = (x-7)^2 + x^2$

$169 = x^2 + 49 - 14x + x^2$

$2x^2 - 14x - 120 = 0$

$x^2 - 7x - 60 = 0$

$x^2 - 12x + 5x - 60 = 0$

$x(x - 12) + 5(x - 12) = 0$

$(x + 5)(x - 12) = 0$

$x = -5, 12$

Since, side cannot be negative,

So, $x = -5$ is rejected

$\therefore$ $x = 12$

$BC = x = 12$ cm

$AB = x - 7 = 12 - 7 = 5$ cm

23. $(a - b)x^2 + (b - c)x + (c - a) = 0$

As roots of equation are equal,

Discriminant (D) = 0

$(b - c)^2 - 4(a - b)(c - a) = 0$

$(b^2 + c^2 - 2bc) - 4(ac - a^2 - bc + ab) = 0$

$\Rightarrow 4a^2 + b^2 + c^2 - 4ac + 2bc - 4ab = 0$

$\Rightarrow (2a)^2 + b^2 + c^2 - 4ac + 2bc - 4ab = 0$

$\Rightarrow (-2a + b + c)^2 = 0$

$\Rightarrow -2a + b + c = 0$

$\Rightarrow 2a = b + c$

24. Let the sides of two squares be x and y

Area of square with side $x = x^2$

Area of square with side $y = y^2$

Perimeter of square with side x = 4x

Perimeter of square with side y = 4y

According to question,

$$x^2 + y^2 = 468 \quad \text{(i)}$$
$$4x - 4y = 24$$
$$\text{i.e. } x - y = 6 \quad \text{(ii)}$$

From (ii), $x = 6 + y$

On putting in (i), we get

$$(6 + y)^2 + y^2 = 468$$
$$36 + y^2 + 12y + y^2 = 468$$
$$2y^2 + 12y - 432 = 0$$
$$y^2 + 6y - 216 = 0$$
$$y^2 - 12y + 18y - 216 = 0$$
$$y(y - 12) + 18(y - 12) = 0$$
$$(y - 12)(y + 18) = 0$$
$$y = 12, -18$$

As side cannot be negative,

$y = -18$ is rejected

$\therefore \quad y = 12$

So, $\quad x = 6 + y$
$= 6 + 12$
$= 18$

So, sides of two squares are 12m and 18m respectively.

25. $$a^2 x^2 - 3abx + 2b^2 = 0$$
$$(ax)^2 - 2\left(\frac{3}{2}\right)abx + 2b^2 = 0$$
$$(ax)^2 - 2ax\left(\frac{3b}{2}\right) + 2b^2 = 0$$
$$(ax)^2 - 2ax\left(\frac{3b}{2}\right) + \left(\frac{3b}{2}\right)^2 + 2b^2 - \left(\frac{3b}{2}\right)^2 = 0$$
$$\left(ax - \frac{3b}{2}\right)^2 + 2b^2 - \frac{9}{4}b^2 = 0$$
$$\left(ax - \frac{3b}{2}\right)^2 - \frac{b^2}{4} = 0$$
$$\left(ax - \frac{3b}{2}\right)^2 = \frac{b^2}{4}$$
$$ax - \frac{3b}{2} = \pm\frac{b}{2}$$

$$ax - \frac{3b}{2} = \frac{b}{2} \qquad ax - \frac{3b}{2} = -\frac{b}{2}$$
$$ax = \frac{4b}{2} = 2b \qquad ax = -\frac{b}{2} + \frac{3b}{2} = b$$
$$x = \frac{2b}{a} \qquad x = \frac{b}{a}$$

26. $$\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0$$

$$\Rightarrow \quad \frac{2x(2x+3) + (x-3) + 3x + 9}{(x-3)(2x+3)} = 0$$
$$\Rightarrow \quad 2x(2x + 3) + (x - 3) + 3x + 9 = 0$$
$$\Rightarrow \quad 4x^2 + 6x + x - 3 + 3x + 9 = 0$$
$$\Rightarrow \quad 4x^2 + 10x + 6 = 0$$
$$\Rightarrow \quad 4x^2 + 4x + 6x + 6 = 0$$
$$\Rightarrow \quad 4x(x + 1) + 6(x + 1) = 0$$
$$\Rightarrow \quad (4x + 6)(x + 1) = 0$$
$$\Rightarrow \quad 4x + 6 = 0 \quad \text{or} \quad x + 1 = 0$$
$$\Rightarrow \quad x = -\frac{3}{2} \quad \text{or} \quad x = -1$$

27. Let the three consecutive natural numbers be $x - 1$, x and $x + 1$.

According to equation,

$$x^2 = [(x + 1)^2 - (x - 1)^2] + 60$$
$$x^2 = x^2 + 1 + 2x - x^2 - 1 + 2x + 60$$
$$x^2 = 4x + 60$$
$$x^2 - 4x - 60 = 0$$
$$x^2 - 10x + 6x - 60 = 0$$
$$x(x - 10) + 6(x - 10) = 0$$
$$(x + 6)(x - 10) = 0$$
$$x = -6 \text{ or } 10$$

As x is a natural number,

$$x = -6 \text{ or } 10$$

As x is a natural number,

$x = -6$ is rejected

So, $\quad x = 10$

$\therefore$ The three numbers 9, 10, 11.

28. Let the time taken by smaller tap to fill tank completely = x hours

$\therefore$ Time taken by larger tap to fill tank completely = x – 8 hours

According to question,

$$\frac{1}{x} + \frac{1}{x-8} = \frac{5}{48}$$

$$\frac{x-8+x}{x(x-8)} = \frac{5}{48}$$

$$\frac{2x-8}{x(x-8)} = \frac{5}{48}$$

$$48\,(2x - 8) = 5x\,(x - 8)$$

$$96x - 384 = 5x^2 - 40x$$

$$5x^2 - 136x + 384 = 0$$

$$5x^2 - 16x - 120x + 384 = 0$$

$$x\,(5x - 16) - 24\,(5x - 16) = 0$$

$$(x - 24)\,(5x - 16) = 0$$

$$x = 24 \quad \text{or} \quad \frac{16}{5}$$

For x = 24

Time taken by smaller tap = 24 hours

Tap taken by larger tap = x – 8

= 24 – 8

= 16 hours

For $x = \frac{16}{5}$

Time taken by larger pipe = x – 8

$$= \frac{16}{5} - 8$$

$$= -\frac{24}{5}$$

Since time cannot be negative,

$x = \frac{16}{5}$ is rejected.

29. $9x^2 - 63x - 162 = 0$

$$\text{Discriminant (D)} = (-63)^2 - 4(9)(-162)$$

$$= 3969 + 5832$$

$$= 9801$$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{63 \pm \sqrt{9801}}{18}$$

$$= \frac{63 \pm 99}{18}$$

$$x = \frac{63+99}{18} \quad \text{or} \quad x = \frac{63-99}{18}$$

$$x = 9 \quad \text{or} \quad x = -2$$

30. Let the larger part be x.

$\therefore$ Smaller part = 16 – x

According to question,

$$2(x)^2 = (16 - x)^2 + 164$$

$$2x^2 = 256 + x^2 - 32x + 164$$

$$x^2 + 32x - 420 = 0$$

$$x^2 + 42x - 10x - 420 = 0$$

$$x\,(x + 42) - 10\,(x + 42) = 0$$

$$(x - 10)\,(x + 42) = 0$$

$$x = 10 \quad \text{or} \quad -42$$

$x = -42$ is rejected as $x < 0$.

$\therefore$ x = 10

So, the required parts are 10 and 6.

Section D

31. $$\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$$

$$\frac{1}{2a+b+2x} - \frac{1}{2x} = \frac{1}{b} + \frac{1}{2a}$$

$$\frac{2x-2a-b-2x}{2x\,(2a+b+2x)} = \frac{2a+b}{2ab}$$

$$\frac{-2a-b}{2x\,(2a+b+2x)} = \frac{2a+b}{2ab}$$

$$\frac{-1}{2x\,(2a+b+2x)} = \frac{1}{2ab}$$

$$\frac{-1}{x\,(2a+b+2x)} = \frac{1}{ab}$$

$$x\,(2a + b + 2x) + ab = 0$$

$$2x^2 + 2ax + bx + ab = 0$$
$$2x(x + a) + b(x + a) = 0$$
$$(2x + b)(x + a) = 0$$
$$x = \frac{-b}{2}, -a$$

32. Let number of books = x

$\therefore$ Cost of each book = $\frac{80}{x}$

According to question,

$$\frac{80}{x+4} = \frac{80}{x} - 1$$
$$\frac{80}{x} - \frac{80}{x+4} = 1$$
$$\frac{1}{x} - \frac{1}{x+4} = \frac{1}{80}$$
$$\frac{x+4-x}{x(x+4)} = \frac{1}{80}$$
$$\frac{1}{x(x+4)} = \frac{1}{320}$$
$$x^2 + 4x - 320 = 0$$
$$x^2 - 16x + 20x - 320 = 0$$
$$x(x - 16) + 20(x - 16) = 0$$
$$(x - 16)(x + 20) = 0$$
$$x = 16 \quad \text{or} \quad x = -20$$

Since, number of books cannot be negative, $x = 16$

So, number of books = 16

33. Let original duration of flight = x hours

Speed of an aircraft = $\frac{2800}{x}$ km/hr

If time increased by 30 minutes

i.e. $\frac{1}{2}$ hour, speed = $\frac{2800}{x+\frac{1}{2}}$

According to question,

$$\frac{2800}{x+\frac{1}{2}} = \frac{2800}{x} - 100$$
$$\frac{2800}{x} - \frac{2800}{\frac{2x+1}{2}} = 100$$
$$\frac{2800}{x} - \frac{5600}{2x+1} = 100$$
$$\frac{28}{x} - \frac{56}{2x+1} = 1$$
$$\frac{1}{x} - \frac{2}{2x+1} = \frac{1}{28}$$
$$\frac{2x+1-2x}{x(2x+1)} = \frac{1}{28}$$
$$\frac{1}{x(2x+1)} = \frac{1}{28}$$
$$2x^2 + x - 28 = 0$$
$$2x^2 + 8x - 7x - 28 = 0$$
$$2x(x + 4) - 7(x + 4) = 0$$
$$(x + 4)(2x - 7) = 0$$
$$x = -4, \frac{7}{2}$$

Since, time cannot be negative,

$$x = \frac{7}{2} = 3.5 \text{ hours}$$

34. Let speed of stream = x km/hr

Speed of boat in still water = 20 km/hr

Speed of boat upstream = (20 – x) km/hr

Speed of boat downstream = (20 + x) km/hr

According to equation,

$$\frac{48}{20-x} = \frac{48}{20+x} + 1$$
$$\frac{1}{20-x} - \frac{1}{20+x} = \frac{1}{48}$$
$$\frac{20+x-20+x}{(20-x)(20+x)} = \frac{1}{48}$$
$$\frac{2x}{(20-x)(20+x)} = \frac{1}{48}$$
$$96x = 400 - x^2$$
$$x^2 + 96x - 400 = 0$$

$$x^2 + 100x - 4x - 400 = 0$$
$$x(x + 100) - 4(x + 100) = 0$$
$$(x - 4)(x + 100) = 0$$
$$x = 4, -100$$

Being the speed, x can not be negative.

So, $x = -100$ is rejected

$\therefore \quad x = 4$

Speed of stream = 4 km/hr

35. $$\frac{1}{2x-3} + \frac{1}{x-5} = 1$$
$$\frac{x-5+2x-3}{(2x-3)(x-5)} = 1$$
$$\frac{3x-8}{2x^2-10x-3x+15} = 1$$
$$2x^2 - 13x + 15 = 3x - 8$$
$$2x^2 - 16x + 23 = 0$$
$$\text{Discriminant (D)} = (-16)^2 - 4(2)(23)$$
$$= 256 - 184$$
$$= 72$$
$$x = \frac{-b \pm \sqrt{D}}{2a}$$
$$= \frac{16 \pm \sqrt{72}}{4}$$
$$= \frac{16 \pm 6\sqrt{2}}{4}$$
$$= \frac{8 \pm 3\sqrt{2}}{2}$$

36. Let present age of sister be x years

$\therefore$ age of girl = 2x years

According to question,
$$(x + 4)(2x + 4) = 160$$
$$2x^2 + 12x + 16 - 160 = 0$$
$$2x^2 + 12x - 144 = 0$$
$$x^2 + 6x - 72 = 0$$
$$x^2 + 12x - 6x - 72 = 0$$
$$x(x + 12) - 6(x + 12) = 0$$
$$(x - 6)(x + 12) = 0$$
$$x = 6, -12$$

As age cannot be negative,
$$x = -12 \text{ is rejected}$$
$$\therefore \quad x = 6$$
$$\text{Age of sister} = 6 \text{ years}$$
$$\text{Age of girl} = 2x$$
$$= 2(6)$$
$$= 12 \text{ years}$$

37. Let number of articles be x

$\therefore$ Cost of production of each article = 2x + 3

According to question,
$$x(2x + 3) = 90$$
$$2x^2 + 3x - 90 = 0$$
$$2x^2 - 12x + 15x - 90 = 0$$
$$2x(x - 6) + 15(x - 6) = 0$$
$$(2x + 15)(x - 6) = 0$$
$$x = \frac{-15}{2} \quad \text{or} \quad x = 6$$

Being number of articles, x cannot be negative.

$\therefore \quad x = 6$

Number of articles = 6

Cost of production of each article
$$= 2x + 3$$
$$= 12 + 3$$
$$= \$\ 15$$

38. Let Shefali's marks in English be x..

$\therefore$ Shefali's marks in Mathematics = 30 – x

According to question,
$$(30 - x + 2)(x - 3) = 210$$

$$(32 - x)(x - 3) = 210$$
$$32x - 96 - x^2 + 3x = 210$$
$$x^2 - 35x + 306 = 0$$
$$x^2 - 17x - 18x + 306 = 0$$
$$x(x - 17) - 18(x - 17) = 0$$
$$(x - 17)(x - 18) = 0$$
$$x = 17 \quad \text{or} \quad x = 18$$

If $x = 17$

Shefali's marks in English = 17

Shefali's marks in Mathematics = 30 – 17 = 13

If $x = 18$

Shefali's marks in English = 18

Shefali's marks in Mathematics = 30 – 18

= 12

39. Let speed of train = x km/hr

Distance covered = 360 km

So, time taken = $\frac{360}{x}$

According to question,

$$\frac{360}{x+5} = \frac{360}{x} - 1$$
$$\frac{360}{x} - \frac{360}{x+5} = 1$$
$$\frac{x+5-x}{x(x+5)} = \frac{1}{360}$$
$$\frac{5}{x(x+5)} = \frac{1}{360}$$
$$x^2 + 5x - 1800 = 0$$
$$x^2 - 40x + 45x - 1800 = 0$$
$$x(x - 40) + 45(x - 40) = 0$$
$$(x - 40)(x + 45) = 0$$
$$x = 40, -45$$

Being speed of train, $x = -45$ is rejected.

$\therefore$ Speed of train = 40 km/hr

40. Let breadth of rectangular mango grove = x m

$\therefore$ Length = $2x$

According to question,

$$2x(x) = 800$$
$$2x^2 = 800$$
$$x^2 = 400$$
$$x = \pm 20$$

Being a dimension, x cannot be negative.

$\therefore$ $x = 20$

So, Breadth = 20 m

Length = $2x$ = 40 m

Chapter 05 Arithmetic Progressions

MULTIPLE CHOICE QUESTIONS

1. (b) $a_n = 3n + 7$

$$a_{n+1} = 3(n+1) + 7 = 3n + 10$$

So, $d = a_{n+1} - a_n$

$$= 3n + 10 - 3n - 7$$

$$= 3$$

2. (c) $a = 1, a_n = 11$

$S_n = 36$

We know that $S_n = \frac{n}{2}(a + a_n)$

$$36 = \frac{n}{2}(1 + 11)$$

$$n = \frac{36 \times 2}{12}$$

$$= 6$$

3. (b) $S_n = 2n^2 + 5n$

$$a_n = S_n - (S_{n-1})$$

$$= (2n^2 + 5n) - [2(n-1)^2 + 5(n-1)]$$

$$= 2n^2 + 5n - 2n^2 - 2 + 4n - 5n + 5$$

$$= 4n + 3$$

4. (d) We can write reverse AP as

185, ..., 13, 9, 5

Such that a = 185, d = –4

So, $a_9 = 185 + (9-1)(-4)$

$$= 185 - 32$$

$$= 153$$

5. (a) 18, a, b, – 3 are in AP.

$\therefore$ $a - 18 = b - a = -3 - b$

$$a - 18 = b - a$$

$$2a - b = 18 \quad \text{(i)}$$

$$b - a = -3 - b$$

$$a - 2b = 3 \quad \text{(ii)}$$

Solving (i) and (ii), we get

$$a = 11, b = 4$$

So, $a + b = 11 + 4$

$$= 15$$

WORKSHEET 1

Section A

1. k + 9, 2k – 1 and 2k + 7 are in A.P. if

$$(2k - 1) - (k + 9) = (2k + 7) - (2k - 1)$$

$$k - 10 = 8$$

$$k = 18$$

2. $S_n = 3n^2 + 5n$

$$S_{20} = 3(20)^2 + 5(20)$$

$$= 3(400) + 100$$

$$= 1200 + 100$$

$$= 1300$$

3. Consider AP : 2, 4, 6, 8, ..., n

Here a = 2, d = 2

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{n}{2}[4 + (n-1)^2]$$

$$= \frac{n}{2}[2n + 2]$$

$$= n(n+1)$$

4. A.P.: $-5, -\frac{5}{2}, 0, \frac{5}{2}, \ldots$.

$a_n = a + (n-1)d, \; d = -\frac{5}{2} + 5 = \frac{5}{2}$

$\therefore a_{25} = -5 + (25-1)\frac{5}{2}$

$= -5 + 24\left(\frac{5}{2}\right)$

$= -5 + 60$

$= 55$

5. $S_p = ap^2 + bp$

$a_p = S_p - S_{p-1}$

$= (ap^2 + bp) - [a(p-1)^2 + b(p-1)]$

$= ap^2 + bp - [ap^2 + a - 2ap + bp - b]$

$= ap^2 + bp - ap^2 - a + 2ap - bp + b$

$= 2ap - a + b$

$\therefore \quad a_{p+1} = 2a(p+1) - a + b$

$= 2ap + 2a - a + b$

$= 2ap + a + b$

So, $\quad d = a_{p+1} - a_p$

$= 2ap + a + b - 2ap + a - b$

$= 2a$

6. $a_n = n^2 + 1$

$a_1 = 1 + 1 = 2$

$a_2 = 2^2 + 1 = 5$

7. $a_n = \frac{3n-2}{4n+5}$

$a_1 = \frac{3-2}{4+5} = \frac{1}{9}, \quad a_2 = \frac{6-2}{8+5} = \frac{4}{13}$

$a_3 = \frac{9-2}{17} = \frac{7}{17}$

So, sequence is $\frac{1}{9}, \frac{4}{13}, \frac{7}{17}, \ldots$

8. $a_n = 3n - 2$

$a_1 = 3 - 2 = 4$

$a_2 = 6 - 2 = 4$

$a_3 = 9 - 2 = 7$ so on

So, sequence is 1, 4, 7, ...

$a_2 - a_1 = 4 - 1 = 3$

$a_3 - a_2 = 7 - 4 = 3$

As difference between the terms is same, so, the given sequence is in A.P.

$a_{10} = 30 - 2 = 28$

9. A.P : 18, 16, 14, ...

$S_n = 0$

$\frac{n}{2}[2a + (n-1)d] = 0$

$\frac{n}{2}[36 + (n-1)(-2)] = 0$

$n(36 - 2n + 2) = 0$

$n(38 - 2n) = 0$

$n = \frac{38}{2} = 19$

10. $a_4 = 0 \Rightarrow a + 3d = 0 \Rightarrow a = -3d$

To prove : $\quad a_{25} = 3a_{11}$

Consider $\quad a_{25} = a + (25-1)d$

$= a + 24d$

$= -3d + 24d$

$= 21d$

$a_{11} = a + 10d$

$= -3d + 10d$

$= 7d$

So, $\quad a_{25} = 3a_{11}$

11. A. P: 6, 13, 20, ..., 216

$a_n = a + (n-1)d$

$216 = 6 + (n-1)7$

$210 = 7(n-1)$

$30 = n - 1$

$n = 31$

So, 216 is 31st term of an A.P.

So, 16th term is the middle term

$$a_{16} = 6 + (16 - 1)\,7$$
$$= 6 + 7\,(15)$$
$$= 6 + 105$$
$$= 111$$

12. Consider 9, 12, 15, 18, ...

$$a_2 - a_1 = 12 - 9 = 3$$
$$a_3 - a_2 = 15 - 12 = 3$$
$$a_4 - a_3 = 18 - 15 = 3$$

As difference between the terms is same,

So, the terms are in A.P.

$$a_{16} = a + 15d$$
$$= 9 + 15\,(3)$$
$$= 9 + 45$$
$$= 54$$
$$a_n = a + (n - 1)\,d$$
$$= 9 + (n - 1)\,3$$
$$= 9 + 3n - 3$$
$$= 3n + 6$$

13. $S_5 + S_7 = 167$

$$\frac{5}{2}\,[2a + (5 - 1)\,d] + \frac{7}{2}\,[2a + (7 - 1)\,d] = 167$$
$$\Rightarrow \quad 5a + \frac{5}{2}\,(4d) + 7a + \frac{7}{2}\,(6d) = 167$$
$$\Rightarrow \quad 5a + 10d + 7a + 21d = 167$$
$$\Rightarrow \quad 12a + 31d = 167 \qquad \text{(i)}$$
$$S_{10} = 235$$
$$\frac{10}{2}\,[2a + (10 - 1)\,d] = 235$$
$$2a + 9d = 47 \qquad \text{(ii)}$$

Multiplying equation (ii) by 6 and subtracting (ii) from (i), we get

$$(12a + 54d) - (12a + 31d)$$
$$= 282 - 167$$
$$23d = 115$$
$$d = \frac{115}{23} = 5$$

From (ii), $2a + 9(5) = 47$

$$2a + 45 = 47$$
$$a = 1$$

So, AP is a, a + d, a + 2d, ...

i.e. 1, 6, 11, ...

14. A.P.: 5, 15, 25, ...

Let n^{th} term of an AP be 130 more than its 31^{st} term

i.e. $a_n = 130 + a_{31}$

$$5 + (n - 1)\,10 = 130 + 5 + (31 - 1)\,10$$
$$5 + 10n - 10 = 135 + 300$$
$$10n = 435 + 5$$
$$10n = 440$$
$$n = 44$$

So, 44^{th} term of an AP is 130 more than its 31^{st} term

15.
$$a_s + a_9 = 72$$
$$a + 4d + a + 8d = 72$$
$$2a + 12d = 72$$
$$a + 6d = 36 \qquad \text{(i)}$$
$$a_7 + a_{12} = 97$$
$$a + 6d + a + 11d = 97$$
$$2a + 17d = 97 \qquad \text{(ii)}$$

On multiplying (i) by 2 and subtracting (ii) from (i), we get

$$(2a + 17d) - (2a + 12d) = 97 - 72$$
$$5d = 25$$
$$d = 5$$

From (i), $a = 36 - 6\,(5)$

$$= 36 - 30 = 6$$
$$a = 6$$

So, AP is a, a + d, a + 2d, ...

i.e. 6, 11, 16, ...

16. Consider AP : 7, 14, 21, ..., 497

$$a_n = a + (n - 1)d$$
$$497 = 7 + (n - 1)7$$
$$497 - 7 = 7(n - 1)$$
$$70 = \frac{490}{7} = n - 1$$
$$n = 71$$

17. $S_7 = 49$

$$\frac{7}{2}[2a + 6d] = 49$$
$$2a + 6d = 14$$
$$a + 3d = 7 \quad \text{(i)}$$

Also, $S_{17} = 289$

$$\frac{17}{2}[2a + 16d] = 289$$
$$2a + 16d = 34$$
$$a + 8d = 17 \quad \text{(ii)}$$

On subtracting (ii) from (i), we get

$$(a + 3d) - (a + 8d) = 7 - 17$$
$$-5d = -10$$
$$d = 2$$

From (i), $a = 7 - 3d$
$$= 7 - 6$$
$$= 1$$

So, $S_n = \frac{n}{2}[2a + (n - 1)d]$
$$= \frac{n}{2}[2 + (n - 1)2]$$
$$= n[1 + (n - 1)]$$
$$= n^2$$

18. Let Sn and S_n^{I} be sum of a terms of two A.P.

$$\frac{S_n}{S_n^{I}} = \frac{7n+1}{4n+27}$$

$$\frac{\frac{n}{2}\left[2a+(n-1)d\right]}{\frac{n}{2}\left[2a^{I}+(n-1)d^{I}\right]} = \frac{7n+1}{4n+27}$$

$$\frac{2a+(n-1)d}{2a^{I}+(n-1)d^{I}} = \frac{7n+1}{4n+27}$$

$$\frac{a+\left(\frac{n-1}{2}\right)d}{a^{I}+\left(\frac{n-1}{2}\right)d^{I}} = \frac{7n+1}{4n+27}$$

$$\frac{a+(m-1)d}{a^{I}+(m-1)d^{I}} = \frac{7(2m-1)+1}{4(2m-1)+27}$$

$$\frac{a_m}{a_m^{I}} = \frac{14m-6}{8m+23}$$

19. Let the digits be a – d, a, a + d

$$a - d + a + a + d = 15$$
$$3a = 15$$
$$a = 5$$

Also, $100(a - d) + 10a + a + d$
$$= [100(a + d) + 10a + a - d] - 594$$

$\therefore$ $100a - 100d + 11a + d$
$$= 100a + 100d + 11a - d - 594$$
$$0 = 200d - 2d - 594$$
$$198d = 594$$
$$d = 3$$

So, number $= 100(a + d)\, 10a + a - d$
$$= 100(8) + 50 + 2$$
$$= 852$$

20. $a_p = q \Rightarrow a + (p - 1)d = q \quad \text{(i)}$

$a_q = p \Rightarrow a + (q - 1)d = p \quad \text{(ii)}$

On subtracting (ii) from (i), we get

$$(p - 1)d - (q - 1)d = q - p$$
$$d[p - 1 - q + 1] = q - p$$
$$d = \frac{q-p}{p-q} = -1$$

From (i), $a + (p - 1)(-1) = q$
$$a = p - 1 + q$$

So, $a_n = a + (n - 1)d$

$= (p - 1 + q) + (n - 1)(-1)$

$= p - 1 + q - n + 1$

$= p + q - n$

21. Here, $a2 - a1 = 19\frac{1}{4} - 20$

$= \frac{77}{4} - 20$

$= \frac{77 - 80}{4} = -\frac{3}{4}$

$a_3 - a_2 = 18\frac{1}{2} - 19\frac{1}{4}$

$= \frac{37}{2} - \frac{77}{4}$

$= \frac{74 - 77}{4}$

$= \frac{-3}{4}$

as $a_3 - a_2 = a_2 - a_1$

i.e._difference between the terms is same, so, the given sequence forms an A.P.

Here, $a = 20, d = \frac{-3}{4}$

$a_n < 0$

$a + (n - 1) d < 0$

$20 + (n - 1)\left(\frac{-3}{4}\right) < 0$

$\frac{-3}{4}(n - 1) < -20$

$n - 1 > -20\left(\frac{-4}{3}\right)$

$n - 1 > \frac{80}{3}$

$n > \frac{80}{3} + 1 = \frac{83}{3} = 27.67$

So, $n = 28$

So, a_{28} is the first negative term

22. For S_1, $a = 1$, $d = 1$

So, $S_1 = \frac{n}{2}[2 + (n - 1)(1)]$

$\frac{n}{2}(n + 1)$

For S_2, $a = 1$, $d = 2$

So, $S_2 = \frac{n}{2}[2 + (n - 1)(2)]$

$= n[1 + n - 1]$

$= n^2$

For S_3, $a = 1$, $d = 3$

$S3 = \frac{n}{2}[2 + (n - 1)3]$

$= \frac{n}{2}[3n - 1]$

Consider $S_1 + S_3 = \frac{n}{2}(n + 1) + \frac{n}{2}(3n - 1)$

$= \frac{n^2}{2} + \frac{n}{2} + \frac{3n^2}{2} - \frac{n}{2}$

$= \frac{4n^2}{2}$

$= 2n^2$

$= 2S_2$

23. A.P.: a, 7, b, 23, c

As the terms are in A.P.,

$7 - a = b - 7 = 23 - b = c - 23$

As $7 - a = b - 7$

$a + b = 14$ (i)

As $b - 7 = 23 - b$

$2b = 30$

$b = 15$

From (i), $a = 14 - b = 14 - 15$

$a = -1$

As $23 - b = c - 23$

$23 - 15 = c - 23$

$c = 31$

24. Let the four parts be

$a - 3d, a - d, a + d, a + 3d$ such that

$a - 3d + a - d + a + d + a + 3d = 32$

$4a = 32$

$a = 8$

Also, $\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{7}{15}$

i.e. $\frac{(8-3d)(8+3d)}{(8-d)(8+d)} = \frac{7}{15}$

$$\frac{64-9d^2}{64-d^2} = \frac{7}{15}$$

$$960 - 135d^2 = 448 - 7d^2$$

$$512 = 128\, d^2$$

$$d^2 = 4$$

$$d = \pm 2$$

For a = 8, d = 2

the four parts are

a – 3d, a – d, a + d, a + 3d

i.e. 8 – 6, 8 – 2, 8 + 2, 8 + 6

i.e. 2, 6, 10, 14

For a = 8, d = – 2

the four parts are

a – 3d, a – d, a + d, a + 3d

i.e. 8 + 6, 8 + 2, 8 – 2, 8 – 2, 8 – 6

i.e. 14, 10, 6 and 2

25. Let the policeman catches the thief in n minutes.

Uniform speed of thief = 100 m/min

As after are minute a policeman runs after the thief to catch him.

So, distance travelled by thief.

= 100 (n + 1) minutes

Given that speed of policeman increases by 10m/min.

speed of policeman forms an AP:

100 m/min, 110 m/min, 120 m/min, ...

So, distance travelled by policeman

$= S_n$

$= \frac{n}{2}[2a + (n-1)\, d]$

$= \frac{n}{2}[200 + (n-1)\, 10]$

$= n\,[100 + 5n - 5]$

$= n\,(95 + 5n)$

Distance travelled by thief

= Distance travelled by policeman

$$100\,(n+1) = n\,(95+5n)$$

$$100n + 100 = 95n + 5n^2$$

$$5n^2 - 5n - 100 = 0$$

$$n^2 - n - 20 = 0$$

$$n^2 - 5n + 4n - 20 = 0$$

$$n\,(n-5) + 4\,(n-5) = 0$$

$$(n+4)\,(n-5) = 0$$

$$n = -4 \text{ or } n = 5$$

As n cannot be negative, n = 5

26. Consider the sequence formed by all three digit numbers which leaves a remainder 3, when divided by 4: 103, 107, 111, 115, ..., 999.

The above sequence forms an A.P. with a = 103 and common difference d = 4

$$a_n = a + (n-1)d$$

$$999 = 103 + (n-1)\,4$$

$$4\,(n-1) = 999 - 103$$

$$4\,(n-1) = 896$$

$$n - 1 = 224$$

$$n = 225$$

The middle term is $\left(\frac{n+1}{2}\right)^{th}$ term

i.e. $\left(\frac{225+1}{2}\right) = 113^{th}$ term

$$a_{113} = 103 + (113-1)\,4$$

$$= 103 + 112\,(4)$$

$$= 103 + 448$$

$$= 551$$

Sum of all terms before middle term

$= S_{112}$

$= \frac{112}{2}$ [2 (103) + (112 – 1) 4]

= 56 [206 + 144]

= 56 (650)

= 36,400

$S_{225} = \frac{225}{2}$ [2 (103) + (225 – 1) 4]

$= \frac{225}{2}$ [206 + 896]

$= \frac{225}{2}(1102)$

= 123975

So, sum of terms after the middle term

$= 123975 – (S_{112} + 551)$

= 123975 – 36400 – 551

= 87024

27. Given: $S_m = S_n$

To prove: $S_{m+n} = 0$

$S_m = \frac{m}{2}[2a + (m – 1)d]$

$S_n = \frac{n}{2}[2a + (n – 1)d]$

As $S_m = S_n$

$\therefore \frac{m}{2}[2a + (m – 1)d] = \frac{n}{2}[2a + (n – 1)d]$

$m [2a + (m – 1)d] = n [2a + (n – 1)d]$

$2am + md (m – 1)] = 2na + nd (n – 1)$

$2am + m^2d – md = 2an + n^2d – nd$

$2am – 2an + m^2d – n^2d – md + nd = 0$

$2a (m – n) + d (m^2 – n^2) – d (m – n) = 0$

$(m – n) [2a + (m + n) d – d) = 0$

$(m – n) [2a + (m + n – 1) d] = 0$

As $m \neq n, 2a + (m + n – 1) d = 0$

Consider

$S_{m+n} = \frac{m+n}{2}$ [2a + (m + n – 1) d]

$= \left(\frac{m+n}{2}\right)(0)$

= 0

So, sum of its (m + n) terms is zero.

28. AP = –12, –, –6, ... , 21

If 1 is added to each term,

A.P. becomes –12 + 1, –9 + 1, –6 + 1, ..., 21 + 1

i.e. – 11, –8, –5, ..., 22

We know that

$a_n = a + (n – 1) d$

$22 = –11 + (n – 1) (3)$

$\frac{33}{3} = n – 1$

$n = 12$

$S_{12} = \frac{12}{2}[2 (–11) + (12 – 1) 3]$

= 6 [–22 + 33]

= 6 (11)

= 66

29. Let the prizes be a, a – 20, a – 40, ...

$S_{10} = 1600$

$\frac{10}{2}[2a + (10 – 1) (– 20)] = 1600$

$5 (2a – 180) = 1600$

$2a – 180 = 320$

$2a = 500$

$a = 250$

So, the prize are 250, 230, 210, 190, 170, 150, 130, 110, 90.

30. First term = a

Second term = b

last term (a_n) = c

To prove: $S_n = \frac{(a+c)(b+c-2a)}{2(b-a)}$

Solution: d = b – a

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

or

$$= \frac{n}{2}[a + a_n]$$

$$= \frac{n}{2}[a + c] \qquad \text{(i)}$$

We know that $a_n = c$

i.e. $a + (n-1)(b-a) = c$

$$(n-1) = \frac{c-a}{b-a}$$

$$n = \frac{c-a}{b-a} + 1$$

$$= \frac{c+b-2a}{b-a} \qquad \text{(ii)}$$

On putting (ii) in (i), we get

$$S_n = \frac{1}{2}(a+c)\left(\frac{c+b-2a}{b-a}\right)$$

$$= \frac{(a+c)(b+c-2a)}{2(b-a)}$$

31. Given that Raghav buys a shop for 120000.

He pays half of the amount in cash

$$= \frac{1}{2} \times 120000$$

$= 60000.$

Balance amount to be paid = 120000 – 60000

= 60000.

Given that amount of each installment = 5000.

He agrees to pay the balance in 12 annual installments with interest of 12%.

1. Amount of the first installment

$$= 5000 + \frac{12}{100} \times 60000$$

$= 5000 + 600 \times 12$

$= 5000 + 7200$

$= 12200.$

2. Amount of the second installment

$$= 5000 + \frac{12}{100} \times (60000 - 5000)$$

$$= 5000 + \frac{12}{100} \times 55000$$

$= 5000 + 550 \times 12$

$= 5000 + 6600$

$= 11600.$

So, the amount paid for installment is 12200, 11600.......It forms an AP.

The 1st term a = 12200

Common Difference d = 11600 – 12200

= –600.

The number of terms n = 12.

We know that sum of n terms

$$= \frac{n}{2}(2a + (n-1) \times d)$$

Therefore the total cost of the shop

$$= 60000 + \frac{12}{2}(2(12200) + (12-1) \times (-600))$$

$= 60000 + 6\,(24400 - 6600)$

$= 60000 + 6 \times 17800$

$= 60000 + 106800$

$= 166800.$

The total cost of the shop = 166800.

32. 4^{th} term = a + 3d

8^{th} term = a + 7d

Sum of the 4th term and 8th term

= a + 3d + a + 7d = 24.

$\Rightarrow$ 2a + 10d = 24

Take 2 common from the equation......

a + 5d = 12................(1)

Sum of 6th term and 10^{th} term = 44

$\Rightarrow$ a + 5d + a + 9d = 44

2a + 14d = 44

Take 2 common from equation....

a + 7d = 22.................(2)

By Elimination Method:-

$a + 5d = 12$

$a + 7d = 22$

$2d = 10$

$d = 5$

Substitute d = 5 in eq (1)

$a + 5d = 12$

$a + 5(5) = 12$

$a = 12 - 25$

$a = -13$

- The 1st term 3 term are –13, –13 + 5, –13 + 10
- –13, –8, –3.

WORKSHEET 2

Section A

1. Consider an AP: 12, 18, 24, ..., 96

$$a_n = a + (n - 1) d$$
$$96 = 12 + (n - 1) 6$$
$$96 - 12 = 6 (n - 1)$$
$$n - 1 = \frac{84}{6}$$
$$n - 1 = 14$$
$$n = 15$$

2. $S_q = 2q + 3q^2$

$$S_{q-1} = 2 (q - 1) + 3 (q - 1)^2$$
$$= 2q - 2 + 3q^2 + 3 - 6q$$
$$= 3q^2 - 4q + 1$$
$$a_q = s_q - s_{q-1}$$
$$= 2q + 3q^2 - 3q^2 + 4q - 1$$
$$= 6q - 1$$
$$a_{q+1} = 6q + 6 - 1 = 6q + 5$$
$$\therefore\ d = a_{q+1} - a_q = 6q + 5 - 6q + 1$$
$$= 6$$

3. Consider AP: 1, 3, 5, 7, ..., n

with a = 1, d = 2

$$Sn = \frac{n}{2}[2a + (n - 1) d]$$
$$= \frac{n}{2}[2 + (n - 1) 2]$$
$$= n [1 + n - 1)$$
$$= n^2$$

4. As terms are in AP,

$$13 - (2p + 1) = (5p - 3) - 13$$
$$13 - 2p - 1 = 5p - 3 - 13$$
$$12 + 16 = 7p$$
$$7p = 28$$
$$p = 4$$

5. First term = a

Second term = b

Last term (a_n) = 2a

Common difference (d) = b – a

$$S_n = \frac{n}{2}[2a + (n - 12d] \text{ or } \frac{n}{2}[a + a_n]$$
$$S_n = \frac{n}{2}[a + 2a]$$
$$= \frac{3a}{2} n \qquad \text{(i)}$$

As $a_n = 2a$

$$a + (n - 1) d = 2a$$
$$a + (n - 1) (b - a) = 2a$$
$$(n - 1) (b - a) = a$$
$$n - 1 = \frac{a}{b - a}$$
$$n = \frac{a}{b - a} + 1$$
$$n = \frac{b}{b - a} \qquad \text{(ii)}$$

On putting (ii) in (i), we get

$$S_n = \frac{3a}{2}\frac{b}{(b - a)}$$
$$= \frac{3ab}{2(b - a)}$$

6. $\frac{l}{m}, \frac{l+m}{m}, \frac{l+2m}{m}, ...$

Here, $a = \frac{l}{m}$

$d = \frac{l+m}{m} - \frac{l}{m} = \frac{l}{m} + 1 - \frac{l}{m} = 1$

$a_n = a + (n-1)d$

$= \frac{l}{m} + (n-1)\,1$

$= \frac{l}{m} + n - 1$

7. Let 184 be the n^{th} term of

AP = 3, 7, 11, ... with

$a = 3$

$d = 7 - 3 = 4$

$a_n = a + (n-1)d$

$184 = 3 + (n-1)\,4$

$184 - 3 = 4(n-1)$

$n - 1 = \frac{181}{4}$

$n = \frac{185}{4}$ which is not a natural number.

So, 184 is not a term of AP: 3, 7, 11, ...

8. Consider AP: 254, ..., 14, 9, 4

with $a = 254$

$d = 9 - 14 = -5$

So, $a_{10} = 254 + (10-1)(-5)$

$= 254 - 45$

$= 209$

9. $a_1 = 4$

$a_n = 4a_{n-1} + 3, n > 1$

$a_2 = 4a_1 + 3 = 16 + 3 = 19$

$a_3 = 4a_2 + 3 = 4(19) + 3 = 79$

$a_4 = 4a_3 + 3 = 4(79) + 3 = 319$

$a_5 = 4a_4 + 3 = 4(319) + 3 = 1279$

$a_6 = 4a_5 + 3 = 4(1279) + 3$

$= 5119.$

10. $a_n = 4n + 5$

$a_1 = 4 + 5 = 9$

$a_2 = 4(2) + 5 = 13$

$a_3 = 4(3) + 5 = 17$

$a_4 = 4(4) + 5 = 21$

$a_2 - a_1 = 13 - 9 = 4$

$a_3 - a_2 = 17 - 13 = 4$

$a_4 - a_3 = 21 - 17 = 4$

As difference between the terms is same, the sequence defined by $a_n = 4n + 5$ is an A.P. such that $d = 4$.

11. A.P : 27, 24, 21, ...

Let sum of n terms of the A.P. be 0.

Here, first term (a) = 27

Common difference (d) = 24 – 27

= –3

$S_n = 0$

$\frac{n}{2}[2a + (n-1)d] = 0$

$\frac{n}{2}[54 + (n-1)(-3)] = 0$

$n(54 - 3n + 3) = 0$

$n(18 - n + 1) = 0$

$18 - n + 1 = 0$

$n = 19$

So, sum of 19 terms is 0.

12. $\frac{S_m}{S_n} = \frac{m^2}{n^2}$

To prove : $\frac{a_m}{a_n} = \frac{2m-1}{2n-1}$

As $\frac{S_m}{S_n} = \frac{m^2}{n^2}$

$\therefore \frac{\frac{m}{2}\left[2a+(m-1)d\right]}{\frac{n}{2}\left[2a+(n-1)d\right]} = \frac{m^2}{n^2}$

$$\frac{2a+(m-1)d}{2a+(n-1)d} = \frac{m}{n}$$

$$\frac{a+\frac{(m-1)}{2}d}{a+\frac{(n-1)}{2}d} = \frac{m}{n}$$

On replacing m by 2m – 1 and n by 2n – 1 on both sides of equation, we get

$$\frac{a+(m-1)d}{a+(n-1)d} = \frac{2m-1}{2n-1}$$

13. $S_n = \frac{3n^2}{2}+\frac{5n}{2}$

We know that $a_n = S_n - S_{n-1}$

So, $a_{25} = S_{25} - S_{24}$

$$= \left[\frac{3}{2}(625+\frac{5}{2}(25)\right]-\left[\frac{3}{2}(576)+\frac{5}{2}(24)\right]$$

$$= \frac{1875}{2}+\frac{125}{2}-\frac{1728}{2}-\frac{120}{2}$$

$$= \frac{1875+125-1728-120}{2}$$

= 76

14. We know

$M = \frac{(n+1)}{2}$ th observation for n = odd

Therefore the 6th term of this AP is 30

Therefore A6 = 30

a + 5d = 30

Therefore we need to find S11

Therefore S11 $= \frac{n}{2}(2a + (n-1)d)$

$= \frac{11}{2}(2a + 10d)$

$= \frac{11}{2}(2(a + 5d)$

Replacing value

$= \frac{11}{2}(2(30))$

= 11 × 30

= 330

15. A = – 7

d = 5

T18 = a + 17d

= – 7 + 17 × 5

= – 7 + 85

= 78

General term = a + (n – 1)d

= – 7 + (n – 1) 5

= – 7 + 5n – 5

= 5n – 12

16. $a_{10} = 52$

∴ $a + 9d = 52$ (i)

$a_{17} = 20 + a_{13}$

$a + 16d = 20 + a + 12d$

$4d = 20$

$d = 5$

From (i), $a + 9(5) = 52$

$a + 45 = 52$

$a = 7$

So, AP is a, a + d, a + 2d, ...

i.e. 7, 12, 17, ...

17. $a_9 = -32$

$a + 8d = -32$ (i)

Also, $a_{11} + a_{13} = -94$

$a + 10d + a + 12d = -94$

$2a + 22d = -94$

$a + 11d = -47$ (ii)

On subtracting (i) from (ii), we get

$a + 11d - a - 8d = -47 + 32$

$3d = -15$

$d = -5$

From (i), $a = -32 - 8d$

$= -32 - 8(-5)$

$= -32 + 40$

$= 8$

18. $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{30} = \frac{30}{2}[2a + 29d] \Rightarrow S_{30} = 30a + 435d$.(i)

$S_{20} = \frac{20}{2}[2a + 19d] \Rightarrow S_{20} = 20a + 190d$

$S_{10} = \frac{10}{2}[2a + 9d] \Rightarrow S_{10} = 10a + 45d$

$3(S_{20} - S_{10}) = 3[20a + 190d - 10a - 45d)$

$= 3[10a + 145d]$

$= 30a + 435d = S_{30}$ [From (i)]

Hence, $S_{30} = 3(S_{20} - S_{10})$ Hence proved.

19. $a_{14} = 2a_8$

$a + 13d = 2[a + 7d]$

$a + 13d = 2a + 14d$

$-d = a$

$a_6 = -8$

$a + 5d = -8$

$-d + 5d = -8$ (As $a = -d$)

$4d = -8$

$d = -2$

So, $a = -d = 2$

We know that $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore S_{20} = \frac{20}{2}[2(2) + (20-1)(-2)]$

$= 10[4 - 38]$

$= 10(-34)$

$= -340$

20. First term (a) = 7

Last term (a_n) = 49

$S_n = 420$

We know that $S_n = \frac{n}{2}[a + a_n]$

$420 = \frac{n}{2}[7 + 49]$

$n = \frac{420}{28} = 15$

Now, $a_n = 49$

$a + (n-1)d = 49$

$7 + (15-1)d = 49$

$14d = 42$

$d = 3.$

21. $a_2 + a_7 = 30$

$a + d + a + 6d = 30$

$2a + 7d = 30$ (i)

Also, $a_{15} = 2a_8 - 1$

$a + 14d = 2[a + 7d] - 1$

$a + 14d = 2a + 14d - 1$

$0 = a - 1$

$a = 1$

From (i), $2(1) + 7d = 30$

$7d = 28$

$d = 4$

So, A.P. is a, a + d, a + 2d, ...

i.e. 1, 5, 9, ...

22. AP : $18, 15\frac{1}{2}, 13, ..., -49\frac{1}{2}$

i.e. $18, \frac{31}{2}, 13, ..., -\frac{99}{2}$

Here, first term (a) = 18

Common difference (d) $= \frac{31}{2} - 18$

$= \frac{31-36}{2} = -\frac{5}{2}$

Last term (a_n) $= -\frac{99}{2}$

$a + (n-1)d = -\frac{99}{2}$

$$18 + (n - 1)\left(-\frac{5}{2}\right) = -\frac{99}{2}$$

$$-\frac{5}{2}(n-1) = -\frac{99}{2} - 18$$

$$-\frac{5}{2}(n-1) = \frac{-99-36}{2}$$

$$-\frac{5}{2}(n-1) = -\frac{135}{2}$$

$$n - 1 = -\frac{135}{2} \times \frac{2}{-5} = 27$$

$$n = 28$$

So, number of terms (n) = 28

We know that $S_n = \frac{n}{2}[2a + (n-1)d]$

$$S_{28} = \frac{28}{2}\left[36 + (28-1)\left(\frac{-5}{2}\right)\right]$$

$$= 14\left[36 - \frac{135}{2}\right]$$

$$= \left(\frac{72-135}{2}\right)$$

$$= 7(-63)$$

$$= -441$$

23. $a_n = -4n + 15$

$a_1 = -4 + 15 = 11$

$a_2 = -4(2) + 15 = -8 + 15 = 7$

$a_3 = -12 + 15 = 3.$

So, First term (a) = 11

Common difference (d) = 7 – 11 = – 4

We know that

$$S_n = \frac{n}{2}[2a + (n - 11d]$$

$$S_{20} = \frac{20}{2}[2(11) + (20-1)(-4)]$$

$$= 10[22 - 76]$$

$$= 10(-54)$$

$$= -540$$

24. $a_8 = 31$

$a + 7d = 31$ (i)

$a_{15} = a_{11} + 16$

$a + 14d = a + 10d + 16$

$4d = 16$

$d = 4$ (ii)

From (i), $a + 28 = 31$

$a = 3$

So, A.P. is a, a + d, a + 2d, ...

i.e. 3, 7, 11, ...

25. $a_{15} = 3 + 2a_7$

$a + 14d = 3 + 2(a + 6d)$

$a + 14d = 3 + 2a + 12d$

$0 = a - 2d + 3$ (i)

Also, $a_{10} = 41$

$a + 9d = 41$ (ii)

On subtracting (i) from (ii), we get

$a + 9d - a + 2d = 41 + 3$

$11d = 44$

$d = 4$

From (ii), $a + 9(4) = 41$

$a = 41 - 36$

$= 5$

We know that $a_n = a + (n-1)d$

$= 5 + (n-1)4$

$= 4n + 1$

26. Consider an AP = 504, 511, 518,...., 896

Here, first term (a) = 504

Common difference (d) = 511 – 504 = 7

Last term (a_n) = 896

As $a_n = 896$

$a + (n-1)d = 896$

$$504 + (n - 1)\,7 = 896$$
$$7(n - 1) = 392$$
$$n - 1 = 56$$
$$n = 57$$

We know that $S_n = \frac{n}{2}[2a + (n-1)\,d]$

$$\therefore \quad S_{57} = \frac{57}{2}[2(504 + (57-1)7]$$
$$= \frac{57}{2}[1008 + 392]$$
$$= 39900$$

27. First term (a) = 5

Let d be the common difference

$$S_4 = \frac{1}{2}[S_8 - S_4]$$

i.e. $\frac{4}{2}[2a + (4-1)\,d]$

$$= \frac{1}{2}\left\{\frac{8}{2}\left[2a+(8-1)\,d\right]-\frac{4}{2}\left[2a+(4-1)\,d\right]\right\}$$

i.e. $\quad 2(2a + 3d) = 2(2a + 7d) - (2a + 3d)$

$$4a + 6d = 4a + 14d - 2a - 3d$$
$$4a + 6d = 2a + 11d$$
$$2a = 5d$$
$$d = \frac{2a}{5} = \frac{2}{5}(5) = 2$$

So, common difference (d) = 2

28. A.P: 3, 9, 15, ..., 99

Here, first term (a) = 3

Common difference (d) = 9 – 3 = 6

Last term (a_n) = 99

$$a + (n-1)d = 99$$
$$3 + (n-1)6 = 99$$
$$6(n-1) = 96$$
$$n - 1 = 16$$
$$n = 17$$

We know that $S_n = \frac{n}{2}[2a + (n-1)d]$

$$S_{17} = \frac{17}{2}[6 + (17-1)\,6]$$
$$= \frac{17}{2}[6 + 96]$$
$$= 867$$

29. First term (a) = 8

Last term (a_n) = 350

Common difference (d) = 9

As $\quad a_n = 350$

$$a + (n-1)d = 350$$
$$8 + (n-1)\,9 = 350$$
$$9(n-1) = 342$$
$$n - 1 = \frac{342}{9} = \frac{114}{3} = 38$$
$$n - 1 = 38$$
$$n = 39$$

We know that

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$S_{39} = \frac{39}{2}[16 + (39-1)\,9]$$
$$= \frac{39}{2}[16 + 342]$$
$$= 6981$$

30. Let the first term of an AP be 'a' and common difference be 'd'.

$$S_{10} = -150$$
$$\frac{11}{2}[2a + (10-1)\,d] = -150$$
$$5(2a + 9d) = -150$$
$$2a + 9d = -30 \quad \text{(i)}$$

Also, $\quad S_{20} - S_{10} = -550$

$$\frac{20}{2}(2a + 19d) - \frac{10}{2}(2a + 9d) = -550$$
$$10(2a + 19d) - 5(2a + 9d) = -550$$

$$2(2a + 19d) - (2a + 9d) = -110$$
$$4a + 38d - 2a - 9d = -110$$
$$2a + 29d = -110 \quad \text{(ii)}$$

On subtracting (ii) from (i), we get

$$2a + 9d - 2a - 29d = -30 + 110$$
$$-20d = 80$$
$$d = -4$$

From (i), $2a + 9(-4) = -30$

$$2a = -30 + 36$$
$$2a = 6$$
$$a = 3$$

So, A.P. is a, a + d, a + 2d, ...

i.e. 3, 3 – 4, 3 – 8, ...

i.e. 3, –1, –5, ...

Section D

31. Let A, D be first term and common difference respectively.

$$S_p = a \Rightarrow \frac{p}{2}[2A + (p-1)D] = a$$

$$S_q = b \Rightarrow \frac{q}{2}[2A + (q-1)D] = b$$

$$S_r = c \Rightarrow \frac{r}{2}[2A + (r-1)D] = c$$

Consider

$$\frac{a}{p}(q-r) + \frac{b}{q}(r-p) + \frac{c}{r}(p-q)$$

$$= \frac{1}{p}\frac{p}{2}[2A + (p-1)D](q-r) + \frac{1}{q}\frac{q}{2}[2A + (q-1)D)(r-p) + \frac{1}{r}\frac{r}{2}[2A + (r-1)D](p-q)$$

$$= \frac{1}{2}[2A + (p-1)D](q-r) + \frac{1}{2}[2A + (q-1)D](r-p) + \frac{1}{2}[2A + (r-1)D](p-q)$$

$$= [A(q-r) + A(r-p) + A(p-q)] + \frac{D}{2}[(p-1)(q-r) + (q-1)(r-p) + (r-1)(p-q)]$$

$$= A(q - r + r - p + p - r) + \frac{D}{2}[(p-1)(q-r) + (q-1)(r-p) + (r-1)(p-q)]$$

$$= 0 + \frac{D}{2}[pq - pr - q + r + qr - qp - r + p + rp - rq - p + q]$$

$$= 0 + 0$$

$$= 0$$

32. Let a and d be the first term and common term of an A.P.

$$a_m = \frac{1}{n}$$

$$a + (m-1)d = \frac{1}{n} \quad \text{(i)}$$

Also, $a_n = \frac{1}{m}$

$$a + (n-1)d = \frac{1}{m} \quad \text{(ii)}$$

On subtracting (i) from (ii), we get

$$a + (n-1)d - a - (m-1)d = \frac{1}{m} - \frac{1}{n}$$

$$d(n - 1 - m + 1) = \frac{n-m}{mn}$$

$$d(n-m) = \frac{n-m}{mn}$$

$$d = \frac{1}{mn}$$

From (i), $a + (m-1)\frac{1}{mn} = \frac{1}{n}$

$$a + (m-1)\frac{1}{mn} = \frac{1}{n}$$

$$a + \frac{1}{n} - \frac{1}{mn} = \frac{1}{n}$$

$$a = \frac{1}{mn}$$

Consider

$$S_{mn} = \frac{mn}{2}\left[\frac{2}{mn} + (mn-1)\frac{1}{mn}\right]$$

$$= \frac{mn}{2}\left[\frac{2}{mn}+1-\frac{1}{mn}\right]$$

$$= \frac{mn}{2}\left[\frac{1}{mn}+1\right]$$

$$= \frac{mn}{2}\left[\frac{mn+1}{mn}\right]$$

$$= \frac{1}{2}(mn + 1)$$

33. Length of each step = 50 m

Width of each step $= \frac{1}{2}$ m

Height of first step $= \frac{1}{4}$ m

Height of second step $= \frac{1}{4}+\frac{1}{4}=\frac{1}{2}$ m

Height of third step $= \frac{3}{4}$ m so on.

Volume of concrete required to build the first step $(V_1) = 50 \times \frac{1}{2} \times \frac{1}{4}$ m^3

Volume of concrete required to build the second step $(V_2) = 50 \times \frac{1}{2} \times \left(2\times\frac{1}{4}\right)$

$(V_3) = 50 \times \frac{1}{2} \times \frac{3}{4}$ m^3 and so on.

Total volume of concrete

$$= V_1 + V_2 + V_3 + ... + V_{15}$$

$$= \left(50\times\frac{1}{2}\times\frac{1}{4}\right)+\left[50\times\frac{1}{2}\times\left(2\times\frac{1}{4}\right)\right]$$

$$+\left(50\times\frac{1}{2}\times3\times\frac{1}{4}\right)+....+\left[50\times\frac{1}{2}\times\left(15\times\frac{1}{4}\right)\right]$$

$$= \left(50\times\frac{1}{2}\right)\left[\frac{1}{4}+\frac{2}{4}+\frac{3}{4}+....+\frac{15}{4}\right]$$

$$= 25\left[\frac{1}{4}+\frac{2}{4}+\frac{3}{4}+....+\frac{15}{4}\right]m^3$$

$$= \frac{25}{4}(1 + 2 + ... + 15)\ m^3$$

$$= \frac{25}{4} \times \frac{15}{2}(1 + 15) = 750\ m^3$$

34. Let the first term and common difference of an A.P. be a and d respectively.

Let S and S^{1} be the sum of odd terms and even terms of A.P.

$$S = a_1 + a_3 + a_5 + ... + a_{2n+1}$$

$$= \frac{n+1}{2}(a_1 + a_{2n+1})$$

$$= \frac{n+1}{2}[a + a + (2n + 1 - 1) d]$$

$$= (n + 1)(a + nd)$$

$$S^1 = a_2 + a_4 + a_6 + ... + a_{2n}$$

$$S^1 = \frac{n}{2}[2a + 2nd]$$

$$= n(a + nd)$$

Consider $\frac{S}{S^1} = \frac{(n+1)(a+nd)}{n(a+nd)}$

$$= \frac{n+1}{n}$$

35. Consider 1, 2, 3, ..., 999, 1000

This sequence forms an AP with first term (a) = 1 and common difference (d) = 1

We know that

$$S_n = \frac{n}{2}[2a + (n - 1) d]$$

$$S_{1000} = \frac{1000}{2}[2 + (1000 - 1) 1]$$

$$= 500(2 + 999)$$

$$= 500(1001)$$

$$= 500500$$

Now consider list of numbers divisible by 2: 2, 4, 6, 8, ..., 1000

This sequence also forms an AP with a = 2, d = 2, $n = \frac{1000}{2} = 500$

$$S_{500} = \frac{500}{2}[2(2) + (500 - 1) 2]$$

$$= 250(4 + 499(2))$$

$$= 250500$$

Again, consider list of numbers divisible by 5: 5, 10, 15, ..., 1000

Here, $a = 5, d = 5, n = \frac{1000}{5} = 200$

$$S_{200} = \frac{200}{2}[10 + (200 - 1)\,5]$$
$$= 100\,[10 + 5\,(199)]$$
$$= 100500$$

Now, we will consider list of numbers divisible by both 2 and 5 i.e. $2 \times 5 = 10$

10, 20, 30, ..., 1000

This list of numbers form an AP with

$a = 10, d = 10, n = \frac{1000}{10} = 100$

$$S_{100} = \frac{100}{2}[20 + (100 - 1)\,10]$$
$$= 50\,(20 + 990)$$
$$= 50500$$

Therefore, sum of numbers which are either divisible by 2 or 5

$$= S_{200} + S_{500} - S_{100}$$
$$= 100500 + 250500 - 50500$$
$$= 300500$$

So, sum of numbers from 1 to 1000 that are neither divisible by 2 nor by 5

$$\text{2 nor by S} = S_{1000} - 300500$$
$$= 500500 - 300500$$
$$= 200000$$

36. Suppose the work is completed in n days

Consider an AP: 150, 146, 142, ...

Here, First term (a) = 150

Common difference (d) = – 4

Total number of workers who worked all the n days. = S_n

$$= \frac{n}{2}[2\,(150) + (n - 1)\,(-4)]$$
$$= \frac{n}{2}(300 - 4n + 4)$$
$$= \frac{n}{2}[304 - 4n]$$
$$= n\,(152 - 2n)$$

If the workers did not drop,

work would have been finished in (n – 8) days such that 150 workers work on each day.

∴ Total number of workers who worked all the n days = 150 (n – 8)

$$\therefore \quad n(152 - 2n) = 150\,(n - 8)$$
$$152\,n - 2n^2 = 150n - 1200$$
$$152n - 150n = 2n^2 - 1200$$
$$2n^2 - 2n - 1200 = 0$$
$$n^2 - n - 600 = 0$$
$$n^2 - 25n + 24n - 600 = 0$$
$$n\,(n - 25) + 24\,(n - 25) = 0$$
$$(n + 24)\,(n - 25) = 0$$
$$n = -24, n = 25$$

Being the number of days, n cannot be negative, so, n = 25

∴ Work would be completed in 25 days

37. Consider the sequence: 200, 250, 300, ...

This sequence form an AP with first term (a) = 200 and common difference (d) = 50

We know that

$$S_n = \frac{n}{2}[2a + (n - 1)\,d]$$
$$S_{30} = \frac{30}{2}[2\,(200) + (30 - 1)\,50]$$
$$= 15\,[400 + 1450]$$
$$= 27{,}750$$

∴ The contractor has to pay $ 27,750 as penalty, if he has delayed the work by 30 days.

38. Consider AP: 20, 19, 18, ...

Here, First term (a) = 20

Common difference (d) = – 1

Let 200 logs be placed in n rows

$\therefore\ S_n = 200$

$$\frac{n}{2}[2(20) + (17 - 1)(-1)] = 200$$

$$\frac{n}{2}[40 - n + 1] = 200$$

$$n(41 - n) = 400$$

$$-n^2 + 41n - 400 = 0$$

$$n^2 - 41n + 400 = 0$$

$$n^2 - 16n - 25n + 400 = 0$$

$$n(n - 16) - 25(n - 16) = 0$$

$$(n - 16)(n - 25) = 0$$

$$n = 16 \text{ or } 25$$

If $n = 25$,

$$a_{25} = 20 + (25 - 1)(-1)$$

$$= 20 - 24$$

$$= -4 \quad \text{not possible}$$

So, $n = 16$

So, 200 logs are placed in 16 rows.

$$a_{16} = 20 + (16 - 1)(-1)$$

$$= 20 - 15 = 5$$

So, there are 5 logs in the top row.

39. Given : a^2, b^2, c^2 are in A.P.

To prove : $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P.

$\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in AP

if $$\frac{1}{c+a} - \frac{1}{b+c} = \frac{1}{a+b} - \frac{1}{c+a}$$

i.e. $$\frac{(b+c)-(c+a)}{(b+c)(a+c)} = \frac{(a+c)-(a+b)}{(a+b)(a+c)}$$

i.e. $$\frac{b+c-c-a}{(b+c)(a+c)} = \frac{a+c-a-b}{(a+b)(a+c)}$$

i.e. $$\frac{b-a}{(b+c)(a+c)} = \frac{c-b}{(a+b)(a+c)}$$

i.e. $$\frac{b-a}{b+c} = \frac{c-b}{a+b}$$

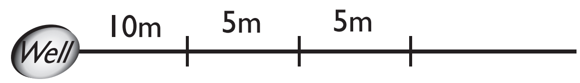
i.e. $(b - a)(a + b) = (c - b)(b + c)$

i.e. $ab + b^2 - a^2 - ab = bc + c^2 - b^2 - bc$

i.e. $b^2 - a^2 = c^2 - b^2$

$\therefore$ a^2, b^2, c^2 are in A.P.

40.



Distance covered by gardener to water 1st tree and return to the initial position

$$= 10 \text{ m} + 10 \text{ m} = 20 \text{ m}$$

Distance covered by gardener to water 2nd tree and return to initial position

$$= 15 \text{ m} + 15 \text{ m} = 30 \text{ m}$$

Distance covered by gardener to water 3rd tree and return to initial position.

$$= 20 \text{ m} + 20 \text{ m} = 40 \text{ m}$$

So, we get an AP: 20, 30, 40, ...

With first term (a) = 20

difference (d) = 10

Total distance covered by the gardener

$$= S_{25}$$

$$= \frac{25}{2}[2(20) + (25 - 1)10]$$

$$= \frac{25}{2}[40 + 240]$$

$$= \frac{25}{2} \times 280$$

$$= 25 \times 140$$

$$= 3500 \text{ m}$$

$\therefore$ Total distance covered by the gardener to water all trees = 3500 m

Chapter 06

Triangles

MULTIPLE CHOICE QUESTIONS

1. Δ ABC ~ Δ PQR

$$\therefore \frac{ar\Delta ABC}{ar\Delta PQR} = \left(\frac{BC}{QR}\right)^2$$

$$\Rightarrow \frac{9}{16} = \frac{(4.5)^2}{(QR)^2}$$

$$\Rightarrow (QR)^2 = \frac{4.5\times 4.5\times 16}{9}$$

$$\Rightarrow QR = \frac{4.5\times 4}{3}$$

= 1.5 x 4

= 6 cm

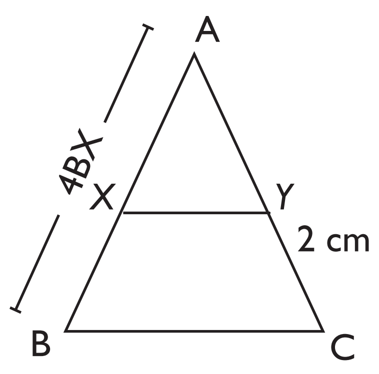
2. We know that ratio of area of two similar triangles is equal to square of ratio of their Corresponding sides (say x and y)

$$\Rightarrow \frac{9}{16} = \left(\frac{x}{y}\right)^2$$

$$\Rightarrow \frac{x}{y} = \frac{3}{4}$$

Option (a)

3.



AS *XY* II BC, so by basic proportionality theorem

$$\frac{AX}{BX} = \frac{AY}{YC}$$

$$\frac{AX}{BX} + 1 = \frac{AY}{YC} + 1$$

$$\frac{AB}{BX} = \frac{AC}{CY}$$

$$\frac{4BX}{BX} = \frac{AC}{2} \quad (\therefore AB = 4BX)$$

$$4 = \frac{AC}{2}25$$

AC = 8 cm

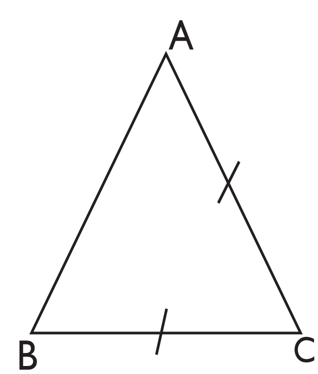
So, AY = AC – CY

= 8 – 2

= 6 cm

Option (d)

4.



$AB^2 = 2\,AC^2$

$= AC^2 + AC^2$

$= AC^2 + BC^2 +$

[∴ AC = BC]

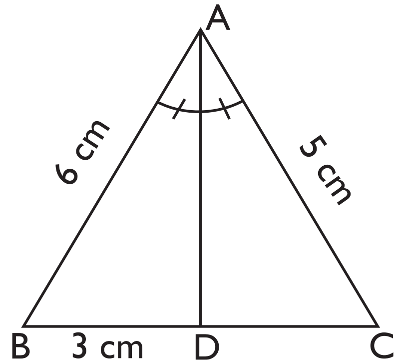
∴ $AB^2 = Ac^2 + BC^2$

∴ ΔABC is a triangle right angled at C

i.e. ∠C = 90°

Option (c)

5.



AS AD bisects ∠BAC

∴ $\frac{AB}{BD} = \frac{AC}{CD}$

[By internal angle bisector theorem]

⇒ $\frac{6}{3} = \frac{5}{CD}$

⇒ CD = $\frac{3 \times 5}{6}$ = 2.5 cm

Option (b)

WORKSHEET 1

Section A

1. Δ ABC ~ Δ DEF

In Δ ABC, ∠A + ∠B + ∠C = 180°

57° + ∠B 73° = 180°

∠B + 130° = 180°

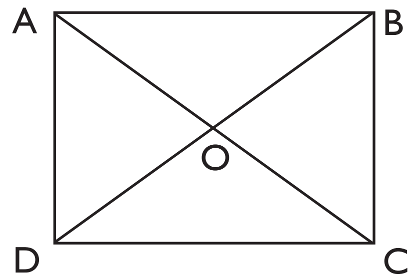
∠B = 108° – 130°

= 50°

∴ ∠E = ∠B = 50°

[Corresponding angles of similar triangles are equal.]

2.



AC = 30 cm

BD = 40 cm

OA = OC = $\frac{1}{2}$ AC = 15 cm

OB = OD = $\frac{1}{2}$ BD = 20 cm

In Δ AOB, ∠AOB = 90°

(Diagonals of rhombus bisect each other at 90°)

$AB^2 = AO^2 + OB^2$ (Pythagoras theorem)

$= (15^2) + (20^2)$

= 225 + 400

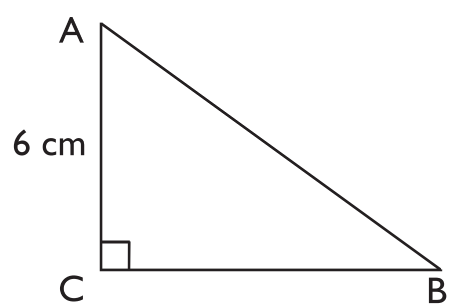
= 625

AB = 25 cm

∴ AB = BC = CA = AD = 25 cm

(All sides of rhombus are equal)

3.



In Δ ABC,

AC = BC = 6 cm (AS Δ ABC is isosceles)

Also, ∠C = 90°

∴ $AB^2 = AC^2 + BC^2$ (Pythagoras theorem)

$= 6^2 + 6^2$

= 36 + 36

$AB^2 = 72$

$AB = 6\sqrt{2}$ cm

4. As, $\Delta DEF \sim \Delta ABC$

$$\frac{DE}{AB} = \frac{EF}{BC} = \frac{DF}{AC}$$

$$\frac{DE}{3} = \frac{4}{2} = \frac{DF}{2.5}$$

$$\frac{DE}{3} = \frac{4}{2}$$

$$DE = \frac{12}{2} = 6 \text{ cm}$$

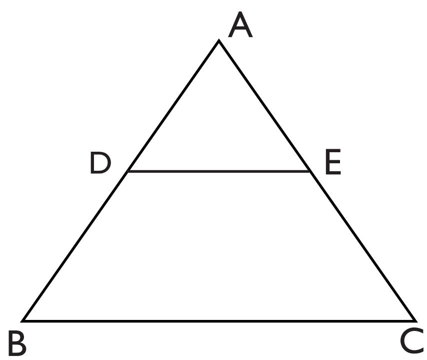
$$\frac{4}{2} = \frac{DF}{2.5}$$

$$DF = \frac{4 \times 2.5}{2} = 5 \text{ cm}$$

Perimeter of Δ DEF = DF + EF + DF

= 6 + 4 + 5 = 15 cm

5.



Let AE = x cm

$\therefore$ CE = AC – AE = 5.6 – x cm

As DE II BC,

$$\frac{AD}{DB} = \frac{AE}{CE}$$

(By Basic proportionality theorem)

$$\frac{3}{5} = \frac{x}{5.6 - x}$$

5x = 3 (5.6 -x)

5x = 16.8 - 3x

8x = 16.8

x = 2.1 cm

$\therefore$ AE = x =2.1 cm

6. We know that ratio of the areas of two similar triangles is equal to the square of their altitudes.

$\therefore$ Ratio of areas = $\left(\frac{2}{3}\right)^2$ = 4.9

7. Given: abc~def

Find: Area of def

$$\frac{\text{Area of def}}{\text{Area of abc}} = \frac{\text{ef}}{\text{Bc}}$$

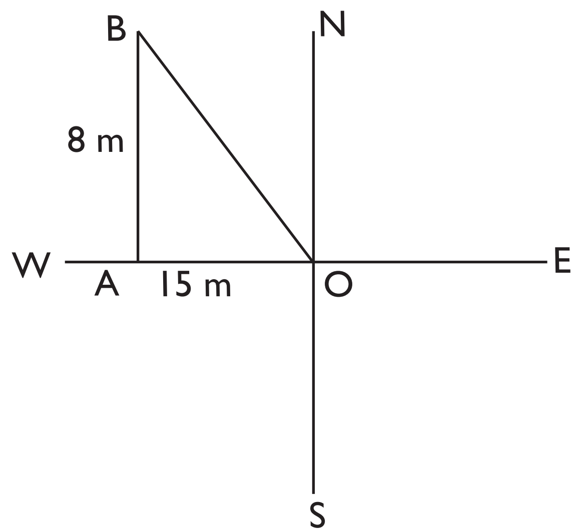
$$\frac{\text{Area of def}}{54} = \frac{4}{3}$$

Area of def = $4 \times \frac{54}{3}$

Area of def = 4 × 18

Area of def = 72cm

8.



In ΔBAO, $\angle$BAO = 90°

$OB^2 = AB^2 + AO^2$ (Pythagoras theorem)

$= 8^2 + 15^2$

= 64 + 225

= 289

$\therefore$ OB = 17m

Section B

9. $\Delta ABC \sim \Delta DEF$,

$$\frac{ar\Delta ABC}{ar\Delta DEF} = \frac{BC^2}{EF^2}$$

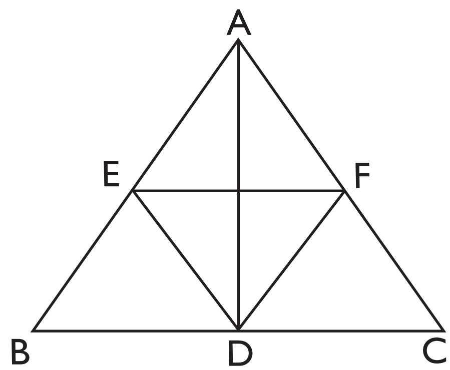
(In two similar triangles, the ratio of their areas is the square of ratio of their sides)

$$\frac{64}{121} = \frac{BC^2}{(15.4)^2}$$

$$(BC)^2 = \frac{64}{121} \times \quad 15.4 \times 15.4$$

$\therefore$ BC = 11.2 cm

10.



In ΔADB, DE is bisector of $\angle$ ADB

$$\frac{BD}{BE} = \frac{AD}{AE}$$

i.e. $\frac{BD}{AD} = \frac{BE}{AE}$ (i)

In ΔADB, DF is bisector of $\angle$ADC

$$\frac{CD}{CF} = \frac{AD}{AF}$$

i.e. $\frac{CD}{AD} = \frac{CF}{AF}$

$\frac{BD}{AD} = \frac{CF}{AF}$ (ii)

(As AD is median $\therefore$ BD = CD)

From (i) and (ii), we get

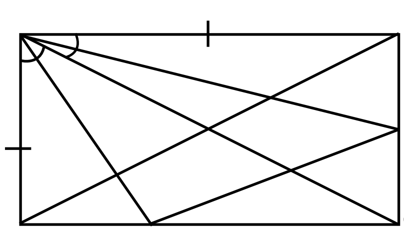
$$\frac{BE}{AE} = \frac{CF}{AF}$$

$$\frac{AE}{BE} = \frac{AF}{CF}$$

So, by converse of Basic proportionality theorem

EF II BC

11.



In Δ ADC, AF bisects $\angle$ DAC

$$\therefore \quad \frac{CF}{DF} = \frac{AC}{AD}$$

$= \frac{AC}{AB}$ (i) (As AB = AD)

In ΔABC, AE bisects $\angle$ BAC

$\frac{CE}{BE} = \frac{AC}{AB}$ (ii)

From (i), (ii)

$$\frac{CF}{DF} = \frac{CE}{BE}$$

$\therefore$ EF II BD

(By converse of Basic proportionaly theorem)

12. In ΔAOB ~ ΔCOD

$\angle$AOB = $\angle$COD (Vertically opposite angles)

$\frac{AO}{OC} = \frac{BO}{DO}$ (Given)

$\therefore$ ΔAOB ~ ΔAOB (SAS)

So, $\frac{AO}{OC} = \frac{BO}{OD} = \frac{AB}{DC}$

(Corresponding sides of similar triangles are proportional)

$$\frac{1}{2} = \frac{5}{CD}$$

CD = 10 cm

13. In Δ KPN and Δ KLM,

$\angle$ K = $\angle$ K (Common)

$\angle$ KNP = $\angle$ KML = 46° (Given)

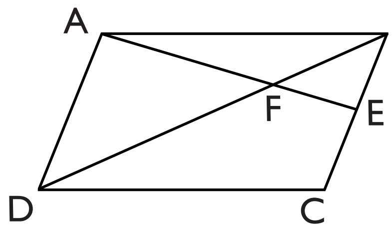
$\therefore$ ΔKPN ~ ΔKLM (AA similarity criterion)

$$\frac{KP}{KL} = \frac{PN}{LM} = \frac{KN}{KM}$$

$$\frac{x}{a} = \frac{c}{b+c}$$

$$x = \frac{ac}{b+c}$$

14.



In ΔAFD and ΔBEF

(Alternate intetior angles)

$\angle AFD = \angle BFE$

(Vertically opposite angles)

$\therefore \Delta AFD \sim \Delta EFB$

So, $\frac{EF}{FA} = \frac{FB}{DF}$

(Corresponding sides of similar triangles are proportional.)

DF x EF = FB x FA

15. As DE II AC, So in ΔABC

$\frac{BC}{CP} = \frac{BE}{EC}$ (i)

(Basic proportionality theorem)

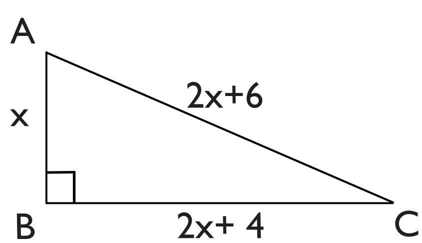
Also, $\frac{BE}{EC} = \frac{BC}{CP}$ (ii) (Given)

From (i), (ii), we have $\frac{BD}{AD} = \frac{BC}{CP}$

$\therefore$ DC II AP

(By converse of Basic proportionality theorem.)

16.



Let the shorter side be x m

$\therefore$ Hypotenuse $= 2x + 6$

Also, Third side $= 2x + 6 - 2$

$= 2x + 4$

In ΔABC, $AC^2 = AB^2 + BC^2$

$(2x + 6)^2 = x^2 + (2x + 4)^2$

$4x^2 + 36 + 24x = x^2 + 4x^2 + 16 + 16x$

$O = x^2 - 20 - 8x$

$x^2 - 8x - 20 = 0$

$x^2 - 10x + 2 (x^2 - 10) = 0$

$(x^2 - 10) (x - 2) = 0$

$x = 10, -2$

Being a side, $x = -2$ is rejected

$\therefore x = 10$

So, AB = 10 m

BC = 2x + 4 = 24 m

AC = 2x + 6 = 26 m

17. We know that diagonals of rhombus bisect each other at 90°.

Let AC = 24 cm

BD = 10 cm

$AO = OC = \frac{1}{2} AC = 12$ cm

$BO = OD = \frac{1}{2} BD = 5$ CM

In ΔAOB,

$AB^2 = BO^2 + AO^2$

$= 5^2 + 12^2$

$= 25 + 144$

$= 169$

$\therefore$ AB = 13 cm

As all sides of rhombus are equal,

AB = BC = CD = AD = 13 cm

18. In ΔABC, DE II BC

$\frac{AD}{DB} = \frac{AE}{EC}$

[Basic proportionality theorem]

$$\frac{x}{x-2} = \frac{x-2}{x-1}$$

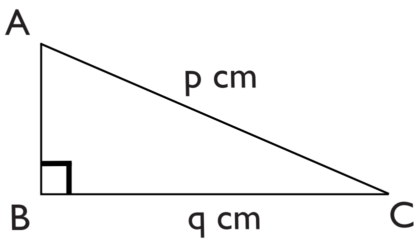
$$x(x-1) = (x-2)(x+2)$$

$$x2 - x = x2 - 4$$

$$x = 4$$

Section C

19.



In Δ ABC, $AC^2 = AB^2 + BC^2$

$P^2 = AB^2 + q^2$

$AB^2 = p^2 - q^2$

$= (p - q)\ (p + q)$

$= 1\ (p + q)$

$AB^2 = p + q$

$AB = \sqrt{p+q}$

20. $\frac{QT}{PR} = \frac{QR}{QS}$ (Given)

In ΔPQR, ∠1 = ∠2

∴ PQ = PR

[sides opposite to equal angles are equal]

So, $\frac{QT}{PR} = \frac{QR}{QS}$

Also, ∠Q = ∠Q (common)

∴ ΔPQS ~ ΔTQR

[By SAS Similarity cretirion]

21. In ΔCBQ and ΔCAP,

∠BCQ = ∠ACP (common)

∠QBC = ∠PAC = 90°

(PA and QB are perpendicular)

∴ ΔCBQ ~ ΔCAP (AA Similarity creterion)

$$\frac{BC}{AC} = \frac{BQ}{AP} = \frac{CQ}{CP}$$

[Corresponding sides of similar triangles are proportional]

$$\frac{BC}{AC} = \frac{Z}{x} \qquad \text{(i)}$$

In ΔABQ and ΔACR,

∠BAQ = ∠CAR (common)

∠ABQ = ∠ACR = 90°

(BQ and RC are perpendicular)

∴ ΔABQ ~ ΔACR (AA Similarity creterior)

$$\frac{AB}{AC} = \frac{BQ}{CR} = \frac{AQ}{AR}$$

$$\frac{AB}{AC} = \frac{Z}{Y} \qquad \text{(ii)}$$

From (i),

$$1 - \frac{BC}{AC} = 1 - \frac{z}{x}$$

$$\frac{AC - BC}{AC} = \frac{x - z}{x}$$

$$\frac{AB}{AC} = \frac{x - z}{x} \qquad \text{(iii)}$$

From (ii) and (iii)

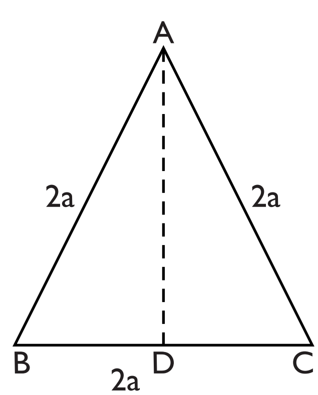
$$\frac{AB}{AC} = \frac{z}{y} = \frac{x - z}{x}$$

$$\frac{z}{y} = 1 - \frac{z}{x}$$

$$\frac{z}{x} + \frac{z}{y} = 1$$

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$$

22.



Draw AD $\perp$ BC

In ΔADB and ΔADC

AB = AC = 2a (Given)

AD = AD
(Common)

$\angle ADB = \angle ADC$

= 90° (By Construction)

$\therefore$ $\Delta ADB \cong \Delta ADC$ (RHS)

$\therefore$ BD = DC $= \frac{1}{2} BC$

= a (CPCT)

In ΔADC, right angled at D

$AC^2 = AD^2 + DC^2$

$(2a)^2 = AD^2 + a^2$

$AD^2 = 4a^2 - a^2 = 3a^2$

$AD = \sqrt{3}\,a$

So, length of the altitude of an equilateral triangle $= \sqrt{3}\,a$ cm

23. In ΔAOB, XY || AB

$\therefore \frac{OX}{AX} = \frac{OY}{BY}$

(i) [Basic Proportionality theorem]

In ΔAOC, XZ || AC

$\therefore \frac{OZ}{ZC} = \frac{OX}{AX}$

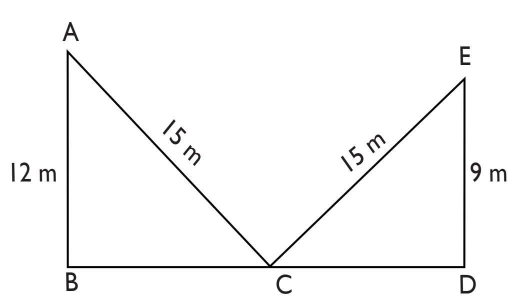
(ii) [Basis Proportionality theorem]

By (i) and (ii), $\frac{OY}{BY} = \frac{OZ}{ZC}$

$\therefore$ YZ || BC

[By Converse of Basic proportionality theorem]

24.



Let AC = CE denotes the ladder

In ΔABC, $AC^2 = AB^2 + BC^2$

$15^2 = 12^2 + BC^2$

$225 - 144 = BC^2$

$BC^2 = 81$

BC = 9 m

In ΔCDE, $CE^2 = DE^2 + CD^2$

$15^2 = 9^2 + CD^2$

$225 - 81 = CD^2$

$144 = CD^2$

$12 = CD$

So, BD = BC + CD

= 9 + 12 = 21 m

Section D

25. In ΔXPQ and ΔXYZ,

$\frac{XP}{PY} = \frac{XQ}{XZ} = 3$ (Given)

$\angle X = \angle X$ (Common)

$\therefore \Delta XPQ \sim \Delta XYZ$

(SAS Similarity creterion)

So, $\frac{ar\,\triangle XPQ}{ar\,\triangle XYZ} = \left(\frac{XP}{XY}\right)^2 = \left(\frac{PQ}{YZ}\right)^2 = \left(\frac{XQ}{XZ}\right)^2$

[Ratio of area of two similar triangles is equal to square of their corresponding sides]

$\frac{ar\,\triangle XPQ}{32} = \left(\frac{XP}{XY}\right)^2 = \left(\frac{3}{4}\right)^2$

ar $\Delta XPQ = \frac{9}{16} \times 32$ $\left[\begin{array}{l} \frac{XP}{Py} = 3 \\ \frac{Py}{XP} = \frac{1}{3} \\ \frac{Py}{XP} + 1 = \frac{1}{3} + 1 \\ \frac{XY}{XP} = \frac{4}{3} \end{array}\right]$

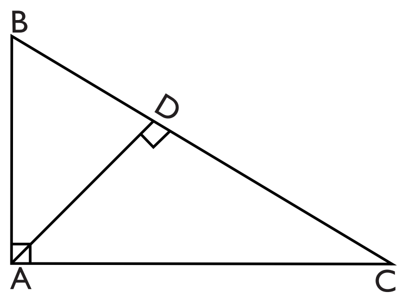
= 18 cm^2

area of quadrilateral PYZQ

= ar ΔXYZ – ar ΔXPQ

= 32 – 18

= 14 cm^2

26. 

In ΔABC, right angled at B,

We need to prove $AC^2 = AB^2 + BC^2$

Draw $BD \perp AC$

We know that if a perpendicular drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.

So, ΔCBA and ΔCDB

[Corresponding sides of similar triangles are proportionals

$$\frac{CB}{CD} = \frac{CA}{CB}$$

$CB^2 = CA \times CD$ (i)

Also, ΔABC and ΔADB

$$\frac{AB}{AD} = \frac{BC}{BD} = \frac{AC}{AB}$$

$$\frac{AB}{AD} = \frac{AC}{AB}$$

$AB^2 = AC \times AD$ (ii)

From (i) and (ii),

$AB^2 + BC^2 = AC \times AD + AC \times CD$

$= AC (AD + CD)$

$= AC \times AC$

$= AC^2$

$\therefore$ $AB^2 + BC^2 = AC^2$

27. As $XY \parallel AC$

$\angle BXY = \angle A$ (Corresponding angles)

$\angle BYX = \angle C$ (Corresponding angles)

$\therefore$ $\Delta ABC \sim \Delta XBY$ (AA Similarity Creterion)

So, $\dfrac{ar\,\triangle ABC}{ar\,\triangle XBY} = \left(\dfrac{AB}{XB}\right)^2$ (i)

[Ratio of areas of two similar triangles is equal to square of ratio of their corresponding sides]

Also, ar ΔABC = 2 ar (XBY)

i.e. $\dfrac{ar\,\triangle ABC}{ar\,\triangle XBY} = \dfrac{2}{1}$ (ii)

From (i) and (ii),

$$\left(\frac{AB}{XB}\right)^2 = \frac{2}{1}$$

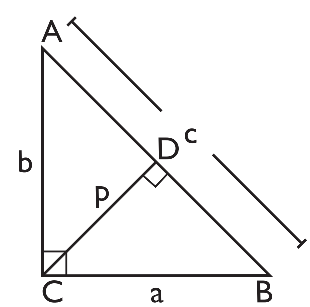
$$\frac{AB}{XB} = \frac{\sqrt{2}}{1}$$

$$\frac{XB}{AB} = \frac{1}{\sqrt{2}}$$

$$\therefore\ 1 - \frac{XB}{AB} = 1 - \frac{1}{\sqrt{2}}$$

$$\frac{AB - XB}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}}$$

$$\frac{XB}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}} = \frac{2-\sqrt{2}}{2}$$

28. 

In ΔACB, right angled at C such that $CD \perp AB$.

We know that if a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other

So, $\Delta BDC \sim \Delta BCA$

$$\therefore \frac{BD}{BC} = \frac{DC}{CA} = \frac{BC}{BA}$$

i.e. $\frac{p}{b} = \frac{a}{c}$

$pc = ab$

$$\Rightarrow p = \frac{ab}{c}$$

$$\Rightarrow p^2 = \frac{a^2b^2}{c^2}$$

$$\Rightarrow \frac{1}{p^2} = \frac{c^2}{a^2b^2}$$

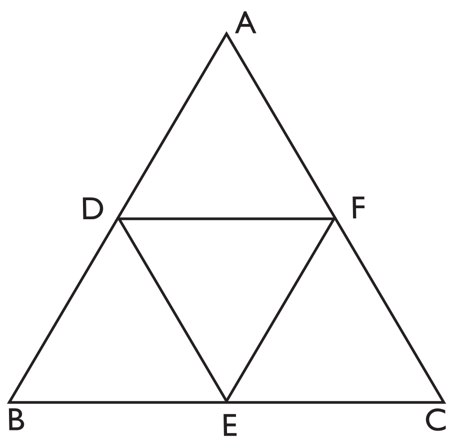
In $\Delta ACB, AC^2 + BC^2 = AB^2 b^2 + a^2 = c^2$

$$\frac{1}{p^2} = \frac{a^2+b^2}{a^2b^2}$$

$$\frac{1}{p^2} = \frac{a^2}{a^2b^2} + \frac{b^2}{a^2b^2}$$

$$\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

29.



Given that:-

ΔABC in whichD, E, F are the mid points of sides AB, BC and CA respectively.

To prove:- each of the triangles are similar to the original triangle, i.e.,

$\Delta ADF \sim \Delta ABC$

$\Delta BDE \sim \Delta ABC$

$\Delta CEF \sim \Delta ABC$

Proof:-

Consider the ΔADF and ΔABC

Since D and F are the mid points of AB and AC respectively.

$\therefore$ DF // BC

$\Rightarrow \angle AFD = \angle B$ (Corresponding angles are equal)

Now, in ΔADF and ΔABC, we have

$\angle ADF = \angle B$(Corresponding angles)

$\angle A = \angle A$(Common)

By AA similar conditions,

$\Delta ADF \sim \Delta ABC$

Similarly, we have

$\Delta BDE \sim \Delta ABC$

$\Delta CEF \sim \Delta ABC$

$\therefore$ EF // AB

$\Rightarrow$ EF // AD..........(1)

And, DE // AC

$\Rightarrow$ DE // AF..........(2)

From eqn (1) and (2), we have

ADEF is a parallelogram.

Similarly, BDFE is a parallelogram.

Now, in ΔABC and ΔDEF

$\angle A = \angle FED$ ($\because$ Opposite angles of parallelogram)

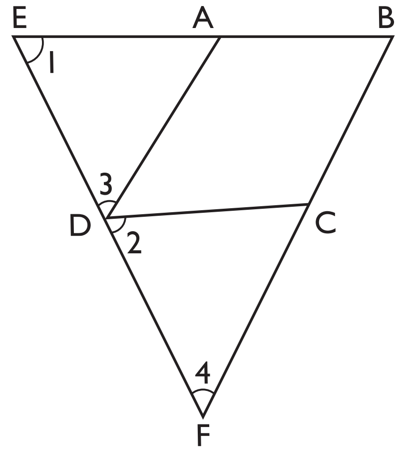
$\angle B = \angle DFE$ ($\because$ Opposite angles of parallelogram)

Therefore, by AA similar condition

$\Delta ABC \sim \Delta DEF$

Hence proved that each of the triangles are similar to the original triangle.

30.



Consider ΔEDA and ΔEFB

$\angle 1 = \angle 2$ (Common)

$\angle 3 = \angle 4$

[Corresponding angles as AD || BF]

$\therefore$ ΔEDA ~ ΔEFB

(AA Similarity Creterion)

$$\therefore \frac{DA}{FB} = \frac{EA}{EB}$$

[Corresponding sides of similar triangles proportional]

$$\Rightarrow \frac{DA}{AE} = \frac{FB}{BE} \quad \text{(i)}$$

Consider ΔEDA and ΔDFC

$\angle 1 = \angle 2$ (Corresponding angles as BE || CD)

$\angle 3 = \angle 4$ (Corresponding angles as AD || BF)

$\therefore$ ΔEDA ~ ΔDFC (AA Similarity Creterion)

$$\therefore \frac{ED}{DF} = \frac{DA}{FC} = \frac{EA}{DC}$$

[Corresponding sides of similar triangles are proportional]

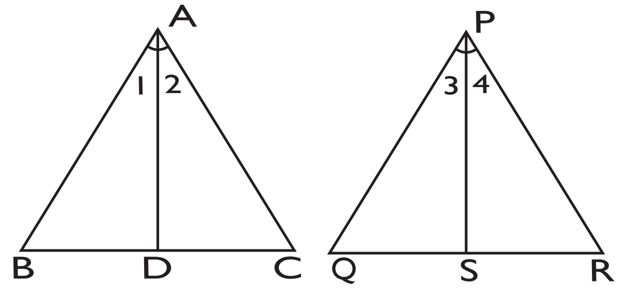
$$\text{i.e.} \quad \frac{DA}{FC} = \frac{EA}{DC}$$

$$\Rightarrow \frac{DA}{AE} = \frac{FC}{CD} \quad \text{(ii)}$$

From (i) and (ii),

$$\frac{DA}{AE} = \frac{FB}{BE} = \frac{FC}{CD}$$

31.



Given : AD and PS are bisectors of $\angle$A and $\angle$P respectively. Such that

$$\frac{BD}{DC} = \frac{QS}{SR}$$

To prove = ΔABC ~ΔPQR

Proof In ΔABC, AD is bisector of $\angle$A

$$\therefore \quad \frac{AB}{BD} = \frac{AC}{CD}$$

$$\text{i.e.} \quad \frac{AB}{AC} = \frac{BD}{CD} \quad \text{(i)}$$

In ΔPQR, PS is bisector of $\angle$P

$$\therefore \quad \frac{PQ}{QS} = \frac{PR}{RS}$$

$$\text{i.e.} \quad \frac{PQ}{PR} = \frac{QS}{RS} \quad \text{(ii)}$$

$$\text{Also,} \quad \frac{BD}{DC} = \frac{QS}{SR} \quad \text{(iii)}$$

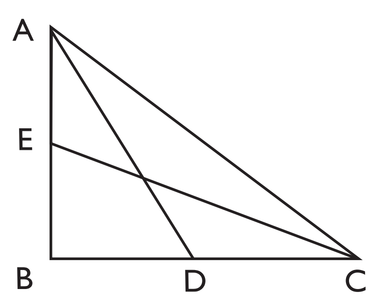
From (i), (ii), (iii), we get

$$\frac{AB}{AC} = \frac{PQ}{PR} \Rightarrow \frac{AB}{PQ} = \frac{AC}{PR}$$

Also, $\angle A = \angle P$ (Given)

$\therefore$ ΔABC ~ ΔPQR

32.



ΔABC is a right triangle right-angled at B

$\therefore \quad AD^2 = AB^2 + BD^2$

(By Pythagoras tearoom)

$\Rightarrow$ $AD^2 = AB^2 + \left(\frac{BC}{2}\right)^2$ [$\because$ BD = DC]

$\Rightarrow$ $AD^2 = AB^2 + \frac{1}{4} BC^2$ (i)

Also, ΔBCE is a right triangle right angled at B

$\therefore$ $CE^2 = BC^2 + BE^2$

$\Rightarrow$ $CE^2 = BC^2 + \left(\frac{AB}{2}\right)^2$ [$\because$ BE = EA]

$\Rightarrow$m$CE^2 = BC^2 + \frac{1}{4} AB^2$ (ii)

On adding (i) and (ii), we get

$AD^2 + CE^2 = \frac{5}{4} (AB^2 + BC^2)$

$\Rightarrow$ $AD^2 + CE^2 = \frac{5}{4} AC^2$

[As ΔABC is right triangle
$\therefore AC^2 = AB^2 + BC^2$]

$\Rightarrow$ $\left(\frac{3\sqrt{5}}{2}\right)^2 + CE^2 = \frac{5}{4}(25)$

$\Rightarrow$ $CE^2 = \frac{125}{4} - \frac{45}{4} = 20$

$\therefore$ CE = $\sqrt{20}$ cm = $2\sqrt{5}$ cm

WORKSHEET 2

Section A

1. $\Delta ABC \sim \Delta RPQ$

$\therefore \frac{AB}{RP} = \frac{BC}{PQ} = \frac{AC}{RQ}$

[Corresponding sides of similar triangles are proportional]

$\therefore \frac{3}{6} = \frac{5}{10} = \frac{6}{RQ}$

$RQ = \frac{6 \times 10}{5} = 12$ cm

2. DABC ~ DDEF

$\therefore \frac{ar\,\Delta ABC}{ar\,\Delta DEF} = \left(\frac{AB}{DE}\right)^2$

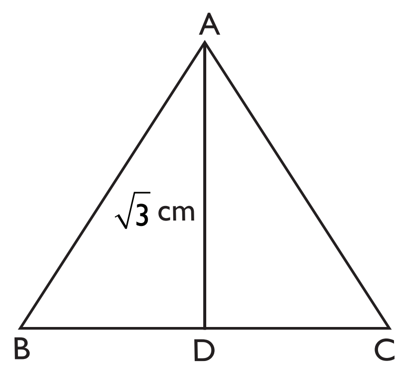
[Ratio of areas of similar triangles is Proportional to the square of ratio of ratio of their corresponding sides]

$\Rightarrow$ $\frac{169}{121} = \frac{(26)^2}{DE}$

$\Rightarrow$ $DE^2 = \frac{(26)^2 \times 121}{169}$

$\Rightarrow$ $DE = \frac{26 \times 11}{13} = 22$ cm

3.



ΔABC is equilateral and AD is the Median such that AD = $\sqrt{3}$ cm

In an equilateral triangle, median and altitude are same

$\therefore$ AD $\perp$ BC

Also, DC $= \frac{1}{2}$ AC

[As AD is the Median]

$AC^2 = AD^2 + CD^2$

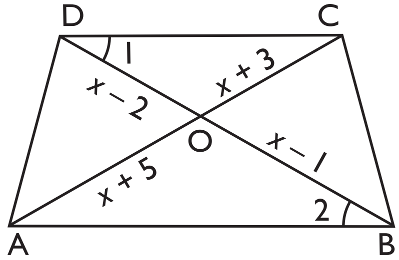
$AC^2 = \left(\sqrt{3}\right)^2 + \left(\frac{1}{2}AC\right)^2$

$AC^2 = 3 + \frac{1}{4} AC^2$

$\frac{3}{4} AC^2 = 3$

$AC^2 = 4$

AC = 2 cm

4. 

In ΔCOD and ΔAOB,

$\angle 1 = \angle 2$

[Corresponding angles as AB || CD]

$\angle AOB = \angle COD$

[Vertically opposite angles]

$\therefore \quad \Delta COD \sim \Delta AOB$

$$\therefore \quad \frac{CO}{AO} = \frac{OD}{OB} = \frac{CD}{AB}$$

[Corresponding sides of similar triangles are proportional.]

$$\frac{x+3}{x+5} = \frac{x-2}{x-1}$$

$\Rightarrow \quad (x + 3)(x – 1) = (x – 2)(x + 5)$

$\Rightarrow \quad x^2 + 2x – 3 = x^2 + 3x – 10$

$\Rightarrow \quad 7 = x$

5. In ΔSPT and ΔQPR,

$\angle PST = \angle PQR$

[Corresponding angles as ST || QR]

$\angle PTS = \angle PRQ$

$\therefore \quad \Delta SPT \sim \Delta QPR$

[AA Similarity Creterion]

$$\therefore \quad \frac{ar\triangle PST}{ar\triangle PQR} = \left(\frac{PT}{PR}\right)^2$$

[Ratio of areas of two similar triangles is equal to square of ratio of their corresponding sides]

$$= \left(\frac{PT}{PT+TR}\right)^2$$

$$= \left(\frac{2}{2+4}\right)^2$$

$$= \left(\frac{2}{6}\right)^2$$

$$= \frac{1}{9}$$

6. DE || BC

$$\therefore \quad \frac{AD}{BD} = \frac{AC}{CE}$$

(Basic Proportionality theorem)

$$\Rightarrow \quad \frac{BD}{AD} = \frac{CE}{AE}$$

$$\Rightarrow \quad \frac{BD}{AD} + 1 = \frac{CE}{AE} + 1$$

$$\Rightarrow \quad \frac{BD+AD}{AD} = \frac{CE+AE}{AE}$$

$$\frac{AB}{AD} = \frac{AC}{AE}$$

Also, $\angle A = \angle A$ (Common)

$\therefore \Delta ADE \sim \Delta ABC$

$$\Rightarrow \quad \frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC}$$

[Corresponding sides of similar triangles are proportional]

$$\Rightarrow \quad \frac{AD}{AB} = \frac{DE}{BC}$$

$$\Rightarrow \quad \frac{1.5}{6} = \frac{DE}{8}$$

$$\Rightarrow \quad DE = \frac{1.5\times 8}{6} = 2 \text{ cm}$$

7. As MN || AB,

$$\frac{CM}{AM} = \frac{CN}{BN}$$

[Basic proportionality theorem]

$$\frac{2}{4} = \frac{BC-BN}{BN}$$

$$\frac{1}{2} = \frac{7.5 - BN}{BN}$$

$\therefore$ BN = 15 – 2BN

$\Rightarrow$ 3BN = 15

BN = 5 cm

8. We know that ratio of area of two similar triangles is equal to square of ratio of their corresponding sides.

So, Ratio of corresponding sides

$$= \sqrt{\frac{25}{64}} = \frac{5}{8}$$

9. DE || BC

$$\Rightarrow \frac{AD}{DB} = \frac{AE}{CE}$$

[Basic proportionality theorem]

$$\Rightarrow \frac{DB}{AD} + 1 = \frac{CE}{AE} + 1$$

$$\Rightarrow \frac{AB}{AD} = \frac{AC}{AE}$$

Also, $\angle A = \angle A$ (Common)

$\therefore$ $\Delta ADE \sim \Delta ABC$

(SAS Similarity Creterion)

$$\therefore \frac{ar\triangle ADE}{ar\triangle ABC} = \left(\frac{DE}{BC}\right)^2$$

$$\frac{ar\triangle ADE}{81} = \frac{\left(\frac{2}{3}BC\right)^2}{BC^2}$$

$$\frac{ar\triangle ADE}{81} = \frac{4}{9}$$

$$\text{ar } \Delta ADE = \frac{4}{9} \times 81 = 36 \text{ cm}^2$$

10. Considers $AC^2 + BC^2$

$= AC^2 + AC^2$ ($\because$ AC = BC)

$= 2AC^2$

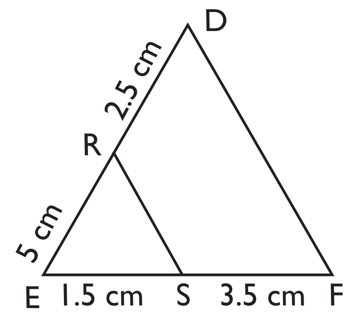
$= AB^2$

$\therefore$ ΔABC is right angled triangle

[As we know that in a triangle, if square of one side is equal to sum of the squares of other two sides then the angle opposite the first side is a right angle.]

Section B

11.



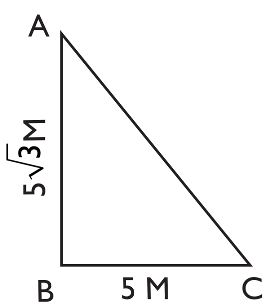
In a triangle ΔDEF, R and S are two points on the sides DE and EF respectively. ER=5cm, RD=2.5cm, FS=3.5 and SD=1.5cm.

$$\therefore \frac{ER}{RD} = \frac{5}{2.5} = \frac{2}{1} \text{ and } \frac{FS}{SD} = \frac{3.5}{1.5} = \frac{7}{3}$$

$$\because \frac{ER}{RD} \neq \frac{FS}{SD}$$

$\therefore$, RS is not parallel to DF.

12.



In ΔABC, right angled at B

$AC^2 = AB^2 + BC$

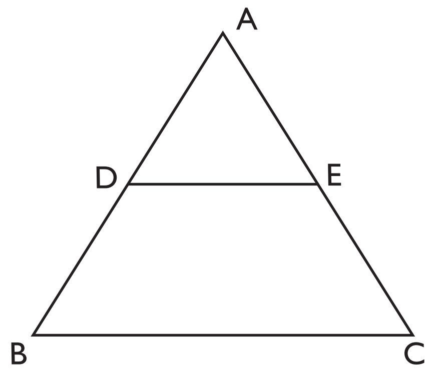
$= \left(5\sqrt{3}\right)^2 + (5)^2$

$= 75 + 25$

$= 100$

$\therefore$ AC = 10 M

13.



As DE || BC

$\frac{AD}{DB} = \frac{AE}{CE}$

$\frac{BD}{AD} = \frac{CE}{AE}$

$\Rightarrow \frac{BD}{AD} + 1 = \frac{CE}{AE} + 1$

$\Rightarrow \frac{AB}{AD} = \frac{AC}{AE}$

$\Rightarrow \frac{AD}{AB} = \frac{AE}{AC}$ (i)

Also, $\angle A = \angle A$ (Common)

$\therefore$ ΔADE ~ ΔABC (SAS Similarity Creterion)

$\therefore \frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC}$

[Corresponding sides of similar triangles are proportional]

$\Rightarrow \frac{AD}{AD+BD} = \frac{AE}{AC}$

$\Rightarrow \frac{AD}{AD+3AD} = \frac{4.5}{AC}$

$\Rightarrow \frac{AD}{4AD} = \frac{4.5}{AC}$

$\Rightarrow$ AC = 4.5 × 4 = 18 cm

Also, $\frac{AD}{AB} = \frac{AE}{AC}$ (From (i))

$\frac{AD}{AD+BD} = \frac{AE}{18}$

$\frac{AD}{AD+3AD} = \frac{AE}{18}$

$\frac{1}{4} = \frac{AE}{18}$

AE $= \frac{18}{4} = \frac{9}{2}$ = 4.5 cm

14. Consider ΔABC with sides as

AB = (a – 1) cm

BC = $\left(2\sqrt{a}\right)$ cm

AC = (a + 1) cm

Consider $AB^2 + BC^2$

$= (a-1)^2 + \left(2\sqrt{a}\right)^2$

$= a^2 + 1 - 2a + 4a$

$= a^2 + 2a + 1$

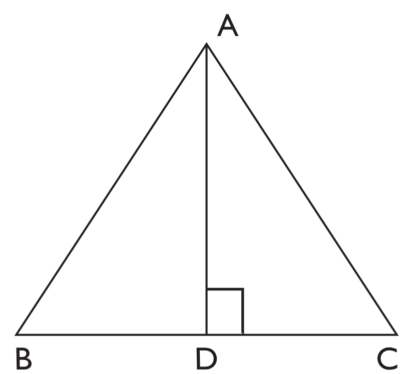
$= (a+1)^2$

$= AC^2$

$\therefore$ ΔABC is a right angled triangle

[As we know that in a triangle if square of one side is equal to the sum of squares of other two sides, then the angle opposite the first side is a right angle i.e. triangle is right angled]

15.



Draw AD $\perp$ BC

In ΔADB and ΔADC

AD = AD (Common)

AB = AC (ΔABC is equilateral)

$\angle$ADB = $\angle$ADC = 90° (By Construction)

$\therefore\quad \Delta ADB \cong \Delta ADC \quad (RHS)$

$\Rightarrow\quad CD = \frac{1}{2} BC = \frac{1}{2} 3\sqrt{3}$ cm [CPCT]

In ΔADC,

$AC^2 = AD^2 + CD^2$

$(3\sqrt{3})^2 = AD^2 + \left(\frac{3\sqrt{3}}{2}\right)^2$

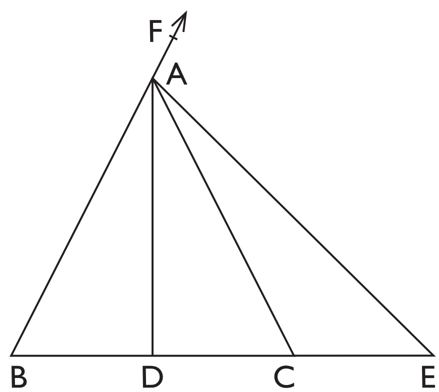
$AD^2 = 27 - \frac{27}{4}$

$= \frac{108-27}{4}$

$= \frac{81}{4}$

$\therefore AC = \frac{9}{2} = 4.5$ cm

16.



To prove = $\frac{BD}{BE} = \frac{CD}{CE}$

As AD bisects $\angle BAC$,

$\frac{AB}{BD} = \frac{AC}{CD}$ [Interior angle bisector theorem]

$\therefore\quad \frac{CD}{BD} = \frac{AC}{AB}$ (i)

Also, AE bisects $\angle CAF$

$\therefore\quad \frac{BE}{AB} = \frac{CE}{AC}$

$\Rightarrow\quad \frac{BE}{CE} = \frac{AB}{AC}$

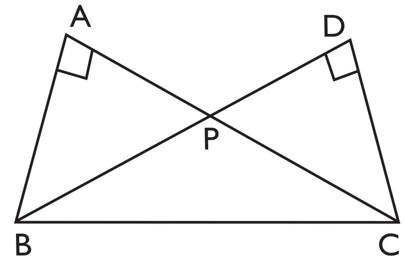
$\Rightarrow\quad \frac{CE}{BE} = \frac{AC}{AB}$ (ii)

From (i) and (ii)

$\frac{CD}{BD} = \frac{CE}{BE}$

$\Rightarrow\quad \frac{BD}{BE} = \frac{CD}{CE}$

17.



To prove: AP × PC = BP × PD

Consider ΔAPB and ΔDPC

$\angle BAP = \angle CDP = 90°$ (Given)

$\angle APB = \angle DPC$

(Vertically opposite angles)

$\therefore$ DAPB ~ ΔDPC (AA similarity crelerion)

$\therefore\quad \frac{AP}{DP} = \frac{PB}{PC} = \frac{AB}{DC}$

(Corresponding sides of similar triangles are proportional)

$\Rightarrow\quad \frac{AP}{DP} = \frac{PB}{PC}$

$\Rightarrow$ AP × PC = BP × PD

18. Consider ΔQPM and ΔRSM

$\angle QPM = \angle RSM = 90°$

$\angle QMP = \angle RMS$

(Vertically opposite angles)

$\therefore\quad \Delta QPM \sim \Delta RSM$ (AA similarity Creterion)

$\therefore\quad \frac{QP}{RS} = \frac{PM}{SM} = \frac{QM}{RM}$

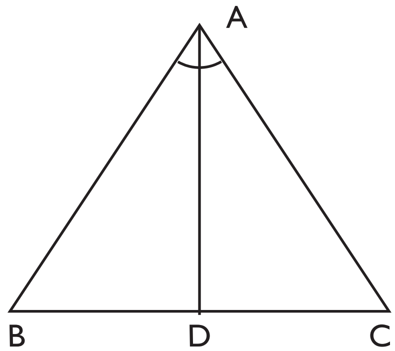
[Corresponding sides of similar triangles are proportional]

i.e. $\frac{PM}{SM} = \frac{QM}{RM}$

$$\frac{3}{4} = \frac{QM}{6}$$

$$QM = \frac{3 \times 6}{4} = \frac{3 \times 3}{2} = 4.5 \text{ cm}$$

19.



AD bisects $\angle A$ So, by Interior angle bisector theorem

$$\frac{AB}{AC} = \frac{BD}{DC}$$

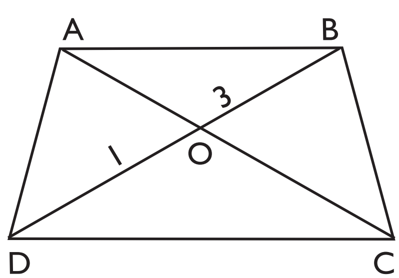
$$\frac{AB}{AC} = \frac{BD}{DC} = 1$$

[$\because$ BD = DC as D is a midpoint of BC]

AB = AC

$\therefore$ ΔABC is an isosceles.

20.



Here,AC divides the diagonal BD in the ratio 1 : 3

Consider ΔAOB and ΔCOD

$\angle$BAO = $\angle$DCO

(Alternate interior angles as AB || CD)

$\angle$AOB = $\angle$COD

(Vertically opposite angles)

$\therefore$ ΔAOB ~ ΔCOD

(AA similarity creterion)

$$\therefore \quad \frac{AO}{CO} = \frac{OB}{OD} = \frac{AB}{CD}$$

[Corresponding sides of similar triangles are proportional.]

$$\Rightarrow \quad \frac{OB}{OD} = \frac{AB}{CD}$$

$$\Rightarrow \quad \frac{3}{1} = \frac{AB}{CD}$$

$\Rightarrow$ AB = 3CD

Section C

21. In ΔADE and ΔABC,

$\angle$A = $\angle$A (Common)

$\angle$ADE = $\angle$ABC (Given)

$\therefore$ ΔADE ~ ΔABC (AA similarity createrion)

$$\Rightarrow \quad \frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC}$$

[Corresponding sides of similar triangles are proportional]

$$\therefore \quad \frac{AD}{AB} = \frac{DE}{BC}$$

$$\Rightarrow \quad \frac{7.6}{AE+BE} = \frac{DE}{8.4}$$

$$\Rightarrow \quad \frac{7.6}{7.2+4.2} = \frac{DE}{8.4}$$

$$\Rightarrow \quad \frac{7.6}{11.4} = \frac{DE}{8.4}$$

$$\Rightarrow \quad DE = \frac{7.6 \times 8.4}{11.4}$$

= 5.6 cm

22. In ΔABC, LM || BC

$$\therefore \quad \frac{AM}{BM} = \frac{AL}{CL}$$

(i) [Basic proportionality theorem]

In ΔADC, $LN \parallel CD$

$\therefore \quad \frac{AN}{DN} = \frac{AL}{CL}$

(ii) [Basic proportionality theorem]

From (i) and (ii), $\frac{AM}{BM} = \frac{AN}{DN}$

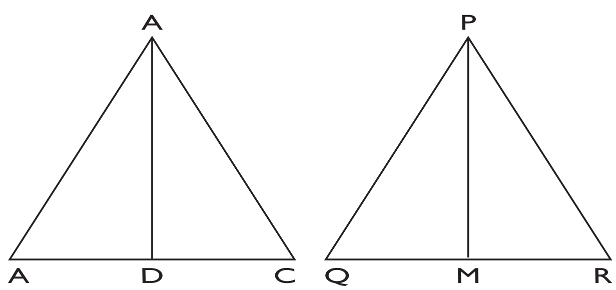
$\Rightarrow \quad \frac{BM}{AM} = \frac{DN}{AN}$

$\Rightarrow \quad \frac{BM}{AM} + 1 = \frac{DN}{AN} + 1$

$\Rightarrow \quad \frac{AB}{AM} = \frac{AD}{AN}$

$\Rightarrow \quad AM \times AD = AB \times AN$

23.



In ΔABD and ΔPQM

$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{BC}{QR}$ (Given)

$\Rightarrow \quad \frac{AB}{PQ} = \frac{AD}{PM} = \frac{BD}{QM}$

$\Rightarrow \quad \frac{AB}{PQ} = \frac{AD}{PM} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR}$

(As AD and PM are the medians)

$\therefore \quad \Delta ABD \sim \Delta PQM$

(SSS similarity creterion)

$\therefore \quad \angle B = \angle Q$

[Corresponding angles of similar triangles are equal]

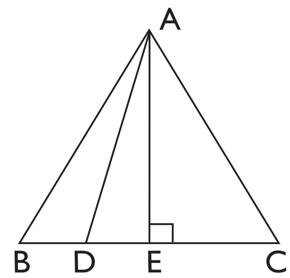
Now, In ΔABC and ΔPQR

$\frac{AB}{PQ} = \frac{BC}{QR}$ (Given)

$\angle B = \angle Q$ (Proved)

$\therefore \quad \Delta ABC \sim \Delta PQR$ (SSS similarity creterion)

24.



Let $AB = BC = AC = a$

$\therefore \quad BD = \frac{BC}{4} = \frac{a}{4}$

Draw $AE \perp BC$

$\therefore \quad BE = EC = \frac{a}{2}$

[In Equilateral triangle altitude is same as Median]

In right angled triangle ΔAED,

$AD^2 = DE^2 + AE^2$ (i)

Now, $DE = BE - BD$

$= \frac{a}{2} - \frac{a}{4}$ $\left[\because BD = \frac{1}{4} = \frac{a}{4}\right]$

$= \frac{a}{4}$ (ii)

In ΔAEC,

$AC^2 = AE^2 + CE^2$

$a^2 = AE^2 + \left(\frac{a}{2}\right)^2$

$AE^2 = a^2 - \frac{a^2}{4} = \frac{3a^2}{4}$ (iii)

On putting (ii), (iii) in (i), we get

$AD^2 = \left(\frac{a}{4}\right)^2 + \left(\frac{3a}{4}\right)^2$

$= \frac{a^2}{16} + \frac{3a^2}{4}$

$= \frac{a^2 + 12a^2}{16}$

$$= \frac{13a^2}{16}$$

$16\,AD^2 = 13\,a^2$

$16AD^2 = 13\,BC^2$

25. As ΔABC is isosceles,

$AB = AC$

$\therefore \angle B = \angle C$

(Angles opposite to equal sides are equal)

In ΔADB and ΔEFC

$\angle ADB = \angle EFC$

(AS $EF \perp AC$ and $AD \perp CD$)

$\angle B = \angle C$ (Proved)

$\therefore \Delta ADB \sim \Delta EFC$ (AA similarity creterion)

$$\therefore \frac{AD}{EF} = \frac{BD}{FC} = \frac{AB}{EC}$$

i.e. $\dfrac{AD}{EF} = \dfrac{AB}{EC}$

$\Rightarrow AD \times EC = AB \times EF$

26. In ΔABC and ΔADE,

$\angle A = \angle A$ (Common)

$\angle ACB = \angle AED = 90°$

(As $DE \perp AB$ and ΔABC is right angled at C)

$\therefore \Delta ABC \sim \Delta ADE$

(By AA Similarity creterion)

$$\Rightarrow \frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE}$$

[Corresponding sides of similar triangles are proportional]

In ΔABC, $\angle C = 90°$

$\therefore AB^2 = AC^2 + BC^2$

[By Pythagoras theorem]

$= (3 + 2)^2 + 12^2$

$= 25 + 144$

$= 169$

$\therefore AB = 13$ cm

As $\dfrac{AB}{AD} = \dfrac{BC}{DE}$

$$\therefore \frac{13}{3} = \frac{12}{DE}$$

$$\therefore DE = \frac{12 \times 3}{13} = \frac{36}{13} \text{ cm}$$

Also, $\dfrac{BC}{DE} = \dfrac{AC}{AE}$

$$\Rightarrow \frac{12}{\frac{36}{13}} = \frac{5}{AE}$$

$$\Rightarrow \frac{12 \times 13}{36} = \frac{5}{AE}$$

$$\Rightarrow AE = \frac{5 \times 36}{12 \times 13}$$

$$= \frac{15}{13} \text{ cm}$$

27. As $\Delta NSQ \cong \Delta MTR$,

$\angle NQS = \angle MRT$ (CPCT)

$\Rightarrow PQ = PR$ (i)

(Sides opposite to equal angles are equal)

Also, as $\angle 1 = \angle 2$

$\therefore PS = PT$ (ii)

(Sides opposite to equal angles are equal.)

On Subtracting (ii) from (i), we get

$PQ – PS = PR – PT$

$QS = TR$ (iii)

From (ii) and (iii),

$$\frac{PS}{QS} = \frac{PT}{TR} \Rightarrow \frac{PS}{PQ} = \frac{PT}{PR}$$

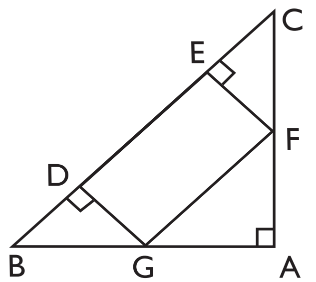
Also, $\angle P = \angle P$ (Common)

$\therefore \quad \Delta PST \sim \Delta PQR$

(SAS similarity creterion)

Section D

28.



In ΔAFG and ΔDBG,

$\angle AGF = \angle DBG$

(Corresponding angles as $GF \parallel BC$)

$\angle GAF = \angle BDG = 90°$ ($\because$ DE FG is a square)

$\therefore \Delta AFG \sim \Delta DBG$ (i)

(AA similarity creterion)

In ΔAGF and ΔEFC,

$\angle AFG = \angle CEF = 90°$

$\angle AFG = \angle ECF$

(Corresponding angles as $GF \parallel BC$)

$\therefore \quad \Delta AGF \sim \Delta EFC$ (ii)

(AA similarity creterion)

From (i), (ii), we get

$\Delta DBG \sim \Delta EFC$

$\Rightarrow \quad \frac{BD}{EF} = \frac{DG}{EC}$

[Corresponding sides of similar triangles are proportional]

$\Rightarrow \quad \frac{BD}{DE} = \frac{DE}{EC}$

[As DEFG is a square, EF = DE and DG = DE]

$\Rightarrow \quad DE^2 = BD \times EC$

29. In ΔAOD, MO bisects $\angle AOD$,

So, by interior angle bisector theorem,

$\frac{AO}{OD} = \frac{AM}{DM}$ (i)

In ΔBOC, NO bisects $\angle BOC$

So, by interior angle bisector theorem,

$\frac{BO}{CO} = \frac{BN}{CN}$

$\Rightarrow \quad \frac{CO}{BO} = \frac{CN}{BN}$ (ii)

We know that AO = OD $\Rightarrow \frac{AO}{OD} = 1$

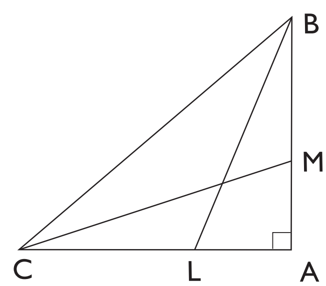
and CO = BO $\Rightarrow \frac{CO}{BO} = 1$

(Radii of same circle)

So, From (i) and (ii), we get

$\frac{AM}{DM} = \frac{CN}{BN}$

30.



In ΔABC, $BC^2 = AB^2 + AC^2$

(By Pythagoras theorem)

In ΔABL, $BL^2 = AB^2 = AL^2$

$= AB^2 + \left(\frac{1}{2}AC\right)^2$

[As L is a midpoint of AC $\therefore AL = \frac{1}{2}AC$]

$BL^2 = AB^2 + \frac{AC^2}{4}$

$4BL^2 = 4AB^2 + AC^2$ (i)

In ΔCMA, $CM^2 = AC^2 + AM^2$

$= AC^2 + \left(\frac{1}{2}AB\right)^2$

$$= AC + \frac{AB^2}{4}$$

[As M is a midpoint of AB $\therefore$ AM = $\frac{1}{2}AB$]

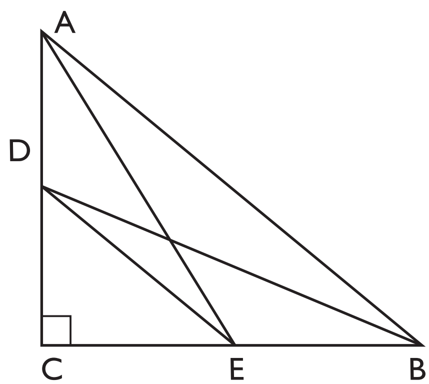
$\Rightarrow$ $4\ CM^2 = 4AC^2 + AB^2$ (ii)

From (i), (ii), we get

$4(BL^2 + CM^2) = 5\ AB^2 + 5\ AC^2$

$= 5\ BC^2$

31.



To prove : $AE^2 + BD^2 = AB^2 + DE^2$

Proof In $\Delta ACE, AE^2 = AC^2 + CE^2$ (i)

(By Pythagoras theorem)

In $\Delta DCB, BD^2 = DC^2 + BC^2$ (ii)

(By Pythagoras theorem)

In $\Delta ABC, AB^2 = AC^2 + BC^2$ (iii)

(By Pythagoras theorem)

In $\Delta DCE, DE^2 = DC^2 + CE^2$ (iv)

(By Pythagoras theorem)

Consider $AE^2 + BD^2$

$= AC^2 + CE^2 + DC^2 + BC^2$ (By ① and ②)

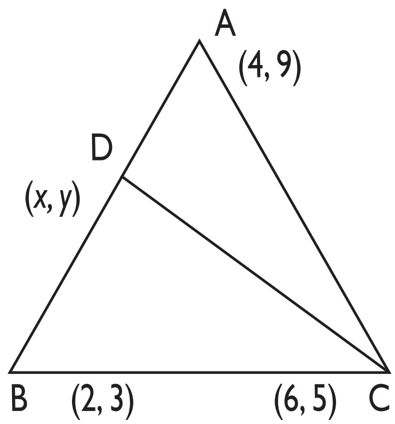
$= (AC^2 + BC^2) + (CE^2 + DC^2)$

$= AB^2 + DE^2$ (By ③,④)

Chapter 07 Cordinate Geometry

MULTIPLE CHOICE QUESTIONS

1.



$D(x, y) = \left(\frac{4+2}{2}, \frac{9+3}{2}\right) = (3, 6)$

So, $CD = \sqrt{(6-3)^2 + (5-6)^2}$

$= \sqrt{9+1}$

$= \sqrt{10}$ units

Option (b)

2. As A, B and C are collinear

$\therefore$ x (– 4 + 5) – 3 (– 5 – 2) + 7 (2 + 4) = 0

x + 21 + 42 = 0

x = – 63

Option (c)

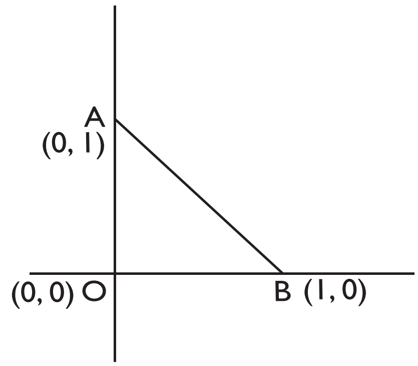
3. $(2, p) = \left(\frac{6-2}{2}, \frac{-5+11}{2}\right)$

= (2, 3)

$\Rightarrow$ p = 3

Option (b)

4.



In DAOB,

$AB^2 = AO^2 + OB^2$

$1^2 + 1^2$

2

$AB = \sqrt{2}$

Perimeter = AO + OB + AB

$= 1 + 1 + \sqrt{2}$

Option (d)

WORKSHEET 1

Section A

1. $\text{Centroid} = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

$(1, 4) = \left(\frac{4 - 9 + x_3}{3}, \frac{-3 + 7 + y_3}{3}\right)$

$(1, 4) = \left(\frac{-5 + x_3}{3}, \frac{4 + y_3}{3}\right)$

$\frac{-5 + x_3}{3} = 1$

$x_3 - 5 = 3$

$x_3 = 8$

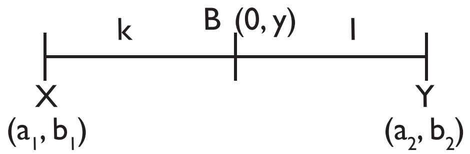
$\frac{4 + y_3}{3} = 4$

$y_3 + 4 = 12$

$y_3 = 8$

So, third vertex is (8, 8)

2.



Let the ratio be k : l

$$\text{So, } (0, y) = \left(\frac{ka_2 + a_1}{k+1}, \frac{kb_2 + b_1}{k+1}\right)$$

$$\frac{ka_2 + a_1}{k+1} = 0$$

$$ka_2 + a_1 = 0$$

$$ka_2 = -a_1$$

$$k = \frac{-a_1}{a_2}$$

3. Distance $= \sqrt{\left(\frac{2}{5}+\frac{8}{5}\right)^2 + (2-2)^2}$

$$= \sqrt{\left(\frac{2}{5}+\frac{8}{5}\right)^2 + 0}$$

$$= \sqrt{\left(\frac{2+8}{5}\right)^2}$$

= 2 sq. units

4. Let Point on y - axis be (0, y).

$$\sqrt{(6-0)^2 + (5-y)^2} = \sqrt{(-4-0)^2 + (3-y)^2}$$

$$\sqrt{36+25+y^2-10y} = \sqrt{16+9+y^2-6y}$$

$$\sqrt{61+y^2-10y} = \sqrt{25+y^2-6y}$$

$$61 + y^2 - 10y = 25 + y^2 - 6y$$

$$36 = 4y$$

$$y = 9$$

So, point on y - axis which is equidistant from point A (6, 5) and B (– 4, 3) is (0, 9)

5. As point A (0, 2) is equidistant from the points B (3, P) and C (P, 5), So,

$$\sqrt{(3-0)^2 + (P-2)^2} = \sqrt{(P-0)^2 + (5-2)^2}$$

$$\sqrt{9 + (P-2)^2} = \sqrt{P^2 + 9}$$

$$(P-2)^2 = P^2$$

$$P^2 + 4 - 4P = P^2$$

$$4P = 4$$

$$P = 1$$

6. $\sqrt{(4-1)^2 + (K-0)^2} = 5$

$$\sqrt{3^2 + K^2} = 5$$

On Squarring both sides, we get

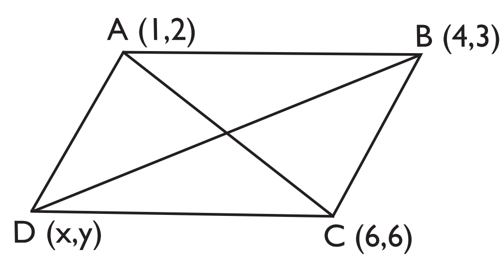
$$9 + K^2 = 25$$

$$K^2 = 25 - 9 = 16$$

$$K^2 = 16$$

$$K = \pm 4$$

7.



We know that diagonals of a parallelogram bisect each other

$$\therefore \left(\frac{1+6}{2}, \frac{2+6}{2}\right) = \left(\frac{4+x}{2}, \frac{3+y}{2}\right)$$

$$\left(\frac{7}{2}, 4\right) = \left(\frac{4+x}{2}, \frac{3+y}{2}\right)$$

$$\therefore \frac{7}{2} = \frac{4+x}{2} \text{ and } 4 = \frac{3+y}{2}$$

7 = 4 + x and 8 = 3 + y

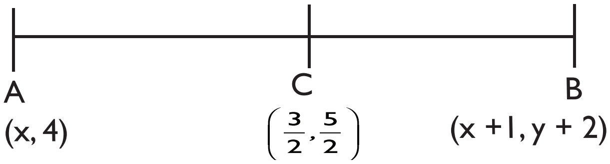
x = 3 and y = 5

So, coordinates of fourth uertex

= (x, y)

= (3, 5)

8.



As C is a midpoint of AB,

$$\left(\frac{x+x+1}{2}, \frac{4+y+2}{2}\right) = \left(\frac{3}{2}, \frac{5}{2}\right)$$

$$\left(\frac{2x+1}{2}, \frac{y+6}{2}\right) = \left(\frac{3}{2}, \frac{5}{2}\right)$$

$$\therefore \frac{2x+1}{2} = \frac{3}{2} \text{ and } \frac{y+6}{2} \; \frac{5}{2}$$

$2x + 1 = 3$ and $y + 6 = 5$

$2x = 2$ and $y = 5 - 6$

$x = 1$ and $y - 1$

Section B

9. Let y – coordinate by v

$\therefore$ x – coordinate = 2v

So, point P is (2v, v)

PQ = PR

$$\sqrt{(2-2v)^2+(-5-v)^2} = \sqrt{(3-2v)^2+(6-v)^2}$$

On squaring both sides, we get

$$(2 - 2v)^2 + (-5 - v)^2 = (-3 - 2v) + (6 - v)^2$$

$$\therefore 4 + 4v^2 - 8v + 25 + v^2 + 10v$$

$$= 9 + 4v^2 = 12v + 36 + v^2 - 12v$$

$\Rightarrow$ $5v^2 + 2v + 29 = 5v^2 + 45$

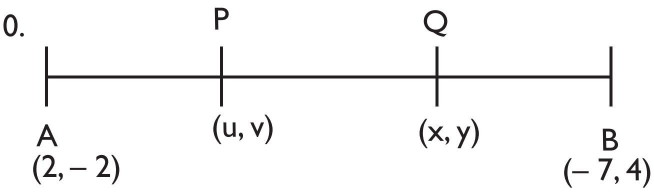
$\Rightarrow$ $2v = 45 - 29$

$2v = 16$

$v = 8$

So, Point P is (2v, v) i.e. (16, 8)

10.



Point P divides AB in ratio 1 : 2

$$\text{So, P (u,v)} = \left(\frac{1(-7)+2(2)}{3}, \frac{1(4)+2(-2)}{3}\right)$$

$$= \left(\frac{-7+4}{3}, \frac{4-4}{3}\right)$$

$$= (-1, 0)$$

Point Q divides AB in ratio 2 : 1

$$\text{So, Q (x, y)} = \left(\frac{2(-7)+1(2)}{3}, \frac{2(4)+11(-2)}{3}\right)$$

$$= \left(\frac{-14+2}{3}, \frac{8-2}{3}\right)$$

$$= \left(\frac{-12}{3}, \frac{6}{3}\right)$$

$$= (-4, 2)$$

11. Let A(3, 0), B(6, 4) and C(–1, 3) be the given points

$$AB = \sqrt{(6-3)^2+(4-0)^2}$$

$$\Rightarrow AB = \sqrt{(3)^2+(4)^2}$$

$$\Rightarrow AB = \sqrt{9+16}$$

$$\Rightarrow AB = \sqrt{25}$$

$$BC = \sqrt{(-1-6)^2+(3-4)^2}$$

$$\Rightarrow BC = \sqrt{(-7)^2+(-1)^2}$$

$$\Rightarrow BC = \sqrt{49+1}$$

$$\Rightarrow BC = \sqrt{50}$$

$$AC = \sqrt{(-1-3)^2+(3-0)^2}$$

$\Rightarrow AC = \sqrt{(-4)^2 + (3)^2}$

$\Rightarrow AC = \sqrt{16+9}$

$\Rightarrow AC = \sqrt{25}$

$\Rightarrow AB^2 = \left(\sqrt{25}\right)^2$

$\Rightarrow AB^2 = 25$

$\Rightarrow AC^2 = 25$

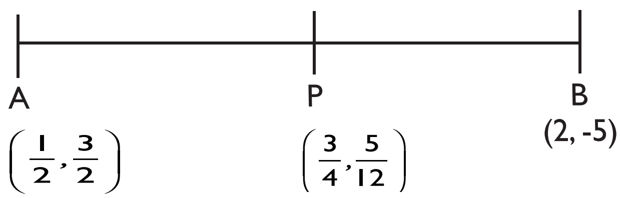
$\Rightarrow BC^2 = \left(\sqrt{50}\right)^2$

$\Rightarrow BC^2 = 50$

Since $AB^2 + AC^2 = BC^2$ and AB = AC

$\therefore$ ABC is a right angled isosceles triangle.

12.



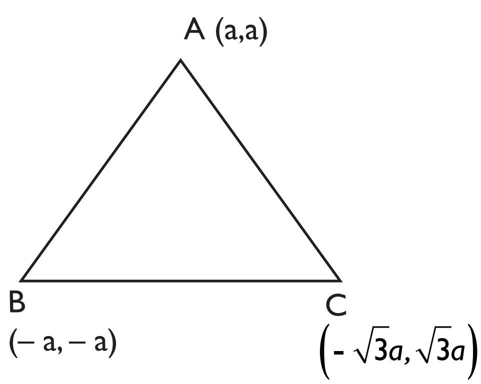
Let point P divides AB in ratio k : 1

$$\text{So, } \left(\frac{3}{4},\frac{5}{12}\right) = \left(\frac{2k+\frac{1}{2}}{k+1},\frac{-5k+\frac{3}{2}}{k+1}\right)$$

$\frac{3}{4} = \frac{2k+\frac{1}{2}}{k+1}$	$\frac{5}{12} = \frac{-5k+\frac{3}{2}}{k+1}$
$\Rightarrow$ 3k + 3 = 8k + 2	5k + 5 = – 60k + 18
$\Rightarrow$ 1 = 5k	65k = 13
$k = \frac{1}{5}$	$k = \frac{1}{5}$

So, point P divides AB in ratio 1 : 5

13.



$AB = \sqrt{(-a-a)^2 + (-a-a)^2}$

$= \sqrt{4a^2 + 4a^2}$

$= \sqrt{8a^2}$

$= 2\sqrt{2}a$ Units

$BC = \sqrt{(-\sqrt{3}a + a)^2 + (\sqrt{3}a + a)^2}$

$= \sqrt{3a^2 + a^2 - 2\sqrt{3}a^2 + 3a^2 + 2\sqrt{3}a^2}$

$= \sqrt{8a^2}$

$= 2\sqrt{2}a$

$AC = \sqrt{(-\sqrt{3}a - a)^2 + (\sqrt{3}a - a)^2}$

$= \sqrt{3a^2 + a^2 + 2\sqrt{3}a^2 + 3a^2 + a^2 - 2\sqrt{3}a^2}$

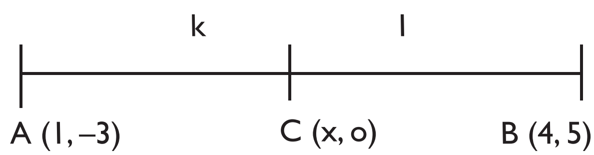
$= \sqrt{3a^2 + a^2 + 3a^2 + a^2}$

$= \sqrt{8a^2}$

$= 2\sqrt{2}a$

As AB = AC, ΔABC is an equilateral triangle.

14.



Let point C (x, o) divides AB in ratio k : 1

So,

$$(x, o) = \left(\frac{4k+1}{k+1},\frac{5k-3}{k+1}\right)$$

$$\therefore \frac{5k-3}{k+1} = 0$$

5k – 3 = 0

$$k = \frac{3}{5}$$

So, x – axis divides the line segment joining point (1, – 3) and (4, 5) in ratio 3 : 5

15. $\sqrt{(9-x)^2+(10-4)^2} = 10$

$81 + x^2 - 18x + 36 = 100$

$x^2 - 18x + 17 = 0$

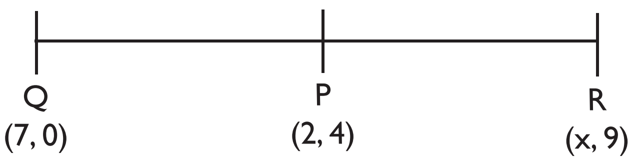
$x^2 - 17x - x + 17 = 0$

$x(x - 17) - 1(x - 17) = 0$

$(x - 1)(x - 17) = 0$

$x = 1, 17$

16.



PQ = PR

$\Rightarrow \sqrt{(2-7)^2+(4-2)^2} = \sqrt{(x-2)^2+(9-4)^2}$

$\Rightarrow \sqrt{25+16} = \sqrt{x^2+4-4x+25}$

$\Rightarrow \sqrt{41} = \sqrt{x^2-4x+29}$

On squarring both sides, we get

$41 = x^2 - 4x + 29$

$0 = x^2 - 4x - 12$

$0 = x^2 - 6x + 2x - 12$

$0 = x(x - 6) + 2(x - 6)$

$0 = (x + 2)(x - 6)$

$x = -2$ or 6

$PQ = \sqrt{(7-2)^2+(0-4)^2}$

$= \sqrt{5^2+(-4)^2}$

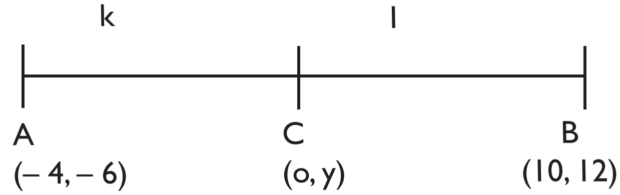
$= \sqrt{25+16}$

$= \sqrt{41}$

Section C

17. Let y – axis divides the line segment joining the points (– 4, – 6) and (10, 12) in ratio k : l

Point on y – axis must be of form (o, y)



$(o, y) = \left(\frac{10k+(-4)}{k+1}, \frac{12k-6}{k+1}\right)$

$(o, y) = \left(\frac{10k-4}{k+1}, \frac{12k-6}{k+1}\right)$

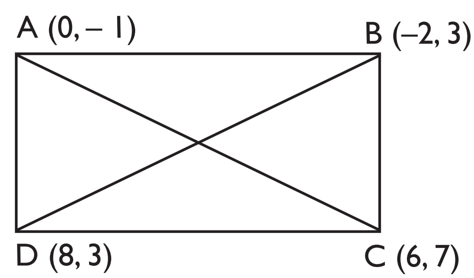
$\Rightarrow \frac{10k-4}{k+1} = 0$

$\Rightarrow 10k = 4$

$k = \frac{2}{5}$

So, ratio is $2 : 5$

18.



$AB = \sqrt{(-2-0)^2+(3+1)^2}$

$= \sqrt{4+16}$

$= \sqrt{20}$

$= 2\sqrt{5}$ Unit

$CD = \sqrt{(-8-0)^2+(3+1)^2}$

$= \sqrt{4+16}$

$= \sqrt{20}$

$= 2\sqrt{5}$ Unit

$\therefore$ AB = CD

$AD = \sqrt{(-8-0)^2+(3+1)^2}$

$= \sqrt{64+16} = \sqrt{80} = 4\sqrt{5}$ Units

$$BC = \sqrt{(6+2)^2 + (7-3)^2}$$

$$= \sqrt{64+16}$$

$$= \sqrt{80}$$

$$= 4\sqrt{5} \text{ Units}$$

$\therefore$ AD = BC

As AB = CD and AD = BC,

So, ABCD is a parallelogram

$$AC = \sqrt{(6-0)^2 + (7+1)^2}$$

$$= \sqrt{36+64}$$

$$= \sqrt{100}$$

$$= 10 \text{ Units}$$

$$BD = \sqrt{(8+2)^2 + (3-3)^2}$$

$$= \sqrt{100}$$

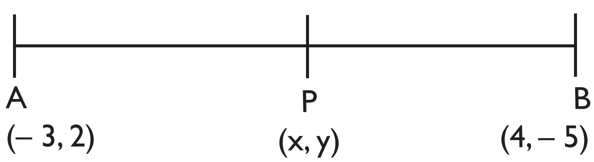
$$= 10 \text{ Units}$$

So, AC = BD

$\therefore$ ABCD is a parallelogram in which both diagonals are equal.

So, ABCD is a rectangle.

19.



As point P is equidistant from A and B,

AP = BP

$$\sqrt{(x+3)^2 + (y-2)^2} = \sqrt{(4-x)^2 + (-5-y)^2}$$

On squarring both sides, we get

$(x + 3)^2 + (y – 2)^2 = (4 – x)^2 + (– 5 – y)^2$

$\underline{x^2} + \underline{9} + \underline{6x} + \underline{y^2} + \underline{4} – \underline{4y}$

$= 16 + \underline{x^2} – \underline{8x} + 25 + \underline{y^2} + \underline{10}y$

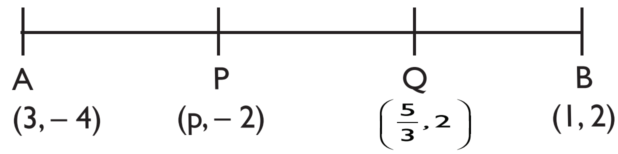
$14x – 14y + 13 = 41$

$14x – 14y – 28 = 0$

$x – y = 2$

$\therefore \quad y = x – 2$

20.



Point P divides AB in ratio 1 : 2

So,

$$p\,(p, -2) = \left(\frac{1(1)+2(3)}{3}, \frac{1(2)+2(-4)}{3}\right)$$

$$p\,(p, -2) = \left(\frac{7}{3}, -2\right)$$

$$\therefore\ p = \frac{7}{3}$$

Point Q divides AB in ratio 2 : 1

So,

$$Q\left(\frac{5}{3}, q\right) = \left(\frac{2(1)+1(3)}{3}, \frac{2(2)+1(-4)}{3}\right)$$

$$Q\left(\frac{5}{3}, q\right) = \left(\frac{5}{3}, 0\right)$$

$\therefore \quad q = 0$

21. As the points A (3p + 1, p), B (p + 2, p – 5) and C (p + 1, – p) are collinear,

area of ΔABC = 0

i. e. $\frac{1}{2}$ [(3p + 1) (p – 5 + p) + (p + 2) (– p – p) + (p + 1) (p – p + 5)] = 0

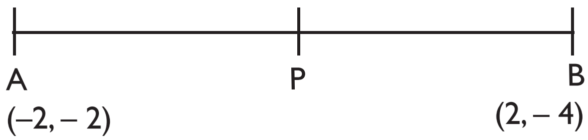
$\Rightarrow$ [(3p + 1) (2p – 5)–2p (p + 2) + 5 (p + 1)] = 0

$\Rightarrow$ [6p² – 15p + 2p – 5 – 2p² – 4p + 5p + 5] = 0

$\Rightarrow$ [4p² – 12p] = 0

p = 0, 3

22.



$AP = \frac{3}{7} AB$

$\Rightarrow \quad AP = \frac{3}{7}(AP + BP)$

$\Rightarrow \quad 7\,AP = 3\,AP + 3\,BP$

$\Rightarrow \quad 4\,AP = 3\,BP$

$\Rightarrow \quad \frac{AP}{PB} = \frac{3}{4}$

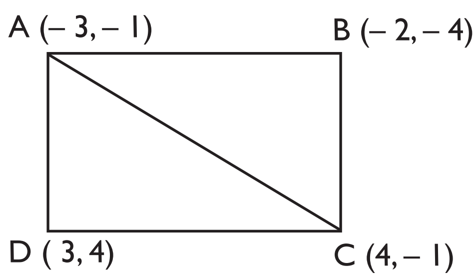
Let point P be (x, y), using section formula,

$$(x, y) = \left(\frac{3(2)+4(-2)}{7}, \frac{3(-4)+4(-2)}{7}\right)$$

$$= \left(\frac{6-8}{7}, \frac{-12-8}{7}\right)$$

$$= \left(\frac{-2}{7}, \frac{-20}{7}\right)$$

23.



Join AC

Area of ΔACD

$= \frac{1}{2}[-3(-1-4) + 4(4+1) + 3(-1+1)]$

$= \frac{1}{2}[-3(-5) + 20]$

$= \frac{1}{2}[15 + 20]$

$= \frac{35}{2}$ sq. Units

Area of ΔACD

$= \frac{1}{2}[(-3)(-4-1) - 2(-1+1) + 4(-1+4)]$

$= \frac{1}{2}[-3(-3) - 2(0) + 4(3)]$

$= \frac{1}{2}[9 + 12] = \frac{21}{2}$ sq. Units

So, area of quadrilateral ABCD

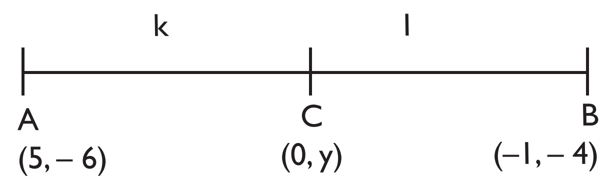
= area of ΔACD + area of ΔABC

$= \frac{35}{2} + \frac{21}{2}$

$= \frac{56}{2}$

= 28 sq. Units

24. Let y – axis divides the line segment joining points A (5, – 6), B (– 1, – 4) in ratio k : 1. Point on y – axis is of from (0, y).



By section formula,

$$(0, y) = \left(\frac{-k+5}{k+1}, \frac{-4k-6}{k+1}\right)$$

$0 = \frac{-k+5}{k+1}$

$k = 5$

So, y – axis divides AB in ratio 5 : 1

Also, $y = \frac{-4k-6}{k+1}$

$= \frac{-20-6}{5+1}$

$= \frac{-26}{6}$

$= \frac{-13}{3}$

So, C (0, y) = $(0, \frac{-13}{3})$

Section D

25. Consider points $(x_1, y_1) = (t, t-2)$

$= (x_2, y_2) = (t+2, t-2)$

$= (x_3, y_3) = (t + 3, t)$

Area of triangle

$$= \frac{1}{2}[x_1(y_2,y_3) + x_2(y_3,y_1) + x_3(y_1,-y_2)]$$

$$= \frac{1}{2}[t(t - 2 - t) + (t + 2)(t - t + 2) + (t + 3)(t - 2 - t + 2)]$$

$$= \frac{1}{2}[t(-2) + (t + 2)(2)]$$

$$= \frac{1}{2}[-2t + 2t + 4]$$

$$= \frac{1}{2}(4)$$

= 2 sq. Units

So, area of triangle is independent of t.

26. $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{3}$

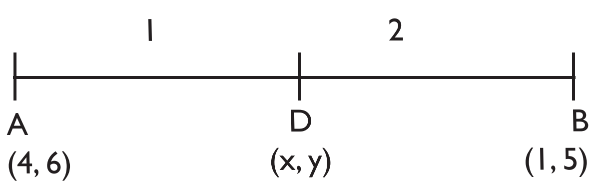
$$\Rightarrow \frac{AD}{AB} = \frac{AC}{AE} = \frac{3}{1}$$

$$\Rightarrow \frac{AD}{AB} - 1 = \frac{AC}{AE} - 1 = 3 - 1$$

$$\Rightarrow \frac{BD}{AD} = \frac{CE}{AE} = 2$$

$$\therefore \frac{AD}{BD} = \frac{AE}{CE} = \frac{1}{2}$$

For coordinates of D



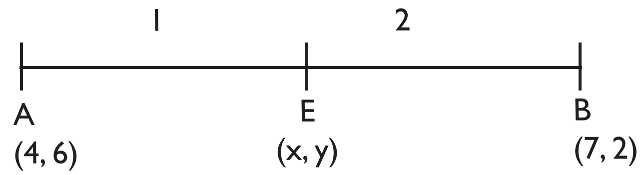
By section formula,

$$(x, y) = \left(\frac{1(1)+2(4)}{3}, \frac{1(5)+2(6)}{3}\right)$$

$$(x, y) = \left(\frac{9}{3}, \frac{17}{3}\right)$$

$$= \left(3, \frac{17}{3}\right)$$

For coordinates of E

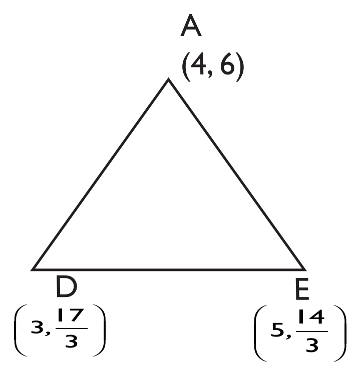


By section formula,

$$(x, y) = \left(\frac{1(7)+2(4)}{3}, \frac{1(2)+2(6)}{3}\right)$$

$$= \left(\frac{7+8}{3}, \frac{2+12}{3}\right)$$

$$= \left(5, \frac{14}{3}\right)$$



ar ΔADE

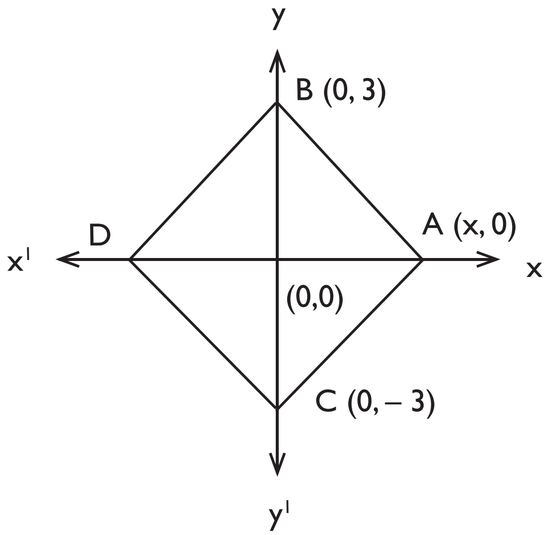
$$= \frac{1}{2}\left[4\left(\frac{17}{3}, \frac{-14}{3}\right) + 3\left(\frac{14}{3}, -6\right) + 5\left(6 - \frac{17}{3}\right)\right]$$

$$= \frac{1}{2}\left[4\left(\frac{3}{3}\right) + 3\left(\frac{14-18}{3}\right) + 5\left(\frac{18-17}{3}\right)\right]$$

$$= \frac{1}{2}\left[4 + (-4) + \frac{5}{3}\right]$$

$$= \frac{5}{6} \text{ sq. Units}$$

27.



Let coordinates of B be (0, y)

As (0, 0) is a Midpoint of BC

$\therefore$ (0, 0) = $\left(\frac{0+0}{2}, \frac{y-3}{2}\right)$

(0, 0) = $\left(\frac{0}{2}, \frac{y-3}{2}\right)$

(0, 0) = $\left(0, \frac{y-3}{2}\right)$

$\frac{y-3}{2} = 0$

y = 3

So, point B is (0, 3)

Let coordinates of point A be (x, 0)

Using distance formula,

AB = $\sqrt{(x-0)^2+(0-3)^2}$

= $\sqrt{x^2+9}$

BC = $\sqrt{(0-0)^2+(-3-3)^2}$

= $\sqrt{36}$

= 6

As ΔABC is equilateral,

AB = BC

i.e. $\sqrt{x^2+9}$ = 6

$x^2 + 9 = 36$

$x^2 = 27$

$x = \pm 3\sqrt{3}$

$\therefore$ Coordinates of point A are $(3\sqrt{3}, 0)$

As BACD is a rhombus and diagonals of rhombus bisect each other. So, 0D = 0A = 3 $\sqrt{3}$ units

$\therefore$ Point D is $(-3\sqrt{3}, 0)$

28. Area of triangle = 5 sq. units

As third vertex leis on y = x + 3,

So, it must be of form (x, x + 3)

Let $(x_1,y_1) = (2, 1)$

$(x_2,y_2) = (3, -2)$

$(x_3,y_3) = (x, x + 3)$

Area of triangle

$= \frac{1}{2}\ [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3\ (y_1 - y_2)]$

$5 = \frac{1}{2}[2(-2 - x - 3) + (x + 3 - 1) + x(1 + 2)]$

10 = [2 (– 5 – x) + 3 (x + 2) + 3x]

10 = [– 10 2x + 3x + 6 + 3x]

10 = [4x – 4]

$\therefore$ ± 10 = 4x – 4

4x – 4 = 10	4x – 4 = – 10
4x = 10	4x = – 6
$x = \frac{7}{2}$	$x = \frac{-3}{2}$
So, third vertex is	So, third vertex is
(x, x + 3)	(x, x + 3)
$= \left(\frac{7}{2}, \frac{7}{2}+3\right)$	$= \left(\frac{-3}{2}, \frac{-3}{2}+3\right)$
$= \left(\frac{7}{2}, \frac{13}{2}\right)$	$= \left(\frac{-3}{2}, \frac{3}{2}\right)$

29. Let $(x_1, y_1) = (a, a^2)$

$(x_2, y_2) = (b, b^2)$

$(x_3, y_3) = (c, c^2)$

Consider Area of triangle

$= \frac{1}{2}$ [x_1 (y_2, – y_3) + x_2 (y_3, y_1) + x_3 (y_1, – y_2)]

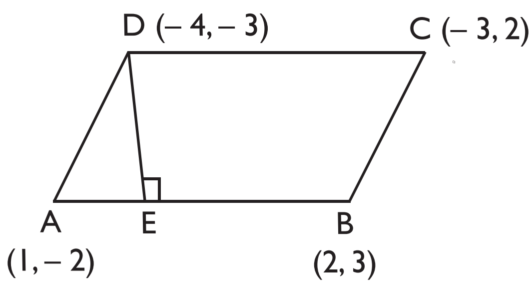
$= \frac{1}{2}$ [a ($b^2 - c^2$) + b ($c^2 - a^2$) + c ($a^2 - b^2$)]

$= \frac{1}{2}$ [$ab^2 - ac^2 + bc^2 - a^2b + a^2c - b^2c$]

$= \frac{1}{2}$ [ab (b – a) + ac (a – c) + bc (c – b)]

Here, it is clear that area of triangle is 0 if a = b = c

but it is given that a ≠ b ≠ c

30. 

Let be the height of parallelogram ABCD.

For ΔABD,

Let (x_1, y_1) = (1, – 2)

(x_2, y_2) = (2, 3)

(x_3, y_3) = (– 4, – 3)

area of ΔABD

$= \frac{1}{2}$ [x_1 (y_2, – y_3) + x_2 (y_3, y_1) + x_3 (y_1, – y_2)]

$= \frac{1}{2}$ [1 (3 + 3) + 2 (– 3 + 2) – 4 (– 2 – 3)]

$= \frac{1}{2}$ [6 – 2 + 20]

= 12 sq. units

For ΔBCD,

Let (x_1, y_1) = (2, 3)

(x_2, y_2) = (– 3, 2)

(x_3, y_3) = (– 4, – 3)

area of ΔBCD

$= \frac{1}{2}$ [x_1 (y_2, – y_3) + x_2 (y_3, y_1) + x_3 (y_1, – y_2)]

$= \frac{1}{2}$ [2 (2 + 3) + – 3 (– 3 – 3) – 4 (3 – 2)]

$= \frac{1}{2}$ [10 + 18 – 4]

= 12 sq. units

Area of parallelogram ABCD

= area of ΔABC + area of ΔBCD

= 12 + 12

= 24 sq. units

We know that area of parallelogram

= base x height

24 = AB x height

By Distance formula,

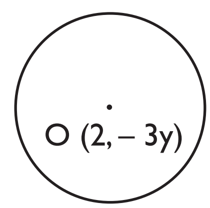
AB = $\sqrt{(2-1)^2 + (3+2)^2}$

= $\sqrt{1+25}$

= $\sqrt{26}$ units

∴ 24 = $\sqrt{26}$ x height

height = $\frac{24}{\sqrt{26}}$ units = $\frac{12}{13}$ $\sqrt{26}$ units

31. 

Let the center be 0 (2, – 3y)

As points A and B lie on a circle,

AO = BO

$\sqrt{(2+1)^2 + (-3y-y)^2} = \sqrt{(2-5)^2 + (-3y-7)^2}$

$\sqrt{9+16y^2} = \sqrt{9+9y^2+49+42y}$

On squarraig both sides, we get

$9 + 16y^2 = 9y^2 + 42y + 58$

$7y^2 - 42y - 49 = 0$

$y^2 - 6y - 7 = 0$

$y^2 - 7y + y - 7 = 0$

$y(y-7) + (y-7) = 0$

$(y+1)(y-7) = 0$

$y = -1, 7$

When y = 1

$A = (-1, y) = (-1, -1)$

$O = (2, 3)$

So,

radius = AO

$= \sqrt{(2+1)^2 + (3+1)^2}$

$= \sqrt{9+16}$

$= \sqrt{25}$

= 5 units

When y = 7

$A = (-1, y)$

$= (-1, 7)$

$O = (2, -3y)$

$= (2, -21)$

So,

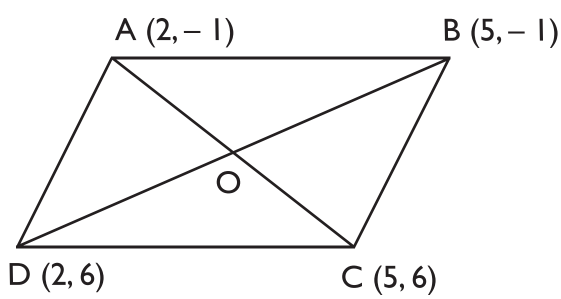
ratio = AO

$= \sqrt{(2+1)^2 + (-2-7)^2}$

$= \sqrt{9+784}$

$= \sqrt{793}$ units

32.



By distance formula,

$AC = \sqrt{(5-2)^2 + (6+1)^2}$

$= \sqrt{9+49} = \sqrt{58}$ units

$BD = \sqrt{(2-5)^2 + (6+1)^2}$

$= \sqrt{9+49}$

$= \sqrt{58}$

So, AC = BD

Also, By Midpoint formula,

Midpoint of AC $= \left(\frac{2+5}{2}, \frac{-1+6}{2}\right)$

$= \left(\frac{7}{2}, \frac{5}{2}\right)$

Midpoint of BD $= \left(\frac{5+2}{2}, \frac{-1+6}{2}\right)$

$= \left(\frac{7}{2}, \frac{5}{2}\right)$

So, Midpoint of AC = Midpoint of BD.

So, AC and BD bisects each other.

WORKSHEET 2

Section A

1. Let P (x, y) be the point equidistant from the point A (5, 1), B (–3, – 7) and C (7, – 1)

$\therefore$ PA = PB = PC

PA = PB

$\Rightarrow \sqrt{(5-x)^2 + (1-y)^2}$

$= \sqrt{(-3-x)^2 + (-7-y)^2}$

$\Rightarrow \sqrt{25 + x^2 - 10x + 1 + y^2 - 2y}$

$= \sqrt{9 + x^2 + 6x + 49 + y^2 + 14y}$

On squarring both sides we get

$x^2 + y^2 - 10x - 2y + 26$

$= x^2 + y^2 + 6x + 14y + 58$

$0 = 16x + 16y + 32$

$x + y = -2$ ①

PB = PC

$\Rightarrow \sqrt{(-3-x)^2 + (-7-y)^2}$

$= \sqrt{(7-x)^2 + (-1-y)^2}$

$\Rightarrow \sqrt{9 + x^2 + 6x + 49 + y^2 + 14y}$

$= \sqrt{49 + x^2 + 14x + 1 + y^2 + 2y}$

On squarring both sides, we get

$x^2 + y^2 + 6x + 14y + 58$

$= x^2 + y^2 - 14x + 2y + 50$

$20x + 12y + 8 = 0$

$5x + 3y = -2$ ②

From ①, we get

$x = -2 - y$

On potting in ②, we get

$5(-2 - y) + 3y = -2$

$-10 - 5y + 3y = -2$

$-2 = 8$

$sy = -4$

So, $x = -2 - y$

$= -2 + 4$

$= 2$

So, point (2, – 4) is equidistant

From point A (5, 1), B (–3, – 7) and C (7, – 1)

2. Reflexion of (–3, 4) in X – axis (Q) = (–3, –4)

Reflexion of (– 3, 4) in Y – axis (R) = (3, 4)

So, by distance formula,

$QR = \sqrt{(3+3)^2 + (4+4)^2}$

$= \sqrt{36+64}$

$= \sqrt{100}$

= 10 units

3. As point (3, a) lies on line

$2x - 3y + 5 = 0$

$\therefore \quad 6 - 3a + 5 = 0$

$3a = 11$

$a = \frac{11}{3}$

4. By Distance formula,

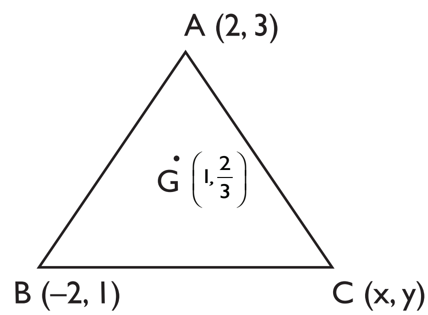
Distance $= \sqrt{(0+6)^2 + (0-8)^2}$

$= \sqrt{(36+64}$

$= \sqrt{100}$

= 10 units

5.



Let $(x_1, y_1) = (2, 3)$

$(x_2, y_2) = (-2, 1)$

$(x_3, y_3) = (x, y)$

Centroid (G) = $\left(1, \frac{2}{3}\right)$

We know that

$$\text{Centroid} = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

$$\left(1, \frac{2}{3}\right) = \left(\frac{2 - 2 + x}{3}, \frac{3 + 1 + y}{3}\right)$$

$$\left(1, \frac{2}{3}\right) = \left(\frac{x}{3}, \frac{4 + y}{3}\right)$$

$\Rightarrow \quad 1 = \frac{x}{3}$ and $\frac{2}{3} = \frac{4+y}{3}$

$\Rightarrow \quad x = 3$ and $y = -2$

6. Let $(x_1, y_1) = (k, 2k)$

$(x_2, y_2) = (3k, 3k)$

$(x_3, y_3) = (3, 1)$

Since the points are collinear, area of triangle is zero

i.e.

$\frac{1}{2}\,[\,x_1 (y_2, - y_3) + x_2 (y_3, y_1) + x_3 (y_1, - y_2)\,] = 0$

$[\,k\,(3k - 1) + 3k\,(1 - 2k) + 3\,(2k - 3k)\,] = 0$

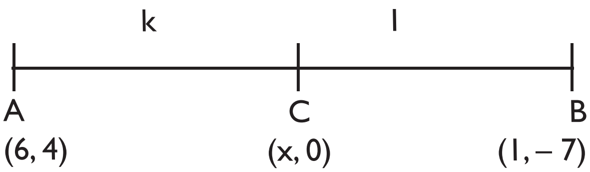
$[\,3k^2 - k + 3k - 6k^2 - 3k\,] = 0$

$[-3k^2 - k] = 0$

$k(3k + 1) = 0$

$k = \dfrac{-1}{3}$

7.



Let the x – axis divides AB in ratio k : 1

Point on x – axis must be of form (x, 0), so, by section formula

$$(x, 0) = \left(\frac{k+6}{k+1}, \frac{7k+4}{k+1}\right)$$

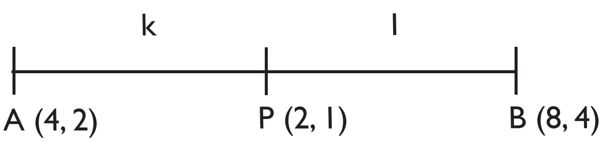
$$(x, 0) = \left(\frac{6+k}{k+1}, \frac{-7k+4}{k+1}\right)$$

$$\therefore \quad \frac{-7k+4}{k+1} = 0$$

$$k = \frac{4}{7}$$

So, x – axis divides line AB in ratio 4 : 7.

8.



Let AP : PB = K : 1

By section formula,

$$P(2, 1) = \left(\frac{8k+4}{k+1}, \frac{4k+2}{k+1}\right)$$

$$\therefore \quad 2 = \frac{8k+4}{k+1}, \quad 1 = \frac{4k+2}{k+1}$$

$8k + 4 = 2k + 2$

$6k = -2$

$$k = \frac{-2}{6} = \frac{-1}{3}$$

$$\therefore \quad \frac{AB}{PB} = \frac{-1}{3}$$

$$\frac{PB}{AP} = -3$$

$$\frac{PB}{AP} + 1 = -3 + 1$$

$$\frac{AP+PB}{AP} = -2$$

$$\frac{AB}{PB} = -2$$

$$\frac{AP}{AB} = \frac{-1}{2}$$

$$AP = \frac{-1}{2} AB$$

9. Consider the two points P(a sin α, –b cos α) and Q (–a cos α, b sin α).

We need to find the distance between P and Q

Let d be the distance PQ.

Thus, by distance formula

$$d = \sqrt{(a \sin\alpha + a\cos\alpha)^2 + (-b\cos\alpha - b\sin\alpha)^2}$$

$$= \sqrt{a^2(\sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha) + b^2(\sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha)}$$

$$= \sqrt{a^2(1 + \sin 2\alpha) + b^2(1 + \sin 2\alpha)}$$

$[\because \sin^2\alpha + \cos^2\alpha = 1 \text{ and } \sin 2\alpha = 2\sin\alpha\cos\alpha]$

$$\therefore \; d = \sqrt{(a^2 + b^2)(1 + \sin 2\alpha)}$$

10. We have to write the condition of three points.

If three points (x_1, y_1), (x_2, y_2) and (x_3, y_3) are collinear, then they will not form a triangle.

In other words, the triangle having vertices (x_1, y_1), (x_2, y_2) and C (x_3, y_3) will have area 0.

The formula to calculate the area of a triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is,

$$A = \frac{1}{2}\begin{vmatrix} x_1 - x_2 & y_1 - y_2 \\ x_2 - x_3 & y_2 - y_3 \end{vmatrix}$$

$$A = \frac{1}{2}|(x_1 - x_2)(y_2 - y_3) - (x_2 - x_3)(y_1 - y_2)$$

Therefore,

$$\frac{1}{2}|\, x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| = 0$$

$$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$$

Therefore, the condition of collinearity of (x_1, y_1), (x_2, y_2) and C (x_3, y_3) is

$$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$$

Section B

11. Let the vertices of triangle be $(x_1, y_1) = (-3, 1)$, $(x_2, y_2) = (0, -2)$ and (x_3, y_3)

Centroid of triangle $(x, y) = (0, 0)$

We Know that

Centroid of triangle

$$(x, y) = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

i.e. $$(0, 0) = \left(\frac{-3 + 0 + x_3}{3}, \frac{1 - 2 + y_3}{3}\right)$$

$$\Rightarrow \quad \frac{-3 + x_3}{3} = 0, \frac{-1 + y_3}{3} = 0$$

$$\Rightarrow \quad x_3 = 3, \quad y_3 = 1$$

So, third vertex is $(x_3, y_3) = (3, 1)$

12. The required ratio would be k : 1.

The coordinates of the point of divisions will be

$$\left(\frac{3k - 4}{k + 1}, \frac{7 + 5}{k + 1}\right)$$

The point which we have identified is on y axis and there the point is zero on x coordinate.

So, $$\frac{3k - 4}{k + 1} = 0$$

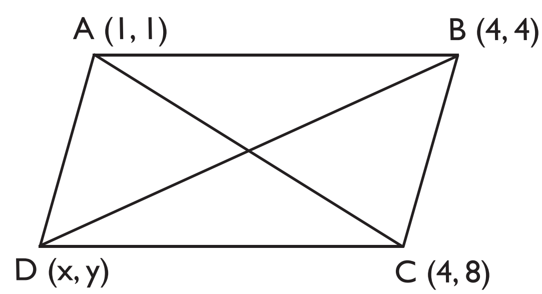
$$3k - 4 = 0$$

$$k = \frac{4}{3}$$

So the required ratio is

$k = \frac{4}{3}$ or 4 : 3

13.



We know that diagonals of parallelogram bisect each other

$$\therefore \quad \left(\frac{1 + 4}{2}, \frac{1 + 8}{2}\right) = \left(\frac{x + 4}{2}, \frac{y + 8}{2}\right)$$

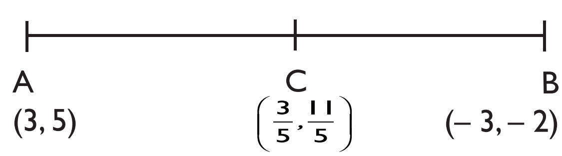
$$(5, 9) = (x + 4, y + 4)$$

$$x + 4 = 5, \quad y + 4 = 9$$

$$x = 1, \quad y = 5$$

So, fourth vertex is (1, 5).

14. Let the point C $\left(\frac{3}{5}, \frac{11}{5}\right)$ divide the line segment joining point A (3, 5) and B (– 3, – 2) in ratio k : 1.



By section formula,

$$\left(\frac{3}{5},\frac{11}{5}\right) = \left(\frac{-3k+3}{k+1},\frac{-2k+5}{k+1}\right)$$

$$\therefore \quad \frac{-3k+3}{k+1} = \frac{3}{5}$$

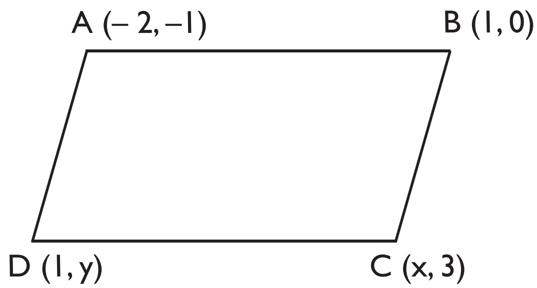
$$5(-3k+3) = 3(k+1)$$

$$-15k + 15 = 3k + 3$$

$$12 = 18k$$

$$k = \frac{2}{3}$$

15.



We know that diagonals of parallelogram bisect each other

$$\therefore \quad \left(\frac{-2k+x}{2},\frac{-1+3}{2}\right) = \left(\frac{1+1}{2},\frac{y+0}{2}\right)$$

$$\Rightarrow \quad \left(\frac{-2+x}{2},1\right) = \left(1,\frac{y}{2}\right)$$

$$\therefore \quad \frac{-2+x}{2} = 1, \; 1 = \frac{y}{2}$$

$$x = 4, \; y = 2$$

16. Given the vertices of a ΔABC are right - angled at A.

$\therefore AB^2 + AC^2 = BC^2$

$AB^2 = (-2 - 0)^2 + (a - 3)^2 = 4 + (a - 3)^2$

$BC^2 = (-1 + 2)^2 + (4 - a)^2 = 1 + (4 - a)^2$

$AC^2 = (-1 - 0)^2 + (4 - 3)^2 = 1 + 1 = 2$

Since $AB^2 + AC^2 = BC^2$

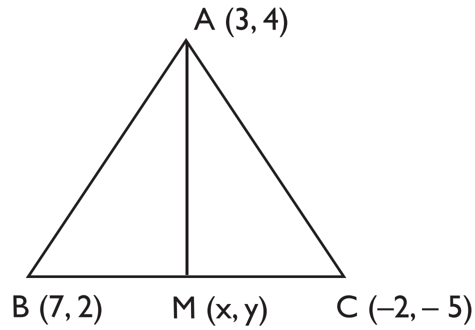
$4 + (a - 3)^2 + 2 = 1 + (4 - a)^2$

$4 + a^2 + 9 - 6a + 2 = 1 + 16 + a2 - 8a$

$2a = 2$

$\therefore a = 1.$

17.



By midpoint formula,

$$M(x, y) = \left(\frac{7-2}{2},\frac{2-5}{2}\right)$$

$$= \left(\frac{5}{2},\frac{-3}{2}\right)$$

By Distance formula,

$$AM = \sqrt{\left(\frac{5}{2}-3\right)^2+\left(\frac{-3}{2}-4\right)^2}$$

$$= \sqrt{\left(\frac{-1}{2}\right)^2+\left(\frac{-3-8}{2}\right)^2}$$

$$= \sqrt{\frac{1}{4}+\frac{121}{4}}$$

$$= \sqrt{\frac{121}{4}} = \sqrt{\frac{61}{2}}$$

18. As point A (x, y) is equidistant from B (6, – 1) and C (2, 3)

$\therefore$ AB = AC

$$\sqrt{(6-x)^2+(-1-y)^2} = \sqrt{(2-x)^2+(3-y)^2}$$

On squarring both sides, we get

$(6 - x)^2 + (- 1 - y)^2 = (2 - x)^2 + (3 - y)^2$

$36 + \underline{x^2} - 12x + 1 + \underline{y^2} + 2y = 4 + \underline{x^2} - 4x + 9 + \underline{y^2} - 6y$

$\therefore -12x + 2y + 37 = - 4x - 6y + 13$

$\Rightarrow \quad 0 = 8x - 8y - 24$

$\Rightarrow \quad 8x - 8y = 24$

$\Rightarrow \quad x - y \quad = 3$

$\Rightarrow \quad x \quad = y + 3$

19. As the points A (2, 1) and B (1, 2) are equidistant from the point C (x, y),

BC = AC

$$\sqrt{(x-1)^2+(y+2)^2} = \sqrt{(x-2)^2+(y-1)^2}$$

On squarring both sides, we get

$(x-1)^2 + (y+2)^2 = (x-2)^2 + (y-1)^2$

$\underline{x}^2 + 1 - 2x + \underline{y}^2 + 4 + 4y = \underline{x}^2 + 4 - 4x + \underline{y}^2 + 1 - 2y$

$-2x + 4y + 5 = -4x - 2y + 5$

$2x + 6y = 0$

$x + 3y = 0$

20. Let the vertices of triangle be

$(x_1, y_1) = (k, 2k)$

$(x_2, y_2) = (3k, 3k)$

$(x_3, y_3) = (3, 1)$

Area of triangle

$= \frac{1}{2}[x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)]$

$= \frac{1}{2}[5 (7 + 4) + 4 (-4 - 2) + 7 (2 - 7)]$

$= \frac{1}{2}[5 (11) + 4 (-6) + 7 (-5)]$

$= \frac{1}{2}[55 - 24 - 35]$

$= \frac{1}{2}[55 - 59]$

$= \frac{4}{2} = 2$ sq. units

Section C

21. PA=PB

Take square both side

$PA^2=PB^2$

Now use distance formula,

$\{x - (a + b)\}^2 + \{y - (b - a)\}^2 = \{x - (a - b)\}^2 + \{y - (a + b)\}^2$

$\Rightarrow \quad x^2 + (a + b)^2 - 2x(a + b) + y^2 + (b - a)^2 - 2y(b - a)y = x^2 + (a - b)^2 - 2x(a - b) + y^2 + (a + b)^2 - 2y(a + b)$

$\Rightarrow \quad 2x(a - b) - 2x(a + b) = 2y(b - a) - 2y(a + b)$

$\Rightarrow \quad 2x\{a - b - a - b\} = 2y\{b - a - a - b\}$

$\Rightarrow \quad 2x(-2b) = 2y(-2a)$

$\Rightarrow \quad bx = ay$

Hence proved.

22. Any point on the x -axis will be the form A(x,0).Let this point divides the line segment Joining (3, –2) and (–7, –1) in the ratio m:n internally.

Thus the coordinate of A is

$$\left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}\right)$$

where (3, –2) and (–7, –1) are (x_1, y_1) and (x_2, y_2) respectively.

Thus the coordinates of

$$A = \frac{(m(-7) + n(3))}{m + n}, \frac{m(-1) + n(-2)}{m + n} = (x,0)$$

Here the y coordinates of A is Zero.

$$\text{Thus}\left(\frac{-m + (-2n)}{m + n}\right) = 0.$$

Hence $-m - 2n = 0$.

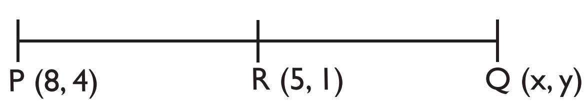
$\Rightarrow -m = 2n$

$\Rightarrow \frac{m}{n} = \frac{2}{-1}$.

$\Rightarrow m : n = 2 : -1$ internally

Thus m : n = 2 : 1 externally.

23.



By mid-point formula,

$$(5, 1) = \left(\frac{8 + x}{2}, \frac{4 + y}{2}\right)$$

$$5 = \frac{8+x}{2}, 1 = \frac{4+y}{2}$$

$x + 8 = 10,\ y + 4 = 2$

$x = 2,\ y = -2$

So, Coordinates of $Q = (x, y)$

$= (2, -2)$

24. Let points be

$(x_1, y_1) = (c, a+b)$

$(x_2, y_2) = (b, b+c)$

$(x_3, y_3) = (a, a+c)$

Area of triangle

$= \frac{1}{2}[x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)]$

$= \frac{1}{2}[c (b+c-a-c) + a (a+c-a-b) + b (a+b-b-c)]$

$= \frac{1}{2}[c (b-a) + a (c-6) + b (a-c)]$

$= \frac{1}{2}[bc - ac + ac - ab + ab - bc]$

$= 0$

As area of triangle = 0

So, points A, B and C are collinear.

25. Let the point be

$(x_1, y_1) = (a, 0)$

$(x_2, y_2) = (0, b)$

$(x_3, y_3) = (1, 1)$

Points are collinear, if area of triangle = 0

i.e. $\frac{1}{2}[x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)] = 0$

$\Rightarrow \frac{1}{2}[a (b-1) + 0 (1+0) + 1 (0-b)] = 0$

$\Rightarrow \frac{1}{2}[ab - a - b] = 0$

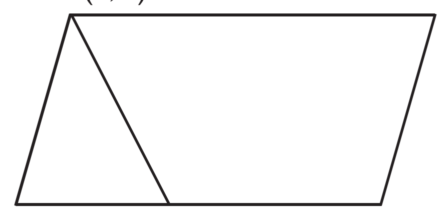
$\Rightarrow ab - a - b = 0$

$\Rightarrow ab = a + b$

$\Rightarrow 1 = \frac{a}{ab} + \frac{b}{ab}$

$\Rightarrow 1 = \frac{1}{a} + \frac{1}{b}$

26.



We know that diagonals of parallelogram bisect each other.

$\therefore$ Midpoint of AC = midpoint of BD

So, by midpoint formula,

$$\left(\frac{6+9}{2}, \frac{1+4}{2}\right) = \left(\frac{x+8}{2}, \frac{y+2}{2}\right)$$

i.e. $\left(\frac{15}{2}, \frac{5}{2}\right) = \left(\frac{x+8}{2}, \frac{y+2}{2}\right)$

i.e. $x + 8 = 15,\ y + 2 = 5$

$x = 7,\ y = 3$

So, point $D = (7, 3)$

Again, by midpoint formula,

$E (u, v) = \left(\frac{6+9}{2}, \frac{1+4}{2}\right)$

$= \left(\frac{7+9}{2}, \frac{3+4}{2}\right) = \left(\frac{16}{2}, \frac{7}{2}\right)$

$= \left(8, \frac{7}{2}\right)$

For area of ΔADE

Let $(x_1, y_1) = (c, a+b)$

$(x_2, y_2) = (b, b+c)$

$(x_3, y_3) = (a, a+c)$

area of ΔADE

$= \frac{1}{2}[x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)]$

$$= \frac{1}{2}\left[6\left(3-\frac{7}{2}\right)+7\left(\frac{7}{2}-1\right)+8(1-3)\right]$$

$$= \frac{1}{2}\left[6\left(\frac{-1}{2}\right)+7\left(\frac{5}{2}\right)+8(-2)\right]$$

$$= \frac{1}{2}\left[-3+\frac{35}{2}-16\right]$$

$$= \frac{1}{2}\left[\frac{35}{2}-19\right]$$

$$= \frac{1}{2}\left[\frac{35-38}{2}\right]$$

$$= \frac{3}{4}$$

27. Given ,

Points = (p + 1, 2p – 2), (p – 1, p) and (p – 3, 2p – 6)

For the given points (x_1, y_1), (x_2, y_2) and (x_3, y_3) to be collinear then

$[x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_3)] = 0$

Here,

$x_1 = p + 1$ $\quad$ $y_1 = 2p - 2$

$x_2 = p - 1$ $\quad$ $y_2 = p$

$x_3 = p - 3$ $\quad$ $y_3 = 2p - 6$

Substituting the values in the formula ,

(p + 1) (p – (2p – 6)) + (p – 1) (2p – 6 – (2p – 2)) + (p – 3) (2p – 2 – (p)) = 0

(p + 1) (p – 2p + 6)) + (p – 1) (2p – 6 – 2p + 2)) + (p – 3) (2p – 2 – p) = 0

(p + 1) (–p + 6) + (p – 1) (–4) + (p – 3) (p – 2) = 0

$-p^2 - p + 6p + 6 - 4p + 4 + p^2 - 3p - 2p + 6 = 0$

– 4p + 16 = 0

4p = 16

Dividing both the sides by 4

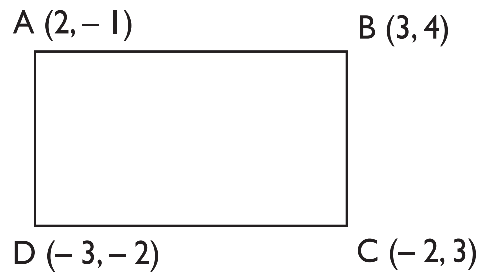
$$\frac{4p}{4} = \frac{16}{4}$$

p = 4

Hence,

For the points to be collinear, p = 4

28.



By Distance formula,

$$AB = \sqrt{(3-2)^2+(4+1)^2}$$
$$= \sqrt{1+25} = \sqrt{26}$$
$$BC = \sqrt{(-2-3)^2+(3-4)^2}$$
$$= \sqrt{25+1} = \sqrt{26}$$
$$CD = \sqrt{(-3+2)^2+(-2-3)^2}$$
$$= \sqrt{1+25}$$
$$= \sqrt{26}$$
$$AD = \sqrt{(-3-2)^2+(-2+1)^2}$$
$$= \sqrt{26}$$

As AB = BC = CD = AD,

ABCD is a rhombus

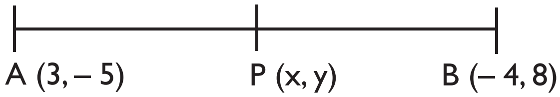
Again, by distance formula,

$$AC = \sqrt{(-2-2)^2+(3+1)^2}$$
$$= \sqrt{16+16}$$
$$= \sqrt{32}$$
$$= 4\sqrt{2} \text{ units}$$
$$BD = \sqrt{(-3-3)^2+(-2-4)^2}$$
$$= \sqrt{(-6)^2+(-6)^2}$$
$$= \sqrt{72}$$
$$= 6\sqrt{2} \text{ units}$$

$\therefore$ AC ≠ BD

As diagonals are not equal, ABCD is a rhombus but not a square.

29.



$\frac{AP}{PB} = \frac{k}{1}$

Let point P be (x, y).

By section formula,

$(x, y) \quad = \left(\frac{-4k+3}{k+1}, \frac{8k-5}{k+1}\right)$

$(x, + y) = \left(\frac{-4k+3}{k+1}, \frac{8k-5}{k+1}\right)$

$\therefore \quad x = \frac{-4k+3}{k+1}, \quad y = \frac{8k-5}{k+1}$

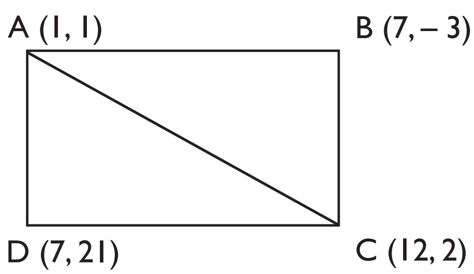
As point P lies on line x + y = 0

$\therefore \quad \left(\frac{-4k+3}{k+1}\right) + \left(\frac{8k-5}{k+1}\right) = 0$

$\Rightarrow \quad 4k - 2 = 0$

$\Rightarrow \quad k = \frac{1}{2}$

30.



Area of ΔABC

$= \frac{1}{2}$ [1 (– 3 – 2) + 7 (2 – 1) + 12 (1 + 3)]

$= \frac{1}{2}$ [– 5 + 7 + 48]

$= \frac{1}{2}$ [50]

= 25 sq. units

Area of ACD

$= \frac{1}{2}$ [1 (2 – 21) + 12 (21 – 1) + 7 (1 – 2)]

$= \frac{1}{2}$ [– 19 + 12 (20) – 7]

$= \frac{1}{2}$ [– 26 + 240]

$= \frac{1}{2}$ [214]

= 107 sq. units

So, area of quadrilateral ABCD

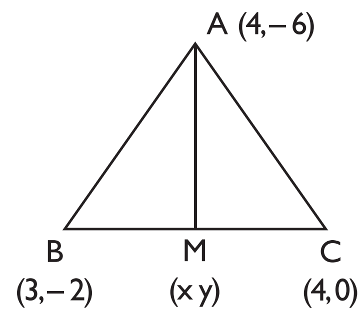
= area of ΔABC + area of ΔACD

= 25 + 107

= 132 sq. units

Section D

31.



Let AM be the median such that point M is (x, y)

$(x, y) \quad = \left(\frac{3+5}{3}, \frac{-2+2}{3}\right)$

(x, y) = (4, 0)

So, point M (x, y) = (4, 0)

Area of ΔAMB

$= \frac{1}{2}$ [4 (y + 2)+ x (– 2 + 6) + 3 (– 6 – y)]

$= \frac{1}{2}$ [4y + 8 – 2x + 6 – 18 – 3y]

$= \frac{1}{2}$ [4x + y – 10]

$= \frac{1}{2}$ [4 (4) + 0 – 10]

= 3 sq. units

Area of ΔAMC

$= \frac{1}{2}$ [4 (y – 2)+ x (2 + 6) + 5 (– 6 – y)]

$= \frac{1}{2}$ [4y – 8 + 8x – 30 – 5y]

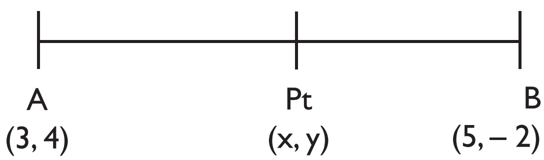
$= \frac{1}{2}$ [8x – y – 38]

$= \frac{1}{2}$ [8 (4) – 0 – 38]

= 3 sq. units

So, median divides the triangle into two triangle of equal area.

32.



PA = PB

$$\sqrt{(-3-x)^2+(4-y)^2} = \sqrt{(5-x)^2+(-2-y)^2}$$

On squarring both sides, we get

$(3 – x)^2 + (4 – y)^2 = (5 – x)^2 + (– 2 – y)^2$

$\Rightarrow$ 9 + $\underline{x}^2$ – 6x + 16 + $\underline{y}^2$ – 8y = 25 + $\underline{x}^2$ – 10 + 4 + $\underline{y}^2$ + 4y

$\Rightarrow$ – 6x – 8y + 25 = – 10x + 4y + 29

$\Rightarrow$ 4x – 12y – 4 = 0

$\Rightarrow$ x – 3y = 1 ①

Also, area of ΔPAB = 10

$\therefore$ $\frac{1}{2}$ [x (4 + 2) + 3 (– 2 – y) + 5 (y – 4)] = 10

$\Rightarrow$ [6x – 6 – 3y + 5y – 20] = 20

$\Rightarrow$ [6x + 2y – 26] = 20

$\Rightarrow$ [3x + y – 13] = 10

$\Rightarrow$ 3x + y – 13 = ± 10

$\Rightarrow$ 3x + y = 23 ② or 3x + y = 3 ③

From ①, x = 1 + 3y

So, eq. ② becomes 3 + 9y + y = 23

10y = 20

y = 2

So, x = 1 + 3y

= 1 + 6

= 7

So, P (x, y) = (7, 2)

On putting x = 1 + 3y in ③, we get

3 (1 + 3y) + y = 3

3 + 9y + y = 3

10y = 0

y = 0

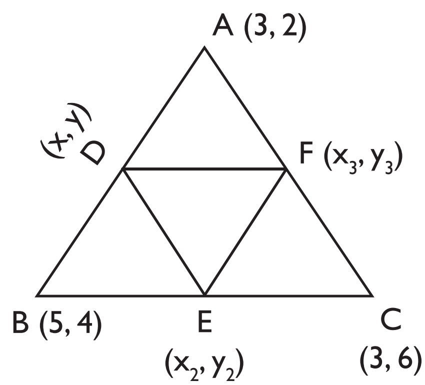
So, x = 1 + 3y

= 1 + 0

= 1

So, P (x, y) = (1, 0)

33.



By midpoint formula,

D (x_2, y_2) $= \left(\frac{3+5}{2}, \frac{2+4}{2}\right)$

D (x_2, y_2) = ($\underline{4, 3}$)

Again, E (x_2, y_2) $= \left(\frac{5+3}{2}, \frac{4+6}{2}\right)$

= ($\underline{4, 5}$)

F (x_3, y_3) $= \left(\frac{3+3}{2}, \frac{2+6}{2}\right)$

= ($\underline{3, 4}$)

Area of ΔDEF

$= \frac{1}{2}$ [x_1 $(y_2, – y_3)$ + x_2 $(y_3, – y_1)$ + x_3 $(y_1, – y_2)$]

$= \frac{1}{2}\ [\, 4\,(5-4) + 4\,(4-3) + 3\,(3-5)\,]$

$= \frac{1}{2}\ [\, 4 + 4 - 6\,] = 1$ sq. unit

34. Let $A(x_1 = -2, y_1 = 5)$, $B(x_2 = k, y_2 = -4)$ and $C(x_3 = 2k+1, y_3 = 10)$ be the vertices of the triangle, so

Area of (ΔABC)

$= \frac{1}{2}\ [x_1\,(y_2 - y_3) + x_2\,(y_3 - y_1) + x_3\,(y_1 - y_2)]$

$53 = \frac{1}{2}\ [\,(-2)\,(-4-10) + k\,(10-5) + (2k+1)\,(5+4)]$

$\Rightarrow \quad 53 = \frac{1}{2}\ [\,28 + 5k + 9\,(2k+1)]$

$\Rightarrow \quad 28 + 5k + 18k + 9 = 106$

$\Rightarrow \quad 37 + 23k = 106$

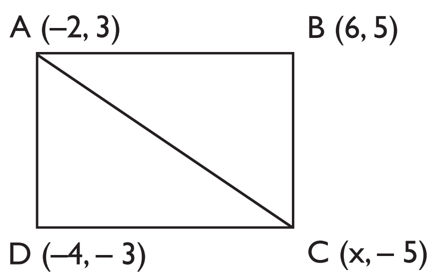
$\Rightarrow \quad 23k = 106 - 37 = 69$

$\Rightarrow \quad k = \frac{69}{23} = 3$

Hence, k = 3

The value of k is 3.

35.



Area of quadrilateral ABCD = 80 sq. units

i.e. area of ΔABC + area of ΔACD = 80

i.e. $\frac{1}{2}\ [\,-2\,(5+5) + 6\,(-5-3)\,]\ +$

$\frac{1}{2}\ [\,-2\,(-5+3) + x\,(-3-3)\,] = 80$

$\Rightarrow \frac{1}{2}\ [\,4 - 6x - 32\,] = 160$

$\Rightarrow \ [\,-6x - 28\,] = 160$

$\Rightarrow \ -6x - 28 = \pm 160$

$-6x - 28 = 160$

$6x = -188$

$x = \frac{-94}{3}$

$-6x - 28 = -160$

$-6x = -132$

$x = 22$

36. Let D (x, y) be the Circumcentre.

We know that Circumcentre of a triangle is equidistant from each of the vertices.

Let the vertices be $A\,(x_1, y_1) = (8, 6)$, $B\,(x_2, y_2) = (8, -2)$ and $C\,(x_3, y_3) = (2, -2)$.

So, AD = BD

$\sqrt{(8-x)^2 + (6-y)^2} = \sqrt{(8-x)^2 + (-2-y)^2}$

On squarring both sides, we get

$(8-x)^2 + (6-y)^2 = (8-x)^2 + (-2-y)^2$

$(6-y)^2 = (-2-y)^2$

$36 + y^2 - 12y = 4 + y^2 + 4y$

$32 = 16y$

$y = 2$

Also, BD = CD

$\sqrt{(8-x)^2 + (-2-y)^2} = \sqrt{(2-x)^2 + (-2-y)^2}$

$(8-x)^2 + (-2-y)^2 = (2-x)^2 + (-2-y)^2$

$64 + \underline{x^2} - 16x + 4\,\underline{y^2} + 4y = 4 + \underline{x^2} - 4x + 4 + \underline{y^2} + 4y$

$\Rightarrow -16x + 4y + 68 = -4x + 4y + 8$

$\Rightarrow 12x = 60$

$\Rightarrow \quad x = 5$

So, Circumcentre is (x, y) = (5, 2)

Circumradius

= AD

$= \sqrt{(8-x)^2 + (6-y)^2}$

$= \sqrt{(8-5)^2 + (6-2)^2}$

$= \sqrt{9+16}$

$= \sqrt{25}$

$= 5$

37. By ,midpoint formula,

$$C\ (x, y) = \left(\frac{O+2a}{3}, \frac{26+O}{3}\right)$$

$$= (a, b)$$

Using distance formula, we have

$$BC = \sqrt{(a-o)^2+(b-2b)^2}$$

$$= \sqrt{a^2+b^2}$$

$$OC = \sqrt{(a-o)^2+(b-o)^2}$$

$$= \sqrt{a^2+b^2}$$

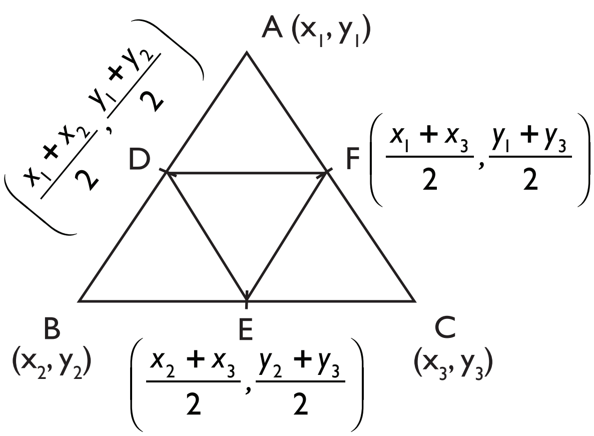
$$AC = \sqrt{(a-2a)^2+(b-o)^2}$$

$$= \sqrt{a^2+b^2}$$

So, BC = CO = AC

∴ Point C is equidistant from the vertices O, and B.

38.



By midpoint formula,

D is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

E is $\left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right)$

F is $\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$

Area of ΔABC

$$\frac{1}{2}\ [\, x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)\,]$$

Consider ar ΔDEF

$$= \frac{1}{2}\left[\begin{array}{l}\left(\frac{x_1+x_2}{2}\right)\left[\left(\frac{y_2+y_3}{2}\right)-\left(\frac{y_1+y_3}{2}\right)\right]\\ +\left(\frac{x_2+x_3}{2}\right)\left[\left(\frac{y_1+y_3}{2}\right)-\left(\frac{y_1+y_3}{2}\right)\right]\\ +\left(\frac{x_1+x_3}{2}\right)\left[\left(\frac{y_1+y_2}{2}\right)-\left(\frac{y_2+y_3}{2}\right)\right]\end{array}\right]$$

$$= \frac{1}{8}\left[\begin{array}{l}(x_1+x_2)(y_2-y_1)\\ +(x_2+x_3)(y_3-y_2)\\ +(x_1+x_3)(y_1-y_3)\end{array}\right]$$

$$= \frac{1}{8}\left[\begin{array}{l}x_1[(y_2-y_1)+(y_1-y_3)]\\ +x_2[(y_2-y_1)+(y_3-y_2)]\\ +x_3[(y_3-y_2)+(y_1-y_3)]\end{array}\right]$$

$$= \frac{1}{8}\left[\begin{array}{l}x_1(y_2-y_3)+x_2(y_3-y_1)\\ +x_3(y_1-y_2)\end{array}\right]$$

$$= \left(\frac{1}{4}\right)\left(\frac{1}{2}\right)[x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)]$$

$$= \frac{1}{4} \text{ area of } \Delta ABC$$

39. Using formula for area of triangle,

ar ΔDBC = $\frac{1}{2}$ [x (5 + 2) – 3 (– 2 – 3x) + 4 (3x – 5)]

$$= \frac{1}{2}\ [\, 7x + 6 + 9x + 12x - 20\,]$$

$$= \frac{1}{2}\ [\, 28x - 14\,]$$

$= [14x - 7]$ ①

Using formula for area of triangle,

$$= \frac{1}{2}[6(5+2) - 3(-2-3) + 4(3-5)]$$

$$= \frac{1}{2}[42 + 15 - 8]$$

$$= \frac{1}{2}[49] \text{ sq. units}$$

As $\frac{ar\Delta DBC}{ar\Delta ABC} = \frac{1}{2}$

$$\Rightarrow \frac{|14x-7|}{\frac{|49|}{2}} = \frac{1}{2}$$

$$\Rightarrow \frac{2|14x-7|}{49} = \frac{1}{2}$$

$$\Rightarrow |14x-7| = \frac{49}{4}$$

$$\Rightarrow 14x - 7 = \pm\frac{49}{4}$$

If $14x - 7 = \frac{49}{4}$

$$14x = \frac{49}{4} + 7 = \frac{49+28}{4} = \frac{77}{4}$$

$$\Rightarrow x = \frac{11}{8}$$

If $14x - 7 = \frac{-49}{4}$

$$14x = \frac{-49}{4} + 7 = \frac{-49+28}{4} = \frac{-21}{4}$$

$$\Rightarrow x = \frac{-3}{8}$$

40. As the point (x_1, y_1), (x_2, y_2) and (x_3, y_3) lie on the same line, area of triangle formed by these points is 0.

i.e. $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

$[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

On dividing by $x_1 x_2 x_3$, we get

$$\left[\frac{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)}{x_1 x_2 x_3}\right] = 0$$

$$\left[\frac{y_2 - y_3}{x_2 x_3} + \frac{y_3 - y_1}{x_1 x_3} + \frac{y_1 - y_2}{x_1 x_2}\right] = 0$$

$$\therefore \frac{y_2 - y_3}{x_2 x_3} + \frac{y_3 - y_1}{x_1 x_3} + \frac{y_1 - y_2}{x_1 x_2} = 0$$

Chapter 08

Introduction to Trigonometry

MULTIPLE CHOICE QUESTIONS

1. $\text{Cot } x = \frac{12}{16} = \frac{3}{4}$

$$\frac{\sin x - \cos x}{\sin x + \cos x} = \frac{1 - \cot x}{1 + \cot x}$$

$$= \frac{1 - \frac{3}{4}}{1 + \frac{3}{4}}$$

$$= \frac{1}{4} \times \frac{4}{7}$$

$$= \frac{1}{7}$$

Option (a)

2. $$\frac{x(2)^2(\sqrt{2})^2}{8\left(\frac{1}{\sqrt{2}}\right)^2\left(\frac{\sqrt{3}}{2}\right)^2} = (\sqrt{3})^2 - \left(\frac{1}{\sqrt{3}}\right)^2$$

$$\frac{8x}{3} = 3 - \frac{1}{3} = \frac{8}{3}$$

$$x = 1$$

Option (a)

3. $A + B + C = 180°$

$\Rightarrow \quad B + C = 180° - A$

$$\therefore \quad = \sin\left(\frac{B+C}{2}\right) = \sin\left(\frac{180° - A}{2}\right)$$

$$= \sin\left(90° - \frac{A}{2}\right)$$

$$= \cos\frac{A}{2}$$

Option (b)

4. $$\frac{\tan 30°}{\tan 0° - \cot 30°}$$

$$= \frac{\frac{1}{\sqrt{3}}}{0 - \sqrt{3}}$$

$$= \frac{-1}{3}$$

Option (b)

5. Consider

$(a \sin\theta + b \cos\theta)^2$

$= a^2 \sin^2\theta + b^2 \cos^2\theta + 2ab \sin\theta \cos\theta$

$= a^2 (1 - \cos^2\theta) + b^2 (1 - \sin^2\theta) + 2ab \sin\theta \cos\theta$

$= a^2 + b^2 - a^2 \cos^2\theta - b^2\sin^2\theta + 2ab \sin\theta \cos\theta$

$= a^2 + b^2 - (a^2 \cos^2\theta + b^2 \sin^2\theta) + 2ab \sin\theta \cos\theta$ (i)

Also, $a \cos\theta - b \sin\theta = c$

$\Rightarrow \quad (a \cos\theta - b \sin\theta)^2 = c^2$

$\Rightarrow \quad a^2 \cos^2\theta + b^2 \sin^2\theta - 2ab \sin\theta \cos\theta = c^2$

$\Rightarrow \quad a^2 \cos^2\theta + b^2 \sin^2\theta = c^2 + 2ab \sin\theta \cos\theta$ (ii)

So, $(a \sin\theta + b \cos\theta)^2$

$= a^2 + b^2 + 2ab \sin\theta \cos\theta - c^2 - 2ab \sin\theta \cos\theta$

[From (i) and (ii)]

$= a^2 + b^2 - c^2$

$\therefore$ a sin + b cos $= \pm \sqrt{a^2 + b^2 - c^2}$

Option (b)

WORKSHEET 1

Section A

1. $$\frac{\cos(90° - \theta)\sec(90° - \theta)\tan\theta}{\cos(90° - \theta)\sin(90° - \theta)\cot(90° - \theta)} + \frac{\tan(90° - \theta)}{\cot\theta}$$

$$= \frac{\sin\theta \operatorname{cosec}\theta \tan\theta}{\sec\theta \cos\theta \tan\theta}$$

$$= \frac{\cot\theta}{\cot\theta}$$

$$= \frac{1}{1} + \frac{1}{1}$$

$$= 2$$

2. Consider

$$\frac{\tan A \tan B + \tan A \cot B}{\sin A \sec B} - \frac{\sin^2 B}{\cos^2 A}$$

$$= \frac{\tan(90° - B)\tan B + \tan A \cot(90° - A)}{\sin A \sec(90° - A)} - \frac{\sin^2 B}{\cos^2(90° - B)}$$

$$= \frac{\cot B \tan B + \tan^2 A}{\sin A \operatorname{cosec} A} - \frac{\sin^2 B}{\sin^2 B}$$

$$= \frac{1 + \tan^2 A}{1} - 1$$

$$= \tan^2 A$$

3. $$\sqrt{\frac{1 + \sin\theta}{1 - \sin\theta}}$$

$$= \sqrt{\frac{1 + \sin\theta}{1 - \sin\theta} \times \frac{1 + \sin\theta}{1 - \sin\theta}}$$

$$= \sqrt{\frac{(1 + \sin\theta)^2}{1 - \sin^2\theta}}$$

$$= \sqrt{\frac{(1 + \sin\theta)^2}{\cos^2\theta}}$$

$$= \frac{1 + \sin\theta}{\cos\theta}$$

4. $\cos\ (a + b) = 0$

$a + b = 90°$

$\therefore \quad a = 90° - 6$

Consider $\sin\ (a - b) = \sin\ [90° - 26]$

$= \cos 26°$

5. $$\frac{\tan^2 - \sec^2\theta}{\cot^2 - \operatorname{cosec}^2\theta}$$

$$= \frac{\sec^2\theta - \tan^2\theta}{\operatorname{cosec}^2\theta - \cot^2\theta}$$

$$= \frac{1}{1}$$

$$= 1$$

6. $\operatorname{cosec}\theta = 3x \Rightarrow \quad x = \frac{1}{3}\operatorname{cosec}\theta$

$\cot\theta = \frac{3}{x} \quad \Rightarrow \frac{1}{x} = \frac{1}{3}\cot\theta$

consider$x^2 - \frac{1}{x^2} = \frac{1}{9} \quad \operatorname{cosec}^2\theta - \frac{1}{9}\cot^2\theta = \frac{1}{9}$

7. $\tan A = \frac{5}{12}$

Consider $(\sin A + \cos A)\ \text{Sec}\ A$

$= (\sin A + \cos A)\ \frac{1}{\cos A}$

$= \tan A + 1$

$= \frac{5}{12} + 1$

$= \frac{17}{12}$

8. Consider $6\tan^2\theta - \frac{6}{\cos^2\theta}$

$= 6\ (\tan^2\theta - \sec^2\theta)$

$= 6 \quad (1)$

$= 6$

Section B

9. $2\sin^2 30° - 3\cos^2 45° + \tan^2 60°$

$= 2\left(\frac{1}{2}\right)^2 - 3\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\sqrt{3}\right)^2$

$= \frac{1}{2} - \frac{3}{2} + 3$

$= 2$

10. (i) We know that $-1 \le \sin\theta \le 1$

$\therefore \quad 0 \le \sin^2\theta \le 1$

If $\sin\theta = x + \frac{1}{x}$,

On squaring both sides, we get

$\sin^2\theta = x^2 + \frac{1}{x^2} + 2$

Here, R H S $= x^2 + \frac{1}{x^2} + 2 > 2$

but Maximum value of $\sin^2\theta$ is

$\therefore \quad \sin^2\theta$ is $\ne x + \frac{1}{x}$

(ii) As $(a - b)^2 \ge 0$

$\Rightarrow \quad a^2 + b^2 - 2ab \ge 0$

$\Rightarrow \quad a^2 + b^2 \ge 2ab$

$\therefore \quad \cos\theta = \frac{a^2 + b^2}{2ab} \ge \frac{2ab}{2ab} = 1$

$\Rightarrow \quad \cos\theta \ge 1$

if $\cos = 1$

$\frac{a^2 + b^2}{2ab} = 1$

$a^2 + b^2 = 2ab$

$(a - b)^2 = 0$

$a = b$

but a and b are distvied

$\therefore \quad \cos\theta > 1$

but $-1 \le \cos\theta \le 1$

So, $\cos\theta \ne \frac{a^2 + b^2}{2ab}$

11. (i) $2\sin 3x = \sqrt{3}$

$\sin 3x = \frac{\sqrt{3}}{2}$

$3x = 60°$

$x = 20°$

(ii) $2\sin\frac{x}{2} = 1$

$\sin\frac{x}{2} = \frac{1}{2} = \sin 30°$

$\frac{x}{2} = 30°$

$x = 60°$

12. $\text{Sin}\theta + \sin^2\theta = 1$

$\Rightarrow \quad \sin\theta = 1 - \sin^2\theta$

$\Rightarrow \quad \sin\theta = \cos^2\theta \qquad$ (i)

$\Rightarrow \quad \tan\theta = \cos\theta$

Consider $\cos^2\theta + \cos^4\theta$

$= \tan^2\theta + \tan 4\theta$

$= \tan^2\theta\,(1 + \tan^2\theta)$

$= \tan^2\theta \sec^2\theta$

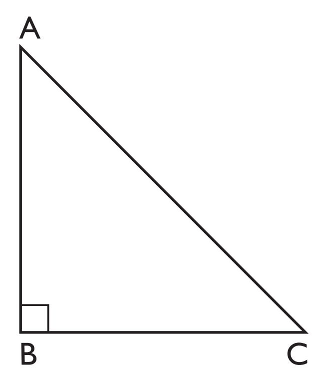
$= \tan^2\theta \frac{1}{\sin\theta} \qquad$ By (i)

$= \frac{\sin\theta}{\cos^2\theta}$

$= \frac{\sin\theta}{\sin\theta} \qquad$ By (i)

$= 1$

13.



$\tan A = \frac{1}{\sqrt{3}}$

$$\frac{BC}{AB} = \frac{1}{\sqrt{3}}$$

Let BC $= k$, AB $= \frac{A + B = 90°}{A – B = 30°}\ k$

$\therefore$ $AC^2 = BC^2 + AB^2 = 4k^2$

$AC = 2k$

Consider

sinA cosC + cosA sinC

$$= \left(\frac{BC}{AC}\right)\left(\frac{BC}{AC}\right) + \left(\frac{AB}{AC}\right)\left(\frac{AB}{AC}\right)$$

$$= \frac{BC^2}{AC^2} + \frac{AB^2}{AC^2}$$

$$= \frac{BC^2 + AB^2}{AC^2}$$

$$= \frac{AC^2}{AC^2}$$

$= 1$

14. Consider

$4 \cot^2 45° – \sec^2 60° + \sin^2 60° – \cos^2 90°$

$$= 4(1)^2 – (2)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 – (0)^2$$

$$= 4 – 4 + \frac{3}{4}$$

$$= \frac{3}{4}$$

15. Consider

$$(\text{cosec}\theta – \cot\theta)^2 = \left(\frac{1}{\sin\theta} – \frac{\cos\theta}{\sin\theta}\right)^2$$

$$= \left(\frac{1 – \cos\theta}{\sin\theta}\right)^2$$

$$= \frac{(1 – \cos\theta)^2}{\sin\theta}$$

$$= \frac{(1 – \cos\theta)^2}{1 – \cos^2\theta}$$

$$= \frac{1 – \cos\theta}{1 + \cos\theta}$$

16. Consider

$3 \cos^2 30° + \sec^2 30° + 2 \cos^2 0° + 3 \sin^2 90° – \tan^2 60°$

$$= 3\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{2}{\sqrt{3}}\right)^2 + 2\,(1)^2 + 3\,(1)^2 – \left(\sqrt{3}\right)^2$$

$$= \frac{9}{4} + \frac{4}{3} + 2 + 3 – 3$$

$$= \frac{9}{4} + \frac{4}{3} + 2$$

$$= \frac{27 + 16 + 24}{12}$$

$$= \frac{67}{12}$$

Section C

17. $\tan\theta + \cot\theta = 2$

$$\tan\theta + \frac{1}{\tan\theta} = 2$$

$\tan^2\theta – 2\tan\theta + 1 = 0$

$(\tan\theta – 1)^2 = 0$

$\tan\theta – 1 = 0$

$\tan\theta = 1 = \tan 45°$

$\theta = 45°$

Consider $\tan^7\theta + \cot^7\theta$

$= \tan^7(45°) + \cot^7(45°)$

$= 1^7 + 1^7$

$= 1 + 1$

$= 2$

18. Consider

$$\frac{\sin\theta}{1 + \cos\theta} + \frac{1 + \cos\theta}{\sin\theta}$$

$$= \frac{\sin\theta\,(\sin\theta) + (1 + \cos\theta)\,(1 + \cos\theta)}{\sin\theta\,(1 + \cos\theta)}$$

$$= \frac{\sin^2\theta + 1 + \cos^2\theta + 2\cos\theta}{\sin\theta\,(1 + \cos\theta)}$$

$$= \frac{2 + 2\cos\theta}{\sin\theta\,(1 + \cos\theta)}$$

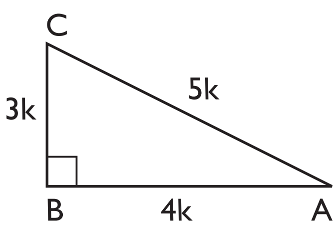
$$= \frac{2 + (1 + \cos\theta)}{\sin\theta\,(1 + \cos\theta)}$$

$$= \frac{2}{\sin\theta} = 2\ \text{cosec}\ \theta$$

19. $\text{Sec } A = \frac{5}{4} = \frac{AC}{AB}$

L H S

$$\frac{3\sin A - 4\sin^3 A}{4\cos^3 A - 3\cos A}$$



$BC^2 = AC^2 - AB^2$

$= 25k^2 - 16k^2$

$= 9k^2$

$\therefore\ BC = 3k$

So, $\frac{3\sin A - 4\sin^2 A}{4\cos^3 A - 3\cos A}$

$$= \frac{3\left(\frac{3}{5}\right) - 4\left(\frac{3}{5}\right)^3}{4\left(\frac{4}{5}\right)^3 - 3\left(\frac{4}{5}\right)}$$

$$= \frac{\frac{9}{5} - \frac{108}{125}}{\frac{256}{125} - \frac{12}{5}}$$

$$= \frac{\frac{9}{5} - \frac{108}{125}}{\frac{256}{125} - \frac{12}{5}}$$

$$= \frac{\frac{225 - 108}{125}}{\frac{256 - 300}{125}}$$

$$= \frac{117}{-44}$$

R H S

$$\frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$$

$$= \frac{3\left(\frac{3}{4}\right) - \left(\frac{3}{4}\right)^3}{1 - 3\left(\frac{3}{4}\right)^2}$$

$$= \frac{\frac{9}{4} - \frac{27}{64}}{1 - \frac{27}{16}}$$

$$= \frac{\frac{144 - 27}{64}}{\frac{16 - 24}{16}}$$

$$= \frac{117}{-44}$$

$$= \frac{-117}{44}$$

So, L H S = R H S

20. $a\cos\theta + b\sin\theta = m$

$a\sin\theta - b\cos\theta = n$

To prove : $a^2 + b^2 = m^2 + n^2$

Proof $a\cos\theta + b\sin\theta = m$

On squarring both sides, we get

$a^2\cos^2\theta + b^2\sin^2\theta + 2ab\sin\theta\cos\theta = m^2$ (1)

$a\sin\theta - b\cos\theta = n$

On squarring both sides, we get

$a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta = n^2$ (2)

On adding (1) and (2), we get

$a^2(\sin^2\theta + \cos^2\theta) + b^2(\sin^2\theta + \cos^2\theta) = m^2 + n^2$

$\Rightarrow\ a^2 + b^2 = m^2 + n^2$

21. $x = a\cos^3\theta$

$y = b\sin^3\theta$

Consider $\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}}$

$$= \left(\frac{a\cos^3\theta}{a}\right)^{\frac{2}{3}} + \left(\frac{b\cos^3\theta}{b}\right)^{\frac{2}{3}}$$

$= \cos^2\theta + \sin^2\theta$

$= 1$

22. $\sin(A + B) = 1 = \sin 90°$

$A + B = 90°$ (1)

$\cos(A - B) = \frac{\sqrt{3}}{2} = \cos 30°$

$A - B = 30°$ (2)

On solving (1) and (2), we get

$$\begin{array}{r} A + B = 90° \\ A - B = 90° \\ \hline 2A = 120° \\ A = 60° \end{array}$$

From (1), $B = 90° - A$

$= 90° - 60°$

$= 30°$

23. Consider

$(1 - \sin\theta + \cos\theta)^2$

$= [(1 - \sin\theta) + \cos\theta]^2$

$= (1 - \sin\theta)^2 + \cos^2\theta + 2\cos\theta\,(1 - \sin\theta)$

$= (1 - \sin\theta)^2 + (1 - \sin^2\theta) + 2\cos\theta(1 - \sin\theta)$

$= (1 - \sin\theta)\,[1 - \sin\theta + 1 + \sin\theta + 2\cos\theta]$

$= (1 - \sin\theta)\,(2\cos\theta + 2)$

$= 2\,(1 + \cos\theta)\,(1 - \sin\theta)$

$=$ R H S

24. LHS $= \dfrac{\tan A + \sin A}{\tan A - \sin A}$

$= \dfrac{\dfrac{\sin A}{\cos A} + \sin A}{\dfrac{\sin A}{\cos A} - \sin A}$

$= \dfrac{\dfrac{\sin A + \sin A\cos A}{\cos A}}{\dfrac{\sin A - \sin A\cos A}{\cos A}}$

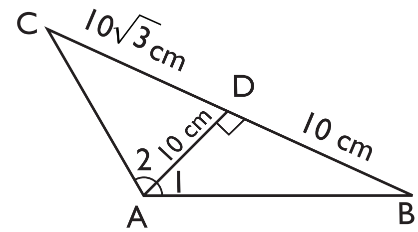
$= \dfrac{\sin A + \sin A\cos A}{\sin A - \sin A\cos A}$

$= \dfrac{\sin A(1 + A\cos A)}{\sin A(1 - A\cos A)} = \dfrac{1 + \dfrac{1}{\sec A}}{1 - \dfrac{1}{\sec A}}$

$= \dfrac{\dfrac{\sec A + 1}{\sec A}}{\dfrac{\sec A - 1}{\sec A}} = \dfrac{\sec A + 1}{\sec A - 1} =$ RHS

Section D

25.



In ΔADB,

$\tan(\angle 1) = \dfrac{BD}{AD} = \dfrac{10}{10} = 1$

$\therefore \quad \angle 1 = 45°$

In ΔADC,

$\tan(\angle 2) = \dfrac{10\sqrt{3}}{10} = \sqrt{3}$

$\angle 2 = 60°$

So, $\angle A = \angle 1 + \angle 2$

$= 45° + 60°$

$= 105°$

26. Consider

L H S $= \dfrac{\sin\theta}{1 - \cos\theta}$

$= \dfrac{\sin\theta\,(1 + \cos\theta)}{(1 - \cos\theta)\,(1 + \cos\theta)}$

$= \dfrac{\sin\theta\,(1 + \cos\theta)}{1 - \cos^2\theta}$

$= \dfrac{\sin\theta\,(1 + \cos\theta)}{\sin^2\theta}$

$= \dfrac{1 + \cos\theta}{\sin\theta}$

$= \dfrac{1}{\sin\theta} + \dfrac{\cos\theta}{\sin\theta}$

$= \text{cosec}\,\theta + \cot\theta$

$= \text{R H S}$

27. Given : $(2\theta + 45°)$ & $(30° + \theta)$ and $\sin(2\theta + 45°) = \cos(30° + \theta)$

$\sin(2\theta + 45°) = \cos(30° + \theta)$

$\sin(2\theta + 45°) = \sin(90°\ \theta(30° - \theta))$

$[\sin(90° - \theta) = \cos\theta]$

$\sin(2\theta + 45°) = \sin(90° - 30° + \theta)$

On equating both sides,

$(2\theta + 45°) = (60° + \theta)$

$2\theta - \theta = 60° - 45°$

$\theta = 15°$

28. Consider

$$\text{L H S} = \frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A}$$

$$= \frac{\frac{\sin A}{\cos A}}{1 - \frac{\cos A}{\sin A}} + \frac{\frac{\cos A}{\sin A}}{1 - \frac{\sin A}{\cos A}}$$

$$= \frac{\sin^2 A}{\cos A\,(\sin A - \cos A)} + \frac{\cos^2 A}{\sin A\,(\cos A - \sin A)}$$

$$= \frac{\sin^2 A}{\cos A\,(\sin A - \cos A)} - \frac{\cos^2 A}{\sin A\,(\cos A - \sin A)}$$

$$= \frac{\sin^3 A - \cos^3 A}{\sin A\cos A\,(\sin A - \cos A)}$$

$$= \frac{(\sin A - \cos A)\,(\sin^2 A + \cos^2 A + \sin A\cos A}{\sin A\cos A\,(\sin A - \cos A)}$$

$$= \frac{1 + \sin A\cos A}{\sin A\cos A}$$

$= 1 + \text{cosec}\,A\sec A$

$= \text{R H S}$

29. To prove :

$$\frac{1}{\text{cosec}A - \cot A} - \frac{1}{\sin A}$$

$$= \frac{1}{\sin A} - \frac{1}{\text{cosec}A + \cot A}$$

i.e. To prove

$$\frac{1}{\text{cosec}A - \cot A} + \frac{1}{\text{cosec}A + \cot A} = \frac{2}{\sin A}$$

Consider

$$\frac{1}{\text{cosec}A - \cot A} + \frac{1}{\text{cosec}A + \cot A}$$

$$= \frac{1}{\frac{1}{\sin A} - \frac{\cos A}{\sin A}} + \frac{1}{\frac{1}{\sin A} + \frac{\cos A}{\sin A}}$$

$$= \frac{\sin A}{1 - \cos A} + \frac{\sin A}{1 + \cos A}$$

$$= \left(\frac{1 + \cos A + 1 - \cos A}{(1 - \cos A)\,(1 + \cos A)}\right)\sin A$$

$$= \left(\frac{2}{1 - \cos^2 A}\right)\sin A$$

$$= \frac{2}{\sin^2 A}\sin A$$

$$= \frac{2}{\sin A}$$

30. $\sin\theta + \cos\theta = p,\ \sec\theta + \text{cosec}\theta = q$

Consider

$q\,(p^2 - 1)$

$= (\sec\theta + \text{cosec}\theta)\ [\sin^2\theta + \cos\theta^2) - 2\,]$

$= (\sec\theta + \text{cosec}\theta)\ [\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1]$

$$= \left(\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right)(2\sin\theta\cos\theta)$$

$$= \left(\frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}\right)(2\sin\theta\cos\theta)$$

$= 2\,(\sin\theta + \cos\theta)$

$= 2p$

31. $\text{Sec}\theta + \tan\theta = p$ (i)

We know that $\text{Sec}^2\theta - \tan^2\theta = 1$

$\Rightarrow \quad (\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$

$\Rightarrow \quad (\sec\theta - \tan\theta)\, p = 1$

$\Rightarrow \quad \sec\theta - \tan\theta = \dfrac{1}{p} \quad \text{(ii)}$

On adding (i) and (ii), we get

$2\sec\theta = p + \dfrac{1}{p}$

$\sec\theta = \dfrac{1}{2}\left(p + \dfrac{1}{p}\right)$

On subtracting (i) from (ii), we get

$-2\tan\theta = \dfrac{1}{p} - p$

$\tan\theta = \dfrac{1}{2}\left(p - \dfrac{1}{p}\right)$

Also,

$$\sin\theta = \frac{\tan\theta}{\sec\theta} = \frac{\frac{1}{2}\left(p - \frac{1}{p}\right)}{\frac{1}{2}\left(p + \frac{1}{p}\right)} = \frac{p^2 - 1}{p^2 + 1}$$

32. $\sin\theta + \cos\theta = \sqrt{2}$

$\Rightarrow \quad \dfrac{1}{\sqrt{2}}\sin\theta + \dfrac{1}{\sqrt{2}}\cos\theta = 1$

$\Rightarrow \quad \cos\dfrac{\pi}{4}\sin\theta + \sin\dfrac{\pi}{4}\cos\theta = 1$

$\Rightarrow \quad \sin\left(\dfrac{\pi}{4} + \theta\right) = 1 = \sin\dfrac{\pi}{2}$

$\Rightarrow \quad \dfrac{\pi}{4} + \theta = \dfrac{\pi}{2}$

$\Rightarrow \quad \theta = \dfrac{\pi}{2} - \dfrac{\pi}{4}$

$\dfrac{\pi}{4}$

Consider

$\tan\theta + \cot\theta$

$= \tan\dfrac{\pi}{4} + \cot\dfrac{\pi}{4}$

$= 1 + 1$

$= 2$

WORKSHEET 2

Section A

1. Consider

$(1 + \cot^2\theta)\sin^2\theta$

$= \left(1 + \dfrac{\cos^2\theta}{\sin^2\theta}\right)\sin^2\theta$

$= \sin^2\theta + \cos^2\theta$

$= 1$

2. Consider

$\operatorname{cosec}^2\theta\,(1 + \cos\theta)(1 - \cos\theta) = x$

$\Rightarrow \quad \operatorname{cosec}^2\theta\,(1 - \cos^2\theta) = x$

$\Rightarrow \quad \operatorname{cosec}^2\theta\,\sin^2\theta = x$

$\Rightarrow \quad \dfrac{1}{\sin^2\theta}\sin^2\theta = x$

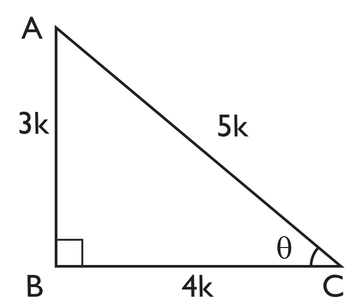
$\Rightarrow \quad 1 = x$

3. $\cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots \cos 179^\circ \cos 188^\circ$

$= \cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots \cos 90^\circ \ldots \cos 179^\circ \cos 188^\circ$

$= 0$

4.



$\cot\theta = 4$

$\cot\theta = \dfrac{4}{3}$

$= \dfrac{BC}{AB}$

Let BC = 4k, AB = 3k

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$= (3k)^2 + (4k)^2$

$= 9k^2 + 16k^2$

$= 25k^2$

$\therefore \quad AC = 5k$

Consider $\dfrac{4\cos\theta - \sin\theta}{2\cos\theta + \sin\theta}$

$$= \frac{4\left(\frac{4}{5}\right) - \left(\frac{3}{5}\right)}{2\left(\frac{4}{5}\right) - \frac{3}{5}}$$

$$= \frac{\frac{16}{5} - \frac{3}{5}}{\frac{8}{5} + \frac{3}{5}}$$

$$= \frac{13}{5} \times \frac{5}{11}$$

$$= \frac{13}{11}$$

5. Consider (secA + tanA) (1 – sinA)

$$= \left(\frac{1}{\cos A} \; \frac{\sin}{\cos A}\right)(1 - \sin A)$$

$$= \frac{(1 + \sin A)(1 - \sin A)}{\cos A}$$

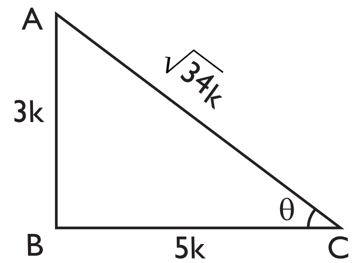
$$= \frac{1 - \sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A}$$

$= \cos A$

6. $3\cos\theta = 5\sin\theta$

$$\frac{3}{5} = \frac{\sin\theta}{\cos\theta}$$

$$\tan\theta = \frac{3}{5} = \frac{AB}{BC}$$



Let AB = 3k

BC = 5k

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$= 9k^2 + 25k^2$

$= 34k^2$

$\Rightarrow \quad AC = \sqrt{34}\,k$

Consider

$$\frac{5\sin\theta - 2\sec^3\theta + 2\cos\theta}{5\sin\theta + 2\sec^3\theta + 2\cos\theta}$$

$$= \frac{5\left(\frac{3}{\sqrt{34}}\right) - 2\left(\frac{\sqrt{34}}{5}\right)^3 + 2\left(\frac{5}{\sqrt{34}}\right)}{5\left(\frac{3}{\sqrt{34}}\right) + 2\left(\frac{\sqrt{34}}{5}\right)^3 - 2\left(\frac{5}{\sqrt{34}}\right)}$$

$$= \frac{\frac{15}{\sqrt{34}} + \frac{10}{\sqrt{34}} - \frac{68}{125}\sqrt{34}}{\frac{15}{\sqrt{34}} + \frac{68}{125}\sqrt{34} - \frac{10}{\sqrt{34}}}$$

$$= \frac{\frac{25\sqrt{34}}{34} - \frac{68}{125}\sqrt{34}}{\frac{5\sqrt{34}}{34} + \frac{68}{125}\sqrt{34}}$$

$$= \frac{\frac{3125\sqrt{34} - 2312\sqrt{34}}{4250}}{\frac{625\sqrt{34} + 2312\sqrt{34}}{4250}}$$

$$= \frac{813\sqrt{34}}{2937\sqrt{34}}$$

$= \frac{271}{979}$

$$\frac{\cos(90° - \theta)\sec(90° - \theta)\tan\theta}{\operatorname{cosec}(90° - \theta)\sin(90° - \theta)\cot(90° - \theta)} + \frac{\tan(90° - \theta)}{\cot\theta}$$

$= \frac{\sin\theta \operatorname{cosec}\theta \tan\theta}{\sec\theta \cos\theta \tan\theta} + \frac{\cot\theta}{\cot\theta}$

$= \frac{1}{1} + 1$

$= 2$

7. $\cos\phi$

$= \cos(180° - (90° + \theta))$

$= \cos(90° - \theta)$

$= \sin\theta$

$= \frac{4}{5}$

8. $\cos^2 17° - \sin^2 73°$

$= \cos^2(90° - 73°) - \sin^2 73°$

$= \sin^2 73° - \sin^2 73°$

$= 0$

9. $\frac{2\tan 30°}{1 + \tan^2 30°}$

$= \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1 + \left(\frac{1}{\sqrt{3}}\right)^2}$

$= \frac{2}{\sqrt{3}} \times \frac{3}{4}$

$= \frac{\sqrt{3}}{2}$

Section B

10. $\tan 2\theta = \cot(\theta + 6°)$

$\Rightarrow \cot(90° - 2\theta) = \cot(\theta + 6°)$

$\Rightarrow (90° - 2\theta) = (\theta + 6°)$

$\Rightarrow 90° - 6° = 3\theta$

$\Rightarrow \frac{84}{3} = \theta$

$\Rightarrow 28° = \theta$

11. $\frac{2\cos 67°}{\sin 23°} - \frac{\tan 40°}{\cot 50°} - \cos 0°$

$= \frac{2\cos(90° - 23°)}{\sin 23°} - \frac{\tan(90° - 50°)}{\cot 50°} - \cos 0°$

$= \frac{2\sin 23°}{\sin 23°} - \frac{\cot 50°}{\cot 50°} - \cos 0°$

$= 2 - 1 - 1$

$= 0$

12. $\sec 4A = \operatorname{cosec}(A - 20°)$

$\operatorname{cosec}(90° - 4A) = \operatorname{cosec}(A - 20°)$
$(\sec\theta = \operatorname{cosec}(90 - \theta)$

Comparing angles

$90° - 4A = A - 20°$

$-4A - A = -20° - 90°$

$-5A = -110°$

$A = \frac{-110°}{-5}$

$A = 22°$

13. $\frac{\cos 70°}{\sin 20°} + \frac{\cos 59°}{\sin 31°} - 8\sin^2 30°$

$= \frac{\cos(90° - 20°)}{\sin 20°} + \frac{\cos(90° - 31°)}{\sin 31°} - 8\left(\frac{1}{2}\right)^2$

$= \frac{\sin 20°}{\sin 20°} + \frac{\sin 31°}{\sin 31°} - \frac{8}{4}$

$= 1 + 1 - 2$

$= 0$

14. $\text{Sin } 75 = \text{Sin } (30 + 45)$

$= \text{Sin } 30 \text{ Cos } 45 + \text{Sin } 45 \text{ Cos } 30$

$= \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \left(\frac{1}{2\sqrt{2}}\right) + \left(\frac{\sqrt{3}}{2\sqrt{2}}\right)$

$= \frac{(1 + \sqrt{3})}{2\sqrt{2}}$

On rationalising :-

$\frac{(1 + \sqrt{3})(2\sqrt{2})}{(2\sqrt{2})(2\sqrt{2})}$

$\frac{(2\sqrt{2} + 2\sqrt{6})}{8}$

$\frac{2(\sqrt{2} + \sqrt{6})}{8}$

$\frac{(\sqrt{2} + \sqrt{6})}{4}$

Therefore $\text{Sin}75 = \frac{(\sqrt{2} + \sqrt{6})}{4}$

15. $\sin\theta = \cos\theta$

$\frac{\sin\theta}{\cos\theta} = 1$

$\tan\theta = 1$

$\therefore \quad \theta = \frac{\pi}{4}$

Consider $2\tan^2\theta + \sin^2\theta - 1$

$= 2\tan^2\frac{\pi}{4} + \sin^2\frac{\pi}{4} - 1$

$= 2(1)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 - 1$

$= 2 - \frac{1}{2} - 1$

$= 1 - \frac{1}{2}$

$= \frac{1}{2}$

16. $\alpha + \beta = 90°$

To prove: $= \sqrt{\cos\alpha \text{ cosec } \beta - \cos\alpha \sin\beta}$

$= \sin\alpha$

Consider $\sqrt{\cos\alpha \sec\alpha - \cos\alpha \cos\alpha}$

$= \sqrt{\cos\alpha \text{ cosec } (90° - \alpha) - \cos\alpha \sin(90° - \alpha)}$

$= \sqrt{\cos\alpha \sec\alpha - \cos\alpha \cos\alpha}$

$= \sqrt{1 - \cos^2\alpha}$

$= \sqrt{\sin^2\alpha}$

$= \sin\alpha$

17. Consider

$2\left(\frac{\cos 58°}{\sin 32°}\right) - \sqrt{3}\left(\frac{\cos 38° \text{ cosec} 52°}{\tan 15° \tan 60° \tan 75°}\right)$

$= 2\left(\frac{\cos(90° - 32°)}{\sin 32°}\right)$

$- \sqrt{3}\left(\frac{\cos(90° - 52°) \text{ cosec} 52°}{\tan(90° - 75°) \tan 60° \tan 75°}\right)$

$= 2\left(\frac{\sin 32°}{\sin 32°}\right) - \sqrt{3}\left(\frac{\sin 52° \text{ cosec} 52°}{\cot 75° \tan 60° \tan 75°}\right)$

$= 2 - \sqrt{3}\left(\frac{1}{\sqrt{3}}\right)$

$= 2 - 1$

$= 1$

18. $\tan\theta + \frac{1}{\tan\theta} = 2$

On squarring both sides, we get

$\tan^2\theta + \frac{1}{\tan^2\theta} + 2\tan\theta\frac{1}{\tan\theta} = 4$

$\tan^2\theta + \frac{1}{\tan^2\theta} + 2 = 4$

$\tan^2\theta + \frac{1}{\tan^2\theta} = 4 - 2$

$\frac{1}{\tan^2\theta} = 2$

19. $\dfrac{2\tan 67°}{\cot 23°} - \dfrac{\sin 40°}{\cos 50°} - \tan 0°$

$= \dfrac{2\tan (90° - 23°)}{\cot 23°} - \dfrac{\sin (90° - 50°)}{\cos 50°} - \tan 0°$

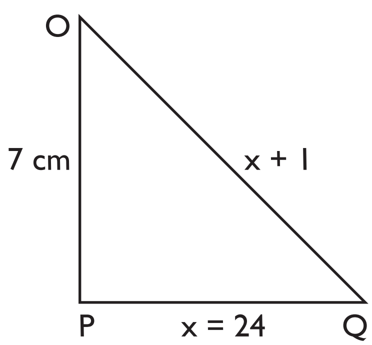
$= \dfrac{2\cot 23°}{\cot 23°} - \dfrac{\cos 50°}{\cos 50°} - \tan 0°$

$= 2 - 1 = 0$

$= 0$

Section C

20.



$OQ - PQ = 1$

Let $PQ = x$

$\therefore$ $OQ = x + 1$

By Pythagoras theorem,

$OQ = OP + PQ$

$(x + 1)^2 = 7^2 + x^2$

$x^2 + 1 + 2x = 49 + x^2$

$2x = 48$

$x = 24$

$\therefore$ $PQ = 24$ cm and $OQ = 25$ cm

$\sin Q = \dfrac{OP}{OQ} = \dfrac{7}{25}$

$\cos Q = \dfrac{PQ}{OQ} = \dfrac{24}{25}$

21. Consider

$(\sec\theta - \tan\theta)^2$

$= \left(\dfrac{1}{\cos\theta} - \dfrac{\sin\theta}{\cos\theta}\right)^2$

$= \left(\dfrac{1 - \sin\theta}{\cos\theta}\right)^2$

$= \dfrac{(1 - \sin\theta)^2}{\cos^2\theta}$

$= \dfrac{(1 - \sin\theta)^2}{1 - \sin^2\theta}$

$= \dfrac{(1 - \sin\theta)(1 - \sin\theta)}{(1 - \sin\theta)(1 + \sin\theta)}$

$= \dfrac{1 - \sin\theta}{1 + \sin\theta}$

22. Consider

$\dfrac{\sec\theta - \tan\theta}{\sec\theta + \tan\theta}$

$= \dfrac{(\sec\theta - \tan\theta)(\sec\theta - \tan\theta)}{(\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}$

$= \dfrac{(\sec\theta - \tan\theta)^2}{\sec^2\theta - \tan^2\theta}$

$= \dfrac{\sec^2\theta + \tan^2\theta - 2\sec\theta\tan\theta}{1}$

$= 1 + \tan^2\theta + \tan^2\theta - 2\sec\theta\tan\theta$

$= 1 - 2\sec\theta\tan\theta + 2\tan^2\theta$

23. $\dfrac{\cos^2 20° + \cos^2 70°}{\sec^2 50° - \cot^2 40°} + 2\,\text{cosec}58°$

$- 2\cot 58° \tan 32° - (4\tan 13° \tan 37° \tan 45° \tan 53° \tan 77°)$

$= \dfrac{\cos^2(90° - 70°) + \cos^2 70°}{\sec^2(90° - 40°) - \cot^2 40°} + 2\,\text{cosec}58°$

$- 2\tan 32° \cot(90° - 32°) - 4\tan(90° - 77°)\tan(90° - 53°)(1)\tan 53° \tan 77°$

$= \dfrac{\sin^2 70° + \cos^2 70°}{\text{cosec}^2 40° - \cot^2 40°} + 2\,\text{cosec}\,58°$

$- 2 \tan 32° \operatorname{cosec} 32° - 4 \cot 77° \cot 53° \tan 53° \tan 77°$

$= 1 + 2 \operatorname{cosec} 58° - 2 \sec 32° - 4$

$= 1 + 2 \operatorname{cosec} (90° - 32°) - 2 \sec 32° - 4$

$= 1 + 2 \sec 32° - 2 \sec 32° - 4$

$= -3$

24. $\operatorname{cosec}\theta + \cot\theta = p \quad \text{(i)}$

Consider

$\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$(\operatorname{cosec}\theta - \cot\theta)(\operatorname{cosec}\theta + \cot\theta) = 1$

$(\operatorname{cosec}\theta - \cot\theta) = \frac{1}{p} \quad \text{(ii)}$

On adding (i) and (ii), we get

$2 \operatorname{cosec}\theta = p + \frac{1}{p}$

$\operatorname{cosec}\theta = \frac{1}{2}\left(p + \frac{1}{p}\right)$

On subtracting (i) and (ii), we get

$-2 \cot\theta = \frac{1}{p} - p$

$\cot\theta = \frac{1}{2}\left(p - \frac{1}{p}\right)$

$\therefore \cos\theta = \frac{\cot\theta}{\operatorname{cosec}\theta}$

$= \frac{\frac{1}{2}\left(p - \frac{1}{p}\right)}{\frac{1}{2}\left(p + \frac{1}{p}\right)}$

$= \frac{p^2 - 1}{p^2 + 1}$

25. $\tan\theta = \frac{1}{\sqrt{7}}$

$\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta}$

$= \frac{(1 + \cot^2\theta) - (1 + \tan^2\theta)}{(1 + \cot^2\theta) + (1 + \tan^2\theta)}$

$= \frac{\cot^2\theta - \tan^2\theta}{2 + \cot^2\theta - \tan^2\theta}$

$= \frac{\left(\sqrt{7}\right)^2 - \left(\frac{1}{\sqrt{7}}\right)^2}{2 + \left(\sqrt{7}\right)^2 + \left(\frac{1}{\sqrt{7}}\right)^2}$

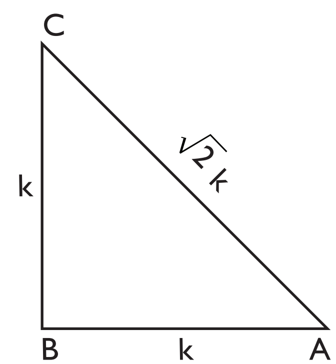
$= \frac{7 - \frac{1}{7}}{2 + 7 + \frac{1}{7}}$

$= \frac{48}{7} \times \frac{7}{64} = \frac{48}{64}$

$= \frac{12}{16}$

$= \frac{3}{4}$

26.



$\operatorname{cosec} A = \frac{\sqrt{2}}{1} = \frac{AC}{BC}$

Let $AC = \sqrt{2}k$

$BC = k$

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$2k^2 = AB^2 + k^2$

$AB^2 = 2k^2 - k^2 = k^2$

$AB = k$

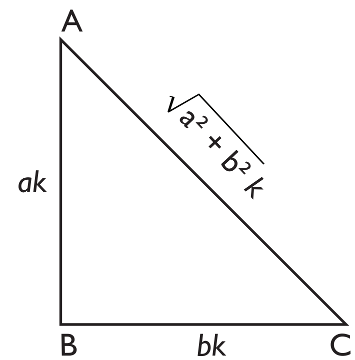
So, $\dfrac{2\sin^2A + 3\cot^2A}{4\tan^2A - \cos^2A}$

$$= \frac{2\left(\frac{1}{\sqrt{2}}\right)^2 + 3(1)^2}{4(1)^2 - \left(\sqrt{2}\right)^2}$$

$$= \frac{1 + 3}{4 - 2}$$

$$= 2$$

27.



$$\sin\theta = \frac{a}{\sqrt{a^2 + b^2}}$$

$$= \frac{AB}{AC}$$

Let $AB = ak$, $AC = \sqrt{a^2 + b^2}\,k$

By Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$(a^2 + b^2)\,k^2 = a^2k^2 + BC^2$$

$$BC^2 = b^2k^2$$

$$BC = bk$$

$$\therefore \quad \cos\theta = \frac{BC}{AC}$$

$$= \frac{b}{\sqrt{a^2 + b^2}}$$

$$\tan\theta = \frac{AB}{BC}$$

$$= \frac{a}{b}$$

28. To prove

$\cos^6A + \sin^6A = 1 - 3\sin^2A\cos^2A$

LHS

$\cos^6A + \sin^6A$

$= (\cos^6A)^3 + (\sin^2A)^3$

$= (\cos^2A + \sin^2A)\,[(\cos^2A)^2 - \cos^2A \times \sin^2A + (\sin^2A)^2]$

(Because, $a^3 + b^3 = (a + b)\,(a^2 - ab + b^2)$)

$= (\cos^2A)^2 - \cos^2A \times \sin^2A + (\sin^2A)^2$

(Because, $\sin^2x + \cos^2x = 1$)

$= (\cos^2A)^2 - \cos^2A \times \sin^2A + (\sin^2A)^2 + 2\cos^2A\sin^2A - 2\cos^2A\sin^2A$

$= (\cos^2A + \sin^2A)^2 - 3\cos^2A\sin^2A$

(Because, $a^2 + 2ab + b^2 = (a + b)^2$)

$= \quad 1 - 3\cos^2A\ \sin^2A$

= RHS

Hemce, proved.

29. $5\ \tan x = 4$

$$\tan x = \frac{4}{5}$$

Consider $\dfrac{5\sin x - 3\cos x}{5\sin x + 2\cos x}$

$$= \frac{\dfrac{5\sin x - 3\cos x}{\cos x}}{\dfrac{5\sin x + 2\cos x}{\cos x}}$$

$$= \frac{5\tan x - 3}{5\tan x + 2}$$

$$= \frac{5\left(\frac{4}{5}\right) - 3}{5\left(\frac{4}{5}\right) + 2}$$

$$= \frac{4 - 3}{4 + 2}$$

$$= \frac{1}{6}$$

Section D

30. $\sec\theta = x + \frac{1}{4x}$

We know that $\sec^2\theta - \tan^2\theta = 1$

$$\tan^2\theta = \sec^2\theta - 1$$
$$= \left(x + \frac{1}{4x}\right)^2 - 1$$
$$= x^2 + \left(\frac{1}{4x}\right)^2 + 2x\left(\frac{1}{4x}\right) - 1$$
$$= x^2 + \left(\frac{1}{4x}\right)^2 - \frac{1}{2}$$
$$= \left(x - \frac{1}{4x}\right)^2$$

So, $\tan^2\theta = \left(x - \frac{1}{4x}\right)$

$\therefore \quad \tan\theta = \pm\left(x - \frac{1}{4x}\right)$

Case 1

$\sec\theta = x + \frac{1}{4x}$

$\tan\theta = x - \frac{1}{4x}$

So,

$\sec\theta + \tan\theta$

$= 2x$

Case 2

$\sec\theta = x + \frac{1}{4x}$

$\tan\theta = -\left(x - \frac{1}{4x}\right)$

So,

$\sec\theta + \tan\theta$

$= x + \frac{1}{4x} - x + \frac{1}{4x}$

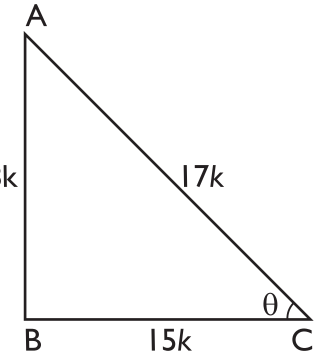
$= \frac{2}{4x}$

$= \frac{1}{2x}$

$\therefore \quad \sec\theta + \tan\theta = 2x + \frac{1}{2x}$

31. $\cot\theta = \frac{15}{8}$

$$= \frac{BC}{AB}$$



Let BC = 15k

AB = 8k

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$= 64k^2 + 225k^2$

$= 289k^2$

$\therefore \quad AC = 17k^2$

(i) Consider $\frac{(2 + 2\sin\theta)(1 - \sin\theta)}{(1 + \cos\theta)(2 - 2\cos\theta)}$

$$= \frac{2(1 + \sin\theta)(1 - \sin\theta)}{2(1 + \cos\theta)(1 - \cos\theta)}$$
$$= \frac{1 - \sin^2\theta}{1 - \cos^2\theta}$$
$$= \frac{\cos^2\theta}{\sin^2\theta}$$
$$= \cot^2\theta$$
$$= \frac{225}{64}$$

(ii) $\frac{\text{cosec}^2\theta - \cot^2\theta}{\text{cosec}^2\theta + \cot^2\theta}$

$$= \frac{1}{\text{cosec}^2\theta + \cot^2\theta}$$
$$= \frac{1}{\left(\frac{17}{8}\right)^2 + \left(\frac{15}{8}\right)^2}$$
$$= \frac{64}{289 + 225}$$
$$= \frac{64}{514}$$

$$= \frac{32}{257}$$

(iii) $\sec^2\theta + \tan^2\theta$

$$= \left(\frac{17}{15}\right)^2 + \left(\frac{8}{15}\right)^2$$

$$= \frac{289}{225} + \frac{64}{225}$$

$$= \frac{353}{225}$$

32. $\tan A = n \tan B$

$$\Rightarrow \frac{\sin A}{\cos A} = n \frac{\sin B}{\cos B} \quad \text{(i)}$$

Also, $\sin A = m \sin B$

$$\Rightarrow \frac{\sin A}{\sin B} = m \quad \text{(ii)}$$

From (i), (ii), we get

$$m = n \frac{\cos A}{\cos B}$$

$$\Rightarrow \cos B = \frac{n}{m} \cos A \quad \text{(iii)}$$

On putting value of $\sin B$ and $\cos B$ from (ii) and (iii) in $\cos^2 B + \sin^2 B = 1$, we get

$$\frac{n^2}{m^2}\cos^2 A + \frac{1}{m^2}\sin^2 A = 1$$

$$n^2 \cos^2 A + \sin^2 A = m^2$$

$$n^2 \cos^2 A + 1 - \cos^2 A = m^2$$

$$n^2 \cos^2 A - \cos^2 A = m^2 - 1$$

$$(n^2 - 1)\cos^2 A = m^2 - 1$$

$$\therefore \quad \cos^2 A = \frac{m^2 - 1}{n^2 - 1}$$

33. $\mathrm{cose\ c}\ \theta - \sin\theta = l$

$\sec\theta - \cos\theta = m$

Consider

$l^2 - m^2 \ (l^2 + m^2 + 3)$

$= (\mathrm{cosec}\theta - \sin\theta)^2 (\sec\theta - \cos\theta)^2[(\mathrm{cosec}\theta - \sin\theta)^2 (\sec\theta - \cos\theta)^2] + 3$

$$= \left(\frac{1}{\sin\theta} - \sin\theta\right)^2 \left(\frac{1}{\cos\theta} - \cos\theta\right)^2 \left[\left(\frac{1}{\sin\theta} - \sin\theta\right)^2 + \left(\frac{1}{\cos\theta} - \cos\theta\right)^2\right] + 3$$

$$= \left(\frac{1 - \sin^2\theta}{\sin\theta}\right)^2 = \left(\frac{1 - \cos^2\theta}{\cos\theta}\right)^2 \left[\left(\frac{1 - \sin^2\theta}{\sin\theta}\right)^2 + \left(\frac{1 - \cos^2\theta}{\cos\theta}\right)^2 + 3\right]$$

$$= \left(\frac{\cos^2\theta}{\sin\theta}\right)^2 \left(\frac{\sin^2\theta}{\cos\theta}\right)^2 \left[\left(\frac{\cos^2\theta}{\sin\theta}\right)^2 + \left(\frac{\sin^2\theta}{\cos\theta}\right)^2 + 3\right]$$

$$= \frac{\cos^4\theta}{\sin^2\theta}\frac{\sin^4\theta}{\cos^2\theta}\left[\frac{\cos^4\theta}{\sin^2\theta} + \frac{\sin^4\theta}{\cos^2\theta} + 3\right]$$

$$= \sin^2\theta\cos^2\theta\left[\frac{\cos^6\theta + \sin^6\theta + 3\sin^2\theta\cos^2\theta}{\sin^2\theta\cos^2\theta}\right]$$

$$= (\sin^2\theta)^3 + (\cos^2\theta)^3 + 3\sin^2\theta\cos^2\theta$$

$$= (\sin^2\theta + \cos^2\theta)(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta) + 3\sin^2\theta\cos^2\theta$$

$$= \sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta$$

$$= (\sin^2\theta + \cos^2\theta)^2$$

$$= 1^2$$

$$= 1$$

34. $\frac{\cos\alpha}{\cos\beta} m$ and $\frac{\cos\alpha}{\sin\beta} = n$

Consider

$(m^2 + n^2)\ \cos^2\beta$

$$= \left(\frac{\cos^2\alpha}{\cos^2\beta} + \frac{\cos^2\alpha}{\sin^2\beta}\right)\cos^2\beta$$

$$= \left(\frac{\cos^2\alpha\sin^2\beta + \cos^2\beta\cos^2\alpha}{\cos^2\beta\sin^2\beta}\right)\cos^2\beta$$

$$= \frac{\cos^2\alpha\sin^2\beta + \cos^2\beta\cos^2\alpha}{\sin^2\beta}$$

$$= \left(\frac{\cos\alpha}{\sin\beta}\right)^2$$

$= n^2$

35. $(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C)$

$= (\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$ (i)

On multiplying both side of (i) by $(\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$, we get

$(\sec^2 A - \tan^2 A)(\sec^2 B - \tan^2 B)(\sec^2 C - \tan^2 C)$

$= (\sec A - \tan A)^2 (\sec B - \tan B)^2 (\sec C - \tan C)^2$

$\Rightarrow 1 = (\sec A - \tan A)^2 (\sec B - \tan B)^2 (\sec C - \tan C)^2$

$\Rightarrow (\sec A - \tan A)^2 (\sec B - \tan B)^2 (\sec C - \tan C)^2 = \pm 1$

Again, Multiplying both sides of (i) by

$(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C)$

we get

$(\sec A + \tan A)^2 (\sec B + \tan B)^2 (\sec C + \tan C)^2$

$= (\sec^2 A - \tan^2 A)(\sec^2 B - \tan^2 B)(\sec^2 C - \tan^2 C)$

$= 1$

$\therefore$ $(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C) = \pm 1$

36. $x \sin^3\theta + y \cos^3\theta = \sin\theta \cos\theta$

$\Rightarrow x \sin^3\theta + \cos^2\theta (y \cos\theta) = \sin\theta \cos\theta$

$\Rightarrow x \sin^3\theta + \cos^2\theta\, x \sin\theta = \sin\theta \cos\theta$

$[\because y \cos\theta = x \sin\theta]$

$\Rightarrow x \sin\theta + (\sin^2\theta + \cos^2\theta) = \sin\theta \cos\theta$

$\Rightarrow x \sin\theta = \sin\theta \cos\theta$

$\Rightarrow x = \cos\theta$

$$\therefore \quad y = \frac{x \sin\theta}{\cos\theta} = \frac{\cos\theta \sin\theta}{\cos\theta} = \sin\theta$$

Also, we know that

$\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow y^2 + x^2 = 1$

$\Rightarrow x^2 + y^2 = 1$

37. $\text{cosec}\theta - \sin\theta = m$

$$\Rightarrow \frac{1}{\sin\theta} - \sin\theta = m$$

$$\Rightarrow \frac{1 - \sin^2\theta}{\sin\theta} = m$$

$$\Rightarrow \frac{\cos^2\theta}{\sin\theta} = m$$

Also, $\sec\theta - \quad = n$

$$\frac{1}{\cos\theta} - \cos\theta = n$$

$$\frac{1 - \cos^2\theta}{\cos\theta} = n$$

$$\frac{\sin^2\theta}{\cos\theta} = n$$

So,

$$\text{L H S} = (m^2 n)^{\frac{2}{3}} + (mn^2)^{\frac{2}{3}}$$

$$= \left(\frac{\cos^4\theta}{\sin^2\theta} \frac{\sin^2\theta}{\cos\theta}\right)^{\frac{2}{3}} + \left(\frac{\cos^2\theta}{\sin\theta} \frac{\sin^4\theta}{\cos^4\theta}\right)^{\frac{2}{3}}$$

$$= (\cos^3\theta)^{\frac{2}{3}} + (\sin^3\theta)^{\frac{2}{3}}$$

$= \cos^2\theta + \sin^2\theta$

$= 1$

38. $a \sec\theta + b \tan\theta + c = 0$

$p \sec\theta + q \tan\theta + r = 0$

To prove:

$(br - qc)^2 - (pc - ar)^2 = (aq - pb)^2$

Consider

$(br - qc)^2 - (pc - ar)^2$

$= [\, b(-p \sec\theta - q \tan\theta) + q(a \sec\theta + b \tan\theta)]^2 - [p(-a \sec\theta - b \tan\theta) + a(p \sec\theta + q \tan\theta)]^2$

$= [- bp \sec\theta - bq \tan\theta + aq \sec\theta + bq \tan\theta]^2 - [- ap \sec\theta - bp \tan\theta + ap \sec\theta + aq \tan\theta]^2$

$= [\sec\theta (aq - bp)]^2 - [(aq - bp) \tan\theta]^2$

$= (aq - bp)^2 (\sec^2\theta - \tan^2\theta)$

$= (aq - bp)^2$

39. $\tan^2\theta = 1 - a^2$

Consider

$(\sec\theta + \tan^3\theta \operatorname{cosec}\theta)$

$= \sqrt{1 + \tan^2\theta} + \tan^2\theta \tan\theta \operatorname{cosec}\theta$

$= \sqrt{1 + \tan^2\theta} + \tan^2\theta \tan\theta \sqrt{1 + \cot^2\theta}$

$= \sqrt{1 + 1 - a^2} + (1 - a^2) \sqrt{1 - a^2} \sqrt{1 + \frac{1}{\tan^2\theta}}$

$= \sqrt{2 - a^2} + (1 - a^2) (\sqrt{1 - a^2}) \sqrt{1 + \frac{1}{1 - a^2}}$

$= \sqrt{2 - a^2} + (1 - a^2) \frac{\sqrt{1 - a^2}}{\sqrt{1 - a^2}} \sqrt{2 - a^2}$

$= \sqrt{2 - a^2} + (1 - a^2) \sqrt{2 - a^2}$

$= \sqrt{2 - a^2} + (1 + 1 - a^2)$

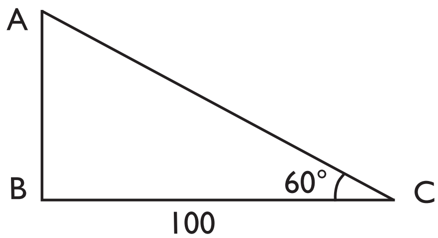
$= \sqrt{2 - a^2} + (2 - a^2)$

$= (2 - a^2)^{\frac{2}{3}}$

Chapter 09 Some Applications of Trigonometry

MULTIPLE CHOICE QUESTIONS

1.



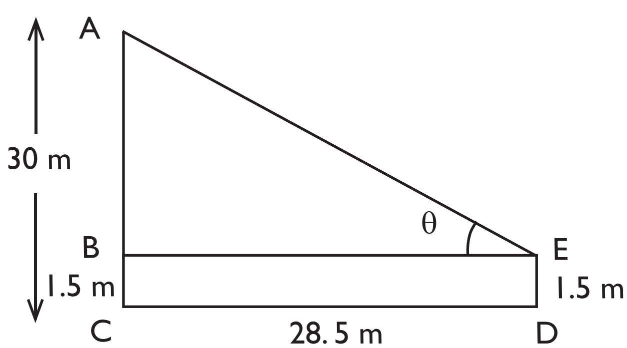
$\tan 60° = \dfrac{AB}{BC}$

$\sqrt{3} = \dfrac{AB}{100}$

$\therefore\ AB = 100\sqrt{3}$ m

Option (d)

2.



$AB = AC - BC$

$= 30 - 1.5$

$= 28.5$ m

$BE = CD = 28.5$ m

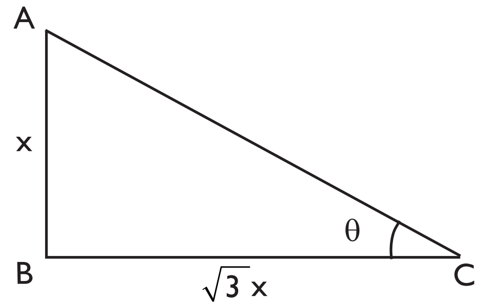
In ΔABE,

$\tan\theta = \dfrac{AB}{BE} = \dfrac{28.5}{28.5} = 1$

$\Rightarrow \theta = 45°$

Option (c)

3.

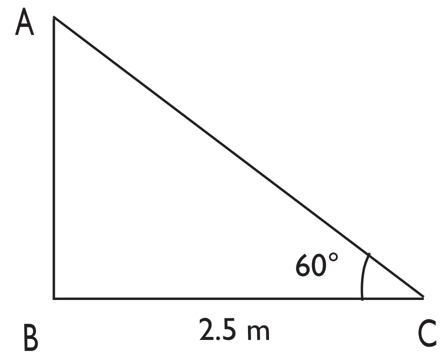


$\tan\theta = \dfrac{AB}{BC} = \dfrac{x}{\sqrt{3}x} = \dfrac{1}{\sqrt{3}}$

$\theta = 30°$

Option (d)

4.



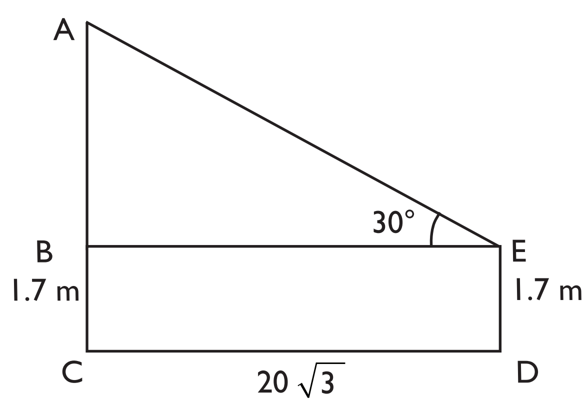
$\cos 60° = \dfrac{BC}{AC}$

$\dfrac{1}{2} = \dfrac{2.5}{AC}$

$AC = 5$ m

Option (b)

5.



In ΔABE, $BE = CD = 20\sqrt{3}$ m

$$\tan 30° = \frac{AB}{BE}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{20\sqrt{3}}$$

$$\therefore \quad AB = 20 \text{ m}$$

So, $AC = AB + BC = 20 + 1.7$

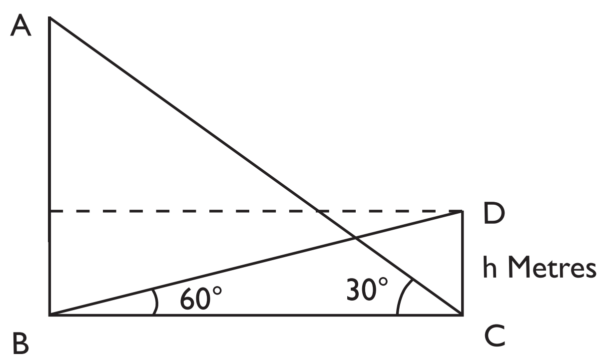
$= 21.7$ m

Option (a)

WORKSHEET 1

Section A

1.



Let AB denotes the tower.

In ΔBCD,

$$\tan 60° = \frac{CD}{BC}$$

$$\sqrt{3} = \frac{h}{BC}$$

$$BC = \frac{h}{\sqrt{3}} \text{ Metre}$$

In ΔABC,

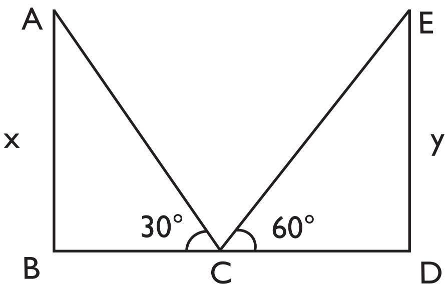
$$\tan 30° = \frac{AB}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{h}$$

$$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{h}$$

$$AB = \frac{h}{3} \text{ Metre}$$

2.



Let AB and DE denote two towers

In ΔABC,

$$\tan 30° = \frac{AB}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{x}{BC}$$

$$\Rightarrow \quad BC = \sqrt{3}$$

In ΔCDE,

$$\tan 60° = \frac{DE}{CD}$$

$$\sqrt{3} = \frac{y}{CD}$$

$$CD = \frac{y}{\sqrt{3}}$$

As $BC = CD$

$$\therefore \quad \sqrt{3}x = \frac{y}{\sqrt{3}}$$

$$3x = y$$

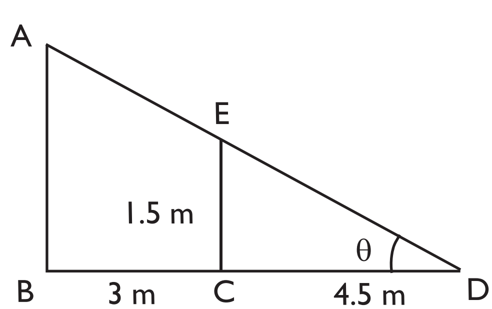
$$\Rightarrow \quad x : y = 1 : 3$$

3. In ΔABC,

$$\tan C = \frac{AB}{BC} = \frac{20}{20\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\therefore \quad C = 30°$$

4.



Let CE denotes the boy and AB denotes a lamp-post.

In ΔDCE,

$$\tan\theta = \frac{CE}{CD} = \frac{1.5}{45} = \frac{1}{3}$$

In ΔABC,

$$\tan\theta = \frac{AB}{BD}$$

$$\frac{1}{3} = \frac{AB}{7.5}$$

$\therefore$ AB $= 2.5$ m

5. In ΔABC,

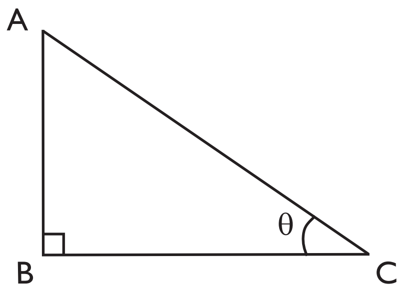
$$\sin 45^\circ = \frac{BC}{AC}$$

$$\frac{1}{\sqrt{2}} = \frac{BC}{150}$$

$$BC = \frac{150}{\sqrt{2}}$$

$$= 75\sqrt{2} \text{ m}$$

6.



Let AB denotes the vertical role and BC denotes the shadow of the role.

Let AB = BC = x

In ΔABC,

$$\tan\theta = \frac{AB}{BC} = \frac{x}{x} = 1$$

$\therefore$ $\theta = 45^\circ$

7. In ΔBAC,

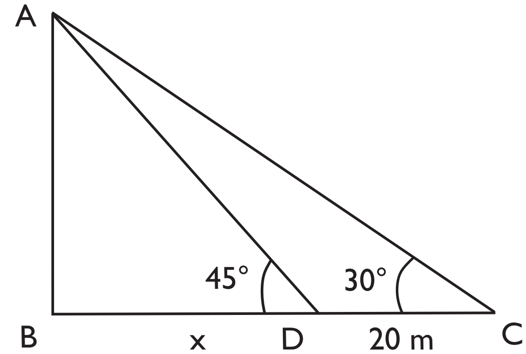
$$\tan\theta = \frac{AB}{AC}$$

$$= \frac{5a\sqrt{3}}{5a}$$

$$= \sqrt{3}$$

$$\theta = 60^\circ$$

8.



Let AB denotes the chimney.

Let BD = x metre

In ΔABC,

$$\tan 30^\circ = \frac{AB}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{x+20}$$

$$AB = \frac{1}{\sqrt{3}}(x+20) \qquad \text{(i)}$$

In ΔABC,

$$\tan 45^\circ = \frac{AB}{BD}$$

$$1 = \frac{AB}{x}$$

$$AB = x \qquad \text{(ii)}$$

From (i) and (ii),

$$AB = x = \frac{1}{\sqrt{3}}(x+20)$$

$$\sqrt{3}x - x = 20$$

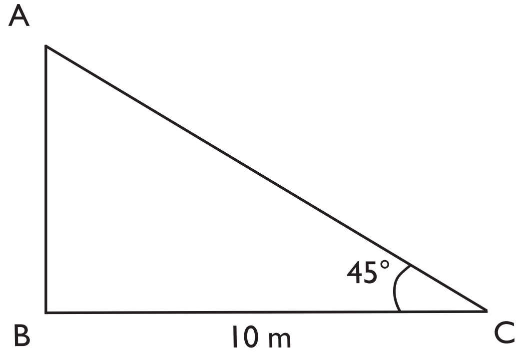
$$x = \frac{20}{\sqrt{3-1}}$$

$$= \frac{20}{2}(\sqrt{3}+1)$$

$$x = 10(\sqrt{3}+1)$$

$\therefore$ $AB = x = 10(\sqrt{3}+1)$

9.



Let AB denotes the tower and BC denotes the shadow

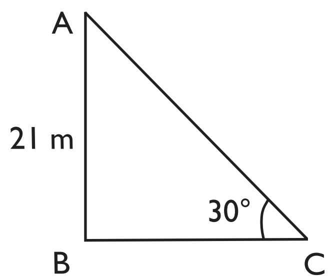
In ΔABC,

$\tan 45^\circ = \dfrac{AB}{BC}$

$1 = \dfrac{AB}{10}$

$AB = 10$ m

10.



Let AC denotes the string of kite.

In ΔABC,

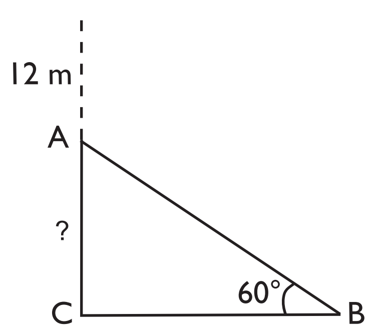
$\sin 30^\circ = \dfrac{AB}{AC}$

$\dfrac{1}{2} = \dfrac{21}{AC}$

$AC = 42$ m

Section B

11.



Let AC be x and AB be 12 – x

$\therefore \sin 60^\circ = \dfrac{AC}{AB}$

$\dfrac{\sqrt{3}}{2} = \dfrac{x}{12 - x}$

$12\sqrt{3} - \sqrt{3}x = 2x$

$12 \times 1.73 = 2x + 1.73x$

$20.76 = 3.73x$

$\dfrac{2076 \times \cancel{100}}{373 \times \cancel{100}} = x$

$5.6\text{m} = x = AC$

12. Let the aeroplane be at B and let the two ships be at C and D such that their angles of depression from B are 60° and 30° respectively.

In ΔCAB, we have,

$\tan 60^\circ = \dfrac{AB}{CA}$

$\Rightarrow \sqrt{3} = \dfrac{1200}{x}$

$\Rightarrow x = \dfrac{1200}{\sqrt{3}} = 400\sqrt{3}$

In ΔBAD, we have

$\tan 30^\circ = \dfrac{AB}{AD}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{1200}{x + y}$

$\Rightarrow x + y = 1200\sqrt{3}$

$\Rightarrow y = 1200\sqrt{3} - x$

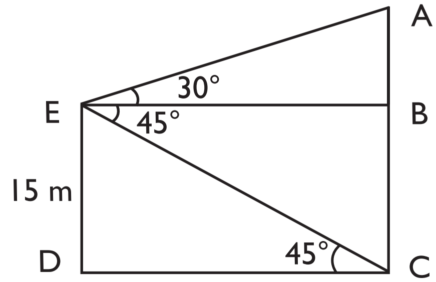
$\Rightarrow y = 1200\sqrt{3} - 400\sqrt{3}$

$= 800\sqrt{3} = 800 \times 1.732$

$= 1385.6$

Hence, the distance between the two ships is 1385.6 metres.

13.



Let the window be at point E and AC be the house.

Let

To find : AC

In ΔCDE,

$\tan 45° = \frac{DE}{CD}$

$1 = \frac{15}{CD}$

$\therefore$ CD = 15 m

$\therefore$ BE = CD = 15 m

In ΔABE,

$\tan 30° = \frac{AB}{BE}$

$\frac{1}{\sqrt{3}} = \frac{AB}{15}$

$\therefore$ AB = $\frac{15}{\sqrt{3}} = 5\sqrt{3}$ m

Also, BC = DE =15 m

$\therefore$ AC = AB = BC

AC = $5\sqrt{3} + 15$

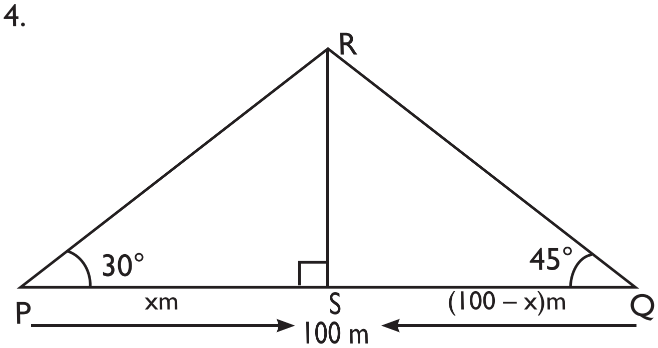
$= 5(\sqrt{3} + 3)$ m

$= 5\ (1.732 + 3)$ m

$= 5 \times 4.732$

$= 23.66$ m

14.



Let AB denotes the three .

Step 1:

Given Data:

In rt. ΔPRS,

x = RS cot 30°

x = RS $\sqrt{3}$

Step 2:

x = $\sqrt{3}$ RS........ (i)

In rt. ΔRSQ,

SQ = RS cot 45°

Step 3:

(100 – x) = RS

x = 100 – RS........ (ii)

Step 4:

Equating (i) and Equation(ii) we have:

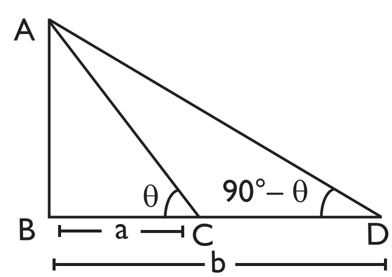
RS = 100 – RS

Step 5:

2.73 RS = 100

RS = 36.63 m

15.



Let AB denotes the tree.

To prove : AB = $\sqrt{ab}$ metres

In ΔABC,

$\tan\theta = \frac{AB}{BC}$

$\tan\theta = \frac{AB}{a}$

$\therefore$ AB $= a \tan\theta$ (i)

In ΔABD,

$\tan(90 - \theta) = \dfrac{AB}{BD}$

$\cot\theta = \dfrac{AB}{b}$

$\therefore$ AB $= b\cot\theta$ (ii)

From (i) and (ii)

AB $= a \tan\theta = b\cot\theta$

$\tan^2\theta = \dfrac{b}{a}$

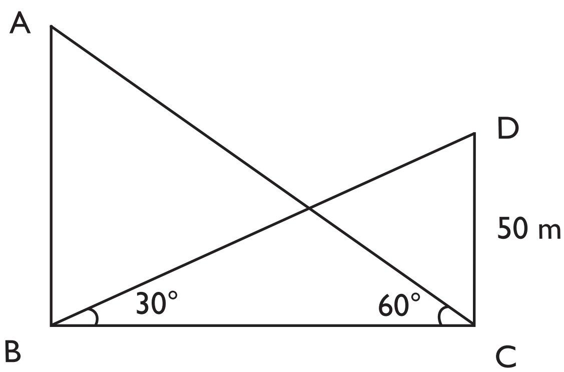
$\therefore$ $\tan\theta = \sqrt{\dfrac{b}{a}}$

So, AB $= a\tan\theta$

$= a\sqrt{\dfrac{b}{a}}$

$= \sqrt{ab}$ metres

16.



Let AB denotes the hill and CD denotes the tower such that CD = 50 m

In ΔBCD,

$\tan 30° = \dfrac{CD}{BC}$

$\dfrac{1}{\sqrt{3}} = \dfrac{50}{BC}$

$\therefore$ BC $= 50\sqrt{3}$ m

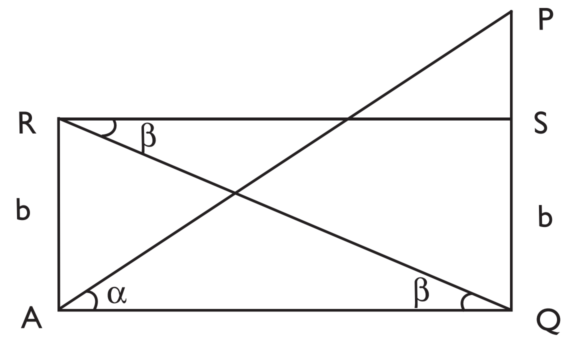
In ΔABC,

$\tan 60° = \dfrac{AB}{BC}$

$\sqrt{3} = \dfrac{AB}{50\sqrt{3}}$

$\therefore$ AB $= 150$ m

17.



Let PQ denotes the tower

To prove : PQ $= b\tan\alpha\cot\beta$

QS = AR = b ft

In ΔRAQ,

$\tan\beta = \dfrac{AR}{AQ}$

$\tan\beta = \dfrac{b}{AQ}$

AQ $= b\cot\beta$

In ΔPQA,

$\tan\alpha = \dfrac{PR}{AQ}$

$\tan\alpha = \dfrac{PS + QS}{b\cos\beta}$

$\tan\alpha = \dfrac{PS + b}{b\cos\beta}$

$\Rightarrow$ PS + b $= b\tan\alpha\cot\beta$

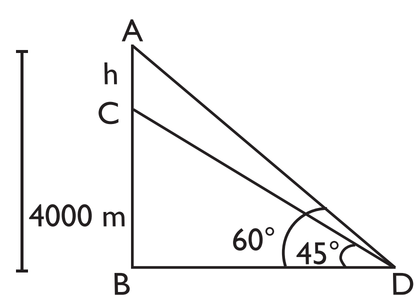
$\Rightarrow$ PS $= b\tan\alpha\cot\beta - b$

So, PQ $= PS + SQ$

$= b\tan\alpha\cot\beta - b + b$

$= b\tan\alpha\cot\beta$

18.



Height of first Airplane = AB = 4000m

Height of another lane = BC

The angles of elevation of two planes from the same point on the ground are 60° and 45°. i.e. $\angle ADB = 60°$ and $\angle CDB = 45°$

Let AC be h

CB = AB – AC = 4000 – h

In ΔABD

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\tan 60° = \frac{AB}{BD}$$

$$\sqrt{3} = \frac{4000}{BD}$$

$$BD = \frac{4000}{\sqrt{3}}$$

BD = 2309.401

In ΔCBD

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\tan 45° = \frac{CB}{BD}$$

$$1 = \frac{4000 - h}{2309.401}$$

2309.401 = 4000 – h

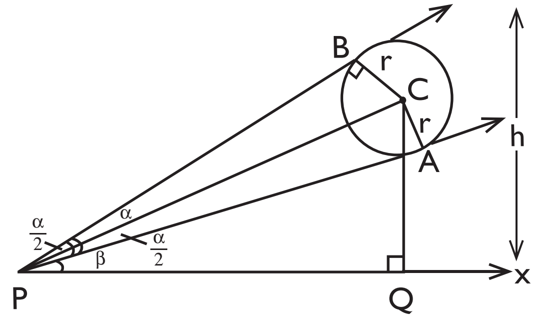
h = 4000 – 2309.401

h = 1690.599

Hence, the vertical distance between the aeroplane at that instant is 1690.599 m.

Section C

19. Let P be the eye of observer. Let PA and PB are tangents to the round balloon.



PX is the horizontal line and CQ $\perp$ PQ. It is given that $\angle APB = \alpha$

$$\therefore\ \angle CPA = \angle CPB = \frac{\alpha}{2}$$

and $\angle CPA = \beta$

Let height of the centre C be h m and CA = CB = r

In right triangle CBP, we have

$$\sin\left(\frac{\alpha}{2}\right) = \frac{BC}{CP}$$

$$\Rightarrow \sin\left(\frac{\alpha}{2}\right) = \frac{r}{CP}$$

$$\Rightarrow CP = \frac{r}{\sin\left(\frac{\alpha}{2}\right)}$$

$$\Rightarrow CP = r \operatorname{cosec} \frac{\alpha}{2}$$

In right triangle CPQ, we have

$$\sin\beta = \frac{CQ}{CP}$$

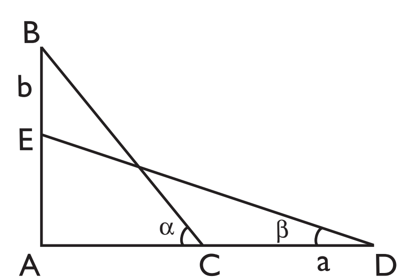
$$\Rightarrow CQ = CP \sin\beta$$

$$\Rightarrow CQ = r \operatorname{cosec} \frac{\alpha}{2} \sin\beta$$

Hence, the height of the centre

$$= r \sin\beta \operatorname{cosec} \frac{\alpha}{2}$$

20.



Let the kite be at point A such that AC is the string of kite.

Let BC be the ladder which slides down a distance b on the wall.

In right triangle ABC, we have

$$\sin \alpha = \frac{AB}{BC} = \frac{AE + EB}{BC}$$

$$\sin \alpha = \frac{AE + b}{BC}$$

But, AE = $\sin \beta \times ED$ (In ΔAED)

So, replacing AE by ED $\sin \beta$, we get

$$\sin \alpha = \frac{ED \sin \beta + b}{BC}$$

$\Rightarrow b = BC \sin \alpha - ED \sin \beta$

As, BC and ED both represent the same ladder.

BC = ED. (length of ladder does not change)

$\Rightarrow BC \sin \alpha - BC \sin \beta = b$

$\Rightarrow BC (\sin \alpha - \sin \beta) = b$ ...(i)

Similarly, in right triangle AED, we have

$$\cos \beta = \frac{AD}{ED} = \frac{AC + CD}{ED}$$

$$\cos \beta = \frac{AC + a}{ED}$$

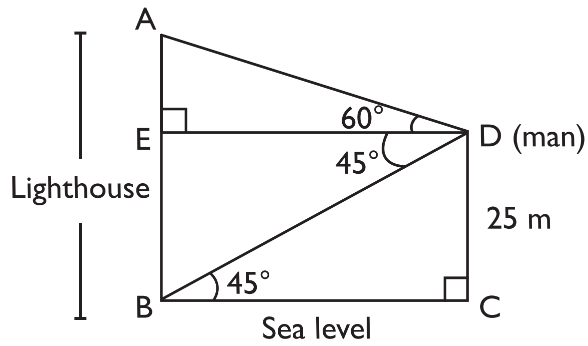
But, AC = BC $\cos \alpha$ (In ΔABC)

So, by replacing AC by BC $\cos \alpha$, we get ED $\cos \beta$ = BC $\cos \alpha$ + a BC (cos, $-\cos \alpha$) = a [$\therefore$ ED = BC] ...(ii) Dividing (ii) by (i), we get

$$\frac{a}{b} = \frac{\cos \beta - \cos \alpha}{\sin \alpha - \sin \beta}$$

$$\frac{a}{b} = \frac{\cos \alpha - \cos \beta}{\sin \beta - \sin \alpha}$$

21.



Let AB represents the light house (A shows its top), BC represents the sea level and D represents the position of the man,

By the below diagram,

In triangle DCB,

$$\tan 45° = \frac{CD}{BC}$$

$1 = \frac{25}{BC} \Rightarrow BC = 25$ m

$\Rightarrow$ ED = 25 meters,

Now, in triangle DAE,

$$\tan 60° = \frac{AE}{ED}$$

$$\sqrt{3} = \frac{AE}{25}$$

$\Rightarrow AE = 25\sqrt{3}$

Hence, the height of lighthouse,

AB = AE + EB

$= 25\sqrt{3} + 25$

$= 25(\sqrt{3} + 1)$ meters.

22. Let the distance between the nearer kilometre stone and the hill be 'a' km. So, the distance between the farther kilometre stone and the hill is '1 + x' km since both are on the same side of the hill.

In triangle APB,

$$\tan 45° = \frac{h}{x}$$

$$\Rightarrow 1 = \frac{h}{x}$$

$\Rightarrow h = x$

In triangle AQB,

$\tan 30° = \frac{h}{1 + x}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{1 + x}$

$\Rightarrow 1 + x = \sqrt{3}\,h$

From equation 1,

$1 + h = \sqrt{3}\,h \Rightarrow 1 = \sqrt{3}\,h - h$

$\Rightarrow h = \frac{1}{\sqrt{3} - 1}$

$\Rightarrow h = 1.365$km

Hence, option A is correct.

Section D

23. Let A and B be the two positions of the ship. Let d be the distance travelled by the ship during the period of observation i.e. AB = d metres.

Let the observer be at O, the top of the lighthouse PO.

It is given that PO = 100 m and the angle of depression from O of A and B are 30° and 45° respectively.

$\therefore\ \angle OAP = 30°$ and $\angle OBP = 45°$

In ΔOPB, we have

$\tan 45° = \frac{OP}{BP}$

$\Rightarrow 1 = \frac{100}{BP}$

$\Rightarrow BP = 100$ m

In ΔOPA, we have

$\Rightarrow \tan 30° = \frac{OP}{AP}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{100}{d + BP}$

$\Rightarrow d + BP = 100\sqrt{3}$

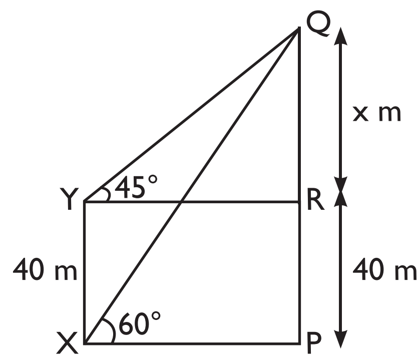
$\Rightarrow d + BP = 100\sqrt{3}$

$\Rightarrow d = 100\sqrt{3} - 100$

$\Rightarrow d = 100(\sqrt{3} - 1) = 100(1.732 - 1) = 73.2$ m.

Hence, the distance travelled by the ship from A to B is 73.2 m.

24.



In YRQ, we have

$\tan 45° = \frac{QR}{YR}$

$\Rightarrow 1 = \frac{x}{YR}$

$\Rightarrow YR = x$

or XP = x   [As YR = XP] ...(1)

Now, In ΔXPQ, we have

$\tan 60° = \frac{PQ}{PX}$

$\Rightarrow \sqrt{3} = \frac{x + 40}{x}$ [Using (1)]

$\Rightarrow \sqrt{3}\,x = x + 40$

$\Rightarrow x(\sqrt{3} - 1) = 40$

$\Rightarrow x = \frac{40}{\sqrt{3} - 1}$

On rationalising the denominator, we get

$x = \frac{40}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1} = \frac{40(\sqrt{3} + 1)}{3 - 1}$

$= 20(\sqrt{3} + 1) = 54.64$ m

So, height of the tower,

PQ = x + 40 = 54.64 + 40 = 94.64 metres

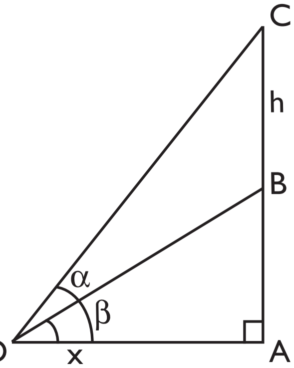
Now, In ΔXPQ, we have

$$\sin 60° = \frac{PQ}{PX}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{94.64}{XQ}$$

$$XQ = \frac{94.64 \times 2}{\sqrt{3}} = \frac{94.64 \times 2 \times \sqrt{3}}{3} = 109.3 \text{ m}$$

25.



Let AB be the tower and BC be the flagstaff. Let OA = x metres, AB = y metres and BC = h metres.

In right ΔOAB,

$$\tan \alpha = \frac{AB}{OA}$$

$$\Rightarrow y = x \tan \alpha \text{ or } x = \frac{y}{\tan \alpha} \quad ...(i)$$

In right ΔOAC,

$$\tan \beta = \frac{y + h}{x}$$

$$\Rightarrow x = \frac{(y + h)}{\tan \beta} \quad ...(ii)$$

From (i) and (ii),

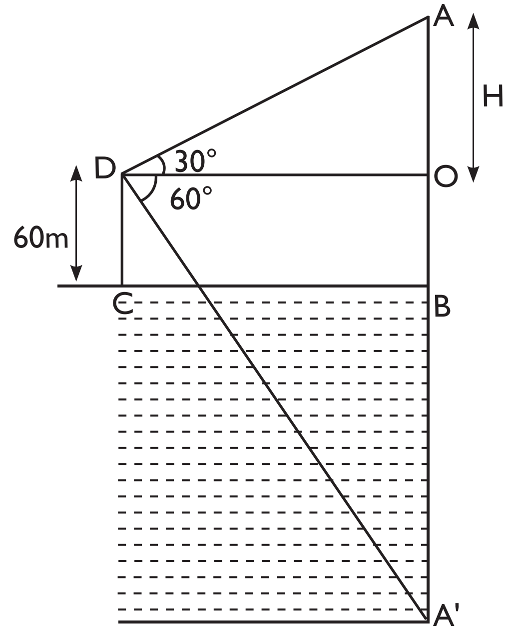
$$\frac{y}{\tan \alpha} = \frac{(y + h)}{\tan \beta}$$

$$y(\tan \beta - \tan \alpha) = h \tan \alpha$$

$$y = \frac{h \tan \alpha}{(\tan \beta - \tan \alpha)}$$

Thus, the height of the tower is $\dfrac{h \tan \alpha}{(\tan \beta - \tan \alpha)}$

26.



Let AO = H

CD = OB = 60 m

A'B = AB = 60 + H

In ΔAOD,

$$\tan 30° = \frac{AO}{OD} = \frac{H}{OD}$$

$$H = \frac{OD}{\sqrt{3}}$$

$$OD = \sqrt{3}H$$

Now, In $\Delta A'OD$,

$$\tan 60° = \frac{OA'}{OD} = \frac{OB + BA'}{OD}$$

$$\sqrt{3} = \frac{60 + 60 + H}{\sqrt{3}H} = \frac{120 + H}{\sqrt{3}H}$$

$$\Rightarrow 120 + H = 3H$$

$$\Rightarrow 2H = 120$$

$$\Rightarrow H = 60\text{m}$$

Thus, height of the cloud from the surface of the lake = AB + A'B = 60 + 60 = 120 m.

27. $\dfrac{AB}{AD} = \tan 30° = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \dfrac{h}{x+y} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow x + y = \sqrt{3}\,h$

$\therefore x = (x + y - y)$

$= \sqrt{3}\,h - h$

$= h(\sqrt{3} - 1)$

Now, $h(\sqrt{3} - 1)$ is covered in 12 min.

So, h will be covered in $\rightarrow$

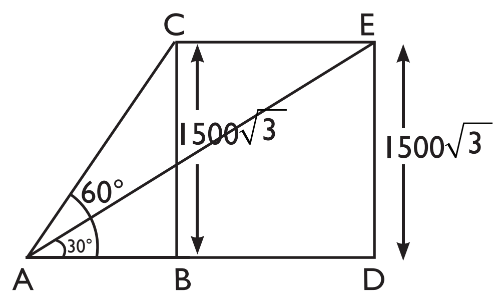
$\left[\dfrac{12}{h(\sqrt{3}-1)} \times h\right]$

$= \dfrac{12}{(\sqrt{3}-1)}$ min

$= \left(\dfrac{1200}{73}\right)$ min

= 16 min, 23 sec

28. Let A be the point of observation, C and E be the two points of the plane. It is given that after 15 seconds angle of changes from 60° to 30°.



i.e. $\angle BAC = 60°$ and $\angle DAE = 30°$. It is also given that height of the jet plane is $1500\sqrt{3}$ m.

i .e. $CB = 1500\sqrt{3}$

[Since jet plane is flying at constant height, therefore, $CB = ED = 1500\sqrt{3}$ m]

$\tan 60° = \dfrac{BC}{AB}$

$\Rightarrow \sqrt{3} = \dfrac{1500\sqrt{3}}{AB}$

$\Rightarrow AB = \dfrac{1500\sqrt{3}}{\sqrt{3}}$

$\Rightarrow AB = 1500\sqrt{3}$ m ...(i)

In right triangle ADE, we have

$\tan 30° = \dfrac{DE}{AD}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{DE}{AB + BD}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{1500\sqrt{3}}{AB + BD}$

$\Rightarrow AB + BD = 1500\sqrt{3} \times \sqrt{3}$

$\Rightarrow AB + BD = 4500$ ...(ii)

Putting the value of (i) in (ii), we get

1500 + BD = 4500

$\Rightarrow BD = 3000$

$\because$ Distance travelled in 15 sec

= CE = BD = 3000 metres,

Now, speed of plane (m/s) = $\dfrac{3000}{15}$ = 200 m/s

Now, speed of plane (km/s) = $\dfrac{200}{1000} \times 3600$

= 720 km/hr

WORKSHEET 2

Section A

1. Let AB be the height of the tower and C be the point.

In right ΔABC,

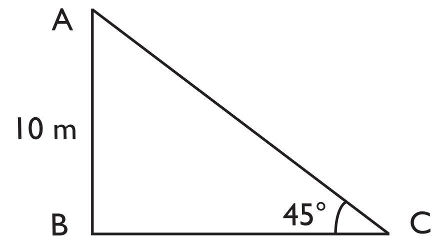
$\tan 30° = \dfrac{AB}{BC}$

AB = BC tan 30°

$AB = \dfrac{20}{\sqrt{3}} = 11.56$m

Therefore, the height of the tower is 11.56m.

2.



Let AB and AC denotes the vertical role and wire respectively.

In $\triangle ABC$,

$\sin 45° = \frac{AB}{AC}$

$\frac{1}{\sqrt{2}} = \frac{10}{AC}$

$\therefore \quad AC = 10\sqrt{2}$ m

3. $BD = AB - AD$

$= 6 - 2.54$

$= 3.46$ m

In $\triangle CBD$,

$\sin 60° = \frac{BD}{CD}$

$\frac{\sqrt{3}}{2} = \frac{3.46}{CD}$

$\therefore \quad CD = \frac{3.46 \times 2}{\sqrt{3}}$

$= \frac{3.46 \times 2}{1.73}$

$= \frac{6.92}{1.732}$

$= 4$ m

4. Let AB = 10 = Height of Pole

And AD be the length of the wire

From $\triangle ABD$,

$\sin 45° = \frac{AB}{AD} \Rightarrow \frac{1}{\sqrt{2}} = \frac{10}{AD} \Rightarrow AD = 10\sqrt{2}$

$\Rightarrow 10 \times 1.414$ (Take $\sqrt{2} = 1.414$)

$= 14.14$m

5. Distance from the foot of ladder to wall = 1.5cm

Angle made by ladder is 60°

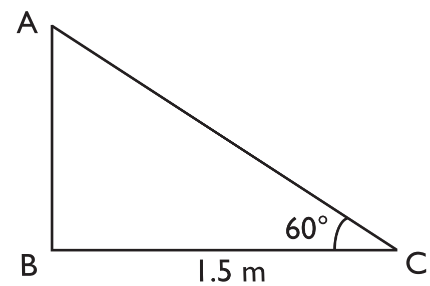
Consider tan 60° =

$$\frac{\text{Height of wall}}{\text{Distance from foot of ladder to wall}}$$

$\sqrt{3} = \frac{\text{Height of wall}}{1.5}$

Height of wall = $1.5\sqrt{3}$

6.



Let AB denotes the wall and AC denotes the ladder.

In $\triangle ABC$,

$\tan 60° = \frac{AB}{BC}$

$\sqrt{3} = \frac{AB}{1.5}$

$\therefore \quad AB = 1.5\sqrt{3}$ m

7. Here is the position of balloon

Now, in $\triangle ABC$,

$\sin 60° = \frac{AB}{AC}$

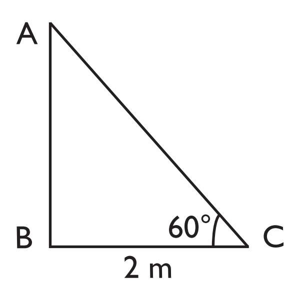
$AB = AC \sin 60°$

$= 215 \times \frac{\sqrt{3}}{2}$

$= 186$ m

$\Rightarrow$ Height of the balloon from the ground is 186 m.

8.



Let AB be the wall and AC be the ladder.

We have,

BC = 2 m and $\angle ACB = 60°$

In ΔABC,

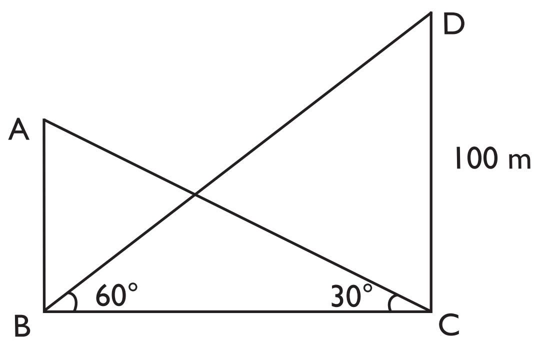
$\cos 60° = \frac{BC}{AC}$

$\Rightarrow \frac{1}{2} = \frac{2}{AC}$

AC = 4 m

Section B

9.



let AB and CD denotes building and tower respectively.

In ΔBCD,

$\tan 60° = \frac{CD}{BC}$

$\sqrt{3} = \frac{100}{BC}$

$BC = \frac{100}{\sqrt{3}}$

$= \frac{100}{3}\sqrt{3}$ m

In ΔCBA,

$\tan 30° = \frac{AB}{BC}$

$\frac{1}{\sqrt{3}} = \frac{AB}{\frac{100\sqrt{3}}{3}}$

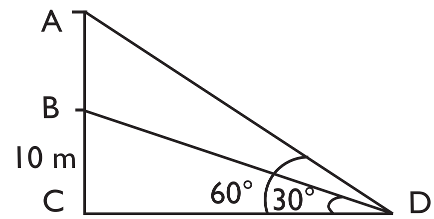
$\frac{1}{\sqrt{3}} = \frac{3\ AB}{100\sqrt{3}}$

$3\ AB = 100$

$AB = \frac{100}{3}$ m

So, height of building $= \frac{100}{3}$ m

10.



Let BC and AB denotes the building and tower respectively.

In ΔBCD

$\tan 30° = \frac{BC}{CD}$

$\frac{1}{\sqrt{3}} = \frac{10}{CD}$

$\therefore\ CD = 10\sqrt{3}$ m

In ΔACD,

$\tan 60° = \frac{AC}{CD}$

$\sqrt{3} = \frac{10 + AB}{10\sqrt{3}}$

$30 = AB + 10$

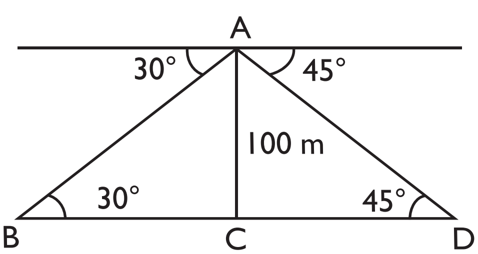
$AB = 30 - 10$

$= 20$ m

So, height of tower = AB

= 20 m

11.



Let AC denotes the tower and the two buses be at points B and D respectively.

To find : BD

In ΔABC,

$$\tan 30° = \frac{AC}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{100}{BC}$$

$$\therefore \quad BC = 100\sqrt{3}$$

In ΔACD,

$$\tan 45° = \frac{AC}{CD}$$

$$1 = \frac{100}{CD}$$

$$CD = 100 \text{ m}$$

$$\text{So, } BD = BC + CD$$

$$= 100\sqrt{3} + 100$$

$$= 100(\sqrt{3} + 1) \text{ m}$$

12. In the first figure and from triangle BCD

$$\sin 30° = \frac{BC}{BD}$$

$$\Rightarrow \frac{1}{2} = \frac{1.5}{BD}$$

$$\Rightarrow BD = 1.5 \times 2$$

$$\Rightarrow BD = 3$$

So length of slide for child below 5 years = 3m

Again in the second figure and from triangle BCD

$$\sin 60° = \frac{BC}{BD}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{3}{BD}$$

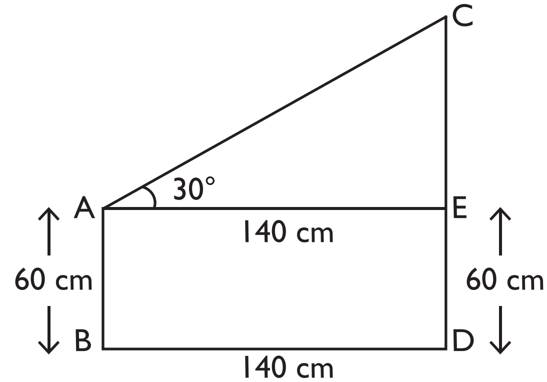
$$\Rightarrow BD = \frac{(3 \times 2)}{\sqrt{3}}$$

$$\Rightarrow BD = \frac{(\sqrt{3} \times \sqrt{3} \times 2)}{\sqrt{3}}$$

$$\Rightarrow BD = 2\sqrt{3}$$

So length of slide for child greater than 5 years $= 2\sqrt{3}$ m

13.



Let AB be the height of second tower and CD be the height of first tower.

Given, BD = AE = 140 m

And AB = DE = 60 m

In ΔAEC,

$$\tan 30° = \frac{\text{Perpendicular}}{\text{Base}} = \frac{CE}{AE}$$

$$\tan 30° = \frac{CE}{140}$$

$$\frac{1}{\sqrt{3}} = \frac{CE}{140}$$

$$CE = \frac{140}{\sqrt{3}}$$

$$CE = \frac{140 \times \sqrt{3}}{(\sqrt{3} \times \sqrt{3})}$$

[Rationalising the denominator]

$$CE = \frac{140\sqrt{3}}{3}$$

Height of the first tower CD = CE + DE

$$= \frac{140\sqrt{3}}{3} + 60$$

$$= \frac{(140 \times 1.73)}{3} + 60 \qquad [\sqrt{3} = 1.73]$$

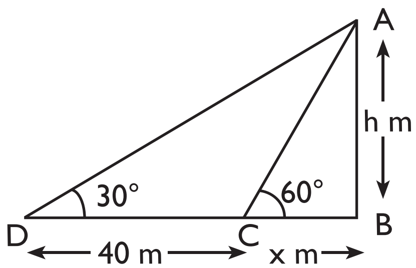
$$= \frac{242.2}{3} + 60$$

$$= 80.73 + 60$$

$$= 140.73 \text{ m}$$

Height of the first tower (CD)=140.73 m

14. AB is the tower and BC is the length of the shadow when the Sun's altitude is 60°, i.e., the angle of elevation of the top of the tower from the tip of the shadow is 60° and DB is the length of the shadow when the angle of elevation is 30°.



Now, let AB be h m and BC be x m. According to the question, DB is 40 m longer than BC.

So, BD = (40 + x) m

Now, we have two triangles ABC and ABD.

In ΔABC,

$\tan 60° = \frac{AB}{BC}$ or $\sqrt{3} = \frac{h}{x}$

$\Rightarrow x\sqrt{3} = h$ ...(i)

In ΔABD,

$\tan 30° = \frac{AB}{BD}$

i.e., $\frac{1}{\sqrt{3}} = \frac{h}{x+40}$ ...(ii)

Using (i) in (ii), we get $(x\sqrt{3})\sqrt{3} = x + 40$,

i.e., $3x = x + 40$

i.e., $x = 20$

So, $h = 20\sqrt{3}$ [From (i)]

Therefore, the height of the tower is $20\sqrt{3}$ m.

Section C

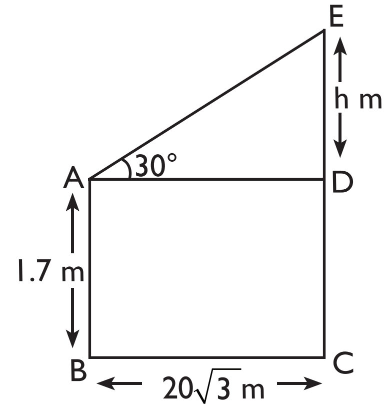
15. Let AB be the height of the observer and EC be the height of the tower.

Given:

AB = 1.7 m $\Rightarrow$ CD = 1.7 m

BC = $20\sqrt{3}$ m

Let ED be h m.



In ΔADE,

$\tan 30° = \frac{ED}{AD}$

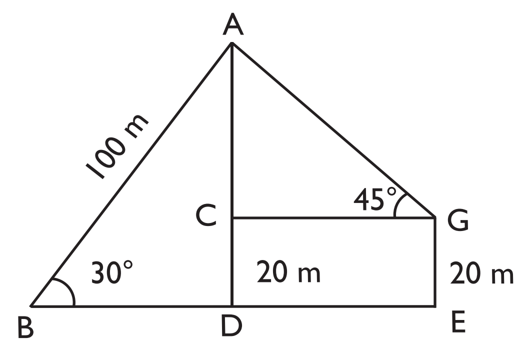
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{20\sqrt{3}}$

$\Rightarrow h = 20$ m

$\therefore$ EC = ED + DC = (h + 1.7)m = 21.7 m

Hence, the height of the tower is 21.7 m.

16.



Distance between girl and bird is $30\sqrt{2}$ m = 42.43 m.

Given: Distance between boy and bird = 100 m

Height of building = 20 m

Angle of elevation boy = 30°

Angle of elevation girl = 45°

To find: Distance between girl and bird

In ΔABC

Using trigonometric ratio, we get

$\sin 30° = \frac{AC}{AB}$

$$\frac{1}{2} = \frac{AC}{100}$$

$$AC = \frac{100}{2}$$

AC = 50 m

$\Rightarrow$ AC = FA + CF (from figure)

50 = FA + 20 ($\because$ CF = ED = 20 m)

FA = 50 – 20

FA = 30 m

Now, In ΔAEF

Using trigonometric ratio, we get

$$\sin 45° = \frac{FA}{AE}$$

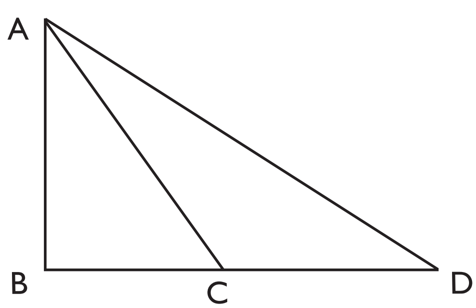
$$\frac{1}{\sqrt{2}} = \frac{30}{AE}$$

$AE = 30\sqrt{2} = 42.43m$

Therefore, distance between girl and bird is

$30\sqrt{2} = 42.43m$

17.



We are given that from the top of tower 96 m high, the angles of depression of two cars on a road at the same level as the base of the tower and on the same side of it are theta and phi, where tan theta = $\frac{3}{4}$ and tan phi = $\frac{1}{3}$

In the figure drawn above, let $\angle ACB = \tan\theta = \frac{3}{4}$ and $\angle ADB = \tan\phi = \frac{1}{3}$ and also the height of the tower = AB = 96 m.

Now, as we know that $\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$

So, in ΔABC,

$$\tan\theta = \frac{AB}{BC}$$

$$\frac{3}{4} = \frac{96}{BC}$$

$$BC = \frac{96 \times 4}{3}$$

$BC = 32 \times 4 = 128$ m

Now, in ΔABD,

$$\tan\phi = \frac{AB}{BD}$$

$$\frac{1}{3} = \frac{96}{BD}$$

$$BD = \frac{96 \times 3}{1}$$

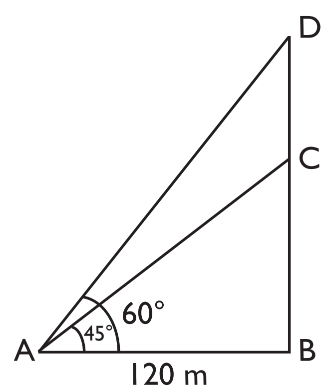
BD = 288 m

So, the distance between two cars =

CD = BD – BC

= 288 m – 128 m = 160 m

18.



Height of the flagstaff = CD

According to the figure,

$$\tan 45° = \frac{CB}{120}$$

$$\Rightarrow 1 = \frac{CB}{120}$$

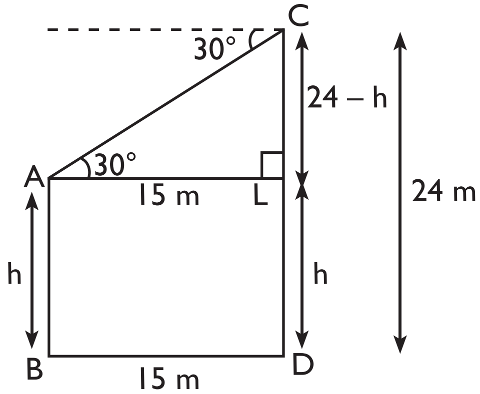
$\therefore$ CB = 120 m

$$\tan 60° = \frac{BD}{120}$$

$$\Rightarrow \sqrt{3} = \frac{BD}{120}$$

$\therefore$ BD = 120 $\times$ 1.73 = 207.6 m

$\therefore$ Height of the flagstaff = CD = 207.6 – 120 = 87.6 m

19. 

Let AB and CD be two poles, where CD = 24 m.

It is given that angle of depression of the top of the pole AB as seen from the top of the pole CD is 30° and horizontal distance between the two poles is 15 m.

$\therefore \angle CAL = 30°$ and BD = 15 m.

To find: Height of pole AB

Let the height of pole AB be h m.

AL = BD = 15 m and AB = LD = h

Therefore, CL = CD – LD = 24 – h

Consider right ΔACL:

$\tan \angle CAL = \frac{\text{Perpendicular}}{\text{Base}} = \frac{CL}{AL}$

$\Rightarrow \tan 30° = \frac{24 - h}{15}$

$\Rightarrow 24 - h = \frac{15}{\sqrt{3}}$

$\Rightarrow 24 - h = 5\sqrt{3}$

$\Rightarrow h = 24 - 5\sqrt{3}$

$\Rightarrow h = 24 - 5 \times 1.732$ [Taking $\sqrt{3} = 1.732$]

$\Rightarrow h = 15.34$

Therefore, height of the pole AB = h m = 15.34 m.

20. Given AB is the tower.

P and Q are the points at distance of 4m and 9m respectively.

From fig, PB = 4m, QB = 9m.

Let angle of elevation from P be α and angle of elevation from Q be β.

Given that α and β are supplementary. Thus, $\alpha + \beta = 90$

In ΔABP,

$\tan \alpha = \frac{AB}{BP}$ ...(i)

In ΔABQ,

$\tan \beta = \frac{AB}{BQ}$

$\tan (90 - \alpha) = AB/BQ$ (Since, $\alpha + \beta = 90$)

$\cot \alpha = \frac{AB}{BQ}$

$1/\tan \alpha = \frac{AB}{BQ}$

So, $\tan \alpha = \frac{BQ}{AB}$ ...(ii)

From (i) and (ii)

$\frac{AB}{BP} = \frac{BQ}{AB}$

$AB^2 = BQ \times BP$

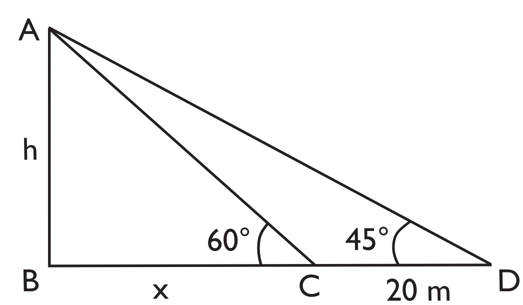
$AB^2 = 4 \times 9$

$AB^2 = 36$

Therefore, AB = 6.

Hence, height of tower is 6m.

Section D

21. 

From the figure h = length of the tower

From the ΔABC

$\Rightarrow h = 1.732\,x$ ------- (1)

From the ΔABD

$\Rightarrow h = 20 + x$ ------ (2)

Equating equation (1) = equation (2)

$1.732\,x = x + 20$

$\Rightarrow 0.732\,x = 20$

$\Rightarrow x = 27.32$ m

Thus the height of the tower is given by

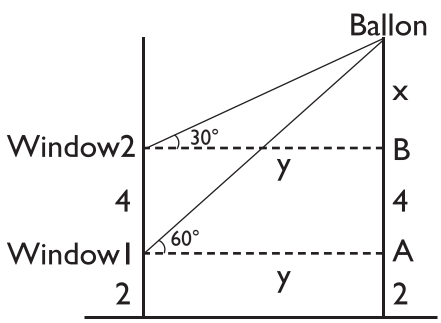
$h = 1.732\,x$

$\Rightarrow h = 1.732 \times 27.32$

$\Rightarrow h = 47.32$ m

This is the value of height of the tower.

22.



$$\tan 30° = \frac{x}{y}$$

$y = x\sqrt{3}$ ...(i)

$$\tan 60° = \frac{x+4}{y}$$

$y = \frac{x+4}{\sqrt{3}}$ ...(ii)

Equating (i) and (ii)

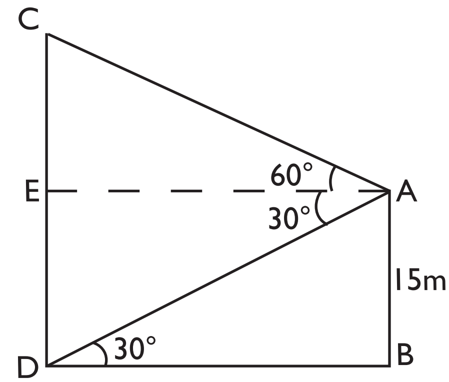
$$x\sqrt{3} = \frac{x+4}{\sqrt{3}}$$

$3x = 3 + 4$

$x = 2$

Height of ballon from the ground = 2 + 4 + 2 = 8 m

23.



AB = ED = 15 m

AE = BD

In ΔADB,

$$\tan 30° = \frac{15}{DB}$$

$DB = 15\sqrt{3}$ m

Or, $AE = 15\sqrt{3}$ m

In ΔACE,

$$\tan 60° = \frac{CE}{AE}$$

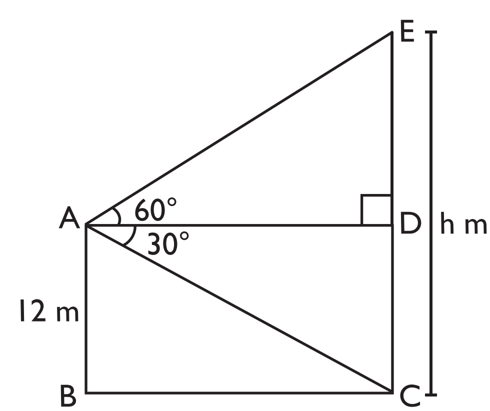
$CE = 15\sqrt{3}$ m = 45 m

So, CD = CE + ED = 60 m

AB is the building.

CD is the tower.

24.



Let AB be the deck of the ship. AB = 10 cm

Suppose CE be the cliff. C and E are the top and bottom of the cliff

Let CE = h m

Given, $\angle EAD = 60°$ and $\angle DAC = 30°$

CD = AB = 12 m

$\therefore$ DE = CE – CD = (h – 12)m

In ΔADE,

$$\tan 60° = \frac{DE}{AD} \qquad \left(\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}\right)$$

$$\therefore \sqrt{3} = \frac{(h-12)m}{AD}$$

$$\Rightarrow AD = \frac{(h-12)}{\sqrt{3}}m = \frac{(h-12)\sqrt{3}}{3}m \qquad ...(i)$$

In ΔADC,

$$\tan 30° = \frac{CD}{AD}$$

$$\therefore \frac{1}{\sqrt{3}} = \frac{12m}{AD}$$

$$\Rightarrow AD = 12\sqrt{3}\text{ m}$$

From (i) and (ii), we have

$$\frac{(h-12)\sqrt{3}}{3}m = 12\sqrt{3}\text{ m}$$

$$\Rightarrow h = 36 + 12 = 48$$

Height of the cliff = 48 m

Distance of cliff from the ship = BC = AD = $12\sqrt{3}$ m

Chapter 10

Circles

MULTIPLE CHOICE QUESTIONS

1. $C - r = 37$

$C = 37 - r$

$C = 2\prod r$

$37 + r = \frac{2 \times 22}{7 \times r}$

$37 + r = \frac{44r}{7}$

$37 = \frac{44r}{7 - r} = \frac{37r}{7}$

$r = \frac{37 \times 7}{37}$

$r = 7$ cm

$C = \frac{2 \times 22}{7 \times 7} = 44$ cm

Option (b)

2. $\pi r_1^2 + \pi r_2^2 = \pi r^2$

$r_1^2 + r_2^2 = r^2$

$5^2 + (12)^2 = r^2$

Option (b)

3. Distance covered in one revolution

$= 2\pi r$

$= 2 \times \frac{22}{7} \times \frac{35}{2}$

$= 110$ cm

Option (b)

4.

Diagonal = BD $= \sqrt{(4 - 0)^3 + (0 - 3)^3}$

$= \sqrt{16 + 9}$

$= \sqrt{25}$

$= 5$

Option (a)

5. Radius $= \frac{18}{2} = 9$ cm

Perimeter $= 2\pi r$

$= 2\pi r\ (9)$

$= 18\pi r$ cm

Option (c)

WORKSHEET 1

Section A

1. Arc length $= \frac{\theta}{360°} 2\pi r$

$3\pi = \frac{\theta}{360} 2\pi \times 6$

$\Rightarrow 3\pi = \frac{\theta\pi}{30}$

$\Rightarrow \theta = \frac{3\pi \times 30}{\pi}$

$= 90°$

2. Diameter $= 14$ cm

$\Rightarrow$ radius $= \frac{14}{2}$ 7 cm

Perimeter of semi – circle protractior

$= 2r + \frac{1}{2}(2\pi r)$

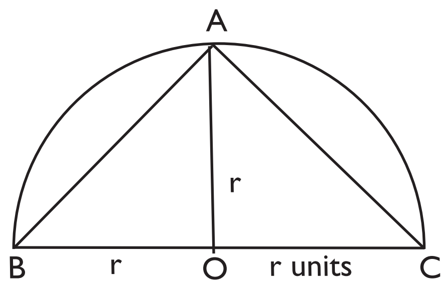
$= 2r + \pi r$

$= 2(7) + \frac{22}{7} \times 7$

$= 14 + 22$

$= 36$ cm

3.



Area of $\Delta BAC = \frac{1}{2} \times BC \times AO$

$= \frac{1}{2} \times 2r \times r$

$= r^2$ sq.units

4. Perimeter of sector

$= 2r + \frac{\theta}{360^\circ} 2\pi r$

$= 2r\left(1 + \frac{\pi\theta}{360^\circ}\right)$

$= 2(10.5)\left(1 + \frac{22}{7} \times \frac{60}{360}\right)$

$= 21\left(1 + \frac{11}{21}\right)$

$= \frac{21 \times 32}{21}$

$= 32$ cm

5. $r = 10$ cm

$\theta = 108^\circ$

area of sector $= \frac{\theta}{360^\circ} \pi r^2$

$= \frac{108}{360} \times \pi(100)$

$= 3\pi 10$

$= 30\pi$ cm^2

6. Distance covered in one revolution

$= 2\pi r$

$= 2 \times \frac{22}{7} \pi$

Number of revolutions in covering a distance of x metres.

$= \frac{x}{2 \times \frac{22}{7} \times r}$

$= \frac{7x}{44r}$

7. Let the diameter and a side be x units.

So, radius of circle $= \frac{x}{2}$ units

$\therefore$ Area of circle $= \pi\left(\frac{x}{2}\right)^2$

$= \frac{\pi x^2}{4}$

Area of equilateral triangle

$= \frac{\sqrt{3}}{4}(\text{side})^2$

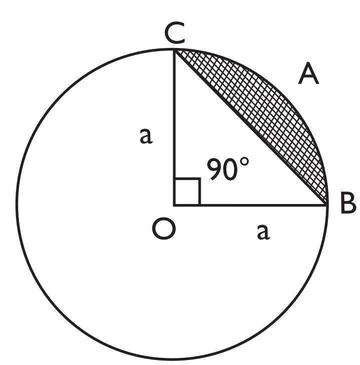
$= \frac{\sqrt{3}}{4} x^2$

$\therefore \frac{\text{Area of circle}}{\text{Area of equilateral triangle}}$

$= \frac{\frac{\pi x^2}{4}}{\frac{\sqrt{3}}{4} x^2}$

$= \frac{\pi}{\sqrt{3}}$

8.



Perimeter of segment ABC

= BC + length of arc $\overset{\frown}{BAC}$

In ΔBOC,

$BC^2 = OC^2 + OB^2$

(By Pythagoras theorem)

$BC^2 = a^2 + a^2$

$= 2a^2$

$\therefore$ BC = $\sqrt{2}\,a$

Also length of arc $\overset{\frown}{BAC}$

$= \frac{90}{360} \times 2 \times \frac{22}{7} \times a$

$= \frac{11a}{7}$

So, Perimeter of segment ABC

$= \sqrt{2}\,a + \frac{11a}{7}$

Section B

9. We know that AD = AF

BD = BE

CE = CF

Let AD = AF = x

BD = BE = y

CE = CF = z

Then x + y = 12

y + z = 8

x + z = 10

On Solving above equation we get x = 7, y = 5, z = 3

So AD = 7, BE = 5, CF = 3

10. BP = AP = 5 cm

(The lengths of tangents drawn from an external point to a circle are equal.)

$\therefore \angle PAB = \angle PBA$

(Angle opposite to equal sides are equal.)

In PAB,

$\angle P + \angle PAB + \angle PBA = 180°$

(Angle sum property)

$60° + 2\angle PAB = 180°$

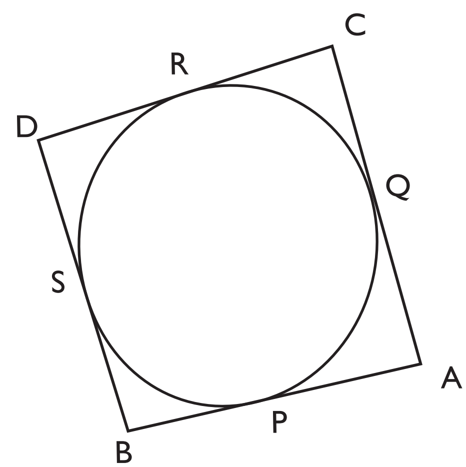
$2\angle PAB = 120°$

$\angle PAB = 60°$

$\therefore \angle PAB = \angle PBA = 60°$

$\therefore$ AB = PA = PB = 5 cm

11.



~~AP~~ + ~~PB~~ + ~~CR~~ + ~~RD~~ = ~~BQ~~ + ~~CQ~~ + ~~AS~~ + ~~DS~~

As we know that the length of tangents drawn from an external point to a circle are equal,

AP = AS (i)

BP = BQ (ii)

CR = CQ (iii)

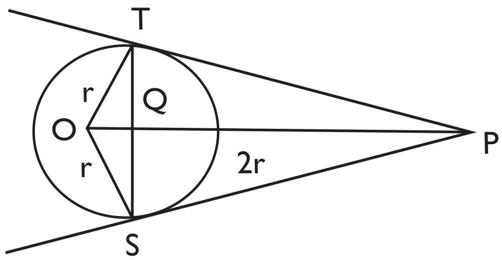
DR = DS (iv)

On adding both sides of (i), (ii), (iii) and (iv), we get

AP + BP + CR + DR = AS + BQ + CQ + DS

$\therefore$ (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

AB + CD = BC + AD

12. 

$\angle TOP = \theta$

As we know that radius is perpendicular to the tangent at the point of contact.

$\angle OTP = 90°$

So, in ΔOTP

$\cos\theta = \frac{OT}{OP} = \frac{r}{2r} = \frac{1}{2}$

$\therefore \cos = \frac{1}{2} = \cos 60°$

$\Rightarrow \theta = 60°$

$\therefore \angle TOS = 60° + 60° = 120°$

As OT = OS

$\Rightarrow \angle OTS = \angle OST$

(Angles opposite to equal sides are equal.)

In $\angle OTS$,

$\angle TOS + \angle OTS + \angle OST = 180°$

$120° + 2\angle OTS = 180°$

$2\angle OTS = 60°$

$\angle OTS = 30°$

$\therefore \angle OTS = \angle OST = 30°$

13. In ΔOTP and ΔOSP

OT = OS (Radii of same circle)

OP = PO (Common)

PT = PS

(The lengths of tangents drawn from an external point to a circle are equal.)

$\therefore \Delta OTP \cong \Delta OSP$ (SSS)

$\therefore \angle OPS = \angle OPT$

$= \frac{1}{2}\angle TPS$ (CPCT)

$= \frac{1}{2}$ (120°)

$= 60°$

In ΔOSP,

$OS \perp PS$

(Radius is perpendicular to the tangent at the point of contact.)

$\cos(\angle OPS) = \frac{PS}{OP}$

$\Rightarrow \cos 60° = \frac{PS}{OP}$

$\Rightarrow \frac{1}{2} = \frac{PS}{OP}$

$\Rightarrow OP = 2\,PS$

14. In ΔOAP,

OA = 6 cm

AP = 8 cm

$\therefore OP^2 = OA^2 + AP^2$

(By Pythagoras theorem)

$= 6^2 + 8^2$

$= 36 + 64$

$= 100$

$\Rightarrow OP = 10$ cm

Now, In ΔOBP,

$OP^2 = OB^2 + BP^2$

$10^2 = 4^2 + BP^2$

(By Pythagoras theorem)

$100 = 16 + BP^2$

$BP^2 = 100 - 16$

= 84

$\therefore$ BP = $2\sqrt{21}$ cm

15. $\angle OAC = 90°$ (as radius $\perp$ tangent)

$\angle BOC = \angle OAC + \angle ACO$

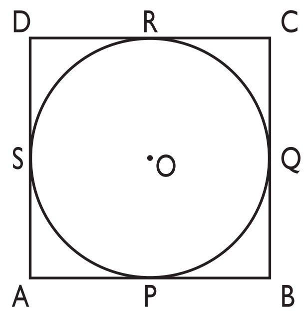
(Exterior angle property)

$130° = 90° + \angle ACO$

$\angle ACO = 130° - 90° = 40°$

Section C

16.



A rhombus is a parallelogram with all equal sides.

In ∥gm ABCD

AB = CO and AD = BC

Hence AP = AS; BP = BQ; CR = CQ; DR = DS

Adding we get AP + BP + CR + DR = AS + BQ + CQ + DS

(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

AB + CD = AD + BC

AB + AB = AD + AD

2 AB = 2 AD

So AB = AD and AB = CD and AD = BC

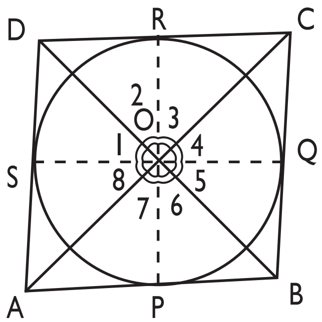
So AB = CD = AD = CD

So ABCD is ∥gm with equal sides.

$\therefore$ ABCD is a rhombus.

$\therefore$ Proved.

17.



Const: Join OP, OQ, OR and OS.

Proof: Since, the two tangents drawn from an external point to a circle subtend equal angles at the centre.

$\therefore$ $\angle 1 = \angle 2, \angle 3 = \angle 4, \angle 5 = \angle 6, \angle 7 = \angle 8$

Since sum of all the angles subtended at a point is 360°.

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8$

$= 360°$

$\Rightarrow 2\angle 2 + 2\angle 3 + 2\angle 6 + 2\angle 7 = 360°$

$\Rightarrow 2(\angle 2 + \angle 3 + \angle 6 + \angle 7) = 360°$

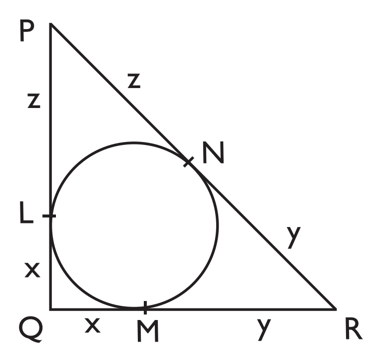
$\Rightarrow \angle 2 + \angle 3 + \angle 6 + \angle 7 = 180°$

$\Rightarrow (\angle 6 + \angle 7) + (\angle 2 + \angle 3) = 180°$

$\Rightarrow \angle AOB + \angle COD = 180°$

Similarly, we can prove $\angle AOD + \angle BOC = 180°$

18.



QL = QM

RM = RN

PL = PN

We know that the tangents drawn to a circle from an external point are equal in length.

Let QL = QM = x

Let RM = RN = y

Let PL = PN = z

Consider PQ + QR + PR = 60

$\Rightarrow x + z + x + y + z + y = 60$

$\Rightarrow 2x + 2y + 2z = 60$

$\Rightarrow x + y + z = 30$

PQ = 20

$x + z = 20$

$\therefore$ RN = 10 cm

Also, QR = 16

$x + y = 16$

$\therefore z = 30 - (x + y)$

$= 30 - 16$

$= 14$ cm

$\therefore$ PL = 14 cm

Again, PR = 24

$y + z = 24$

$\therefore x = 30 - (y + z)$

$= 30 - 24$

$= 6$

$\therefore$ QM = 6 cm

19. Given: a circle with centre o to which XY and XY' are tangents.

TP – $\angle AOB = 90°$

Const: Join OY, OY' and OC

Proof: In ΔOYA and ΔOCA

OY = OC [radii]

OA=OA [common]

AY = AC [tangents]

$\Rightarrow$ By SSS

$\Delta OYA \approx \Delta OCA$

$\Rightarrow \angle OY'A = \angle OCA$ [CPCT]---------------(1)

|| ly $\Delta OY'B \approx \Delta OCB$

$\Rightarrow \angle Y'BO = \angle CBO$ [CPCT]---------------(2)

$\angle YAB + \angle Y'BA = 180°$[co-interior angles]

$2\angle OAB + 2\angle OBA = 180°$ ---[from (1) and (2)]

$2(\angle OAB + \angle OBA) = 180°$

$\angle OAB + \angle OBA = 90°$---------------------(3)

In ΔAOB

$\angle OAB + \angle OBA + \angle AOB = 180°$
[Angle sum property of a triangle]

$90° + \angle AOB = 180°$

$\angle AOB = 180° - 90°$

$\angle AOB = 90°$

$\therefore$ Hence Proved

20. GIVEN: a circle with centre o to which XY and XY' are tangents

TP – $\angle AOB = 90°$

CONST.: join OY, OY' and OC

PROOF: In ΔOYA and ΔOCA

OY = OC [radii]

OA=OA [common]

AY = AC [tangents]

$\Rightarrow$ By SSS

$\Delta OYA \approx \Delta OCA$

$\Rightarrow \angle OY'A = \angle OCA$ [CPCT]---------------(1)

|| ly $\Delta OY'B \approx \Delta OCB$

$\Rightarrow \angle Y'BO = \angle CBO$ [CPCT]-----------------(2)

$\angle YAB + \angle Y'BA = 180°$[co-interior angles]

$2\angle OAB + 2\angle OBA=180°$-------[from (1) and (2)]

$2(\angle OAB + \angle OBA) = 180°$

$\angle OAB + \angle OBA = 90°$---------------------(3)

In ΔAOB

$\angle OAB + \angle OBA + \angle AOB = 180°$

[Angle sum property of a triangle]

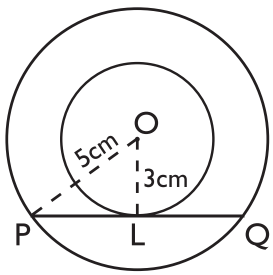
$90° + \angle AOB = 180°$

$\angle AOB = 180° - 90°$

$\angle AOB = 90°$

$\therefore$ Hence Proved

21.



Let O be the common centre of the two concentric circle.

Let PQ be a chord of the larger circle which touches the smaller circle at L.

Join OL and OP.

Since, the tangent at any point of a circle is perpendicular to the radius through the point jof contact.

Therefore,

$\angle OLP = 90°$

Now,

In ΔOLP, we have

$OP^2 = OL^2 + PL^2$

[Using Pythagoras theorem]

$\Rightarrow (5)^2 = (3)^2 + PL^2$

$\Rightarrow 25 = 9 + PL^2$

$\Rightarrow PL^2 = 16$

$\Rightarrow PL = 4$ cm

Since, the perpendicular from the centre of a circle to a chord bisects the chord.

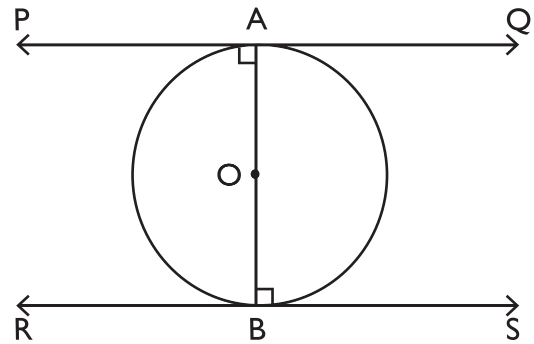
Therefore,

PL = LQ = 4 cm

$\therefore PQ = 2\,PL = 2 \times 4 = 8$ cm

Hence, the required length = 8 cm.

22.



Let AB be the diameter of a circle, with centre O. The tangents PQ and RS are drawn at point A and B, respectively.

We know that a tangent at any point of a circle is perpendicular to the radius through the point of contact.

$\therefore OA \perp PQ$ and $OB \perp RS$

$\Rightarrow \angle OBR = 90°$

$\angle OBS = 90°$

$\angle OAP = 90°$

$\angle OAQ = 90°$

We can observe the following:

$\angle OBR = \angle OAQ$ and $\angle OBS = \angle OAP$

Also, these are the pair of alternate interior angles.

Since alternate angles are equal, the lines PQ and RS are parallel to each other.

Hence, proved.

Section D

23. Clearly $\angle OPT = 90°$

Applying Pythagoras in ΔOTP, we have

$\Rightarrow OT^2 = OP^2 + PT^2$

$\Rightarrow 13^2 = 5^2 + PT^2$

$\Rightarrow PT^2 = 169 - 25 = 144$

$\Rightarrow PT = 12$ cm

Since, lengths of tangents drawn from a point to a circle are equal. Therefore,

$AP = AE = x$(say)

$\Rightarrow AT = PT - AP = (12 - x)$ cm

Since AB is the tangent to the circle E. Therefore, $OE \perp AB$.

$\Rightarrow \angle OEA = 90°$

$\Rightarrow \angle AET = 90°$

$\Rightarrow AT^2 = AE^2 + ET^2$ [

[Applying Pythagoras Theorem in ΔAET]

$\Rightarrow (12 - x)^2 = x^2 + (13 - 5)^2$

$\Rightarrow 144 - 24x + x^2 = x^2 + 64$

$\Rightarrow 24x = 80$

$\Rightarrow x = \frac{10}{3}$ cm

Similarly, $BE = \frac{10}{3}$ cm

$\therefore AB = AE + BE = \left(\frac{10}{3} + \frac{10}{3}\right)$ cm $= \frac{20}{3}$ cm

24. (i) $PA . PB = (PN - AN)(PN + BN)$

$= (PN - AN)(PN + AN)$ (As $AN = BN$)

$= PN^2 - AN^2$

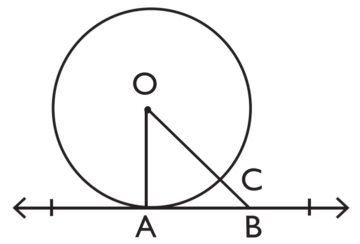
(ii) $PN^2 - AN^2 = (OP^2 - ON^2) - AN^2$ (As $ON \perp PN$)

$= OP^2 - (ON^2 + AN^2)$

$= OP^2 - OA^2$ (As $ON \perp AN$)

$= OP^2 - OT^2$ (As $OA = OT$)

25.



Given: A circlr C (0, r) and a tangent l at point A.

To prove: $OA \perp l$

Construction: Take a point B, other than A. On the tangent l. Join OB. Suppose OB meets the circle in C.

Proof: We know that, among all line segment joining the point O to a point on l, the perpendicular is shortest to l.

OA = OC (Radius of the same circle)

Now, OB = OC + BC

$\therefore OB > OC$

$\Rightarrow OB > OA$

$\Rightarrow OA > OB$

B is an arbitrary point on the tangent l. Thus, OA is shorter than any other line segment joining O to any point on l.

Here, $OA \perp l$

26. We know that $\angle ADO = 90°$ (Since O'D is perpendicular to AC)

$\angle ACO = 90°$ (OC (radius) perpendicular to AC (tangent))

In triangles ADO' and ACO,

$\angle ADO = \angle ACO$ (each 90°)

$\angle DAO = \angle CAO$ (common)

by AA criterion, triangles ADO' and ACO are similar to each other.

$\frac{AO'}{AO} = \frac{DO'}{CO}$

(corresponding sides of similar triangles)

$AO = AO' + O'X + OX$

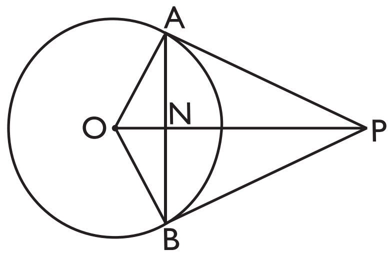
$= 3AO'$ (since $AO' = O'X = OX$ because radii of the two circles are equal)

$$\frac{AO'}{AO} = \frac{AO'}{3AO} = \frac{1}{3}$$

$$\frac{DO'}{CO} = \frac{AO'}{AO} = \frac{1}{3}$$

$$\frac{DO'}{CO} = \frac{1}{3}$$

27.



$OA = 10$ cm

$ON \perp AB$

$AN = NB = \frac{16}{2} = 8$ cm

Pythagoras Theorem

In ΔONA,

$ON^2 + NA^2 = OA^2$

$ON^2 = OA^2 - NA^2$

$ON^2 = 10^2 - 8^2$

$ON^2 = 36$ cm

$ON = 6$ cm

$$\tan\angle AON = \frac{AN}{ON} = \frac{8}{6} = \frac{4}{3}$$

ΔOAP

$$\tan\angle AON = \frac{PA}{OA}$$

$$\frac{4}{3} = \frac{PA}{10}$$

$$\Rightarrow PA = \frac{40}{3} \text{ cm}$$

28. Given $\angle RPQ = 30°$ and PR and PQ are tangents drawn from P to the same circle.

Hence PR = PQ [Since tangents drawn from an external point to a circle are equal in length]

$\therefore \angle PRQ = \angle PQR$ [Angles opposite to equal sides are equal in a Δ]

In ΔPQR

$\angle RQP + \angle QRP + \angle RPQ = 180°$

[Angle sum property of a Δ]

$2\angle RQP + 30° = 180°$

$2\angle RQP = 150°$

$\angle RQP = 75°$

So $\angle RQP = \angle QRP = 75°$

$\angle RQP = \angle RSQ = 75°$

[By Alternate Segment Theorem]

Given, RS || PQ

$\therefore \angle RQP = \angle SRQ = 75°$ [Alternate angles]

$\angle RSQ = \angle SRQ = 75°$

$\therefore$ QRS is also an isosceles triangle.

[Since sides opposite to equal angles of a triangle are equal.]

$\angle RSQ + \angle SRQ + \angle RQS = 180°$

[Angle sum property of a triangle]

$75° + 75° + \angle RQS = 180°$

$150° + \angle RQS = 180°$

$\therefore \angle RQS = 30°$

29. Given: ABCD is a quadrilateral such that $\angle D = 90°$.

BC = 38 cm, CD = 25 cm and BP = 27 cm

BP = BQ = 27 cm

[Tangents from an external point]

BC = 38

$\Rightarrow BQ + QC = 38$

$\Rightarrow 27 + QC = 38$

$\Rightarrow QC = 38 - 27$

$\Rightarrow$ QC = 11 cm

$\therefore$ QC = 11 cm = CR

[Tangents from an external point]

CD = 25 cm

CR + RD = 25

$\Rightarrow$ 11 + RD = 25

$\Rightarrow$ RD = 25 – 11

$\Rightarrow$ RD = 14cm

Also,

RD = DS = 14cm

$\therefore$ OR and OS are radii of the circle.

From tangents R and S, $\angle ORD = \angle OSD = 90°$

Now, ORDS is a square.

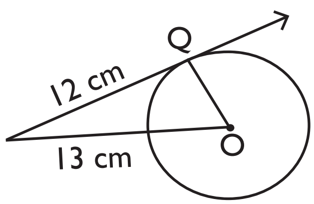
$\therefore$ OR = DS = 14 cm

Thus, radius , r = 14 cm

WORKSHEET 2

Section A

1.



As we know that tangent to a circle is perpendicular to the radius through the point of contact.

$\therefore$ OQ $\perp$ PQ

$\therefore$ $\angle OQP = 90°$

So, In ΔOQP,

$OP^2 = OQ^2 + PQ^2$

$13^2 = OQ^2 + 12^2$

$169 = OQ^2 + 144$

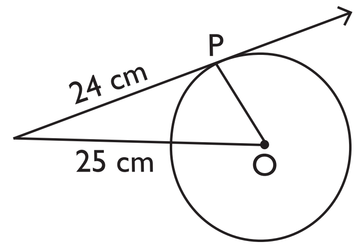
$OQ^2 = 169 - 144$

$= 25$

$\therefore$ OQ = 5 cm

So, radius of circle = 5 cm

2.



As we know that tangent to a circle is perpendicular to the radius through the point of contact.

$\therefore$ OP $\perp$ PQ

i.e. $\angle OPQ = 90°$

In ΔOPQ,

$OQ^2 = OP^2 + PQ^2$

$25^2 = OP^2 + 24^2$

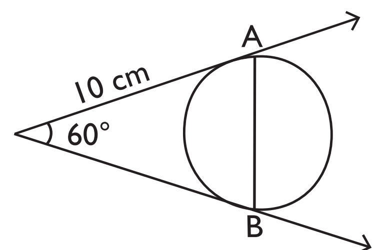
$625 = OP^2 + 576$

$OP^2 = 625 - 576$

$= 49$

$\therefore$ OP = 7 cm

3.



PB = PB

= 10 cm

(Length of tangents drawn from an external point to a circle are equal.)

$\Rightarrow$ $\angle PAB = \angle PBA$

(Angles opposite to equal sides are equal)

In ΔPBA,

$\angle P + \angle PAB + \angle PBA = 180°$

(Angle sum property)

$60° + \angle PAB + \angle PBA = 180°$

$\angle PAB + \angle PBA = 180° - 60°$

$= 120°$

$\Rightarrow \angle PAB = \angle PBA = 60°$

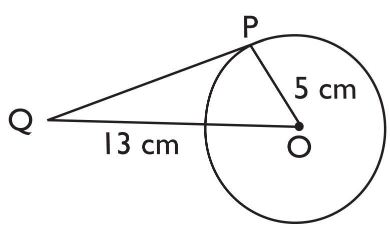
So, $\angle PAB = \angle PBA = \angle P = 60°$

$\Rightarrow$ ΔAPB is equilateral

$\Rightarrow AB = AP = 10$ cm

(Sides of equilateral triangle are equal.)

4.



As we know that tangent to a circle is perpendicular to the radius through the point of contact.

$\therefore$ $OP \perp PQ$

i.e. $\angle OPQ = 90°$

So, In ΔOPQ,

$OQ^2 = OP^2 + PQ^2$

$13^2 = 5^2 + PQ^2$

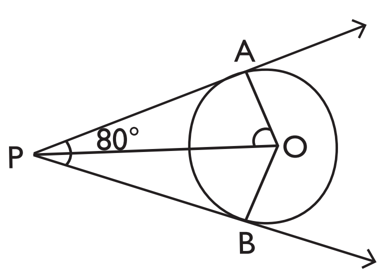
$169 = 25 + PQ^2$

$\therefore PQ^2 = 169 - 25$

$= 144$

$\Rightarrow PQ = 12$ cm

5.



In ΔPOA and ΔPOB,

$PA = PB$

(Length of tangents drawn from an external point to a circle are equal.)

$OP = PO$ (Common)

$OA = OB$ (Radii of same circle)

$\therefore \Delta POA \cong \Delta POB$

(SSS congruence creteria)

$\therefore \angle APO = \angle BPO$ (CPCT)

$= \frac{1}{2} \angle APB$

$= \frac{1}{2} (80°)$

$= 40°$

Also, $OA \perp AP$

i.e. $\angle OAP = °90$

(As tangent is perpendicular to radius through point of contact.)

In ΔOAP,

$\angle OAP + \angle APO + \angle AOP = 180°$

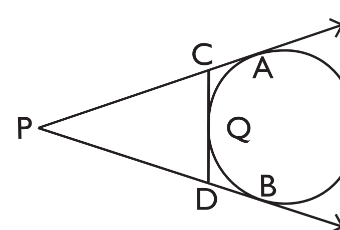
(Angle sum property)

$\therefore 90° + 40° + \angle AOP = 180°$

$\Rightarrow 130° + \angle AOP = 180°$

$\Rightarrow \angle AOP = 180° - 130° = 50°$

6.



$PA = PB$

(Tangents from external point P)

$\Rightarrow PC + CA = 10$

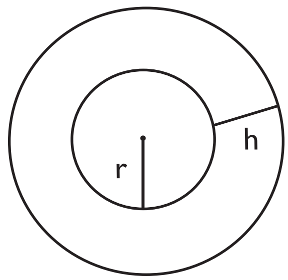
$\Rightarrow PC + CQ = 10$

[∵ CA = CQ (Tangents from external point C)]

$\Rightarrow$ PC + 2 = 10

$\Rightarrow$ PC = 8 cm

7.



Radius of inner circle $= r$

Area of inner circle $= \pi r^2$

Radius of outer circle $= r + h$

Area of outer circle $= \pi (r + h)^2$

So, area of circular path

= area of outer circle

– area of inner circle

$= \pi (r + h)^2 - \pi r^2$

$= \pi (r^2 + h^2 + 2rh - r^2)$

$= \pi (h^2 + 2rh)$

$= \pi h (h + 2r)$

$= \pi h (2r + h)$

8. Area of sector $= \frac{\theta}{360} \pi r^2$

$\therefore\ 20\pi = \frac{\theta}{360} \pi r^2$

$\Rightarrow\ 20 = \frac{\theta}{360} \pi r^2$

$\Rightarrow\ = \frac{\theta}{360}\ \frac{20}{r^2}$ ①

Also, arc length = 5π

$\Rightarrow\ \frac{\theta}{360} 2\pi r = 5\pi$

$\Rightarrow\ \frac{2r\theta}{360} = 5$

$\Rightarrow\ \frac{\theta}{360} = \frac{5}{2r}$ ②

From ①, ②

$\frac{\theta}{360} = \frac{20}{r^2} = \frac{5}{2r}$

$\Rightarrow\ 40r = 5r^2$

$\Rightarrow\ 5r (r - 8) = 0$

$\Rightarrow\ r = 8$

So, radius of circle = 8 cm

9. A circle can have infinitely many tangents

10. Remark: If AB and CD are two common tangents to the two circles of unequal radii then they will always intersect each other.

Given: Two circles with centre's O_1 and O_2. AB and CD are common tangents to the circles which intersect in P.

To prove: AB = CD

Proof:

AP = PC ...(i) (Length of tangents drawn from an external point to the circle are equal)

PB = PD ...(ii)(Length of tangents drawn from an external point to the circle are equal)

Adding (i) and (ii), we get

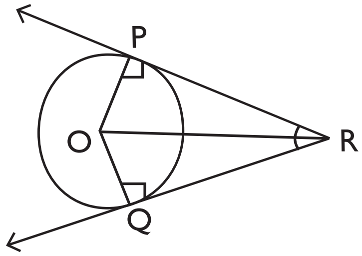
AP + PB = PC + PD

$\Rightarrow$ AB = CD

Section B

11. $\angle ABQ = \frac{1}{2} \angle AOQ = \frac{1}{2} (58^\circ) = 29^\circ$

(∵ Angle subtended by an arc at the centre is double the angle subtended by it on the remaining part of the circle.)

12. 

Join OP and OQ

PR and RQ are tangents to circle at points P and Q respectively.

$\Rightarrow$ OP $\perp$ PR and OQ $\perp$ QR

(As tangent is perpendicular to radius through point of contact.)

In ΔOPR and ΔOQR

OP = OQ

(Radii of same circle)

OR = OR (Common)

$\angle$OPR = $\angle$OQR = 90° (Proved above)

$\therefore$ $\Delta OPR \cong \Delta OQR$

(RHS congruence criteria)

$\Rightarrow \angle ORP = \angle ORQ = \frac{1}{2}\angle PRQ$

$= \frac{1}{2}(120°)$

$= 60°$

In ΔPRO,

$\cos 60° = \frac{PR}{OR}$

$\frac{1}{2} = \frac{PR}{OR}$

$\Rightarrow PR = \frac{1}{2} OR$ (i)

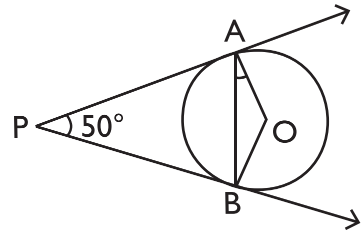
In ΔQRO

$RQ = \frac{1}{2} OR$ (ii)

On adding (i) and (ii), we get

$PR + RQ = \frac{1}{2} OR + \frac{1}{2} OR$

$= RO$

13. 

$\Rightarrow$ PA = PB

(Length of tangents drawn from an external point to a circle are equal.)

$\Rightarrow \angle PAB = \angle PBA$ (i)

(Angles opposite to equal sides are equal.)

In ΔAPB,

$\angle P + \angle PAB + \angle PBA = 180°$

(Angle sum property)

$50° + \angle PAB + \angle PAB = 180°$ (From (i))

$2\angle PAB = 130°$

$\therefore \angle PAB = \angle PBA = 65°$

OA $\perp$ AP

(As tangent is perpendicular to radius through point of contact.)

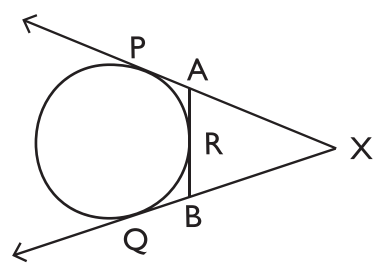
$\Rightarrow \angle OAP = 90°$

$\Rightarrow \angle OAB + \angle PAB = 90°$

$\Rightarrow \angle OAB + 65° = 90°$

$\Rightarrow \angle OAB = 90° - 65°$

$= 25°$

14. 

As we know that lengths of tangents drawn from an exterior point to a circle are equal.

$\therefore$ XP = XQ, AP = AR and BQ = BR

$\Rightarrow$ XP + AP = XB + BQ

$\Rightarrow \quad XP + AR = XB + BR$

15. As we know that lengths of tangents draw from an exterior point to a circle are equal.

$CE = CD = 9$ cm

$BF = BD = 6$ cm

$AE = AF = x$ cm

Also, $OE \perp AC$, $OD \perp BC$ and $OF \perp AB$

(As tangent is perpendicular to radius through point of contact.)

$$\text{Area of } \Delta BOC = \frac{1}{2} \times BC \times OD = \frac{1}{2} \times (9 + 6) \times 3 = \frac{1}{2} \times 15 \times 3 = \frac{45}{2} \text{ cm}^2$$

$$\text{Area of } \Delta AOC = \frac{1}{2} \times AC \times OE = \frac{1}{2} \times (9 + x) \times 3 = \frac{3}{2}(9 + x) \text{ cm}^2$$

$$\text{Area of } \Delta AOB = \frac{1}{2} \times AB \times OF = \frac{1}{2} \times (x + 6) \times 3 = \frac{3}{2}(x + 6) \text{ cm}^2$$

Area of ΔABC = area of ΔBOC + area of ΔAOC + area of ΔAOB

$$54 = \frac{45}{2} + \frac{1}{2}(9 + x) \times \frac{3}{2}(x + 6)$$

$$54 = \frac{45}{2} + \frac{27}{2} + \frac{18}{2} + \frac{3}{2}x + \frac{3}{2}x$$

$$54 = 45 + 3x$$

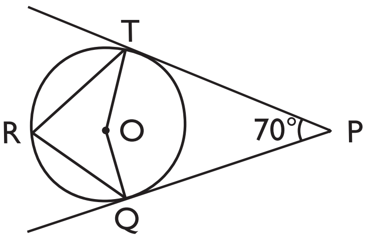
$$9 = 3x$$

$\therefore \quad = 3$

So, $AB = x + 6 = 3 + 6 = 9$ cm

$AC = x + 9 = 3 + 9 = 12$ cm

16.



As we know that tangent is perpendicular to the radius through the point of contact.

$\therefore \quad OT \perp PT$ and $OQ \perp PQ$

i.e. $\angle OTP = \angle OQP = 90°$

In quadrilateral TOQP

$\angle TOQ + \angle OQP + \angle QPT + \angle PTO = 360°$

(Angle sum property of quadrilateral.)

$\Rightarrow \quad \angle TOQ + 90° + 70° + 90° = 360°$

$\Rightarrow \quad \angle TOQ + 250° = 360°$

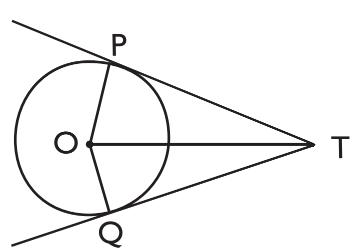
$\Rightarrow \quad \angle TOQ = 360° - 250° = 110°$

$$\angle TRQ = \frac{1}{2} \angle TOQ$$

(Angle subtended an arc at the centre is double the angle subtended by it on the remaining part of the circle.)

$$= \frac{1}{2}(110) = 55°$$

17.



Join OT

OP ⊥ PT

(As tangent is perpendicular to the radius through point of contact)

i.e. ∠OPT = 90°

In ∠OPT,

$OT^2 = OP^2 + PT^2$

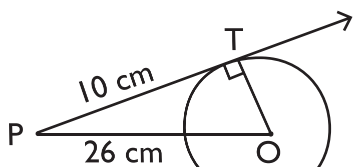
$= 5^2 + 8^2$

$= 25 + 64$

$= 89$

∴ OT $= \sqrt{89}$ cm

18.



OT ⊥ PT i.e. ∠OTP = 90°

(Tangent is perpendicular to radius through point of contact.)

In ΔOTP,

$OP^2 = OT^2 + PT^2$

(By Pythagoras theorem)

$26^2 = OT^2 + 10^2$

$676 = OT^2 + 100$

$OT^2 = 576$

OT = 24 cm

∴ Radius of the circle = 24 cm

19. AE and CE are tangents to the circle with center O,

∴ AE = CE (i)

(∵ Lengths of tangents drawn from an exterior point to a circle are equal.)

Also, DE and BE are tangents to the circle with centre O^2

∴ BE = DE (ii)

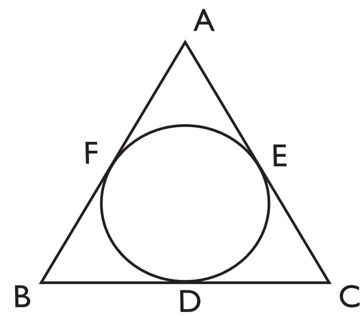
(∵ Lengths of tangents drawn from an exterior point to a circle are equal.)

On adding (i) and (ii), we get

AE + BE = CE + DE

∴ AB = CD

20.



AF = AE (i)

(Lenghts of tangents drawn from an exterior point to a circle are equal.)

ALso, AB = AC (ii)
(Given)

On subtracting (i) from (ii), we get

AB – AF = AC – AE

BF = CE (iii)

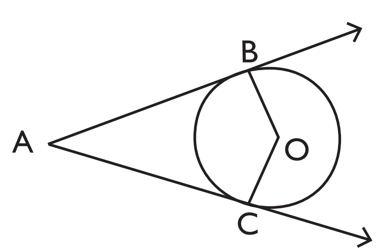
But BF = BD and CE = CD

(Lengths of tangents drawn from an exterior point to a circle are equal.)

∴ BD = CD

Section C

21.



AB and AC are tangents to a circle.

OB $\perp$ AB and OC $\perp$ AC

(Tangent is perpendicular to the radius through the point of contact.)

i.e. $\angle OBA = \angle OCA = 90°$ (i)

In quadrilateral ABOC,

$\angle A + \angle B + \angle O + \angle C = 360°$

(Angle sum property of quadrilateral)

$\Rightarrow \angle A + \angle O + \angle B + \angle C = 360°$

$\Rightarrow \angle A + \angle O + 90° + 90° = 360°$

From (i)

$\Rightarrow \angle A + \angle O + 180 = 360°$

$\Rightarrow \angle A + \angle O = 360° - 180°$

$= 180°$

22. BP and BQ are tangents to the circle

$\therefore$ BP = BQ (i)

(Lenghts of tangents drawn from an exterior point to a circle are equal.)

Also, CP = CR (ii)

(Lenghts of tangents drawn from an exterior point to a circle are equal.)

Consider

Perimeter of ΔABC = AB + BC + AC

= AB + (BP + CP) + AC

= AB + (BQ + CR) + AC

From (i) and (ii),

= AQ + AR

= AQ + AQ

= 2AQ

$\because$ AQ = AR as lengths of tangents drawn from an exterior point to a circle are equal.

$\therefore$ AQ $= \frac{1}{2}$ (Perimeter of ΔABC)

23. Consider ΔOAP and ΔOBP,

AP = BP

(Lenghts of tangents drawn from an exterior point to a circle are equal.)

OP = OP (Common)

AO = BO (Radii of same circle)

$\therefore$ $\Delta OAP \cong \Delta OBP$ (SSS congruence creteria)

$\Rightarrow$ $\angle APO \ \angle BPO$ (CPCT)

Now, Consider ΔACP and ΔBCP,

AP = BP

PC = CP (Common)

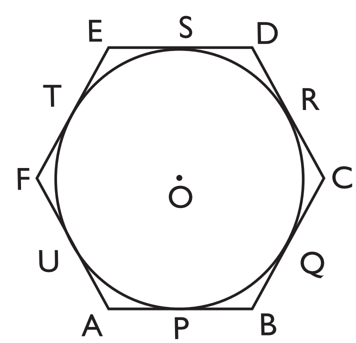
$\angle APC = \angle BPC$ (Proved above)

$\therefore$ $\Delta APC \cong \Delta BCP$ (SSS congruence creteria)

$\Rightarrow$ AC = BC and $\angle ACP = \angle BCP = 90°$ (CPCT)

So, OP is the perpendicular bisector of AB

24.



As we know that lengths of tangents drawn from an external point to a circle are equal

$\therefore$ AP = AU

BP = BQ

CQ = CR

DS = DR

ES = ET

FU = FT

Consider

AB + CD + EF

= (AP + BP) + (CR + DR) + (ET + TF)

= (AU + BQ) + (CQ + DS) + (ES + UF)

= (BQ + QC) + (DS + ES) + (AU + FU)

= BC + DE + AF

25. PR and CR are tangents to circle with centre A

$\therefore$ PR = CR (i)

(Lenghts of tangents drawn from an exterior point to a circle are equal.)

QR and CR are tangent to circle with center B

$\therefore$ QR = CR (ii)

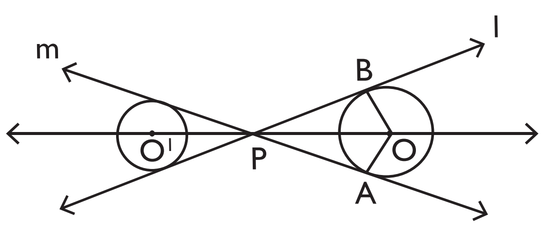
(Lenghts of tangents drawn from an exterior point to a circle are equal.)

From (i) and (ii), we get

PR = QR

$\therefore$ RC bisects PQ

26.



In ΔOPA and ΔOBP,

OA = OB (Radii of circle)

PA = PB

(Lengths of tangents from an external point to a circle are equal.)

OP = PO (Common)

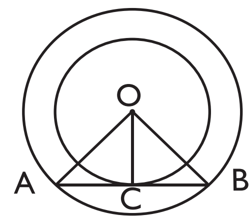
$\therefore$ $\Delta AOP \cong \Delta OBP$ (SSS congruence criteria)

$\Rightarrow$ $\angle APO \cong \angle BPO$ (CPCT)

$\Rightarrow$ OP is the bisector of $\angle APB$

$\therefore$ O lies on the bisector of the angle between l and m.

27.



We know that the radius and tangent are perpendicular at their point of contact.

$\therefore$ $\angle OCA = \angle OCB = 90°$

Now, In ΔOCA and ΔOCB

$\angle OCA = \angle OCB = 90°$

OA = OB (Radii of the larger circle)

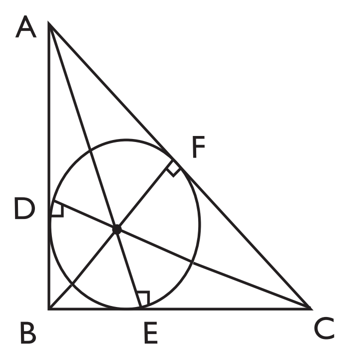
OC = OC (Common)

By RHS congruency

$\Delta OCA \cong \Delta OCB$

$\therefore$ CA = CB

28.



In ΔABC, right angles at B

$AC^2 = AB^2 + BC^2$

$= 24^2 + 10^2$

$= 576 + 100$

$= 676$

$\therefore$ AC $= 26$ cm

Area of $\Delta ABC = \frac{1}{2} \times BC \times AB$

$= \frac{1}{2} \times 10 \times 24$

$= 120 \text{ cm}^2$

Also, $OF \perp AC$, $OE \perp BC$ and $OD \perp AB$

($\because$ Tangent is perpendicular to the radius through point of contact.)

Area of $\Delta BOC = \frac{1}{2} \times BC \times OE$

$= \frac{1}{2} \times 10 \times r$

$= 5r$

a rea of $\Delta AOC = \frac{1}{2} \times 26 \times r$

$= 13r$

area of $\Delta AOB = \frac{1}{2} \times AB \times OD$

$= \frac{1}{2} \times 24 \times r$

$= 12r$

Area of ΔABC = area of ΔBOC

+ area of ΔAOC

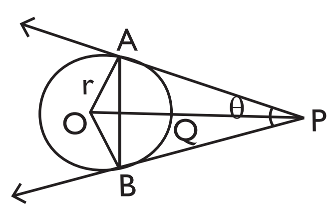
+ area of ΔAOB

$\therefore$ 120 $= 5r + 13r + 12\,r$

120 $= 30r$

4 $= r$

29.



AP is tangent to the circle

$\therefore$ $OA \perp AP$

i.e. $\angle OAP = 90°$

(Tangent is perpendicular to radius through point of contact.)

In ΔOAP,

$\sin\theta = \frac{OA}{OP} = \frac{r}{2r}$

$= \frac{1}{2}$

(OP = Diameter = 2r)

$\therefore$ $\theta = 30°$

$\Rightarrow$ $\angle OPA = 30°$

Similarly, $\angle OPB = 30°$

$\therefore$ $\angle APB = 30° + 30°$

$= 60°$

Also, AP = BP

(Lengths of tangent drawn from an external point to a circle are equal.)

So, In ΔAPB,

$\angle PAB = \angle PBA$ (i)

(Angles opposite to equal sides are equal.)

In ΔAPB,

$\Rightarrow$ $\angle PAB + \angle PBA + \angle APB = 180°$

(Angle sum property)

$\Rightarrow$ $\angle PAB + \angle PAB + 60° = 180°$

$\Rightarrow$ $2\,\angle PAB = 180° - 60°$

$= 120°$

$\Rightarrow$ $\angle PAB = \frac{120°}{2}$

$= 60°$

So, $\angle PAB = \angle PBA = \angle APB = 60°$

$\Rightarrow$ ΔAPB is equilateral.

30. As we know that lengths of tangents drawn from an external point to a circle are equal,

$\therefore$ PD = PF, RF = RE, QD = QE

Consider

Perimeter of ΔPQR

$= PQ + QR + PR$

$= (\underline{PD} + DQ) + (QE + ER) + (\underline{PF} + FR)$

$= (PD + PF) + (RF + RE) + (QD + QE)$

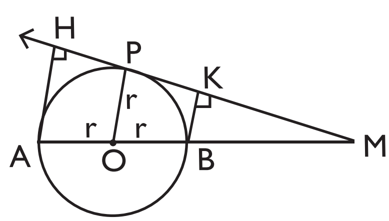
$= (PF + PF) + (RE + RE) + (QD + QD)$

$= 2\,PF + 2\,RE + 2\,QD$

$= 2\,(PF + ER + QD)$

Section D

31.



PM is to circle

$\therefore \angle MPO = 90°$

(Tangent is perpendicular to radius through point of contact)

Let AH = x, BK = y, BM = z

Let r be the radius of circle

In ΔMKB and ΔMHA

$\angle M = \angle M$ (Common)

$\angle MKB = \angle MHA = 90°$

$\therefore \Delta MKB \cong \Delta MHA$

(AA similarity creteria.)

$\Rightarrow = \frac{MK}{MH} = \frac{KB}{HA} = \frac{MB}{MA}$

(Corresponding sides of similar triangles are propertional)

$\Rightarrow \frac{BK}{AH} = \frac{MB}{MA}$

$\Rightarrow \frac{x}{y} = \frac{z}{2r + z}$

$\Rightarrow 2r\,y + y\,z + = x\,z$

$\Rightarrow Z = \frac{2ry}{x - y}$ (i)

Now, In ΔMKB and ΔMPO,

$\angle M = \angle M$ (Common)

$\angle MKB = \angle MPO = 90°$

$\therefore \Delta MKB$ and ΔMPO, (AA similarity creteria.)

$\Rightarrow \frac{MK}{MP} = \frac{BK}{OA} = \frac{MB}{MO}$

(Corresponding sides of similar triangles are proportional.)

$\Rightarrow \frac{BK}{PO} = \frac{BM}{OM}$

$\Rightarrow \frac{y}{r} = \frac{z}{r + z}$

$\Rightarrow yr + yz = rz$

$\Rightarrow z = \frac{ry}{r - y}$ (ii)

From (i) and (ii), we get

$z = \frac{2ry}{x - y} = \frac{ry}{r - y}$

$\Rightarrow \frac{2ry}{x - y} = \frac{ry}{r - y}$

$\Rightarrow \frac{2y}{x - y} = \frac{y}{r - y}$

$\Rightarrow \frac{2}{x - y} = \frac{1}{r - y}$

$\Rightarrow 2r - 2y = x + y$

$\Rightarrow x + y = 2r$

$\Rightarrow AH + BK = AB$ ($\because$ AB = 2r)

32. Consider ΔOEA and ΔOEP

OA = OP (Radii of same circle)

OE = OE (Common)

AE = PE (OE bisects AP)

$\therefore \Delta OEA \cong \Delta OEP$

(SSS congruence creteria)

$\Rightarrow$ $\angle OEA = \angle OEP$ (CPCT)

$\therefore$ $\angle OEA = \Delta OEP = 90°$ (i)

($\angle OEA$ and $\angle OEP$ are linear pair)

Also, $AB \perp BC$ as BC is a tangent to the circle

(Tangent is perpendiculer to the radius through the point of contact.)

$\Rightarrow$ $\angle ABC = 90°$ (ii)

Now, In ΔAEO and ΔABC

$\angle EAO = \angle BAC$ (Common)

$\angle AEO = \angle ABC$

$= 90°$ (From (i) and (ii))

$\Rightarrow$ $\Delta AEO \sim \Delta ABC$

(By SS Similarity creteria)

33. Given: d_1, d_2 ($d_2 > d_1$) be the diameters of two concentric circles and C be the length of a chord of a circle which is tangent to the circle.

To prove: $d_2 = d_1^2 + c^2$

Now,

$OQ = \frac{d_2}{2}$, $OR = \frac{d_1}{2}$ and $PQ = c$

Since PQ istangent to the circle therefore OR is perpendicular to PQ

$\Rightarrow QR = \frac{PQ}{2} = \frac{c}{2}$

Using pythagorus theorm in triangle OQR

$OQ^2 = OR^2 + QR^2$

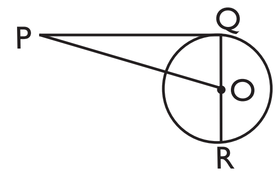
$\Rightarrow \left(\frac{d_2}{2}\right)^2 = \left(\frac{d_1}{2}\right)^2 + \left(\frac{c}{2}\right)^2$

$\Rightarrow \frac{1}{4}(d_2)^2 = \frac{1}{4}(d_1)^2 + (c)^2$

$\Rightarrow d_2^2 = d_1^2 + c^2$

Hence Proved.

34.



$OQ : PQ = 3 : 4$

Let $OQ = 3k$, $PQ = 4k$

PQ is tangent to the circle

$\therefore$ $OQ \perp PQ$

i.e. $\angle OQP = 90°$

(Tangent is perpendicular to radius through point of contact.)

In ΔOQP,

$OP^2 = OQ^2 + PQ^2$

(Pythagoras theorem)

$= (3k)^2 + (4k)^2$

$= 9k^2 + 16k^2$

$= 25k^2$

$\therefore$ $OP = 5k$

Also, Perimeter of ΔPOQ = 60 cm

$\Rightarrow$ $PO + OQ + PQ = 60$

$\Rightarrow$ $5k + 3k + 4k = 60$

$\Rightarrow$ $12k = 60$

$\Rightarrow$ $k = \frac{60}{12}$

$= 5$

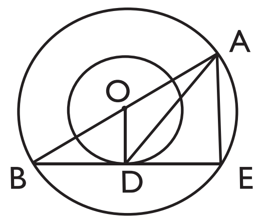
So, $PQ = 4k = 4 \times 5 = 20$ cm

$QR = 20\ Q = 2\ (3k) = 6k$

$= 6 \times 5 = 30$ cm

$OP = 5k = 5 \times 5 = 25$ cm

35.



BE is tangent to circle

$\therefore$ OD $\perp$ BE

i.e. $\angle$ODB = 90°

(Tangent is perpendicular to radius through point of contact)

$\Rightarrow$ BD = DE

(As perpendicular from centre to the chord bisects the chord)

$\Rightarrow$ D is a midpoint of BE

Also, O being the centre is a midpoint of AB

So, By midpoint theorem,

OD || AE and OD $= \frac{1}{2}$ AE

$\therefore$ AE = 20 D

= 2 (8)

= 16 cm

In ΔODB, $\angle$ODB = 90°

$\therefore$ $OB^2 = OD^2 + BD^2$

(By Pythagoras theorem)

$\Rightarrow$ $13^2 = 8^2 + BD^2$

$\Rightarrow$ $169 = 64 + BD^2$

$\Rightarrow$ $BD^2 = 169 - 64$

$\Rightarrow$ $BD^2 = 105$

$\Rightarrow$ BD $= \sqrt{105}$ cm

$\Rightarrow$ DE $= \sqrt{105}$ cm

($\because$ BD = DE)

In ΔAED, $\angle$AED = 90°

$\therefore$ $AD^2 = AE^2 + DE^2$

$= (16)^2 + (\sqrt{105})^2$

= 256 + 105

= 361

$\therefore$ AD = 19 cm

36. BD is tangent to the circle

$\therefore$ OC $\perp$ BD

i.e. $\angle$OCD = 90°

(Tangent is perpendicular to radius through point of contact.)

$\Rightarrow$ $\angle$OCA + $\angle$ACD = 90° (i)

Now, OA = OC

(Being radii of same circle.)

$\therefore$ In ΔAOC,

$\angle$OCA = $\angle$OAC

(Angles opposite to equal sides are equal.)

$\Rightarrow$ $\angle$OCA = $\angle$BAC (ii)

From (i) and (ii), we get

$\angle$BAC + $\angle$ACD = 90°