MATHEMATICS

SECTION-A

1.	(b)	**2.**	(b)
3.	(c)	**4.**	(a)
5.	(b)	**6.**	(d)
7.	(b)	**8.**	(a)
9.	(d)	**10.**	(a)

11. Mid point of AC = Mid point of BD

$$\left(\frac{6+9}{2},\frac{1+4}{2}\right)=\left(\frac{8+p}{2},\frac{2+3}{2}\right)$$

$$\left(\frac{15}{2},\frac{15}{2}\right)=\left(\frac{8+p}{2},\frac{5}{2}\right)$$

So, $\frac{15}{\cancel{2}}=\frac{8+p}{\cancel{2}}$

$\boxed{p=7}$

12. $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

$\frac{4}{k} \quad \frac{2}{6}$

2k = 24

$\boxed{k=12}$

OR

12. $2x^2 + kx + 3 = 0$

$b^2 - 4ac = 0$

$k^2 - 24 = 0$

$k^2 = 24$

$k = \pm\sqrt{24} = \pm 2\sqrt{6}$

13. cos 60° sin 30° – sin 60° cos 30°

$$\frac{1}{2}\times\frac{1}{2}-\frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2}$$

$$\frac{1}{4} = \frac{3}{4}$$

$$\frac{1-3}{4} = \frac{-2}{4} = \frac{-1}{2}$$

14. cos 0° cos 30° – cos 45° cos 60° cos 90°

$$1 \times \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} \times \frac{1}{2} \times 0 = 0$$

15. $\Delta ABC \sim \Delta DEF$

$\angle A = \angle D$

$\angle B = \angle E$

$\angle C = \angle F$

$\angle A + \angle B + \angle C = 180°$

$40 + \angle B + 68 = 180°$

$\boxed{\angle B = 72°}$

16. $\sec\theta = \frac{13}{12} = \frac{H}{B}$

$\sin\theta = \frac{P}{H} = \frac{5k}{13k}$

H = 13 k

B = 12 k

$\boxed{\sin\theta = \frac{5}{13}}$

$= \sqrt{H^2 - B^2}$

$= \sqrt{(13k)^2 - (12k)^2}$

$= \sqrt{169k^2 - 144k^2}$

$= \sqrt{25k^2}$

$\boxed{B = 5k}$

OR

sin 6A = cos (A – 8°)

co (90 – 6A) = cos (A – 8°)

90 – 6A = A – 8

98 = 7A

$\boxed{A = 14}$

17. r = 2 km

$\theta = 120°$

length of Arc = $2\pi r \times \frac{\theta}{360}$

$2 \times \frac{22}{\cancel{7}} \times {}^{3}\cancel{21} \times \frac{120}{360}$

$\cancel{132} \times \frac{1}{\cancel{3}} = 44 cm^2$

18. $P(\text{product of no in } 6) = \frac{\text{Total no of far outcomes}}{\text{Total possible outcomes}}$

$= \frac{4}{36} = \frac{1}{9}$

19. By BPT

$\frac{AD}{DB} = \frac{AE}{EC}$

$\frac{1.5}{3} = \frac{1}{EL}$

$\boxed{EC = 2 \text{ cm}}$

20. 2, 7, 12.......

$T_n = a + (n - 1) d$

$T_{10} = 2 + (10 - 1) 5$

$= 2 + 9 \times 5$

$\boxed{T_{10} = 47}$

21. (i) P (card will be an ace) = $\frac{\text{Total no of favourable}}{\text{Total no of favourable}}$

$= \frac{4}{52}$

$= \frac{1}{13}$

(ii) p (not an ace) = $\frac{\cancel{48}}{\cancel{52}}$

$= \frac{12}{13}$

22. (a) P (no on the card is odd) = $\frac{\text{No of favourable outcome}}{\text{Total no of possible outcome}}$

$= \frac{9}{17}$

(b) P (Sum of no 4, 8 or 12) = $\frac{\cancel{9}}{\cancel{36}} = \frac{1}{4}$

23. Given PQ and RS are tangents and AB is a diameter.

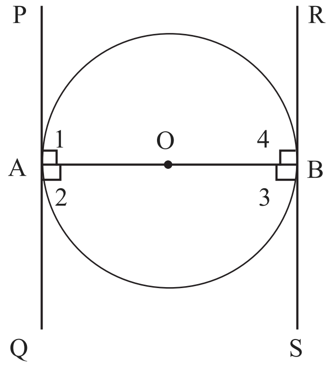
To pwe-: PQ || RS

Proof-: $\angle 1 = \angle 3$ (Alternate Interior)

$\angle 2 = \angle 4$ (Alternate Interior)

So AB is a transversal.

Hence PQ || RS



24. Sin 0° = 0

Sin 30° = $\frac{1}{2}$

Sin 45° = $\frac{1}{\sqrt{2}}$

Sin 60° = $\frac{\sqrt{3}}{2}$

Sin 90° = 1

or

$$\boxed{\text{Cos}A = \sqrt{1-\sin^2 A}}$$

$\tan A = \sqrt{\sec^2 A - 1}$

$= \sqrt{\frac{1-\cos^2 A}{\cos^2 A}}$

$= \sqrt{\frac{\sin^2 A}{1-\sin^2 A}}$

$$\boxed{\tan A = \frac{\sin A}{\sqrt{1-\sin^2 A}}}$$

$\sec A = \frac{1}{\cos A}$

$\sec A = \boxed{\sec A = \frac{1}{\sqrt{1-\sin^2 A}}}$

25. Circumference = 22 cm

$2\pi r = 22$

$2 \times \frac{22}{7} \times r = 22$

$r = \frac{7}{2} = 3.5$ cm

$A = \pi r^2$

$A = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$

$A = \frac{77}{2} = 38.5 \text{ cm}^2$

26. (a) 3 cubic polynomial

(b)

$$\begin{array}{r|l} x+1 & \overline{x^2+2x+1}\;(\;x+1 \\ & \underline{-x^2+x} \\ & \quad x+1 \\ & \quad \underline{-x+1} \\ & \quad\;\; 0 \end{array}$$

SECTION-C

27. $x^2 - 2x - 8$

On compairing with standard Quadratic eqn

$a = 1, b = -2, c = -8$

$x^2 - 2x - 8 = 0$

$x^2 - 4x + 2x - 8 = 0$

$x(x - 4) + 2(x - 4) = 0$

$(x - 4)(x + 2) = 0$

$x = 4, x = -2$

$$\alpha + \beta = \frac{-b}{a} = \frac{-(-2)}{1} = \frac{\text{Coefficient of x}}{\text{Coefficient of } x^2}$$

$$\alpha - \beta = \frac{c}{a} = \frac{-8}{1} = \frac{\text{Cons tan t terms}}{\text{Coefficient of } x^2}$$

28. Construction

29. r = 45 cm and

$$\theta = \frac{360°}{8} = 45°$$

area of sector $\pi r^2 \dfrac{\theta}{360}$

$$= \frac{22}{7} \times 45 \times 45 \times \frac{45}{360}$$

$$= \frac{22275}{28} \text{ cm}^2$$

30. $(\text{Cosec}\theta - \cot\theta)^2$

$\text{Cosec}^2 - 2\,\text{cosec}\theta\cot\theta + \cot^2\theta$

$$\Rightarrow \quad \frac{1}{\sin^2\theta} - 2 \times \frac{1}{\sin\theta} \times \frac{\cos\theta}{\sin\theta} + \frac{\cos^2\theta}{\sin^2\theta}$$

$$\Rightarrow \quad \frac{1 - 2\cos\theta + \cos^2\theta}{\sin^2\theta}$$

$$\Rightarrow \quad \frac{(1 - \cos\theta)^2}{\sin^2\theta}$$

$$\Rightarrow \quad \frac{(1 - \cos\theta)(1 - \cos\theta)}{1 - \cos^2\theta}$$

$$\Rightarrow \quad \frac{(1 - \cos\theta)(1 - \cos\theta)}{(1 - \cos\theta)(1 + \cos\theta)}$$

$\Rightarrow \quad \frac{1-\cos\theta}{1+\cos\theta}$

L.HS

$$\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}$$

$\Rightarrow \quad \frac{\cos^2 A+(1+\sin A)^2}{(1+\sin A)(\cos A)}$

$\Rightarrow \quad \frac{\cos^2 A+2\sin A+\sin^2 A}{(1+\sin A)(\cos A)}$

$\Rightarrow \quad \frac{1+1+2\sin A}{(1+\sin A)(\cos A)}$

$\Rightarrow \quad \frac{2}{\cos A}=2\sec A$ R.H.S

31. Let us suppose that $6-\sqrt{3}$ is rational.

$\Rightarrow 6-\sqrt{3}=\frac{a}{b}$

Where a + b are to prime integers.

$\Rightarrow \frac{6b-a}{b}=\sqrt{3}$ ...(i)

where a, b are integers. for every value of $\frac{6b-a}{b}$ is a rational no. but $\sqrt{3}$ is irrational.

$6-\sqrt{3}$ is irrational.

OR

HCF (616, 32) = ?

616 = 32 × 19 + 8

32 = 8 × 4 + 0

$\Rightarrow$ HCF (616, 32) = 8

$\Rightarrow$ they can march in 0 columns each

32. Given: A circle with centre O with tangent XY at point of contact P.

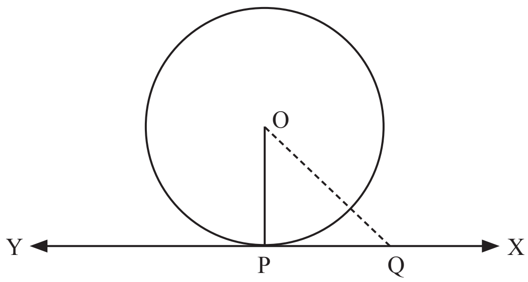
To prove OP $\perp$ XY

Proof Let Q be point on XY connect OQ.

Let touches the circle at R.

Hence OQ > OR

OQ > OP (OP = OR radius)



Hence OP is the smallest line that connects XY.

Hence OP $\perp$ XY

33. A (2, –2) (x_1, y_1) —— P (x, y) —— Q (a, b) —— B (–7, 4) (x_2, y_2)

$\therefore$ P divides AB into 1 : 2 $(m_1 : m_2)$

$$\Rightarrow (x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$(x, y) = \left(\frac{(1)(-7)+(2)(2)}{3}, \frac{(1)(4)+2(-2)}{3}\right)$$

$$(x, y) = \left(\frac{-7+4}{3}, \frac{4-4}{3}\right)$$

$$(x, y) = \left(\frac{-3}{3}, 0\right)$$

$$(x, y) = (-1, 0)$$

Now Q is mid-point of PB

P (–1, 0) B (–7, 4)

$\Rightarrow$ (x_1, y_1) (x_2, y_2)

$$(a, b) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$(a, b) = \left(\frac{-1-7}{2}, \frac{0+4}{2}\right)$$

$$(a, b) = (-4, 2)$$

34. $8x + 5y = 9 \times 2$ (A)

$3x + 2y = 4 \times 5$ (B)

$16x + 10y = 18$

$15x + 10y = 20$

$-$ $-$

$+$ $\boxed{x = -2}$ (1)

put $x = -2$ into (A)

$8(-2) \times 5(y) = 9$

$-16 \times 5y = 9$

$5y = 25$

$\boxed{y = 5}$

Now $y = mx + 3$

$5 = m(-2) + 3$

$\frac{5-3}{-2} = m$

$\boxed{-1 = m}$

SECTION-D

Sol. Let the two consecutive integers are x and x + 1, then

according to the problem,

$(x)^2 + (x + 1)^2 = 365$

$x^2 + x^2 + 2x + 1 = 365$

$2x^2 + 2x - 364 = 0$

$x^2 + x - 182 = 0$

$x^2 + 14x - 13x - 182 = 0$

$x(x + 14) - 13(x + 14) = 0$

$(x + 14)(x - 13) = 0$

By zero product Rule, we have

$x + 14 = 0$ or $x - 13 = 0$

$x = -14$ or $x = 13$

so the two consecutive positive integers, sum of whose squares is 365, are 13 and 14.

Sol. Here, $a_2 = 14$ and $a_3 = 18$

$a + d = 14$ (1) and $a + 2d = 18$ (2)

on subtracting equ (1) from equ (2)

$(a + 2d) - (a + d) = 18 - 14$

$\boxed{d = 4}$

put d = 4 in equ (1), we get

$a + 4 = 14$

$\Rightarrow \boxed{a = 10}$

Now, sum of 51 terms $= \frac{51}{2}(2 \times 10 + (51 - 1)4)$

$S_{51} = \frac{51}{2}(20 + 200)$

$S_{51} = 51 \times 110$

$\boxed{s_{51} = 5610}$

Sol. Here, sum of first 14 terms is 1505

i.e. $S_{14} = 1505$ and first term, $a = 10$

or $\frac{14}{2}(2 \times 10 + (14 - 1)d) = 1505$

$20 + 13d = \frac{3010}{14}$

$20 + 13d = 215$

$13d = 195$

$d = \frac{195}{13}$

$\boxed{d = 15}$

Now,

$a_{25} = a + 24d$

$a_{25} = 10 + 24 \times 15$

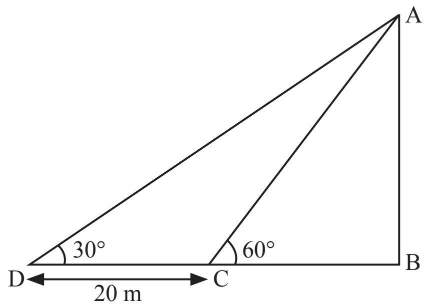
$a_{25} = 10 + 360$

$\boxed{a_{25} = 370}$

Sol. Here, height of T.V. tower, h = AB

width of the canal, x = BC

In Δ ABD,



$$\tan 30° = \frac{AB}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{20+x}$$

$$\text{or } h\sqrt{3} = 20 + x \qquad \text{(i)}$$

In Δ ABC,

$$\tan 60° = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{h}{x}$$

$$h = x\sqrt{3} \qquad \text{(ii)}$$

from (i) and (ii), we get

$$\left(x\sqrt{3}\right)\left(\sqrt{3}\right) = 20 + x$$

$$3x = 20 + x$$

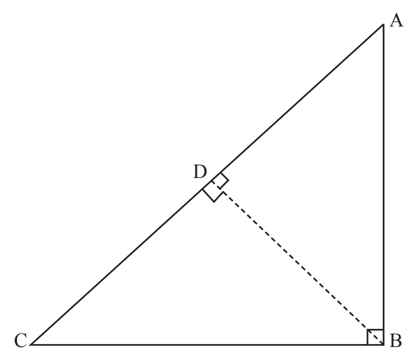
$$2x = 20$$

$$x = 10$$

$$\therefore h = 10\sqrt{3}$$

Hence, the height of the tower, h = $10\sqrt{3}$ m.

the width of the canal, x = 10 m.

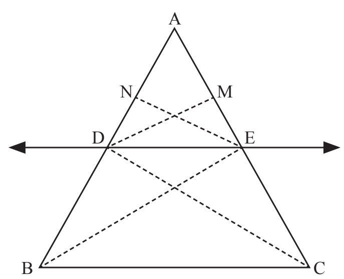


Sol. **Statement:** If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Given: Here, ABC is a triangle in which a line 'l' parallel to side BC intersects other two sides AB and AC at D and E respectively.

To Prove: $\frac{AD}{DB} = \frac{AE}{EC}$

Construction: Join BE and CD and draw DM ⊥ AC and EN ⊥ AB.



Proof: In ΔADC, EN ⊥ AD & DM ⊥ AE

$$\text{ar}(\Delta ADC) = \frac{1}{2} AD \times EM$$

$$\text{or ar}(\Delta ADC) = \frac{1}{2} AE \times DM$$

$$\text{Also, ar}(\Delta BDE) = \frac{1}{2} \times BD \times BN \text{ and}$$

$$\text{ar}(\Delta CED) = \frac{1}{2} \times CE \times DM$$

$$\therefore \frac{\text{ar}(\Delta ADE)}{\text{ar}(\Delta BDE)} = \frac{\frac{1}{2} \times AD \times DM}{\frac{1}{2} \times BD \times EN}$$

$$\Rightarrow \frac{\text{ar}(\Delta ADE)}{\text{ar}(\Delta BDE)} = \frac{AD}{BD}$$

$$\text{Also } \frac{\text{ar}(\Delta ADE)}{\text{ar}(\Delta CED)} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times CE \times DM}$$

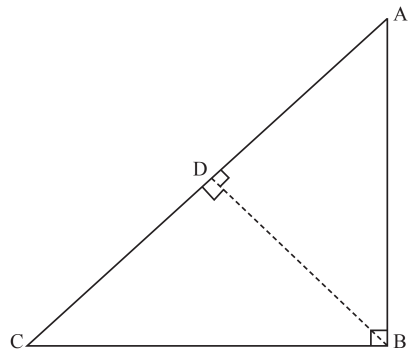
$$\text{or } \frac{\text{ar}(\Delta ADE)}{\text{ar}(\Delta CED)} = \frac{AE}{CE} \qquad (2)$$

As we know that ΔBDE and ΔCED share common base BE and lic between the same parallels DE and BC, then

ar(ΔBDE) = ar(ΔCED) (3)

from equ (1), (2) and (3) we can conclude that

$$\frac{AD}{DB} = \frac{AE}{EC}$$



Sol. **Given:** Here, ABC is a triangle in which ∠ABC = 90°

To Prove: $AC^2 = AB^2 + BC^2$

Construction: Draw BD ⊥ AC

Proof: In ΔABC and ΔADB

∠ABC = ∠ADB = 90°

∠BAC = ∠DAB (common angle)

∴ ΔABC ~ ΔADB (by AA Similarity)

$$\Rightarrow \frac{AB}{AC} = \frac{AD}{AB}$$ (Sides are proportional in similar Δ's)

$\Rightarrow AB^2 = AC \times AD$ (1)

In ΔABC and ΔBDC

∠ABC = ∠BDC = 90°

$\angle BCA = \angle DCB$ (common angle)

$\Delta ABC \sim \Delta BDC$ (By AA Similarity)

$\frac{BC}{DC} = \frac{AC}{BC}$ (Sides are proportional in similar Δ's)

$\Rightarrow BC^2 = AC \times DC$ (2)

On adding equ (1) and (2), we get

$AB^2 + BC2 = AC \times AD \times AC \times DC$

$\Rightarrow AB^2 + BC2 = AC (AD + DC)$

$\Rightarrow AB^2 + BC2 = AC \times AC$

$\Rightarrow \boxed{AB^2 + BC^2 = AC^2}$

Hence, In a right triangle, the square of the hypotenuse is equal to the sum of the squares of other two sides.

Sol. Here, the dimensions of this solid cylinder,

Diameter, $d = 12$ cm $\Rightarrow$ $r = 6$cm

height, $h = 12$

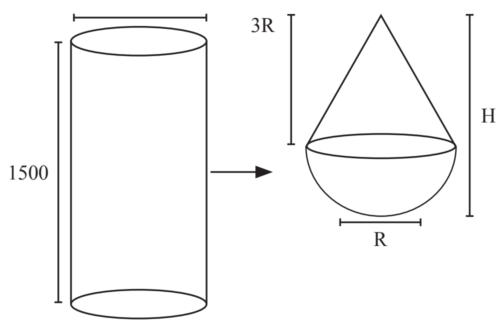
The solid cylinder is melted and recasted into 12 toys in the shape of a right circular cone mounted on a hemisphere

height of the toy = H

radius of Hmisphere = R

height of the cone, h' = 3R

$\therefore$ height of toy, h = 4R



As we know that,

Vol of solid cyl = 12 × vol of 1 toy

$\pi r^2 h = 12 \times \left(\frac{1}{3}\pi R^2 h^1 + \frac{2}{3}\pi R^3\right)$

$\pi r^2 h = 12 \times \frac{12}{3}\pi (R^2 h^1 + 2R^3)$

$r^2 h = 4 \times (R^1 H^1 + 2R^3)$

$= \frac{6 \times 6 \times 15}{4} = (3R^3 + 2R^3)$

$= 3 \times 3 \times 15 = 5R^3$

$R^3 = 3 \times 3 \times 3$

$\therefore$ radius of hemishere = 3 cm

height of the toy, $H = 4R = 4 \times 3 = 12$cm

Sol. Here, the dimensions of sphere

radius, R = 4.2 cm

Also, height of the cylinder = h

radius of the cylinder = 6 cm

As we know that, on melting and recasting

Vol of sphere = Vol of cylinder

$$\frac{4}{3}\pi R^3 = \pi r^2 h$$

or $h = \frac{4R^3}{3r^2}$

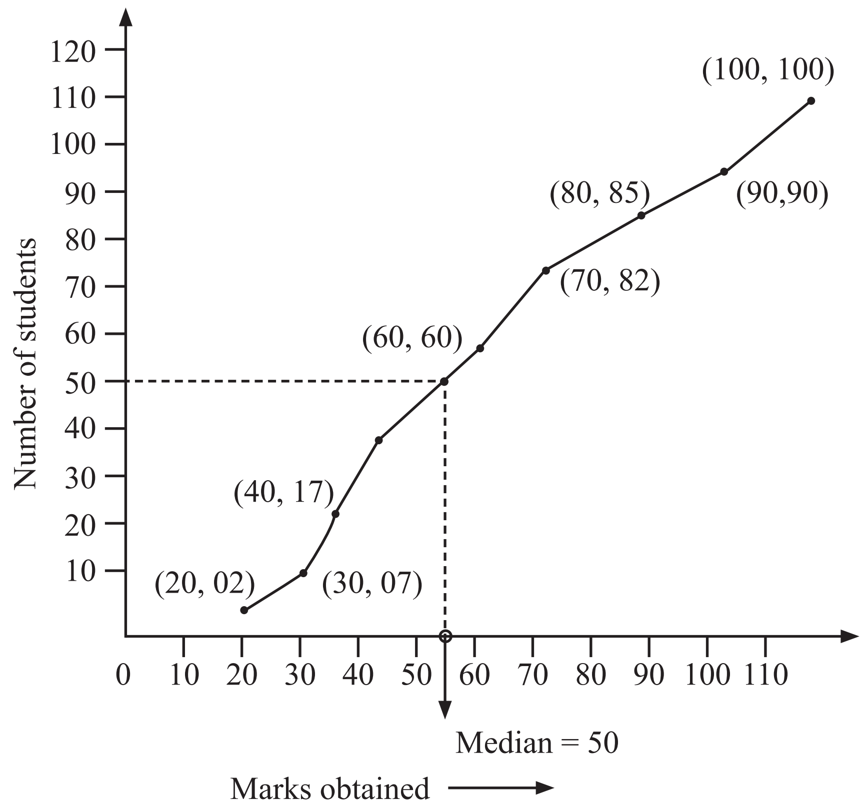
or $h = \frac{4 \times 4.2 \times 4.2 \times 4.2}{3 \times 6 \times 6}$

or $h = 4 \times 0.7 \times 0.7 \times 1.4$

or $h = 3.3$ cm (appron)

Sol. Here, frequency distribution table

Marks obtained	Number of students	(x, 4)
Less than 20	2	(20, 2)
Less than 30	7	(30, 7)
Less than 40	17	(40, 17)
Less than 50	40	(50, 40)
Less than 60	60	(60, 60)
Less than 70	82	(70, 82)
Less than 80	85	(80, 85)
Less than 90	90	(90, 90)
Less than 100	100	(100, 100)



Less tan type O give showing the marks obtained by the number of students.

Answers being written by students may have caused spelling errors.